Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 780823, 10 pages
http://dx.doi.org/10.1155/2014/780823

Research Article

Optimization of Power Allocation for Multiusers in Multi-Spot-Beam Satellite Communication Systems

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Received 13 December 2013; Revised 18 February 2014; Accepted 20 February 2014; Published 24 March 2014

Academic Editor: Changzhi Wu

In recent years, multi-spot-beam satellite communication systems have played a key role in global seamless communication. However, satellite power resources are scarce and expensive, due to the limitations of satellite platform. Therefore, this paper proposes optimizing the power allocation of each user in order to improve the power utilization efficiency. Initially the capacity allocated to each user is calculated according to the satellite link budget equations, which can be achieved in the practical satellite communication systems. The problem of power allocation is then formulated as a convex optimization, taking account of a trade-off between the maximization of the total system capacity and the fairness of power allocation amongst the users. Finally, an iterative algorithm based on the duality theory is proposed to obtain the optimal solution to the optimization. Compared with the traditional uniform resource allocation or proportional resource allocation algorithms, the proposed optimal power allocation algorithm improves the fairness of power allocation amongst the users. Moreover, the computational complexity of the proposed algorithm is linear with both the numbers of the spot beams and users. As a result, the proposed power allocation algorithm is easy to be implemented in practice.

1. Introduction

As an important complement of the terrestrial networks, satellite communication systems provide service to users in several scenarios where terrestrial networks cannot be used. In modern satellite communication systems the multi-spot-beam technique has been widely applied, due to its advantage of concentrating the energy on a small area to provide high data rate to the users and reusing the same frequency to increase the total system capacity [1]. However, due to the limitations of the satellite platform, it is known that the satellite power resources are scarce and expensive. Moreover, the real traffic demands of each use are also different and time varying. As a result, it is necessary to optimize the power allocation to each user to satisfy its traffic demand.

The problem of power allocation in the multi-spot-beam satellite system has been investigated in [2–9]. In [2] the problem of power allocation was formulated as an optimization problem, which is shown to be convex. Then the Lagrangian multipliers were introduced to solve the optimization problem. However, the way to find the optimal Lagrangian multiplier was not provided in [2]. As a result, the methods of bisection and subgradient were applied to search the optimal Lagrangian multipliers in [3, 4]. In order to improve the total system capacity, a method of selecting a small number of active beams was proposed in [5], while keeping the fairness of power allocation amongst the beams. In [6], a joint power and bandwidth allocation algorithm was proposed. The algorithm improved both the total system capacity and the fairness amongst the beams, due to the dynamic allocation of both the power and bandwidth resource. The work in [2–6] proposed power allocation algorithms for the spot beams, without considering the power allocation to each user in the beams. However, for the users they only care about the power allocation to them. Therefore, it is significant to investigate how to allocate the power resources to the different users in different spot beams. In [7] a power allocation algorithm was proposed to stabilize the total system capacity even if the channel model and the specific arrival rates were unknown, as long as the arrival rate

vector was inside the capacity region. When the users were covered by multiple satellites, each of which had multiple queues for downlink traffic, a routing decision was made to maximize the total system throughput. In [8] an optimal power allocation algorithm was proposed to maximize the total system effective capacity in the mobile satellite systems. The main problem in [2–8] is that the allocated capacity to each user is calculated through the Shannon capacity formula. However, the capacity only can be obtained in theory, which cannot be achieved in the practical satellite communication system. Therefore, the proposed power allocation algorithms in these papers may not be the optimal algorithm for the practical systems. In order to overcome this drawback, in [9] a practical capacity formula was applied in the power allocation, aiming to maximize the number of users which are satisfied with the desired quality of service. However, only a heuristic algorithm was proposed without mathematic analysis, and the fairness of power allocation amongst the users was also ignored.

This paper is aimed to fill these gaps, by optimizing the power allocation to each user in the multi-spot-beam satellite communication, according to the practical formula for calculating the allocated capacity to each user. The first step is to calculate the allocated capacity to each user according to the satellite link budget equations, which can be achieved in the practical system. It is found that the allocated capacity to each user is determined by the allocated satellite power, coding and modulation mode, and channel condition. At the same time, the allocated capacity is also constrained by the bandwidth of each user. In order to preciously describe the impact of these factors on the power allocation, the problem of power allocation is mathematically formulated as a nonlinear optimization problem, which is demonstrated as a convex optimization problem. An iterative algorithm based on the duality theory is then proposed to obtain the optimal solution to the optimization. Finally, the impact of the coding and modulation mode adopted by each user, the bandwidth of each spot beam, and the channel conditions of each user on the power allocation results are discussed.

The main contributions of this paper are summarized as follows:

(1) the mathematical formulation of the problem of power allocation for multiple users in the multi-spot-beam satellite communication system according to the practical capacity formula, through a compromise between the maximization of the total system capacity and the fairness of the power allocation amongst the users;

(2) the proposal of an iterative algorithm, which will obtain the optimal solution to the optimization;

(3) the analysis of the impact of the coding and modulation mode, bandwidth of each spot beam, and channel conditions of each user on the power allocation results.

The remainder of this paper is organized as follows. In Section 2, the model of the multi-spot-beam satellite communication system with multiple users is described, and the calculation of the capacity allocated to each user according to the satellite link budget equations is also shown. In Section 3, the problem of power allocation is formulated as a convex optimization problem. Section 4 proposes the iterative algorithm to obtain the optimal solution to the optimization. Section 5 presents the simulation results and analyzes the impact of the coding and modulation mode, bandwidth of each spot beam, and channel conditions of each user on the power allocation result. Section 6 concludes the paper.

