

## Research Article

# Design of the Congestion Control for TCP/AQM Network with Time-Delay

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Received 8 July 2014; Accepted 20 August 2014; Published 25 September 2014

Academic Editor: Housheng Su

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The purpose of this paper is to design congestion controller for TCP/AQM (transmission control protocol/active queue management) networks using model following control; the equilibrium of a class of TCP/AQM networks with time-delay is investigated, and the effect of communication time-delay on the stability is addressed. The features of this design method are bounded property of the internal states of the control system being given and the utility of this control. Such design exhibits important attributes including fast convergence with high accuracy to a desired queue length. Simulation results show that the time-delay nonlinear behavior of the system can be controlled by this method.

## 1. Introduction

In recent years, with the rapid growth of throughput-demanding applications, congestion control has emerged as a major issue in computer and communication network design [1]. So many researchers are seeking some methods to effectively control congestion. TCP congestion control mechanism is used to prevent congestion collapse.

AQM schemes have been proposed to complement the TCP network congestion control [2]. Several mathematical models are developed by some researchers [3–5] and a variety of control theory-based AQM schemes are proposed based on these models. The simulated approaches contain a wide range of variations in network topologies, topological parameters, load and capacity, and traffic mixtures. The outperformance of the PFC-AQM in comparison with the commonest AQM methods such as the RED (random early detection), PI, and REM (random early marking) emphasizes the proper applicability of PFC as an AQM method [6–9]. Based on the system model for congestion control in transmission control protocol TCP/AQM networks, control theory-based approaches are utilized either to analyze or to design the AQM schemes. Based on the system model, several conventional controllers [10–15] are designed as AQM methods in TCP networks.

The design of some communication systems requires the implementation of time-delays within the system. These time-delays can be accomplished with a variety of optics technologies, which could be readily fabricated and integrated into the communication system without significant impacts on the system design [16–19]. Time-delay is very important for the modeling of networks, occurring both in the control of networks and in the control over networks [20]. In the context of communication networks, the term “congestion control” is generally used to refer to the action of regulating various flows within a network. In recent years, intense research efforts are devoted to the application of the Smith predictor for queue length control of ATM (asynchronous transfer mode) networks [21]. The TCP was designed in the late 1980s by Jacobson, which is a critical part of the internet machinery.

The purpose of this paper is to design a congestion controller based on the model following control system (MFCS) [22, 23] control theory. The features of this design method are that bounded property of the internal state of the system, which is given and confirmed on basis of a numerical example of the network congestion system in which the output signal of the control system asymptotically follows the reference model signal in the case of the existence of disturbances.

The paper is organized as follows. In Section 2, the TCP/AQM network in congestion control model is described. In Section 3, controller design of the network congestion system with time-delay is proposed. In Section 4, bounded analysis of control system internal state is shown. Section 5 is the simulation results. The paper is concluded in Section 6.

## 2. The TCP/AQM Networks in Congestion Control Model

In this paper, the network in Figure 1 is considered. The network consists of  $n$  nodes (sender, 1 node (receiver), and 1 bottleneck router. The bottleneck router sends packets from these senders to the receiver. This network topology denotes 1 server machine to multiple client machines in a computer network. TCP is only the communication protocol in Figure 1. Large-scale networks can be simplified as in Figure 1 in case of designing congestion controllers if only one router is bottleneck in the large-scale computer network.

In this approach, we overview the dynamical fluid-flow model developed by [24–29] to describe the behavior of TCP/AQM networks. A simplified version of that system model is considered, which ignores the timeout and slow start mechanism of TCP. The model involves the average value of key network variables and is described by the following coupled nonlinear differential equations with time-delay [25]:

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)}{2} \frac{W(t-R(t))}{R(t-R(t))} p(t-R(t)), \quad (1)$$

$$\dot{q}(t) = \frac{W(t)}{R(t)} N(t) - D(t), \quad (2)$$

$$R(t) = \frac{q(t)}{D(t)} + T_p, \quad (3)$$

where  $W(t)$  is the congestion window size in packets at time  $t$ ,  $q(t)$  is the queue length at the congested router in packets,  $R(t)$  is the RTT (round trip time) which represents the time-delay in TCP dynamics in seconds,  $D(t)$  is the link capacity in packets per second,  $T_p$  is the propagation time-delay in seconds,  $N(t)$  is the number of active TCP connections, and  $p(t)$  is the packet mark/drop probability.

These differential equations in the block diagram of Figure 2 are taken from [24] highlighting TCP window-control and queue dynamics.

