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Research Article

Optimal Replenishment Decisions under Two-Level Trade Credit with Partial Upstream Trade Credit Linked to Order Quantity and Limited Storage Capacity

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This paper extends the previous economic order quantity (EOQ) models under two-level trade credit such as Goyal (1985), Teng (2002), Huang (2003, 2007), Kreng and Tan (2010), Ouyang et al. (2013), and Teng et al. (2007) to reflect the real-life situations by incorporating the following concepts: (1) the storage capacity is limited, (2) the supplier offers the retailer a partially upstream trade credit linked to order quantity, and (3) both the dispensable assumptions that the upstream trade credit is longer than the downstream trade credit $N < M$ and the interest charged per dollar per year is larger than or equal to the interest earned per dollar per year $I_c < I_e$ are relaxed. We then study the necessary and sufficient conditions for finding the optimal solution for various cases and establish a useful algorithm to obtain the solution. Finally, numerical examples are given to illustrate the theoretical results and provide the managerial insights.

1. Introduction

Trade credit financing is a crucial issue and increasingly recognized as important means to increase profitability in a production-inventory system. In practice, the supplier usually allows the wholesaler a fixed permissible delay period for settling the account (i.e., an upstream trade credit) and the wholesaler in turn provides a similar credit period to its customers (i.e., a downstream trade credit). It is well known that the permissible delay in payments has two benefits: (1) it invites new buyers who consider it to be a type of price reduction, and (2) it may be useful as an alternative to price discount because it does not aggravate competitors to decrease their prices and thus introduce permanent price reductions (e.g., [1]).

In 1985, the EOQ model with upstream trade credit was first proposed by Goyal [2]. Following, prolific extensions of his model have been developed by researchers. For example, Aggarwal and Jaggi [3] extended Goyal's [2] model for the deteriorating items. Jamal et al. [4] further generalized Aggarwal and Jaggi's [3] model to allow for shortages. Teng [5] amended Goyal's [2] model by considering the different between unit price and unit cost and found that it makes economic sense for a well-established wholesaler to order less quantity and take the benefits of payment delay more frequently. Chang et al. [6] developed an EOQ model for deteriorating items under supplier's upstream trade credit linked to ordering quantity. Liang and Zhou [7] established a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. There are several interesting and relevant papers related to the trade credits, for example, Huang [8], Ouyang et al. [9], Chen et al. [10] Chung and Huang [11], Hu and Liu [12], Min et al. [13] Giri et al. [14], Khanra et al. [15], Sarkar [16], and so forth. Nerveless, all inventory models described above only considered an upstream trade credit.

Huang [17] extended Goyal's [2] model to establish an EOQ model under two levels of trade credit policy with

the downstream trade credit period N being less than the upstream trade credit period M. Later, Kreng and Tan [18] modified Huang's [17] model by considering the upstream trade credit linked to the ordering quantity. Recently, Ouyang et al. [19] not only complemented the shortcomings in Kreng and Tan [18] on the interest earned and charged, but also relaxed those dispensable assumptions such that the downstream trade credit period is less than the upstream trade credit period. Other interesting and relevant papers related to two-level trade credit such as Teng et al. [20], Liao [21], Goswami et al. [22], Min et al. [23], Ho [24], and others.

In addition, it is observed that the classical inventory models generally deal with single storage facility. The basic assumption in these models is that the manager owns a storage room with unlimited capacity. However, in practice, the manager may purchase a huge quantity of goods at a time for some reasons such as when suppliers provide price discounts for bulk purchases or trade credits to encourage the retailer to buy more. These huge stocks cannot be stored in the existing storage (the own warehouse) with limited capacity. Therefore, a rented warehouse (RW) is needed to store the excess units over the capacity of the own warehouse. An early discussion on the inventory model with two-warehouse was given by Hartely [25]. Further literatures in this direction include Sarma [26], Dave [27], Goswami and Chaudhuri [28], Pakkala and Achary [29], Bhunia and Maiti [30], Benkherouf [31], Yang [32], Huang [33], Lee and Hsu [34], Sett et al. [35], and others.

Consequently, to reflect the real-life situations, this paper extended the previous EOQ models with two-level trade credit such as Goyal [2], Teng [5], Huang [8, 17], Ouyang et al. [19], and Teng et al. [20] by incorporating the following concepts: (1) the storage capacity is limited, (2) the supplier offers the retailer a partial upstream trade credit linked to order quantity, and (3) both the dispensable assumptions of the upstream trade credit is longer than the downstream trade credit $N < M$ and the interest charged per dollar is larger than or equal to the interest earned per dollar $I_c < I_e$ are relaxed.

The rest of this paper is organized as follows. In Section 2, we describe the notation and assumptions adopted throughout this paper. Then, mathematical models are developed to minimize the total costs per year in Section 3 for various cases. In Section 4, we study the necessary and sufficient conditions and establish several theoretical results for finding the optimal solution under various situations. Numerical examples and sensitivity analysis with major parameters are given to illustrate the theoretical results and obtain some managerial insights in Section 5. Finally, conclusions are given in Section 6.

2. Notation and Assumptions

The notation used throughout this paper is as follows:

D: the demand rate per year;

A: the ordering cost per order;

c: the purchasing cost per unit;

p: the selling price per unit, with $p > c$;

h: the unit holding cost per year excluding interest charge in own warehouse (OW);

k: the unit holding cost per year excluding interest charge in rented warehouse (RW), with $k > h$;

I_e: the interest earned per dollar per year;

I_c: the interest charged per dollar per year;

α: the fraction of the delay payments permitted by the supplier if the order quantity is less than the preassign quantity, $0 \leq \alpha \leq 1$;

M: the wholesaler's trade credit period in years offered by the supplier;

N: the retailer's trade credit period in years offered by the wholesaler;

W: the capacity in own warehouse;

T_W: the time interval in which maximum inventory in own warehouse is depleted to zero; that is, $T_W = W/D$;

Q_d: the minimum order quantity at which full delay in payments is permitted;

T_d: the time interval in which the quantity Q_d is depleted to zero, that is, $T_d = Q_d/D$;

T: the length of replenishment cycle in years;

Q: the order quantity, where $Q = DT$;

T^*: the optimal length of replenishment cycle time in years;

Q^*: the optimal order quantity.

The models proposed in this paper are based on the following assumptions.

(1) Demand rate is known and constant.

(2) Time horizon is infinite.

(3) Replenishment is instantaneous and shortages are not allowed.

(4) If the order quantity Q is greater than W, then the wholesaler needs to rent an additional warehouse to hold inventory.

(5) If the wholesaler's order quantity Q is greater than or equal to Q_d, then fully delayed payment is permitted by its supplier. Otherwise, the partially delayed payment is permitted. That is, the wholesaler must take a loan to pay its supplier the partial payment of $(1-\alpha)cQ$ immediately when the order is received and then pay off the loan with entire revenue.

(6) The wholesaler offers a credit period N to every retailer.

(7) During the credit period $M(> N)$, sales revenue is deposited in an interest bearing account with the rate I_e. At the end of the permissible delay M, the wholesaler pays off all units sold, keeps the profit for use in other activities, and starts paying for the interest charges with the rate I_c on loan.

3. Model Formulation

From assumptions, as $Q \geq Q_d$ (i.e., $T \geq T_d$), the full delay in payment is permitted. Otherwise, the partial delay in payment is permitted where the wholesaler must pay the supplier the amount $(1-\alpha)cDT$ immediately when the order is filled and pay the rest at the time M. Furthermore, if the order quantity Q is greater than W, then the wholesaler needs to rent an additional warehouse to hold inventory.

The annual total relevant cost consists of the following elements.

(a) The ordering cost per year (say OC) is

$$\mathrm{OC} = \frac{A}{T}. \tag{1}$$

(b) The holding cost per year excluding interest charges (say HC) is

$$\mathrm{HC} = \begin{cases} \mathrm{HC}_1, & \text{if } Q \leq W \ \ (\text{i.e.}, T \leq T_W), \\ \mathrm{HC}_2, & \text{if } Q > W \ \ (\text{i.e.}, T > T_W), \end{cases} \tag{2}$$

where

$$\mathrm{HC}_1 = \frac{hDT}{2}, \tag{3}$$

$$\begin{aligned} \mathrm{HC}_2 &= \frac{\left[hW(2DT-W)+k(DT-W)^2\right]}{(2DT)} \\ &= \frac{\left[hDT_W(2T-T_W)+kD(T-T_W)^2\right]}{(2T)}. \end{aligned} \tag{4}$$

(c) Interest earned and the interest charged.

As to calculate the interest earned and interest charged, there are two possible cases that should be considered: when $Q \geq Q_d$ (i.e., $T \geq T_d$), the full delay in payment is permitted; otherwise, the partial delay in payment is permitted where the wholesaler must pay the supplier the amount $(1-\alpha)cDT$ immediately when the order is filled and pay the rest at the time M. That is, there are two cases that might arise: (i) full delay in payments ($T \geq T_d$) and (ii) partial delay in payments ($T < T_d$).