2. A Multi-Spot-Beam Satellite Communication System Model

Figure 1 shows the configuration of a multi-spot-beam satellite communication system, where a regenerative satellite payload is considered and the single channel per carrier (SCPC) technique is employed as the access method for the downlink. In this system uplink signal from user is demodulated and decoded to recover the originally transmitted data on the satellite. Then the decoded data to user is reencoded and remodulated using the same or different coding and modulation schemes in the downlink, where different users use different signals at different frequency and bandwidth. This paper proposes solving the problem of power allocation for different users in the downlink.

It is assumed that the system consists of K spot beams B_i, $i \in \{1, \ldots, K\}$, and M users U_i, $i \in \{1, \ldots, M\}$. The set of users which are served by the spot beam B_i is denoted by $\mathcal{N}_{B_i}$. The traffic demand of the ith user is T_i, and the satellite transmitting power allocated to the ith user is P_i. The coding and modulation mode adopted by the ith useris α_i, and the corresponding threshold signal-to-noise ratio per bit for demodulation is $(E_b/N_0)_{\alpha_i}$. It is noted that there are many schemes of the choice for α_i; however, it is beyond the scope of this paper. In order to simplify the problem, it is supposed that each user can only support one kind of coding and modulation mode. When the user is given, the coding and modulation mode adopted by the user is determined. It is meant that α_i is only determined by the ith user. Thus the allocated capacity D_i of the ith user is calculated according to the following equations [1]:

$$D_i = \frac{(C/N_0)_i}{(E_b/N_0)_{\alpha_i}}, \tag{1}$$

where $(C/N_0)_i$ is the downlink carrier power-to-noise power spectral density ratio of the ith user, which can be calculated according to the satellite link budget equation [1], given as follows:

$$\left(\frac{C}{N_0}\right)_i = \frac{P_i \cdot G_S}{L_i \cdot k} \cdot \left(\frac{G}{T}\right)_i, \tag{2}$$

where L_i is the downlink loss of the ith user, which is affected by the channel condition. It mainly consists of free-space loss, rain attenuation, and other losses due to catastrophic failure. $(G/T)_i$ is the gain-to-equivalent noise temperature ratio of the receiving equipment of the ith user. G_S is the transmitting

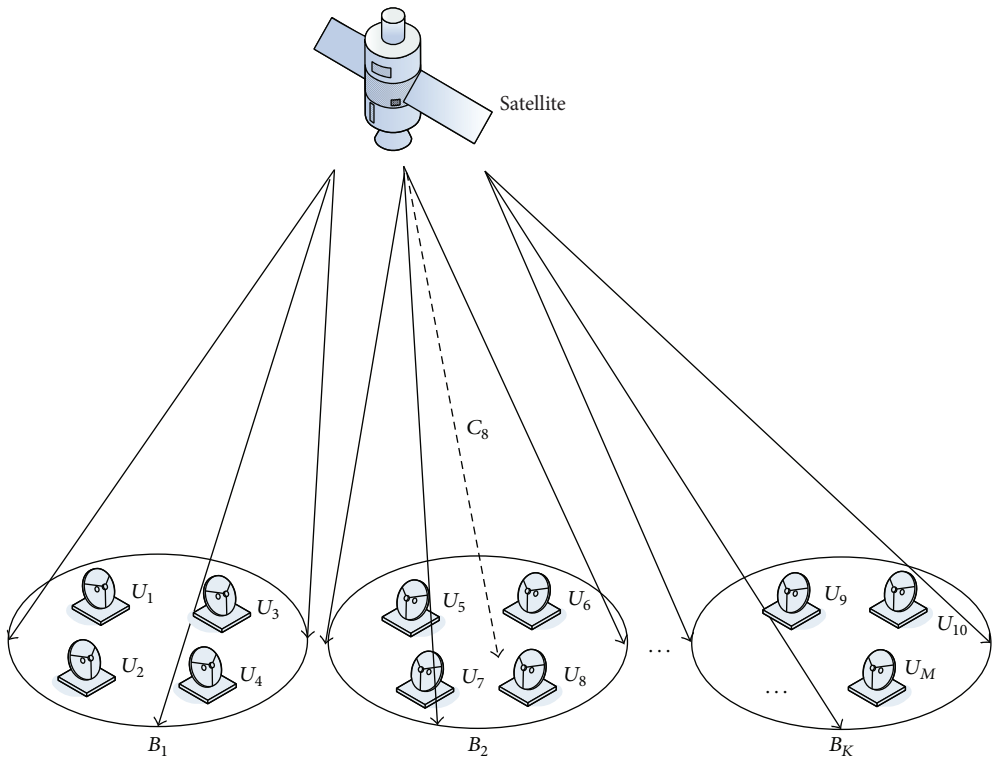

Figure 1: Configuration of a multi-spot-beam satellite communication system.

antenna gain of the satellite. It is assumed that the value of G_S is the same for all the users in this paper. k is Boltzmann's constant, which is 1.379×10^{-23} W/KHz.

It is noted that the interbeam interference from the sidelobes of adjacent spot beams will decrease the capacity of each user. However, the interbeam interference is ignored here, because the very narrow spot beams over a large number of spot beams are considered [10].

According to (1) and (2), it is shown that the capacity allocated to the ith user is determined by the allocated satellite transmitting power, given as

$$D_i = \frac{P_i \cdot G_S \cdot (G/T)_i}{L_i \cdot (E_b/N_0)_{\alpha_i} \cdot k}. \tag{3}$$

It is observed from (3) that the allocated capacity D_i of the ith user is increased as the power allocated to it increases. However, the total satellite power resources are fixed, so the capacity of the system is limited. Moreover, the allocated capacity of each user is also constrained by the bandwidth resources allocated to it, which are also scarce in the system. When the coding and modulation mode adopted by the ith user is given, the bandwidth that needs to be provided to it is expressed as

$$W_i = \frac{D_i \cdot [1 + \rho(\alpha_i)]}{\eta(\alpha_i)}, \tag{4}$$

where $\eta(\alpha_i)$ and $\rho(\alpha_i)$ are the spectral efficiency and roll-off factor of the coding and modulation mode α_i.