Set up a model for nonlinear TCP networks dynamic model. Let  $x_1(t) = q(t) - q_0$ ,  $x_2(t) = \dot{x}_1(t)$ , and  $u(t) = p(t)$ , where  $q_0$  is a desired queue length in the router. Assume that the rate of the change for  $x_1(t)$ ,  $N(t)$ ,  $C(t)$ , and  $R(t)$  is slower than  $W(t)$ ,  $q(t)$ ; (1) can be expressed in the following form [30]:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t-R(t)), \quad (4)$$

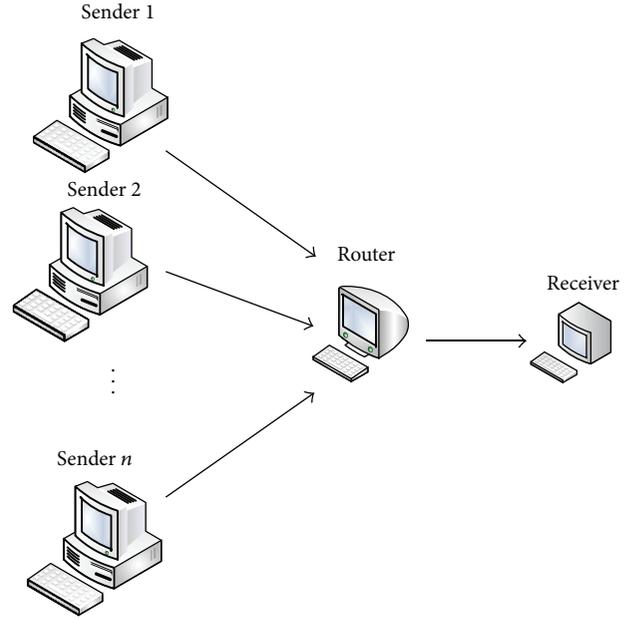


FIGURE 1: The network of the congestion control.

where

$$\begin{aligned} x(t) &= [x_1(t) \ x_2(t)]^T, \\ f(x(t)) &= [f_1(x(t)) \ f_2(x(t))]^T, \\ g(x(t)) &= [g_1(x(t)) \ g_2(x(t))]^T, \\ f_1(x(t)) &= x_2(t), \\ f_2(x(t)) &= \frac{N(t)}{R^2(t)}, \\ g_1(x(t)) &= -\frac{[(x_2(t) + C(t)) + (x_2(t-R(t)) + C(t))]}{2N(t)}. \end{aligned} \quad (5)$$

Therefore, the system is defined by

$$\dot{x}(t) = f(x(t), u(t-R(t))), \quad (6)$$

$$y(t) = [0 \ 1] x(t). \quad (7)$$

## 3. Controller Design of the Network Congestion System with Time-Delay

Based on (6), the system in (4) can be rewritten in equivalent form as follows:

$$\begin{aligned} \dot{x}(t) &= A_{11}x(t) + A_{12}u(t) + f_1(x(t), u(t-R(t))), \\ \dot{u}(t) &= A_{21}x(t) + A_{22}u(t) + g_1(x(t), u(t)) + Bu_1(t), \end{aligned} \quad (8)$$

where  $A_{11} \in R^{n \times n}$ ,  $A_{12} \in R^{n \times l}$ ,  $A_{21} \in R^{l \times n}$ , and  $A_{22} \in R^{l \times l}$ .



Let

$$\begin{aligned}\Gamma_{n+l-1} &= I, \\ \Gamma_{n+l-2} &= A_1 \Gamma_{n+l-1} + \alpha_{n+l-1} I \\ &= A_1 + \alpha_{n+l-1} I \\ &= A_1 - T_r(A_1) I,\end{aligned}\quad (15)$$

where  $\alpha_{n+l-1} = -T_r(A_1)$ .

Now the representations of input-output (12) are given as follows:

$$\begin{aligned}D(p) y(t) &= p^{n+l-2} \left( C \frac{\partial f(x_1(t))}{\partial u^T(t)} B \right) u_1(t) \\ &+ N_1(p) u_1(t) + p^{n+l-2} f_4(x_1(t)) \\ &+ N_{f_2}(p) f_2(x_1(t)),\end{aligned}\quad (16)$$

where

$$\begin{aligned}f_4(x_1(t)) &= C \left\{ \left( \frac{\partial f(x_1(t))}{\partial x^T(t)} - T_r(A_1) I \right) f(x_1(t)) \right. \\ &+ \left( \frac{\partial f(x_1(t))}{\partial u^T(t)} \right) g_1(x_1(t)) \\ &+ \left( \frac{\partial f(x_1(t))}{\partial u^T(t)} - A_{12} \right) \\ &\cdot (A_{21}x(t) + A_{22}u(t)) + (T_r(A_1) I - A_{11}) \\ &\left. \cdot (A_{11}x(t) + A_{12}u(t)) \right\}.\end{aligned}\quad (17)$$

Let  $N_r v_z(t) = (C(\partial f(x_1(t))/\partial u^T(t))B)u_1(t) + f_4(x_1(t))$ .  
We have

$$\begin{aligned}D(p) y(t) &= p^{n+l-2} N_r v_z(t) + N_1(p) u_1(t) \\ &+ N_{f_2}(p) f_2(x_1(t)).\end{aligned}\quad (18)$$

The reference model is given as

$$\begin{aligned}\dot{x}_m(t) &= A_m x_m(t) + B_m r_m(t) \\ y_m(t) &= C_m x_m(t).\end{aligned}\quad (19)$$

Choose a stable polynomial  $T(p)$  which satisfies the following conditions. (1) The degree of  $T(p)$  is  $\rho \geq n_d + 2n - n_m - 2 - \eta_i$ . (2) The coefficient of the maximum degree term of  $T(p)$  is the same as  $D(p)$ .