Case 1 (full delay in payments ($T \geq T_d$)). Based on the values of M, N, and $T+N$, we have the following three alternative situations: (1) $M \geq T+N > N$, (2) $T+N \geq M > N$, and (3) $T+N > N \geq M$. Let us discuss them accordingly.

(1) $M \geq T+N$. In this situation, the wholesaler receives the total revenue at time $T+N$ and is able to pay the supplier the total purchase cost at time M (see Figure 1). Consequently, the interest charged per year (say IC_{11}) is

$$\mathrm{IC}_{11} = 0, \tag{5}$$

and the interest earned per year (say IE_{11}) is

$$\begin{aligned} \mathrm{IE}_{11} &= \frac{pI_eDT^2}{2T} + \frac{pI_e(M-T-N)DT}{T} \\ &= pI_eD(M-N) - \frac{pI_eDT}{2}. \end{aligned} \tag{6}$$

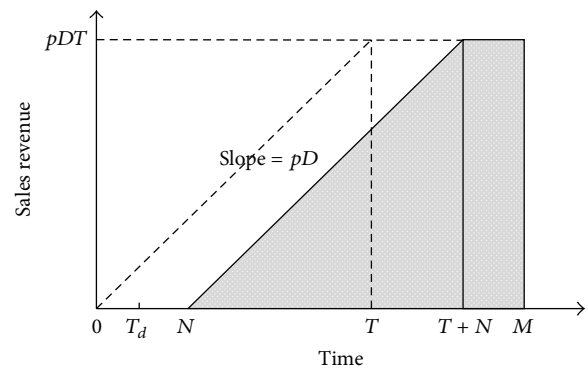

Figure 1: $Q \geq Q_d$ and $N < T+N \leq M$.

(2) $T+N \geq M > N$. In this situation, the wholesaler will sell the items and uses the sales revenue to earn interest at the rate I_e in the interval $[N, M]$ (see Figure 2(a)). On the other hand, the wholesaler receives the pay after N and pays off all units sold at time M and starts paying for the interest charges with the rate I_c on items sold after M (see Figure 2(b)). As a result, the interest charged per year (say IC_{12}) is

$$\mathrm{IC}_{12} = \frac{cI_cD(T+N-M)^2}{2T}, \tag{7}$$

and the interest earned per year (say IE_{12}) is

$$\mathrm{IE}_{12} = \frac{pI_eD(M-N)^2}{2T}. \tag{8}$$

(3) $T+N > N \geq M$. When $M \leq N$, there is no interest earned for the wholesaler. In addition, the wholesaler must finance all items ordered at time M at an interest charged I_c per dollar per year and starts to pay off the loan after time N (see Figure 3). Hence, the interest charged per year (say IC_{13}) is

$$\begin{aligned} \mathrm{IC}_{13} &= \frac{cI_cDT\left[(N-M)+(T+N-M)\right]}{2T} \\ &= cI_cD(N-M) + \frac{cI_cDT}{2}, \end{aligned} \tag{9}$$

and the interest earned per year (say IE_{13}) is

$$\mathrm{IE}_{13} = 0. \tag{10}$$

Consequently, from (5), (7), and (9), the interest charged per year for the case with full delay in payments (say IC_1) is

$$\mathrm{IC}_1 = \begin{cases} 0, & \text{if } M \geq T+N, \\ \dfrac{cI_cD(T+N-M)^2}{(2T)}, & \text{if } T+N \geq M > N, \\ cI_cD(N-M) + \dfrac{cI_cDT}{2}, & \text{if } T+N > N \geq M. \end{cases} \tag{11}$$

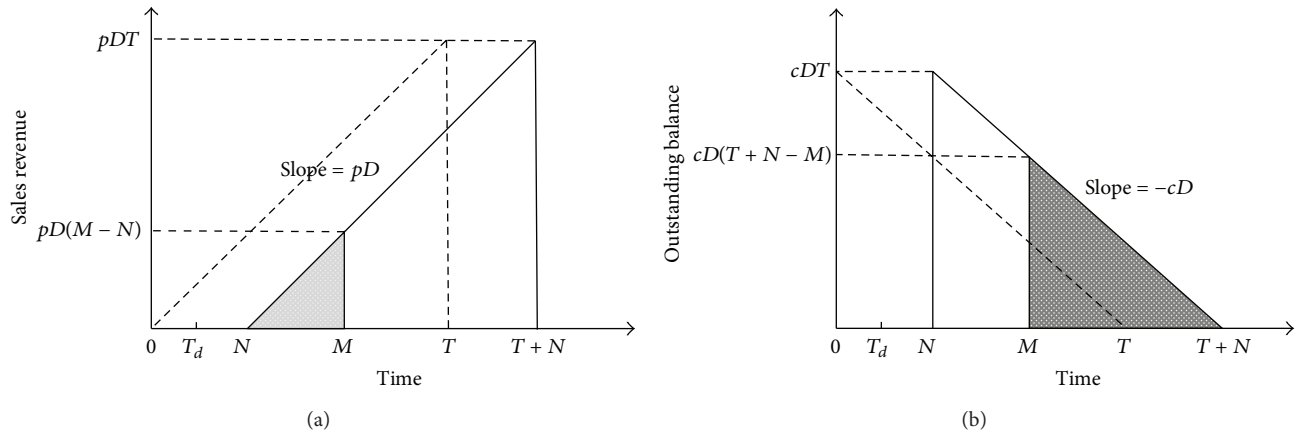

FIGURE 2: $Q \geq Q_d$ and $N < M \leq T + N$.

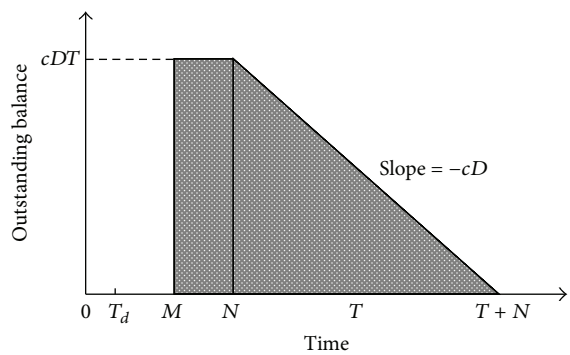

FIGURE 3: $Q \geq Q_d$ and $M \leq N$.

Similarly, from (6), (8), and (10), we have that the interest earned per year for the case with full delay in payments (say IE_1) is

$$\mathrm{IE}_1 = \begin{cases} pI_eD(M-N) - \dfrac{pI_eDT}{2}, & \text{if } M \geq T+N, \\ \dfrac{pI_eD(M-N)^2}{(2T)}, & \text{if } T+N \geq M > N, \\ 0, & \text{if } T+N > N \geq M. \end{cases} \tag{12}$$

Therefore, the total cost per year for the case with full delay in payments (denoted by $\mathrm{TC}_1(T)$) is given by

$$\mathrm{TC}_1(T) = \begin{cases} \mathrm{TC}_{11}(T) & \text{if } T \leq T_W, \\ \mathrm{TC}_{12}(T) & \text{if } T > T_W, \end{cases} \tag{13}$$

where

$$\mathrm{TC}_{11}(T) = \mathrm{OC} + \mathrm{HC}_1 + \mathrm{IC}_1 - \mathrm{IE}_1 = \begin{cases} \mathrm{TC}_{11\text{-}1}(T), & \text{if } M \geq T+N, \\ \mathrm{TC}_{11\text{-}2}(T), & \text{if } T+N \geq M > N, \\ \mathrm{TC}_{11\text{-}3}(T), & \text{if } T+N > N \geq M, \end{cases} \tag{14}$$

$$\mathrm{TC}_{12}(T) = \mathrm{OC} + \mathrm{HC}_2 + \mathrm{IC}_1 - \mathrm{IE}_1 = \begin{cases} \mathrm{TC}_{12\text{-}1}(T), & \text{if } M \geq T+N, \\ \mathrm{TC}_{12\text{-}2}(T), & \text{if } T+N \geq M > N, \\ \mathrm{TC}_{12\text{-}3}(T), & \text{if } T+N > N \geq M, \end{cases} \tag{15}$$

$$\mathrm{TC}_{11\text{-}1}(T) = \frac{A}{T} + \frac{(h+pI_c)DT}{2} - pI_eD(M-N), \tag{16}$$

$$\mathrm{TC}_{11\text{-}2}(T) = \frac{A}{T} + \frac{(h+cI_c)DT}{2} - cI_cD(M-N) + \frac{(cI_c - pI_e)D(M-N)^2}{2T}, \tag{17}$$

$$\mathrm{TC}_{11\text{-}3}(T) = \frac{A}{T} + \frac{(h+cI_c)DT}{2} + cI_cD(N-M), \tag{18}$$

$$\mathrm{TC}_{12\text{-}1}(T) = \frac{A}{T} + \frac{hDT_W(2T-T_W) + kD(T-T_W)^2}{2T} - pI_eD(M-N) + \frac{pI_eDT}{2}, \tag{19}$$

$$\mathrm{TC}_{12\text{-}2}(T) = \frac{A}{T} + \left(\left(hDT_W(2T-T_W) + kD(T-T_W)^2 + (cI_c - pI_e)D(M-N)^2\right) \times (2T)^{-1}\right) + \frac{cI_cDT}{2} - cI_cD(M-N), \tag{20}$$