Let W_{B_i} denote the bandwidth of the ith spot beam. Thus the total bandwidth that can be provided to the users in the ith spot beam cannot exceed W_{B_i}. In other words, the allocated capacity to the users is also constrained by the bandwidth of each spot beam.

3. Mathematical Formulation of the Optimization Problem

In this study, the objective of the power allocation optimization is to minimize the sum of the squared differences between the traffic demand and the capacity allocated to each user, taking account of a trade-off between the maximum total system capacity and the fairness of power allocation amongst the users. Therefore, the optimization problem is formulated as follows:

$$\min_{\{P_i\}} \sum_{i=1}^{M} (T_i - D_i)^2 \tag{5}$$

subject to

$$D_i = \frac{P_i \cdot G_S \cdot (G/T)_i}{L_i \cdot (E_b/N_0)_{\alpha_i} \cdot k} \le T_i \tag{6}$$

$$\sum_{i=1}^{M} P_i \le P_{\text{total}} \tag{7}$$

$$\sum_{i \in \mathcal{N}_{B_j}} W_i \le W_{B_j}. \tag{8}$$

The constraint (6) indicates that the allocated capacity to each user should not exceed the traffic demand of it, in order to avoid the waste of the scarce power resources. Conditions (7)-(8) imply the constraint for the total power of the satellite and the total bandwidth of each spot beam, respectively.

It is seen that the problem is a nonlinear optimization problem with constraints. Moreover, it is obvious that the objective function in (5) is convex and the functions in constrains (6)–(8) are linear. As a result, the problem under consideration is a convex optimization [11].

Due to the nonlinearity of the optimization, it is difficult to obtain the global optimal solution. In order to make the above problem tractable, an iterative algorithm based on the duality theory is proposed in the following section. It is known that if the optimization problem is a convex optimization problem, the duality gap between the primal problem and dual problem is zero, and the optimal value of the dual problem is equal to the optimal value of the primal problem. As a result, the dual problem can be first solved to obtain the optimal dual solution, and the primal optimal solution is then computed by solving the primal problem at the point of the optimal dual solution [11]. Fortunately, it has been proved that the optimization problem studied here is a convex optimization problem; thus the power allocation result obtained by the proposed algorithm is the optimal power allocation for the users in the multi-spot-beam satellite communication system.

4. Proposed Power Allocation Algorithm

As mentioned previously, the proposed power allocation algorithm is based on the duality theory. By introducing nonnegative dual variables λ and $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \ldots, \sigma_K]$ yielded the Lagrangian, given as

$$L(\mathbf{P}, \boldsymbol{\sigma}, \lambda) = \sum_{i=1}^{M} (T_i - D_i)^2 - \lambda \left(P_{\text{total}} - \sum_{i=1}^{M} P_i \right) - \sum_{i=1}^{K} \sigma_i \left(W_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} W_j \right), \tag{9}$$

where $\mathbf{P} = [P_1, P_2, \ldots, P_n]$.

Maximizing (9) with respect to the nonnegative λ and $\boldsymbol{\sigma}$ brings the following function:

$$z(\mathbf{P}) = \max_{\lambda \geq 0, \boldsymbol{\sigma} \geq 0} L(\mathbf{P}, \boldsymbol{\sigma}, \lambda). \tag{10}$$

It is seen that if the optimization variables P_i are satisfied with the constrains (7)-(8), then $\lambda(P_{\text{total}} - \sum_{i=1}^{M} P_i) \geq 0$ and $\sum_{i=1}^{K} \sigma_i (W_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} W_j) \geq 0$. Therefore, (10) will get the maximal value when $\lambda(P_{\text{total}} - \sum_{i=1}^{M} P_i) = 0$ and $\sum_{i=1}^{K} \sigma_i (W_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} W_j) = 0$. As a result, $z(\mathbf{P}) = \sum_{i=1}^{M} (T_i - D_i)^2$. To this end, the primal optimization with constraints is changed into the optimization with no constraints as follows [11]:

$$p = \min_{\mathbf{P}} z(\mathbf{P}) = \min_{\mathbf{P}} \max_{\lambda \geq 0, \boldsymbol{\sigma} \geq 0} L(\mathbf{P}, \boldsymbol{\sigma}, \lambda). \tag{11}$$

In addition, the Lagrange dual function can be obtained from (9) as [11]

$$D(\boldsymbol{\sigma}, \lambda) = \min_{\mathbf{P}} L(\mathbf{P}, \boldsymbol{\sigma}, \lambda) \tag{12}$$

and the dual problem of (11) can be written as

$$d = \max_{\lambda \geq 0, \boldsymbol{\sigma} \geq 0} D(\boldsymbol{\sigma}, \lambda) = \max_{\lambda \geq 0, \boldsymbol{\sigma} \geq 0} \min_{\mathbf{P}} L(\mathbf{P}, \boldsymbol{\sigma}, \lambda). \tag{13}$$

The work in [12] solved the joint spectrum and power allocation in cognitive radio networks and proposed a method to solve the dual problem. Inspired with this paper, the dual problem (13) is decomposed into the following two sequentially iterative subproblems.

Subproblem 1: Power Allocation. Given the dual variables λ and $\boldsymbol{\sigma}$, for any $i = [1, \ldots, M]$, maximizing (9) with respect to P_i brings the following equation:

$$\frac{2 \cdot G_S \cdot (G/T)_i}{L_i \cdot (E_b/n_0)_{\alpha_i} \cdot k} \left(T_i - \frac{P_i^{\text{opt}} \cdot G_S \cdot (G/T)_i}{L_i \cdot (E_b/n_0)_{\alpha_i} \cdot k} \right) = \lambda + \sigma_j \frac{G_S \cdot (G/T)_i \cdot [1 + \rho(\alpha_i)]}{L_i \cdot (E_b/n_0)_{\alpha_i} \cdot k \cdot \eta(\alpha_i)}, \quad i \in \mathcal{N}_{B_j}. \tag{14}$$

The optimized power allocation of the ith user P_i^{opt} can be easily obtained from (14). It is seen from (14) that nonnegative dual variables λ and $\boldsymbol{\sigma}$ guarantee that $T_i \geq D_i$. As a result, the constrain (6) is satisfied.

Subproblem 2: Dual Variables Update. The optimal dual variables can be obtained by solving the problem:

$$\left(\boldsymbol{\sigma}^{\text{opt}}, \lambda^{\text{opt}}\right) = \arg \max_{\boldsymbol{\sigma}, \lambda} \min \left[L\left(\mathbf{P}^{\text{opt}}, \boldsymbol{\sigma}, \lambda\right) \right]. \tag{15}$$

Due to concavity of the dual objective function, here a subgradient (a generalization of gradient) method is applied to update the duality variables, shown as [13]

$$\lambda^{n+1} = \left[\lambda^n - \Delta_\lambda^n \left(P_{\text{total}} - \sum_{i=1}^{M} P_i^{\text{opt}} \right) \right]^+, \tag{16}$$

$$\sigma_i^{n+1} = \left[\sigma_i^n - \Delta_\sigma^n \left(W_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} W_j \right) \right]^+, \tag{17}$$

where $[x]^+ = \max\{0, x\}$, n is the iteration number, and Δ is the iteration step size of each dual variable.

The subgradient method is very suitable for the situation that the dual function is not differentiable. As a result, the method has been widely applied to solve the optimization problem [12–18]. It has proven that the above dual variables update algorithm is guaranteed to converge to the optimal solution as long as the iteration step size chosen is sufficiently small [13]. A common criterion for choosing the iteration step size is that the step size must be square summable, but not absolute summable [13, 18].

Step 1. Set appropriate initial values for the dual variables.
Step 2. Substitute the values of the dual variables into (14), and then calculate the optimized power allocation to each user.
Step 3. Substitute the values of the power of each user which is obtained from step 2, into (16) and (17), and then update the dual variables.
Step 4. If the conditions of $\left|\lambda^{n+1}\left(P_{\text{total}}-\sum_i P_i\right)\right|<\varepsilon$ and $\left|\sigma_i^{n+1}\left(W_{B_i}-\sum_{j\in\mathcal{N}_{B_i}} W_j\right)\right|<\varepsilon$, $\forall i\in\{1,\ldots,K\}$ are satisfied simultaneously, then terminate the algorithm. Otherwise, jump to Step 2.

Algorithm 1: The proposed power allocation algorithm.

Table 1: Parameters of the multi-spot-beam satellite communication system.

Parameter	Value
Beam number	4
User number	20
User number per spot beam	5
Traffic demand of each user	From 1 Mbps to 20 Mbps by step of 1 Mbps
Total satellite power $[P_{\text{total}}]$	20 W
Satellite transmitting antenna gain $[G_S]$	20000
Bandwidth of each spot beam	100 MHz
Gain-to-equivalent noise temperature ratio of the receiving equipment $[G/T]$	20
Downlink loss $[L_i]$	$2e^{21}$
Spectral efficiency of the coding and modulation mode $[\eta(\alpha_i)]$	1.5
Roll-off factor of the coding and modulation mode $[\rho(\alpha_i)]$	1
Threshold signal-to-noise ratio per bit of the coding and modulation mode $\left[(E_b/N_0)_{\alpha_i}\right]$	2.63

The whole process of the proposed power allocation algorithm can be summarized as shown in Algorithm 1.

According to Algorithm 1, it is shown that the computational complexity of step 2 and step 3 is $O(M)$ and $O(2K)$, respectively. Thus the total computational complexity of the algorithm is $O(SM+2SK)$, where S is the number of iterations. It is noted that S is independent of K and M. Therefore, the computational complexity of the proposed algorithm is linear with both the numbers of the spot beams and users, and the proposed algorithm is easy to be implemented in practice.

5. Simulation Results and Analysis

For the simulation, a multi-spot-beam satellite communication system model is set up. It is assumed that the values of downlink loss, gain-to-equivalent noise temperature ratio of the receiving equipment, and coding and modulation mode are the same for all the users. The parameters of the system are shown in Table 1.

5.1. Efficiency of the Proposed Power Allocation Algorithm. The proposed power allocation algorithm is compared with the following two traditional allocation algorithms in order to verify the efficiency of it.

(i) *Uniform Resource Allocation Algorithm.* The power allocated to each user is $P_i = P_{\text{total}}/M$, $i\in\{1,2,\ldots,M\}$. The bandwidth allocated to the user in the same spot beam is $W_j = W_{B_i}/|\mathcal{N}_{B_i}|$, $j\in\mathcal{N}_{B_i}$, where $|\mathcal{N}_{B_i}|$ is the cardinality of the set $\mathcal{N}_{B_i}$.

(ii) *Proportional Resource Allocation Algorithm.* The power allocated to each user is $P_i = T_i\cdot P_{\text{total}}/\sum_{i=1}^{M} T_i$, $i\in\{1,2,\ldots,M\}$. The bandwidth allocated to the user in the same spot beam is $W_j = T_j\cdot W_{B_i}/\sum_{k\in\mathcal{N}_{B_i}} T_k$, $j\in\mathcal{N}_{B_i}$.