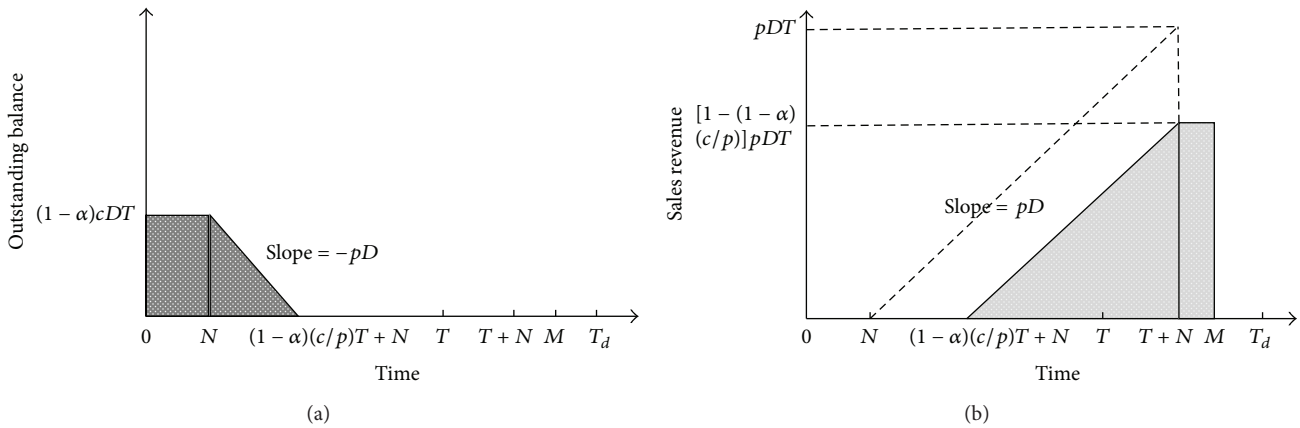

Figure 4: $Q < Q_d$ and $N < T + N \leq M$.

$$\begin{aligned} \mathrm{TC}_{12\text{-}3}(T) &= \frac{A}{T} + \frac{hDT_W(2T - T_W) + kD(T - T_W)^2}{2T} \\ &\quad + \frac{cI_cDT}{2} + cI_cD(N - M). \end{aligned} \tag{21}$$

Note that $\mathrm{TC}_{1i\text{-}1}(M - N) = \mathrm{TC}_{1i\text{-}2}(M - N)$, $i = 1, 2$.

Case 2 (partial delay in payments ($T < T_d$)). In this case, the partial delay in payment is permitted where the wholesaler must take a loan to pay the supplier the amount $(1 - \alpha)cDT$ immediately when the order is filled and pay the rest at the time M. From the constant sales revenue pD, the wholesaler will be able to pay off the loan $(1 - \alpha)cDT$ at the time $(1 - \alpha)(c/p)T + N$. Similar to Case 1, based on the values of M, $(1 - \alpha)(c/p)T + N$, and $T + N$, we have the following three alternative situations: (1) $M \geq T + N$, (2) $T + N \geq M \geq (1 - \alpha)(c/p)T + N$, and (3) $(1 - \alpha)(c/p)T + N \geq M$. Let us discuss them accordingly.

(1) $M \geq T + N$. In this situation, the wholesaler takes a loan to pay the supplier the amount $(1 - \alpha)cDT$ immediately but receives the revenue after N. That is, the wholesaler will pay off the loan from sales revenue at time $(1 - \alpha)(c/p)T + N$ and the interest earned starts from time $(1 - \alpha)(c/p)T + N$ to M (see Figure 4). Consequently, the interest charged per year (say IC_{21}) is

$$\begin{aligned} \mathrm{IC}_{21} &= \frac{(1-\alpha)\,cI_cDT\,\{N + [(1-\alpha)(c/p)T + N]\}}{2T} \\ &= \frac{(1-\alpha)\,cI_cD}{2}\left[2N + (1-\alpha)\left(\frac{c}{p}\right)T\right], \end{aligned} \tag{22}$$

and the interest earned per year (say IE_{21}) is

$$\begin{aligned} \mathrm{IE}_{21} &= \Bigg([p - (1-\alpha)\,c]\,I_eDT \\ &\quad \times \left\{(M - T - N) + \left[M - (1-\alpha)\left(\frac{c}{p}\right)T - N\right]\right\}\Bigg) \\ &\quad \times (2T)^{-1} \\ &= \frac{[1 - (1-\alpha)(c/p)]\,pI_eD}{2} \\ &\quad \times \left[2(M - N) - (1-\alpha)\left(\frac{c}{p}\right)T - T\right]. \end{aligned} \tag{23}$$

(2) $T + N \geq M \geq (1 - \alpha)(c/p)T + N$. In this situation, the wholesaler takes a loan to pay the supplier the amount $(1 - \alpha)cDT$ immediately but receives the revenue after N. That is, the wholesaler will pays off the loan from sales revenue at time $(1 - \alpha)(c/p)T + N$ and the interest earned starts from time $(1-\alpha)(c/p)T+N$ to M (see Figure 5). After M, the wholesaler starts paying for the interest charges with the rate I_c on items sold. As a result, the interest charged per year (say IC_{22}) is

$$\begin{aligned} \mathrm{IC}_{22} &= \frac{(1-\alpha)\,cI_cDT\,\{N + [(1-\alpha)(c/p)T + N]\}}{2T} \\ &\quad + \frac{cI_cD(T + N - M)^2}{2T} \\ &= \frac{(1-\alpha)\,cI_cD}{2}\left[2N + (1-\alpha)\left(\frac{c}{p}\right)T\right] \\ &\quad + \frac{cI_cD(T + N - M)^2}{2T}, \end{aligned} \tag{24}$$

and the interest earned per year (say IE_{22}) is

$$\mathrm{IE}_{22} = \frac{pI_eD[M - N - (1-\alpha)(c/p)T]^2}{2T}. \tag{25}$$

(3) $(1 - \alpha)(c/p)T + N \geq M$. In this situation, there is no interest earned. Due to $T < T_d$, the wholesaler takes a loan to pay the supplier the amount $(1 - \alpha)cDT$ immediately but receives the revenue after N. The loan will be paid off from sales revenue at the time $(1-\alpha)(c/p)T+N$. Furthermore, the wholesaler starts paying for the interest charges with the rate

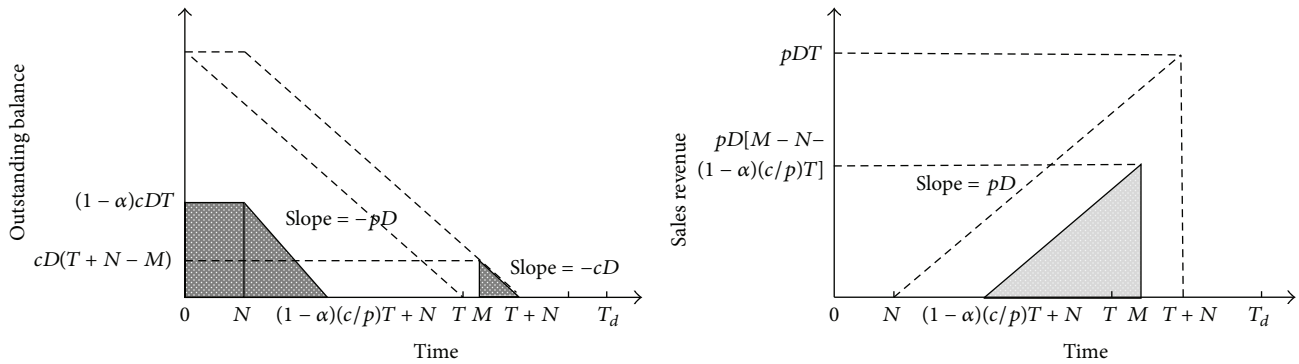

Figure 5: $Q < Q_d$ and $N < (1-\alpha)(c/p)T + N < M \leq T + N$.

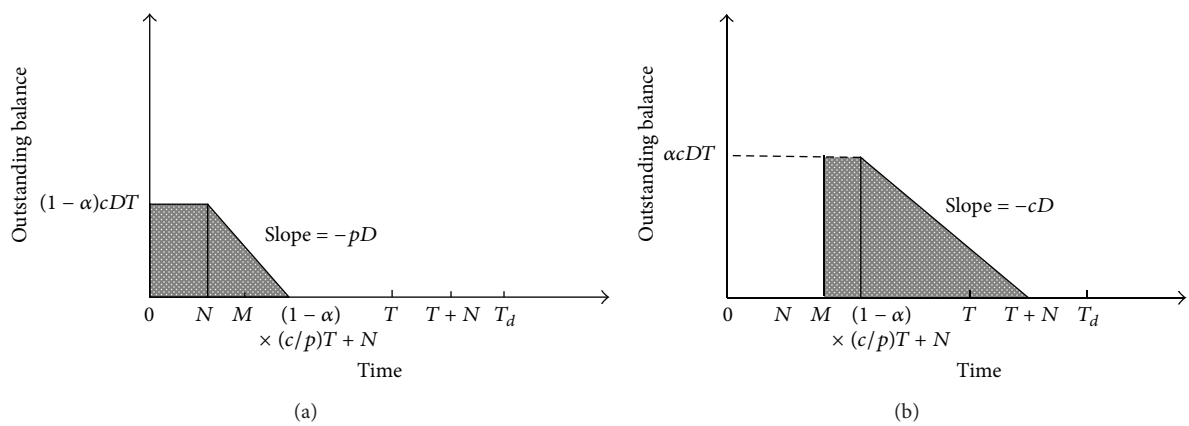

Figure 6: $Q < Q_d$ and $N < M < (1-\alpha)(c/p)T + N \leq T + N$.

I_c on items sold after $(1-\alpha)(c/p)T+N$ (see Figure 6). Hence, the interest charged per year (say IC_{23}) is

$$\begin{aligned}\mathrm{IC}_{23} &= \frac{(1-\alpha)\,cI_cDT\,\{N+[(1-\alpha)(c/p)T+N]\}}{2T} \\ &\quad + \frac{\alpha cI_cDT\,[(1-\alpha)(c/p)T+N-M]}{T} \\ &\quad + \frac{\alpha cI_cDT\,[T-(1-\alpha)(c/p)T]}{2T} \\ &= (1-\alpha)\,cI_cDN + \frac{cI_cD\,(1-\alpha)(c/p)T}{2} \\ &\quad + \frac{\alpha cI_cD\,[T+2(N-M)]}{2},\end{aligned} \tag{26}$$

and the interest earned per year (say IE_{23}) is

$$\mathrm{IE}_{23} = 0. \tag{27}$$

Consequently, from (22), (24), and (26), the interest charged per year for the case with partial delay in payments (say IC_2) is

$$\mathrm{IC}_2 = \begin{cases} \dfrac{(1-\alpha)\,cI_cD}{2}\left[2N+(1-\alpha)\left(\dfrac{c}{p}\right)T\right], \\ \qquad \text{if } M \geq T+N, \\ \dfrac{(1-\alpha)\,cI_cD}{2}\left[2N+(1-\alpha)\left(\dfrac{c}{p}\right)T\right] \\ +\dfrac{cI_cD(T+N-M)^2}{2T}, \\ \qquad \text{if } T+N \geq M \geq (1-\alpha)\left(\dfrac{c}{p}\right)T+N, \\ (1-\alpha)\,cI_cDN + \dfrac{cI_cD\,(1-\alpha)(c/p)T}{2} \\ +\dfrac{\alpha cI_cD\,[T+2(N-M)]}{2}, \\ \qquad \text{if } (1-\alpha)\left(\dfrac{c}{p}\right)T+N \geq M. \end{cases} \tag{28}$$

Similarly, from (23), (25), and (27), we have that the interest earned per year for the case with partial delay in payments (say IE_2) is

$$\mathrm{IE}_2 = \begin{cases} \dfrac{[1-(1-\alpha)(c/p)]\, pI_eD}{2} \\ \times\left[2(M-N)-(1-\alpha)\left(\dfrac{c}{p}\right)T-T\right], \\ \quad \text{if } M \geq T+N, \\ \dfrac{pI_eD[M-N-(1-\alpha)(c/p)T]^2}{2T}, \\ \quad \text{if } T+N \geq M \geq (1-\alpha)\left(\dfrac{c}{p}\right)T+N, \\ 0, \text{if } (1-\alpha)\left(\dfrac{c}{p}\right)T+N \geq M. \end{cases} \tag{29}$$

For convenient, we let $v = (1-\alpha)(c/p)$ where $0 \leq v < 1$. Therefore, the total cost per year for the case with partial delay in payments (denoted by $\mathrm{TC}_2(T)$) is given by

$$\mathrm{TC}_2(T) = \begin{cases} \mathrm{TC}_{21}(T) & \text{if } T \leq T_W, \\ \mathrm{TC}_{22}(T) & \text{if } T > T_W, \end{cases} \tag{30}$$

where

$$\begin{aligned} \mathrm{TC}_{21}(T) &= \mathrm{OC} + \mathrm{HC}_1 + \mathrm{IC}_2 - \mathrm{IE}_2 \\ &= \begin{cases} \mathrm{TC}_{21\text{-}1}(T), & \text{if } M \geq T+N, \\ \mathrm{TC}_{21\text{-}2}(T), & \text{if } T+N \geq M \geq vT+N, \\ \mathrm{TC}_{21\text{-}3}(T), & \text{if } vT+N \geq M, \end{cases} \end{aligned} \tag{31}$$

$$\begin{aligned} \mathrm{TC}_{22}(T) &= \mathrm{OC} + \mathrm{HC}_2 + \mathrm{IC}_2 - \mathrm{IE}_2 \\ &= \begin{cases} \mathrm{TC}_{22\text{-}1}(T), & \text{if } M \geq T+N, \\ \mathrm{TC}_{22\text{-}2}(T), & \text{if } T+N \geq M \geq vT+N, \\ \mathrm{TC}_{22\text{-}3}(T), & \text{if } vT+N \geq M, \end{cases} \end{aligned} \tag{32}$$

$$\begin{aligned} \mathrm{TC}_{21\text{-}1}(T) = {} & \frac{A}{T} + \frac{hDT}{2} + \frac{(1-\alpha)\, cI_cD}{2}(2N+vT) \\ & - \frac{(1-v)\, pI_eD}{2}(2(M-N)-vT-T), \end{aligned} \tag{33}$$

$$\begin{aligned} \mathrm{TC}_{21\text{-}2}(T) = {} & \frac{A}{T} + \frac{hDT}{2} + \frac{(1-\alpha)\, cI_cD}{2}(2N+vT) \\ & + \frac{cI_cD(T+N-M)^2}{2T} \\ & - \frac{pI_eD(M-N-vT)^2}{2T}, \end{aligned} \tag{34}$$

$$\begin{aligned} \mathrm{TC}_{21\text{-}3}(T) = {} & \frac{A}{T} + \frac{(h+cI_cv)\, DT}{2} + (1-\alpha)\, cI_cDN \\ & + \frac{\alpha cI_cD[T+2(N-M)]}{2}, \end{aligned} \tag{35}$$

$$\begin{aligned} \mathrm{TC}_{22\text{-}1}(T) = {} & \frac{A}{T} + \frac{hDT_W(2T-T_W)+kD(T-T_W)^2}{2T} \\ & + \frac{(1-\alpha)\, cI_cD}{2}(2N+vT) \\ & - \frac{(1-v)\, pI_eD}{2}[2(M-N)-vT-T], \end{aligned} \tag{36}$$

$$\begin{aligned} \mathrm{TC}_{22\text{-}2}(T) = {} & \frac{A}{T} + \frac{hDT_W(2T-T_W)+kD(T-T_W)^2}{2T} \\ & + \frac{(1-\alpha)\, cI_cD}{2}(2N+vT) \\ & + \frac{cI_cD(T+N-M)^2}{2T} \\ & - \frac{pI_eD(M-N-vT)^2}{2T}, \end{aligned} \tag{37}$$

$$\begin{aligned} \mathrm{TC}_{22\text{-}3}(T) = {} & \frac{A}{T} + \frac{hDT_W(2T-T_W)+kD(T-T_W)^2}{2T} \\ & + \frac{cI_cvDT}{2} + (1-\alpha)\, cI_cDN \\ & + \frac{\alpha cI_cD[T+2(N-M)]}{2}. \end{aligned} \tag{38}$$

Note that $\mathrm{TC}_{2i\text{-}1}(M-N) = \mathrm{TC}_{2i\text{-}2}(M-N)$ and $\mathrm{TC}_{2i\text{-}2}((M-N)/v) \geq \mathrm{TC}_{2i\text{-}3}((M-N)/v)$ for $M > N$, $i = 1, 2$.

Remark 1. When $\alpha = 1$ (i.e., the supplier offers the full delay in payment regardless of the order quantity), which implies $v = 0$, we have $\mathrm{TC}_{2i\text{-}j}(T) = \mathrm{TC}_{1i\text{-}j}(T)$, for $i = 1, 2$ and $j = 1, 2, 3$.

Remark 2. When $h = k$ (i.e., the capacity in own warehouse is unlimited), we have $\mathrm{TC}_{i2\text{-}j}(T) = \mathrm{TC}_{i1\text{-}j}(T)$, for $i = 1, 2$ and $j = 1, 2, 3$.

Remark 3. When $\alpha = 1$ and $h = k$, we have $\mathrm{TC}_{22\text{-}j}(T) = \mathrm{TC}_{21\text{-}j}(T) = \mathrm{TC}_{12\text{-}j}(T) = \mathrm{TC}_{11\text{-}j}(T)$, for $j = 1, 2, 3$.