Table 2: Total system capacity of the three algorithms when the channel conditions of each user are the same.

Algorithms	$\sum C_i$
Uniform resource allocation	109.1 Mbps
Proportional resource allocation	109.1 Mbps
Proposed optimal power allocation	109.1 Mbps

Figure 2 shows the capacity distributions of the users which are allocated by the three algorithms. Table 2 shows the total system capacities of the three algorithms. It is noted that when the channel conditions of each user are the same, the uniform resource allocation algorithm is a special case of the water-fill algorithm, which can achieve the maximal total system capacity [19]. As shown in Figure 2, the uniform resource allocation algorithm uniformly allocates the resources to each user, regardless of the traffic demand of each user, even resulting in some users being allocated more capacity than that is needed. As a result, this uniform resource allocation algorithm causes a waste of the scarce

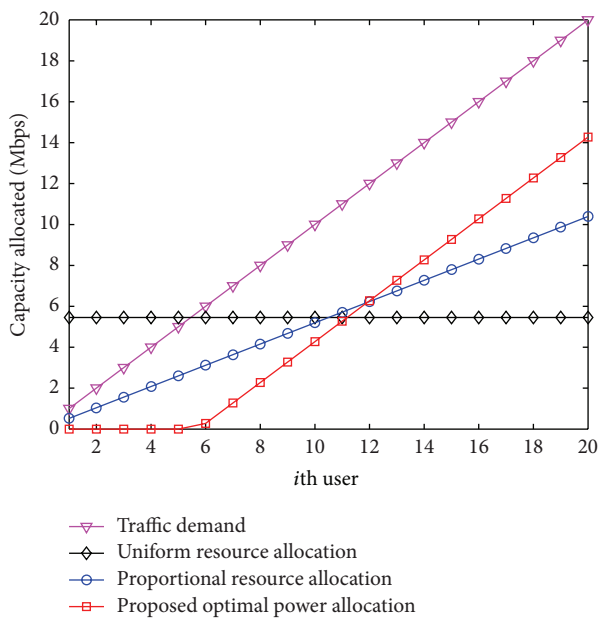

Figure 2: Comparison of the three algorithms in terms of the capacity allocated to each user when the channel conditions of each user are the same.

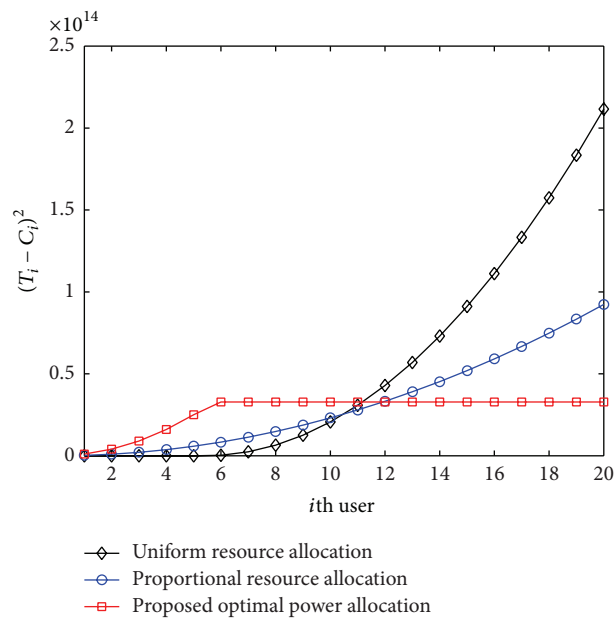

Figure 3: Comparison of the three algorithms in terms of the squared difference between the traffic demand and the capacity allocated to each user when the channel conditions of each user are the same.

Table 3: Sum of $(T_i - C_i)^2$ of the three algorithms when the channel conditions of each user are the same.

Algorithms	$\sum (T_i - C_i)^2$
Uniform resource allocation	$1.134E15$
Proportional resource allocation	$6.627E14$
Proposed optimal power allocation	$5.470E14$

resources. The proportional resource allocation algorithm allocates the power resources to each user only according to its traffic demand. The capacity allocated to each user is linearly increasing, considering the fairness of power allocation amongst the users to some extent. However, it is not the optimal solution to the optimization. In order to get a better fairness, the proposed power algorithms provide more capacity to the users with higher traffic demands and suppress the capacities of the users with lower traffic demands. For example, the algorithm provides no capacity to the five lowest traffic demand users. Although the capacities allocated to each user are different, the total system capacities are the same for the three algorithms, due to the linearity of the capacity function in terms of the allocated power, and the sameness of the channel conditions of each user. The conclusion is also demonstrated by the data in Table 2.

Figure 3 shows the squared difference between the traffic demand and the capacity allocated of each user of the three algorithms. Table 3 presents the sum of the squared differences of the three algorithms. It is shown from Figure 3 that for the uniform and proportional resource allocation algorithms, although the squared difference between the traffic demand and the capacity allocated to the user with low traffic demand is small, however, the squared difference increases rapidly when the traffic demand increases. On the contrast, for the proposed optimal power allocation algorithm, the squared difference between the traffic demand and the capacity allocated to the users with low traffic demand is larger than that of the former two algorithms. However, the squared difference is almost the same from user 6 to user 20. As a result, the total squared difference of the proposed power allocation algorithm is less than that of the former two algorithms, which is also shown in Table 3. In other words, the power allocation result of the proposed algorithm is the best amongst the three algorithms.