Remark 4. (i) When $\alpha = 0$ and $h = k$, the model can be reduced to Ouyang et al. [19].

(ii) When $\alpha = 1$, $Q_d = 0$, and $h = k$, the model is similar to Huang [17] and Teng et al. [20].

(iii) When $Q_d = 0$, $N = 0$, and $h = k$, the model is similar to Huang [8].

(iv) When $\alpha = 1$, $Q_d = 0$, $N = 0$, and $h = k$, the model can be reduced to Teng [5].

(v) When $\alpha = 1$, $Q_d = 0$, $N = 0$, $h = k$, and $p = c$, the model is the same as Goyal [2].

Therefore, our model is in general framework that includes numerous previous models such as Goyal [2], Teng

[5], Huang [8, 17], Ouyang et al. [19], and Teng et al. [20] as special cases.

4. Theoretical Results

Now, we will determine the optimal length of replenishment cycle time in years T^* which minimizes the annual total relevant cost. First, from Remarks 1–3, we can see that the total relevant cost functions of $\mathrm{TC}_{22\text{-}j}(T)$ can be reduced to $\mathrm{TC}_{11\text{-}j}(T)$, $\mathrm{TC}_{12\text{-}j}(T)$, and $\mathrm{TC}_{21\text{-}j}(T)$ as $\alpha = 1$ or/and $h = k$, where $j = 1, 2, 3$. Here we only discuss how to find the optimal length of replenishment cycle time in years T_{22} that minimizes the annual total relevant cost $\mathrm{TC}_{22\text{-}j}(T)$, where $j = 1, 2, 3$. Following, we will develop an iterative algorithm to find the optimal solution T^* for the whole problem.

The first-order necessary condition for $\mathrm{TC}_{22\text{-}1}(T)$ to be minimized is $d\mathrm{TC}_{22\text{-}1}(T)/dT = 0$ which leads to

$$T_{22\text{-}1} = \sqrt{\frac{2A + (k-h)\,DT_W^2}{D\left[k + (1-\alpha)\,cI_c v + \left(1-v^2\right) pI_e\right]}}. \tag{39}$$

Note that $k > h$, $0 \le \alpha \le 1$, and $0 \le v < 1$; and hence $T_{22\text{-}1}$ is well defined. Furthermore, the second-order sufficient condition is

$$\frac{d^2\mathrm{TC}_{22\text{-}1}(T)}{dT^2} = \frac{2A + (k-h)\,DT_W^2}{T^3} > 0. \tag{40}$$

Therefore, $\mathrm{TC}_{22\text{-}1}(T)$ is a convex function of T, and $T_{22\text{-}1}$ in (39) satisfies $d\mathrm{TC}_{22\text{-}1}(T)/dT = 0$. To ensure $M \ge T_{22\text{-}1} + N$, we substitute (39) into the inequality $M \ge T_{22\text{-}1} + N$ and obtain that

$M \ge T_{22\text{-}1} + N$, if and only if

$$\begin{aligned} 2A \le{}& \left[k + (1-\alpha)\,cI_c v + \left(1-v^2\right) pI_e\right] D(M-N)^2 \\ &- (k-h)\,DT_W^2. \end{aligned} \tag{41}$$

On the other hand, if $2A > [k+(1-\alpha)cI_c v+(1-v^2)pI_e]D(M-N)^2 - (k-h)DT_W^2$, then we have

$$\begin{aligned} &\frac{d\mathrm{TC}_{22\text{-}1}(T)}{dT} \\ &= \left[k + (1-\alpha)\,cI_c v + \left(1-v^2\right) pI_e\right] D \\ &\quad - \frac{2A + (k-h)\,DT_W^2}{T^2} \\ &< -\frac{\left[k + (1-\alpha)\,cI_c v + \left(1-v^2\right) pI_e\right] D\left[(M-N)^2 - T^2\right]}{2T^2} \\ &< 0, \end{aligned} \tag{42}$$

for all $T \in (0, M-N]$ which implies that $\mathrm{TC}_{22\text{-}1}(T)$ is a strictly decreasing function of $T \in (0, M-N]$. Therefore, $\mathrm{TC}_{22\text{-}1}(T)$ has a minimum value at the boundary point $T = M - N$.

For notational convenience, we let

$$\begin{aligned} \Delta_1(\alpha, k) \equiv{}& \left[k + (1-\alpha)\,cI_c v + \left(1-v^2\right) pI_e\right] D(M-N)^2 \\ &- (k-h)\,DT_W^2. \end{aligned} \tag{43}$$

Then we have the following result.

Lemma 5. (1) *If $2A \le \Delta_1(\alpha, k)$, then $TC_{22\text{-}1}(T)$ has a minimum value at $T = T_{22-1}$.*

(2) *If $2A > \Delta_1(\alpha, k)$, then $TC_{22\text{-}1}(T)$ has a minimum value at $T = M - N$.*

Similarly, the first-order necessary condition for $\mathrm{TC}_{22\text{-}2}(T)$ to be minimized is $d\mathrm{TC}_{22\text{-}2}(T)/dT = 0$ which leads to

$$T_{22\text{-}2} = \sqrt{\frac{2A + (k-h)\,DT_W^2 + \left(cI_c - pI_e\right) D(M-N)^2}{D\left[k + (1-\alpha)\,cI_c v + cI_c - pI_e v^2\right]}}. \tag{44}$$

Furthermore, the second-order sufficient condition is

$$\begin{aligned} &\frac{d^2\mathrm{TC}_{22\text{-}2}(T)}{dT^2} \\ &\quad = \frac{2A + (k-h)\,DT_W^2 + \left(cI_c - pI_e\right) D(M-N)^2}{T^3}. \end{aligned} \tag{45}$$

To ensure $T + N \ge M \ge vT + N$, we substitute (44) into the inequality $T_{22\text{-}2} + N \ge M \ge vT_{22\text{-}2} + N$, and obtain that

$$\begin{aligned} &T_{22-2} + N \ge M \ge vT_{22-2} + N, \\ &\text{if and only if } \Delta_1(\alpha, k) \le 2A \le \Delta_2(\alpha, k), \end{aligned} \tag{46}$$

where $\Delta_1(\alpha, k)$ is defined as above, and

$$\begin{aligned} \Delta_2(\alpha, k) \equiv{}& (p-c)\,I_c D(M-N)^2 \\ &+ \left(k + cI_c\right) \frac{D(M-N)^2}{v^2} - (k-h)\,DT_W^2. \end{aligned} \tag{47}$$

It is noted that if $2A \ge \Delta_1(\alpha, k)$, then we have

$$\begin{aligned} &2A + (k-h)\,DT_W^2 + \left(cI_c - pI_e\right) D(M-N)^2 \\ &\quad \ge \left[k + (1-\alpha)\,cI_c v + cI_c - pI_e v^2\right] D(M-N)^2 \\ &\quad = \left[k + cI_c + pv^2\left(I_c - I_e\right)\right] D(M-N)^2 \\ &\quad > 0. \end{aligned} \tag{48}$$

Therefore, $T_{22\text{-}2}$ in (44) is well defined and $d^2\mathrm{TC}_{22\text{-}2}(T)/dT^2 > 0$, which implies that $\mathrm{TC}_{22\text{-}2}(T)$ is a convex function of T.

On the other hand, if $2A > \Delta_2(\alpha, k)$, then we have

$$\begin{aligned}\frac{d\mathrm{TC}_{22\text{-}2}(T)}{dT} &= \left[k + cI_c + pv^2\left(I_c - I_e\right)\right]D \\ &\quad - \frac{2A + (k-h)DT_W^2 + \left(cI_c - pI_e\right)D(M-N)^2}{T^2} \\ &< \left[k + cI_c + pv^2\left(I_c - I_e\right)\right]D\left[1 - \frac{(M-N)^2}{v^2T^2}\right] \\ &< 0,\end{aligned} \tag{49}$$

for all $T \in (M-N, (M-N)/v)$ which implies that $\mathrm{TC}_{22\text{-}2}(T)$ is a strictly decreasing function of $T \in [M-N, (M-N)/v]$. Therefore, $\mathrm{TC}_{22\text{-}2}(T)$ has a minimum value at the boundary point $T = (M-N)/v$.

If $2A < \Delta_1(\alpha, k)$, then we have

$$\begin{aligned}\frac{d\mathrm{TC}_{22\text{-}2}(T)}{dT} &= \left[k + cI_c + pv^2\left(I_c - I_e\right)\right]D \\ &\quad - \frac{2A + (k-h)DT_W^2 + \left(cI_c - pI_e\right)D(M-N)^2}{T^2} \\ &> \left[k + cI_c + pv^2\left(I_c - I_e\right)\right]D\left[1 - \frac{(M-N)^2}{T^2}\right] \\ &> 0,\end{aligned} \tag{50}$$

for all $T \in (M-N, (M-N)/v)$ which implies that $\mathrm{TC}_{22\text{-}2}(T)$ is a strictly increasing function of $T \in [M-N, (M-N)/v]$. Therefore, $\mathrm{TC}_{22\text{-}2}(T)$ has a minimum value at the boundary point $T = M-N$.