5.2. Impact of the Spot Beam Bandwidth on the Power Allocation Result. As mentioned above, the capacity allocated to each user is constrained by both the power and bandwidth allocated to it. Due to the limitation of the bandwidth of each spot beam, the capacity allocated to the users in the same spot beam is also constrained. As a result, the power resources allocated to the users are impacted. In order to show the impact of the spot beam bandwidth on the power allocation result, the power allocation results are compared when the bandwidth of each spot beam is set to be 25 MHz, 50 MHz, and 100 MHz, and other parameters of the system stay the same.

From Figure 4, it is obvious that the power allocation results are different when the bandwidth resources of each spot beam are various. When the spot beam bandwidth is 25 MHz, the capacity allocated to each user is constrained by the bandwidth. Although the total system power is 20 W, the total power allocated to all the users is only 13.06 W. As a result, the power resources in the system are wasted and the total system capacity is decreased. When the bandwidth is 50 MHz, the capacities allocated to the users in the last

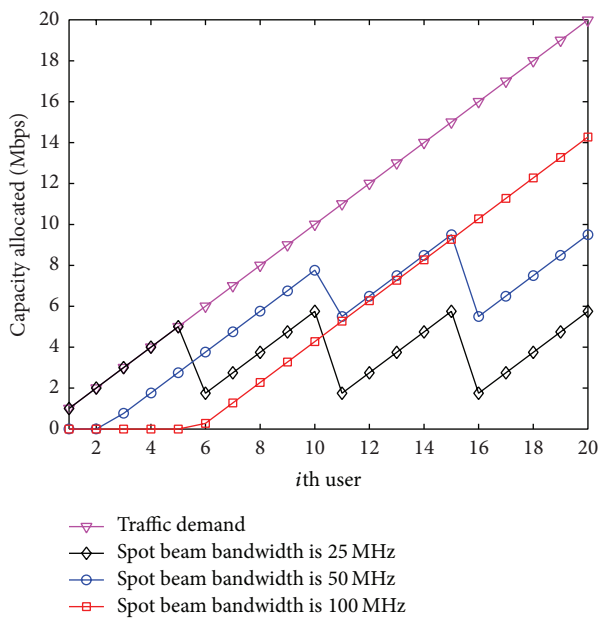

Figure 4: Comparison of the three different spot beams bandwidths in terms of the capacity allocated to each user.

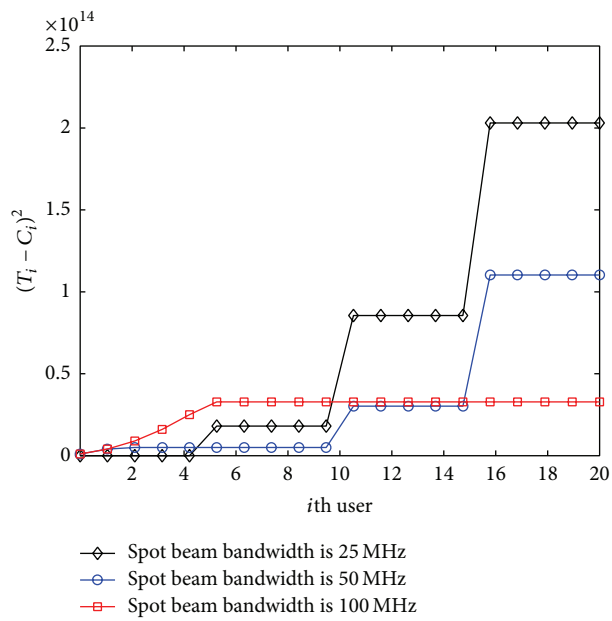

Figure 5: Comparison of the three different spot beams bandwidths in terms of the squared difference between the traffic demand and the capacity allocated to each user.

Table 4: Total system capacity of three different spot beams bandwidths.

Bandwidth of each spot beam	$\sum C_i$
25 MHz	71.25 Mbps
50 MHz	109.1 Mbps
100 MHz	109.1 Mbps

Table 5: Sum of $(T_i - C_i)^2$ of the three different spot beams bandwidths.

Bandwidth of each spot beam	$\sum (T_i - C_i)^2$
25 MHz	15.34E14
50 MHz	7.476E14
100 MHz	5.470E14

two spot beams are constrained by the bandwidth, due to the high traffic demand of users. Thus the power resources will be provided to the users with low traffic demands in the former two spot beams. When the bandwidth of each spot beam is 100 MHz, the system has more than enough bandwidth to be allocated to each user, thus the capacity allocated to each user is limited by the total system power resources. In order to improve the fairness of power allocation amongst the users, the power resources are rarely or never provided to the users with low traffic demand. Although the power resources allocated to each user are different when the spot beam bandwidth is 50 MHz and 100 MHz, the power resources are sufficiently utilized. As a result, the total system capacity is the same, which is also seen from Table 4.

As mentioned in Figure 5 and Table 5, when the bandwidth of each spot beam is lower, more power resources will be provided to the users with low traffic demand. Therefore, it is seen from Figure 5 that the squared difference between traffic demand and allocated capacity to users with low traffic demand is smaller. However, the squared difference is lager for the users with high traffic demand. As a result, the total squared difference is larger when the bandwidth of each spot beam is lower. This conclusion can be also observed from Table 5.

5.3. Impact of the Coding and Modulation Mode of Each User on the Power Allocation Result. It is known that the power efficiency and spectral efficiency of a given coding and modulation mode are usually contradictory to each other. In other word, a higher spectral efficiency coding and modulation code can support more capacity in the limited bandwidth. However, more power must be provided to it to support the coding and modulation mode, due to a higher value of E_b/N_0, resulting in lower power efficiency, and vice versa. It is seen from the analysis in Section 5.2 that when the bandwidth of each spot beam is 25 MHz, the capacity allocated to each other is limited by the bandwidth and the power resources are wasted. In order to solve the problem, a higher bandwidth efficiency coding and modulation mode can be adopted by each user. The capacity allocation results are compared when each user adopts the three different coding and modulation modes, as shown in Table 6.