From above arguments, we have proved the following result.

Lemma 6. (1) *If $\Delta_1(\alpha, k) \le 2A \le \Delta_2(\alpha, k)$, then $TC_{22\text{-}2}(T)$ has a minimum value at $T = T_{22-2}$.*

(2) *If $2A > \Delta_2(\alpha, k)$, then $TC_{22\text{-}2}(T)$ has a minimum value at $T = (M-N)/v$.*

(3) *If $2A < \Delta_1(\alpha, k)$, then $TC_{22\text{-}2}(T)$ has a minimum value at $T = M-N$.*

By using analogous discussions, we can easily obtain the values of T (say $T_{22\text{-}3}$) which minimizes $\mathrm{TC}_{22\text{-}3}(T)$ is

$$T_{22\text{-}3} = \sqrt{\frac{2A + (k-h)DT_W^2}{D\left(k + cI_cv + \alpha cI_c\right)}}. \tag{51}$$

To ensure $vT + N \ge M$, we substitute (51) into the inequality and $vT_{22\text{-}2} + N \ge M$ and obtain that

$$vT_{22\text{-}3} + N \ge M, \qquad \text{if and only if } 2A \ge \Delta_3(\alpha, k), \tag{52}$$

where

$$\Delta_3(\alpha, k) \equiv \left(k + cI_cv + \alpha cI_c\right)\frac{D(M-N)^2}{v^2} - (k-h)DT_W^2. \tag{53}$$

Conversely, if $2A < \Delta_3(\alpha, k)$, then we have

$$\begin{aligned}\frac{d\mathrm{TC}_{22\text{-}3}(T)}{dT} &= -\frac{2A + (k-h)DT_W^2}{2T^2} + \frac{D\left(k + cI_cv + \alpha cI_c\right)}{2} \\ &> \frac{D\left(k + cI_cv + \alpha cI_c\right)\left[(vT)^2 - (M-N)^2\right]}{2v^2T^2} \\ &> 0.\end{aligned} \tag{54}$$

Therefore, $\mathrm{TC}_{22\text{-}3}(T)$ is s strictly increasing function of $T \in [(M-N)/v, \infty)$, which implies that $\mathrm{TC}_{22\text{-}3}(T)$ has a minimum value at the boundary point $T = (M-N)/v$. From above arguments, we have proved the following result.

Lemma 7. (1) *If $2A \ge \Delta_3(\alpha, k)$, then $TC_{22\text{-}3}(T)$ has a minimum value at $T = T_{22-3}$.*

(2) *If $2A < \Delta_1(\alpha, k)$, then $TC_{22\text{-}3}(T)$ has a minimum value at $T = (M-N)/v$.*

It is obvious that $\Delta_2(\alpha, k) > \Delta_1(\alpha, k)$ and $\Delta_2(\alpha, k) > \Delta_3(\alpha, k)$. Consequently, combining Lemmas 5, 6, and 7 and the facts that $\mathrm{TC}_{22\text{-}1}(M-N) = TRC_{22\text{-}2}(M-N)$ and $\mathrm{TC}_{22\text{-}2}((M-N/v)) \ge \mathrm{TC}_{22\text{-}3}((M-N/v))$ for $M > N$, we can obtain a table (Table 1) to determine the optimal length of cycle time T that minimizes the annual total relevant cost $\mathrm{TC}_{22}(T)$ (say T_{22}).

For Remarks 1–3, we can obtain the following tables (Tables 2–4) to determine the optimal lengths of cycle time T that minimize the annual total relevant costs $\mathrm{TC}_{11}(T)$, $\mathrm{TC}_{12}(T)$, and $\mathrm{TC}_{21}(T)$, respectively (say T_{11}, T_{12}, and T_{21}).

Next, we will can establish the following algorithm to determine the optimal length of cycle time T^*.

Algorithm.

Step 1. Compare T_d with T_W. If $T_d \ge T_W$, then go to Step 2; otherwise, go to Step 3.

Step 2

Step 2.1. Calculate the value of T_{12} from Table 3 and compare it with T_d. If $T_{12} > T_d$, then set $T_1 = T_{12}$ and evaluate $\mathrm{TC}_1(T_1) = \mathrm{TC}_{12}(T_{12})$; otherwise, T_{12} is not a feasible solution. Set $\mathrm{TC}_1(T) = +\infty$.

Step 2.2. Calculate T_{22} from Table 1 and compare it with T_d and T_W. If $T_d \ge T_{22} > T_W$, then set $T_2 = T_{22}$ and evaluate $\mathrm{TC}_2(T_2) = \mathrm{TC}_{22}(T_{22})$; otherwise, T_{22} is not a feasible solution. Set $\mathrm{TC}_2(T) = +\infty$.

Step 2.3. Calculate T_{21} from Table 4 and compare it with T_W. If $T_W \ge T_{21}$, then set $T_2 = T_{21}$ and evaluate

Table 1: The optimal length of replenishment cycle time T_{22} under various cases.

Situation	Case	Condition	$\mathrm{TC}_{22}(T_{22})$	T_{22}
$M > N$	$\Delta_1(\alpha, k) > \Delta_3(\alpha, k)$	$2A > \Delta_2(\alpha, k)$	$\mathrm{TC}_{22\text{-}3}(T_{22\text{-}3})$	$T_{22\text{-}3}$
		$\Delta_2(\alpha, k) \geq 2A \geq \Delta_1(\alpha, k)$	$\min\{\mathrm{TC}_{22\text{-}2}(T_{22\text{-}2}), \mathrm{TC}_{22\text{-}3}(T_{22\text{-}3})\}$	$T_{22\text{-}2}$ or $T_{22\text{-}3}$
		$\Delta_1(\alpha, k) \geq 2A \geq \Delta_3(\alpha, k)$	$\min\{\mathrm{TC}_{22\text{-}1}(T_{22\text{-}1}), \mathrm{TC}_{22\text{-}3}(T_{22\text{-}3})\}$	$T_{22\text{-}1}$ or $T_{22\text{-}3}$
		$2A < \Delta_3(\alpha, k)$	$\min\{\mathrm{TC}_{22\text{-}1}(T_{22\text{-}1}), \mathrm{TC}_{22\text{-}3}((M-N)/v)\}$	$T_{22\text{-}1}$ or $(M-N)/v$
	$\Delta_1(\alpha, k) < \Delta_3(\alpha, k)$	$2A \geq \Delta_2(\alpha, k)$	$\mathrm{TC}_{22\text{-}3}(T_{22\text{-}3})$	$T_{22\text{-}3}$
		$\Delta_2(\alpha, k) > 2A > \Delta_3(\alpha, k)$	$\min\{\mathrm{TC}_{22\text{-}2}(T_{22\text{-}2}), \mathrm{TC}_{22\text{-}3}(T_{22\text{-}3})\}$	$T_{22\text{-}2}$ or $T_{22\text{-}3}$
		$\Delta_3(\alpha, k) \geq 2A \geq \Delta_1(\alpha, k)$	$\min\{\mathrm{TC}_{22\text{-}2}(T_{22\text{-}2}), \mathrm{TC}_{22\text{-}3}((M-N)/v)\}$	$T_{22\text{-}2}$ or $(M-N)/v$
		$2A < \Delta_1(\alpha, k)$	$\min\{\mathrm{TC}_{22\text{-}1}(T_{22\text{-}1}), \mathrm{TC}_{22\text{-}3}((M-N)/v)\}$	$T_{22\text{-}1}$ or $(M-N)/v$
$M \leq N$			$\mathrm{TC}_{22\text{-}3}(T_{22\text{-}3})$	$T_{22\text{-}3}$

Table 2: The optimal length of replenishment cycle time T_{11} under various cases.

Situation	Condition	$\mathrm{TC}_{11}(T_{11})$	T_{11}
$M > N$	$2A \geq \Delta_1(1, h)$	$\mathrm{TC}_{11\text{-}2}(T_{11\text{-}2})$,	$T_{11\text{-}2}$
	$2A < \Delta_1(1, h)$	$\mathrm{TC}_{11\text{-}1}(T_{11\text{-}1})$,	$T_{11\text{-}1}$
$M \leq N$		$\mathrm{TC}_{11\text{-}3}(T_{11\text{-}3})$	$T_{11\text{-}3}$

where $T_{11\text{-}1} = \sqrt{2A/[(h+pI_e)D]}$, $T_{11\text{-}2} = \sqrt{[2A+(cI_c-pI_e)D(M-N)^2]/[(h+cI_c)D]}$, and $T_{11\text{-}3} = \sqrt{2A/[(h+cI_c)D]}$.

Table 3: The optimal length of replenishment cycle time T_{12} under various cases.