It is known that when mode 1 is adopted by each user, the power resources are wasted, due to the low spectral efficiency. When mode 2 is adopted by each user, it is seen from Figure 6 that more capacity will be allocated to the users in spot beam 2 to spot beam 4, due to the higher spectral efficiency of the mode and sufficient utilization of the power resource. As a result, the total system capacity is increased. When

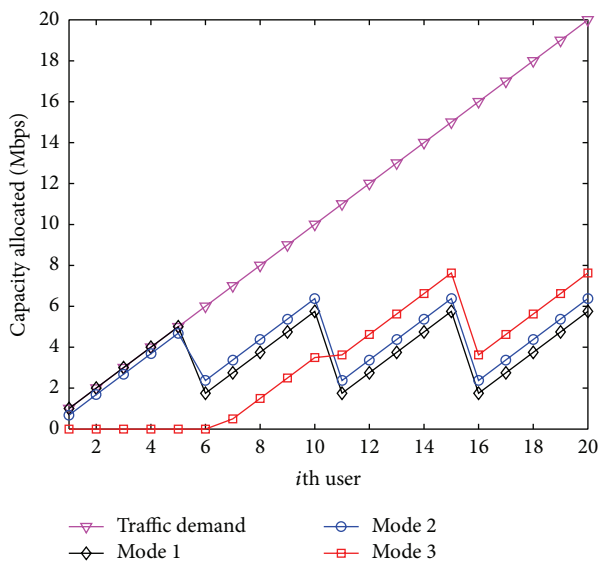

Figure 6: Comparison of the three different coding and modulation modes in terms of the capacity allocated to each user.

Table 6: Threshold signal-to-noise ratio per bit and spectral efficiency of the three coding and modulation modes.

Coding and modulation mode	Threshold signal-to-noise ratio per bit	Spectral efficiency
Mode 1	2.63	1.5
Mode 2	3.63	1.75
Mode 3	4.47	2.15

Table 7: Total system capacity of three different coding and modulation modes.

Adopted coding and modulation mode of each user	$\sum C_i$
Mode 1	71.25 Mbps
Mode 2	79.02 Mbps
Mode 3	64.24 Mbps

mode 3 is adopted, although the spectral efficiency is further improved, the power efficiency is further reduced. Therefore, the capacity allocated to each user is limited by the power resources allocated to it. Due to the low power efficiency, the total system capacity is increased, which is shown in Table 7.

When the spectral efficiency of the coding and modulation mode is higher, the users with high traffic demand in the last several spot beams are provided more capacity due to the higher spectral efficiency, resulting in a lower squared difference as shown in Figure 7. Therefore, the total system squared difference between traffic demand and capacity allocated to the users is smaller, especially for mode 3. This conclusion is obviously seen from Table 8.

5.4. Impact of the Channel Condition of Each User on the Power Allocation Result. It is known that the channel conditions of each user are affected by many kinds of factor, causing that the downlink losses of each user are not the same. In order to

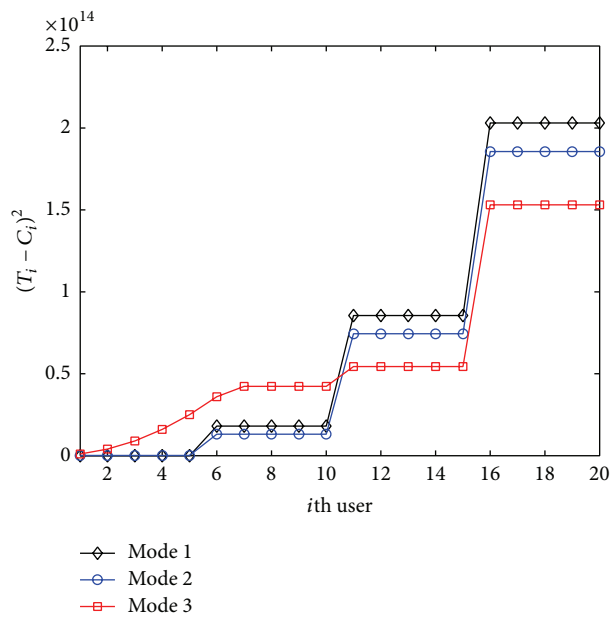

Figure 7: Comparison of the three different spot beams bandwidths in terms of the squared difference between the traffic demand and the capacity allocated to each user.

Table 8: Sum of $(T_i - C_i)^2$ of the three different spot beams bandwidths.

Adopted coding and modulation mode of each user	$\sum (T_i - C_i)^2$
Mode 1	1.533E15
Mode 2	1.366E15
Mode 3	1.298E15

Table 9: Total system capacity of the three algorithms when the channel conditions of each user are not the same.

Algorithms	$\sum C_i$
Uniform resource allocation	63.91 Mbps
Proportional resource allocation	60.73 Mbps
Proposed optimal power allocation	55.10 Mbps

show the impact of channel condition on the power allocation result, the channel conditions of the users in the same spot beam are set to be $2e^{21}$, $3e^{21}$, $4e^{21}$, $5e^{21}$, and $6e^{21}$. Moreover, the traffic demands of the users in the same spot beam are set the same, and the traffic demands of the users in the four different spot beams are set to be 3 Mbps, 8 Mbps, 13 Mbps, and 18 Mbps. The simulation results are shown in Figure 8 and Table 9.

It is seen from Figure 8 that the proposed power allocation algorithm provides more capacity to the users with higher traffic demand, in order to minimize the total system squared difference between the traffic demand and capacity allocated to each user. The proposed algorithm allocates the same capacities to the users in spot beam 3 or 4, which implied that more power resource will be allocated to the users with

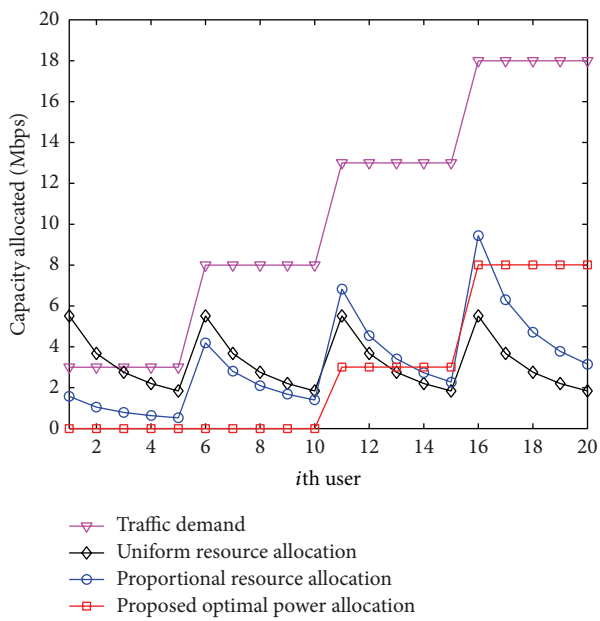

Figure 8: Comparison of the three algorithms in terms of the capacity allocated to each user when the channel conditions of each user are not the same.

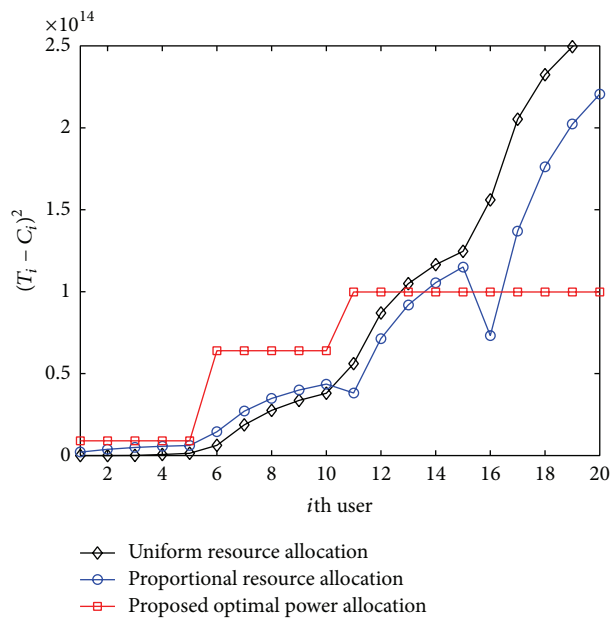

Figure 9: Comparison of the three algorithms in terms of the squared difference between the traffic demand and the capacity allocated to each user when the channel conditions of each user are not the same.

Table 10: Sum of $(T_i - C_i)^2$ of the three algorithms when the channel conditions of each user are not the same.

Algorithms	$\sum (T_i - C_i)^2$
Uniform resource allocation	$1.719E15$
Proportional resource allocation	$1.414E15$
Proposed optimal power allocation	$1.363E15$

worse channel conditions in these two spot beams. As a result, compared with the other two resource allocation algorithms, the total system capacity of the proposed power allocation algorithm is decreased, as clearly shown in Table 9.

As mentioned in Figure 9 and Table 10, the proposed power allocation algorithm provides more capacity to the users with higher traffic demand. Therefore, the squared differences between the traffic demand and capacity allocated to these users are lower. Compared with the other two algorithms, although the squared differences of the users with lower traffic demand are higher, the total squared difference of the proposed power allocation algorithm is lower, as shown in Table 10. As a result, it is observed that the proposed algorithm improves the fairness of power allocation amongst the user at cost of the total system capacity.

6. Conclusion

In the multi-spot-beam satellite system it is crucial for us to improve the power resources utilization efficiency, due to the scarceness of the satellite power resources. To this end, the problem of power allocation was mathematically formulated as a convex optimization problem and an optimal power allocation algorithm was proposed to solve the problem. In the optimization, the capacity allocated to each user was calculated according to satellite link budget equations rather than the Shannon capacity formula. As a result, the capacity allocated to each user can be achieved and the power allocation result is more suitable for the practical multi-spot-beam satellite communication system. Moreover, the computational complexity of proposed algorithm is linear with both the numbers of the spot beams and users. As a result, it can be implemented in the practical system.

It is shown from the simulation results that, compared with the traditional power allocation algorithms, the proposed algorithm improved the fairness of the power allocation amongst the users. Both the coding and modulation mode adopted by each user and the bandwidth of each spot beam have a significant impact on the power allocation result. When the bandwidth of each spot beam is sufficient, more power resources will be provided to the users with higher traffic demand to improve the fairness of power allocation amongst the users. On the contrast, when the bandwidth of each spot beam is limited, more power will be provided to the users with lower traffic demand. Even the satellite power resources are wasted, due to the further reduction of bandwidth of each spot beam. The impact of the coding and modulation mode on the power allocation result is similar to that of the bandwidth of each spot beam. Moreover, the channel conditions of each user also affect the power allocation result. The proposed algorithm provides more resource to the users with the high traffic demand. As a result, if the channel conditions of these high traffic demand users are worse, the total system capacity will be decreased.

Conflict of Interests

The authors declare that they do not have any commercial or associative interest that represents a conflict of interests in connection with the work submitted.

Acknowledgment

The authors would like to thank the project support by the National High-Tech Research & Development Program of China under Grant 2012AA01A508.

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