Situation	Condition	$\mathrm{TC}_{12}(T_{12})$	T_{12}
$M > N$	$2A \geq \Delta_1(1, k)$	$\mathrm{TC}_{12\text{-}2}(T_{12\text{-}2})$,	$T_{12\text{-}2}$
	$2A < \Delta_1(1, k)$	$\mathrm{TC}_{12\text{-}1}(T_{12\text{-}1})$,	$T_{12\text{-}1}$
$M \leq N$		$\mathrm{TC}_{12\text{-}3}(T_{12\text{-}3})$	$T_{12\text{-}3}$

where $T_{12\text{-}1} = \sqrt{[2A+(k-h)DT_W^2]/(k+pI_e)D}$, $T_{12\text{-}2} = \sqrt{[2A+(cI_c-pI_e)D(M-N)^2+(k-h)DT_W^2]/[(k+cI_c)D]}$, and $T_{12\text{-}3} = \sqrt{[2A+(k-h)DT_W^2]/[(k+cI_c)D]}$.

$\mathrm{TC}_2(T_2) = \mathrm{TC}_{21}(T_{21})$; otherwise, T_{21} is not a feasible solution. Set $\mathrm{TC}_2(T) = +\infty$. Go to Step 4.

Step 3

Step 3.1. Calculate T_{12} from Table 3 and compare it with T_W. If $T_{12} > T_W$, then set $T_1 = T_{12}$ and evaluate $\mathrm{TC}_1(T_1) = \mathrm{TC}_{12}(T_{12})$; otherwise, T_{12} is not a feasible solution. Set $\mathrm{TC}_1(T) = +\infty$.

Step 3.2. Calculate T_{11} from Table 2 and compare it with T_d and T_W. If $T_W \geq T_{11} > T_d$, then set $T_1 = T_{11}$ and evaluate $\mathrm{TC}_1(T_1) = \mathrm{TC}_{11}(T_{11})$; otherwise, T_{11} is not a feasible solution. Set $\mathrm{TC}_1(T) = +\infty$.

Step 3.3. Calculate T_{21} from Table 4 and compare it with T_d. If $T_d \geq T_{21}$, then set $T_2 = T_{21}$ and evaluate $\mathrm{TC}_2(T_2) = \mathrm{TC}_{21}(T_{21})$; otherwise, T_{21} is not a feasible solution. Set $\mathrm{TC}_2(T) = +\infty$. Go to Step 4.

Step 4. Find $\mathrm{Min}_{i=1,2}\mathrm{TC}_i(T)$. Let $\mathrm{TC}(T^*) = \mathrm{Min}_{i=1,2}\mathrm{TC}_i(T)$, and then T^* is the optimal solution.

5. Numerical Example

To illustrate the previous results, we use a numerical example as follows.

Example 1. Given A = \$100/order, p = \$80/unit, c = \$50/unit, D = 2500units/year, k = \$12/unit/year, h = \$10/unit/year, I_c = \$0.15/\$/year, I_e = \$0.1/\$/year, M = 0.25 years, and N = 0.25 years, according to Algorithm in the previous section, we obtain the optimal cycle time and the optimal order quantity for different parameters of $\alpha \in \{0.2, 0.5, 0.8\}$, $W \in \{100, 200, 300\}$, and $Q_d \in \{100, 200, 300\}$ as shown in Table 5.

From the results of Table 5, the following observations can be made.

(1) The wholesaler will determine whether to enjoy full or partial delay in payments based on the value of the permitted minimum order quantity with full delay in payments. For the low permitted minimum order quantity with full delay in payments (e.g., $Q_d = 100$), the wholesaler will take the fully permissible delay and pay at the end of M. Otherwise, if the value of Q_d is high enough (e.g., $Q_d \geq 200$), then the wholesaler will take a loan to pay its supplier the partial payment of $(1-\alpha)cQ$ immediately when the order is received.

(2) The wholesaler will determine whether to rent an additional warehouse based on the value of the capacity in own warehouse. That is, when the capacity in own warehouse is low (e.g., $W = 100$), the wholesaler will need to rent an additional warehouse to satisfy more goods in stocks. If the capacity in own warehouse is high enough (e.g., $W \geq 200$), then the wholesaler will no longer rent an additional warehouse.

(3) For the case of partial delay in payments ($Q_d \geq 200$), when the value of the fraction of the delay payments permitted by the supplier increases, all the optimal

Table 4: The optimal length of replenishment cycle time T_{21} under various cases.

Situation	Case	Condition	$\mathrm{TC}_{21}(T_{21})$	T_{21}
$M > N$	$\Delta_1(\alpha, h) > \Delta_3(\alpha, h)$	$2A > \Delta_2(\alpha, h)$	$\mathrm{TC}_{21\text{-}3}(T_{21\text{-}3})$	$T_{21\text{-}3}$
		$\Delta_2(\alpha, h) \geq 2A \geq \Delta_1(\alpha, h)$	$\min\{\mathrm{TC}_{21\text{-}2}(T_{21\text{-}2}), \mathrm{TC}_{21\text{-}3}(T_{21\text{-}3})\}$	$T_{21\text{-}2}$ or $T_{21\text{-}3}$
		$\Delta_1(\alpha, h) \geq 2A \geq \Delta_3(\alpha, h)$	$\min\{\mathrm{TC}_{21\text{-}1}(T_{21\text{-}1}), \mathrm{TC}_{21\text{-}3}(T_{21\text{-}3})\}$	$T_{21\text{-}1}$ or $T_{21\text{-}3}$
		$2A < \Delta_3(\alpha, h)$	$\min\{\mathrm{TC}_{21\text{-}1}(T_{21\text{-}1}), \mathrm{TC}_{21\text{-}3}((M-N)/v)\}$	$T_{21\text{-}1}$ or $(M-N)/v$
	$\Delta_1(\alpha, h) < \Delta_3(\alpha, h)$	$2A \geq \Delta_2(\alpha, h)$	$\mathrm{TC}_{21\text{-}3}(T_{21\text{-}3})$	$T_{21\text{-}3}$
		$\Delta_2(\alpha, h) > 2A > \Delta_3(\alpha, h)$	$\min\{\mathrm{TC}_{21\text{-}2}(T_{21\text{-}2}), \mathrm{TC}_{21\text{-}3}(T_{21\text{-}3})\}$	$T_{21\text{-}2}$ or $T_{21\text{-}3}$
		$\Delta_3(\alpha, h) \geq 2A \geq \Delta_1(\alpha, h)$	$\min\{\mathrm{TC}_{21\text{-}2}(T_{21\text{-}2}), \mathrm{TC}_{21\text{-}3}((M-N)/v)\}$	$T_{21\text{-}2}$ or $(M-N)/v$
		$2A < \Delta_1(\alpha, h)$	$\min\{\mathrm{TC}_{21\text{-}1}(T_{21\text{-}1}), \mathrm{TC}_{21\text{-}3}((M-N)/v)\}$	$T_{21\text{-}1}$ or $(M-N)/v$
$M \leq N$			$\mathrm{TC}_{21\text{-}3}(T_{21\text{-}3})$	$T_{21\text{-}3}$

where $T_{21\text{-}1} = \sqrt{2A/\{D[h + (1-\alpha)cI_c v + (1-v^2)pI_e]\}}$, $T_{21\text{-}2} = \sqrt{[2A + (cI_c - pI_e)D(M-N)^2]/\{D[h + cI_c + (1-\alpha)cI_c v - pI_e v^2]\}}$, and $T_{21\text{-}3} = \sqrt{2A/[D(h + cI_c v + \alpha c I_c)]}$.

Table 5: Optimal solutions of Example 1.

α	W	Q_d	T^*	Q^*	TC (T^*)	Full/partial delay in payments	Rented warehouse
0.2	100	100	$T_{12\text{-}3} = 0.0653$	163.299	2984.34	Full	Yes
		200	$T_{22\text{-}3} = 0.0694$	173.623	6545.00	Partial	Yes
		300	$T_{22\text{-}3} = 0.0694$	173.623	6545.00	Partial	Yes
	200	100	$T_{11\text{-}3} = 0.0676$	169.031	2965.05	Full	No
		200	$T_{21\text{-}3} = 0.0724$	181.071	6511.34	Partial	No
		300	$T_{21\text{-}3} = 0.0724$	181.071	6511.34	Partial	No
	300	100	$T_{11\text{-}3} = 0.0676$	169.031	2965.05	Full	No
		200	$T_{21\text{-}3} = 0.0724$	181.071	6511.34	Partial	No
		300	$T_{21\text{-}3} = 0.0724$	181.071	6511.34	Partial	No
0.5	100	100	$T_{12\text{-}3} = 0.0653$	163.299	2984.34	Full	Yes
		200	$T_{22\text{-}3} = 0.0678$	169.526	5211.12	Partial	Yes
		300	$T_{22\text{-}3} = 0.0678$	169.526	5211.12	Partial	Yes
	200	100	$T_{11\text{-}3} = 0.0676$	169.031	2960.63	Full	No
		200	$T_{21\text{-}3} = 0.0705$	176.261	5180.45	Partial	No
		300	$T_{21\text{-}3} = 0.0705$	176.261	5180.45	Partial	No
	300	100	$T_{11\text{-}3} = 0.0676$	169.031	2960.63	Full	No
		200	$T_{21\text{-}3} = 0.0705$	176.261	5180.45	Partial	No
		300	$T_{21\text{-}3} = 0.0705$	176.261	5180.45	Partial	No
0.8	100	100	$T_{12\text{-}3} = 0.0653$	163.299	2984.34	Full	Yes
		200	$T_{22\text{-}3} = 0.0663$	165.707	3875.57	Partial	Yes
		300	$T_{22\text{-}3} = 0.0663$	165.707	3875.57	Partial	Yes
	200	100	$T_{11\text{-}3} = 0.0676$	169.031	2958.43	Full	No
		200	$T_{21\text{-}3} = 0.0687$	171.815	3847.61	Partial	No
		300	$T_{21\text{-}3} = 0.0687$	171.815	3847.61	Partial	No
	300	100	$T_{11\text{-}3} = 0.0676$	169.031	2958.43	Full	No
		200	$T_{21\text{-}3} = 0.0687$	171.815	3847.61	Partial	No
		300	$T_{21\text{-}3} = 0.0687$	171.815	3847.61	Partial	No

values of T^*, Q^*, and $\mathrm{TC}(T^*)$ decrease. The simple economic explanation for this is that the larger the fraction of the delay payments permitted, the lower the length of replenishment cycle, order quantity, and total relevant cost will be. That is, the wholesaler will reduce the order quantity to enjoy the benefit of delay in payments when the fraction of the delay payments permitted by the supplier increases.

Example 2. This example discusses the influences of changes in wholesaler's and retailer's trade credit periods on T^*, Q^*, and $\mathrm{TC}(T^*)$ of Example 1. For convenience, the case with fixed $\alpha = 0.5$, $W = 100$, and $Q_d = 200$ is taken into account. According to algorithm in the previous section, we obtain the optimal cycle time and the optimal order quantity for different parameters of $M \in \{0.2, 0.25, 0.3\}$ and $N \in \{0.2, 0.25, 0.3\}$ as shown in Table 6.

Table 6: Optimal solutions for various M and N.

M	N	T^*	Q^*	TC(T^*)
0.2	0.2	$T_{22\text{-}3} = 0.0678$	169.526	4742.37
	0.25	$T_{22\text{-}3} = 0.0678$	169.526	5679.87
	0.30	$T_{22\text{-}3} = 0.0678$	169.526	6617.37
0.25	0.2	$T_{22\text{-}2} = 0.0642$	160.469	4241.82
	0.25	$T_{22\text{-}3} = 0.0678$	169.526	5211.12
	0.30	$T_{22\text{-}3} = 0.0678$	169.526	6148.62
0.3	0.2	$T_{22\text{-}1} = 0.0639$	159.693	3556.24
	0.25	$T_{22\text{-}2} = 0.0642$	160.469	4710.57
	0.30	$T_{22\text{-}3} = 0.0678$	169.526	5649.20

Table 7: Effect of changes in major parameters of Example 2.

Parameter	% change	% change in		
		T^*	Q^*	TC(T^*)
A	−20	−10.29	−10.29	−7.74
	−10	−5.01	−5.01	−3.77
	10	4.77	4.77	3.59
	20	9.33	9.33	7.02
p	−20	2.16	2.16	2.00
	−10	1.10	1.10	1.00
	10	−1.14	−1.14	−0.99
	20	−2.31	−2.31	−1.98
c	−20	1.98	1.98	−10.73
	−10	0.96	0.96	−5.37
	10	−0.91	−0.91	5.38
	20	−1.78	−1.78	10.76
D	−20	11.54	−10.77	−13.06
	−10	5.28	−5.24	−6.42
	10	−4.54	5.01	6.24
	20	−8.50	9.80	12.33
k	−20	4.11	4.11	−0.70
	−10	1.94	1.94	−0.34
	10	−1.76	−1.76	0.31
	20	−3.35	−3.35	0.60
h	−20	1.93	1.93	−3.26
	−10	0.97	0.97	−1.63
	10	−0.98	−0.98	1.62
	20	−1.97	−1.97	3.23
I_c	−20	2.24	2.24	−9.44
	−10	1.08	1.08	−4.72
	10	−1.00	−1.00	4.71
	20	−1.92	−1.92	9.41
I_e	−20	2.01	2.01	0.64
	−10	1.01	1.01	0.33
	10	−1.03	−1.03	−0.33
	20	−2.09	−2.09	−0.67

From the results in Table 5, the following observations can be made.

(1) The optimal total cost per year decreases when the value of M increases or the value of N decreases. That is, it is benefit for the wholesaler to lengthen the wholesaler's trade credit period in years offered by the supplier or shorten the retailer's trade credit period in years offered by the wholesaler.

(2) For the high value of M (e.g., $M = 0.3$), the optimal order quantity increases as the value of N increases.

(3) For the high value of N (e.g., $N = 0.2$), the optimal order quantity decreases as the value of M increases.

Example 3. Here we discuss the influences of changes in major parameters A, p, c, D,k, h, I_c, and I_e on T^*, Q^*, and TC(T^*) of Example 2. For convenience, the case with fixed $M = 0.25$ and $N = 0.2$ is taken into account. The sensitivity analysis is performed by changing each of the parameters by −20%, −10%, +10%, and +20%, taking one parameter at a time and keeping the remains unchanged. The computational results are shown in Table 7.

On the basis of the results of Table 7, the following observations can be made.

(1) The optimal length of replenishment cycle T^*, the optimal order quantity Q^*, and the optimal total cost per year TC(T^*) increase with the increase in the value of A.

(2) It is obvious that all the values of T^*, Q^*, and TC(T^*) decrease as the revenue parameter p or I_e increases. That is, both selling price per unit and interest earned per dollar per year have negative effects on the length of replenishment cycle, order quantity, and the annual total relevant cost.

(3) When the value of c, k, h, or I_c decreases, the length of replenishment cycle and order quantity decrease but the total relevant cost increases. The simple economic explanation for this is that the larger cost parameters (purchasing cost, holding cost, and interest charged per dollar per year), the lower the length of replenishment cycle and order quantity, while the larger the annual total relevant cost will be.

(4) The value of T^* decreases while the values Q^* and TC(T^*) increase as the parameter D increases.

6. Conclusions

In this paper, we extended the previous economic order quantity (EOQ) models under two-level trade credit to reflect the following real-life situations: (1) the storage capacity is limited; (2) the supplier offers the retailer a partial upstream trade credit linked to order quantity; (3) the upstream trade credit may be longer than, equal to, or less than the downstream trade credit; and (4) the interest charged per dollar per year may be larger than, equal to, or less than the interest earned per dollar per year. In theoretical results, we studied the necessary and sufficient conditions for finding the optimal solution under various situations in Tables 1–4. Furthermore, we established a useful algorithm to obtain

the optimal solution. Finally, we have provided numerical examples and sensitivity analysis with major parameters to illustrate the proposed model and understand managerial insights. Our model is in general framework that includes numerous previous models such as Goyal [2], Teng [5], Huang [8, 17], Ouyang et al. [19], and Teng et al. [20] as special cases. It is our belief that our work will make some innovational and significant contributions for a wholesaler to determine his/her optimal lot size simultaneously when facing the real-life situations.

Conflict of Interests

None of the authors have financial relationship with other people or organizations that can inappropriately influence our work.

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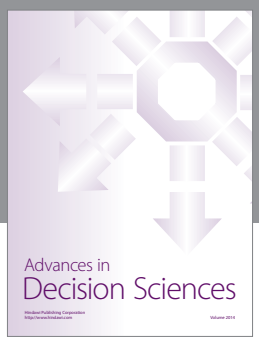

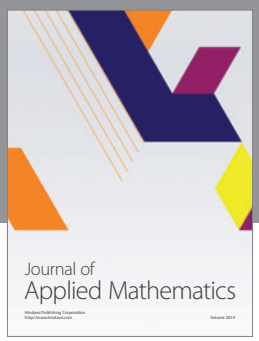

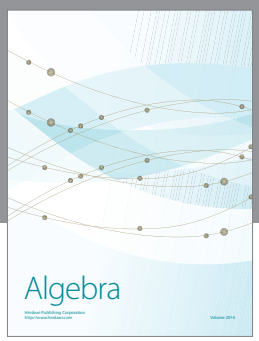

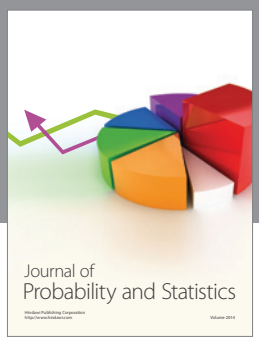

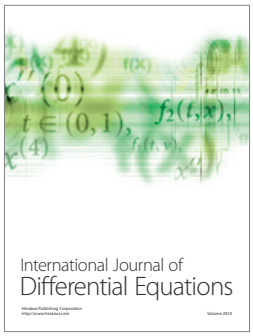

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