Consider the following equation:

$$T(p) D_m(p) = D_d(p) D(p) R(p) + S(p), \quad (20)$$

where the degree of each polynomial is  $\partial T(p) = \rho$ ,  $\partial D_m(p) = n_m$ ,  $\partial D_d(p) = n_d$ ,  $\partial D(p) = n$ ,  $\partial R(p) = \rho + n_m - n_d - n$ , and  $\partial S(p) \leq n_d + n - 1$ .  $T(p)$ ,  $D_m(p)$ ,  $D_d(p)$ ,  $D(p)$ , and  $R(p)$  are monic polynomials.

In this paper, we propose a design of model following control system with disturbances. We can prove that all the internal states are bounded and output error

$$e(t) = y(t) - y_m(t) \quad (21)$$

converges to zero asymptotically. Then the following form is obtained:

$$\begin{aligned}T(p) D_m(p) e(t) &= D_d(p) R(p) \{ p^{n+l-2} N_r v_z(t) \\ &+ N_1(p) u_1(t) + N_{f_2}(p) f_2(x_1(t)) \} \\ &+ S(p) y(t) - T(p) D_m(p) y_m(t),\end{aligned}\quad (22)$$

where  $N_1(p) = \sum_{k=0}^{n+l-3} C_1 \Gamma_k p^k B_1 = N_{f_2}(p) B_1$ .

The control law (controller)  $v_z(t)$  can be obtained by making the right-hand side of (22) equal to zero.

Thus,

$$\begin{aligned}v_z(t) &= -N_r^{-1} Q(p)^{-1} \{ D_d(p) R(p) p^{n+l-2} - Q(p) \} N_r v_z(t) \\ &- N_r^{-1} Q(p)^{-1} D_d(p) R(p) N_{f_2}(p) \\ &\cdot \{ B_1 u_1 + f_2(x_1(t)) \} \\ &- N_r^{-1} Q(p)^{-1} S(p) y(t) + r_m(t), \\ v_m(t) &= N_r^{-1} Q(p)^{-1} T(p) N_m(p) r_m(t).\end{aligned}\quad (23)$$

Therefore,  $v_z(t)$  of (22) is obtained from  $e(t) = 0$ . The model following control system can be realized if the system internal states are bounded.

#### 4. Bounded Analysis of Control System Internal State

Let  $z_1(t) = [x_1^T(t) \ \xi_1^T(t) \ \xi_2^T(t) \ \xi_3^T(t)]^T$ ; then the system is defined by

$$\begin{aligned}\dot{z}_1(t) &= A_s + \psi(z_1(t)), \\ x_1(t) &= C_s z_1(t),\end{aligned}\quad (24)$$

where

$$A_s = \begin{bmatrix} A_1 - B_1 E_2 C_1 & -B_1 H_1 & -B_1 H_2 & -B_1 H_3 \\ -G_1 E_2 C_1 & F_1 - G_1 H_1 & -G_1 H_2 & -G_1 H_3 \\ G_2 C_1 & 0 & F_2 & 0 \\ -G_3 B_1 E_2 C_1 & -G_3 B_1 H_1 & -G_3 B_1 H_2 & F_3 - G_3 B_1 H_3 \end{bmatrix},$$

$$C_s = [I \ 0 \ 0 \ 0],$$

$$\psi(z_1(t)) = f_2(x_1(t)) - B_1 \left( C \frac{\partial f(x_1(t))}{\partial u^T(t)} B \right)^{-1} \cdot (W(x_1(t)) v_z(t) + f_4(x_1(t))),$$

$$\begin{aligned} \psi(z_2(t)) &= G_3 \left( f_2(x_1(t)) \right. \\ &\quad \left. - B_1 \left( C \frac{\partial f(x_1(t))}{\partial u^T(t)} B \right)^{-1} f_4(x_1(t)) \right) \\ &= G_3 B_1 \left( C \frac{\partial f(x_1(t))}{\partial u^T(t)} B \right)^{-1} W(x_1(t)) v_z, \end{aligned}$$

$$C \frac{\partial f(x_1(t))}{\partial u^T(t)} B = N_r + W(x_1(t)). \quad (25)$$

The characteristic polynomial  $|pI - A_s|$  can be calculated as follows:

$$|pI - A_s| = |Q(p)|^2 V_s(p) T(p)^l D_m(p)^l \quad (26)$$

with stable polynomials of  $T(p)$ ,  $D_m(p)$ ,  $|Q(p)|$ , and  $V_s(p)$ . Therefore,  $A_s$  is also a stable system matrix.

Now, the system can be rewritten as

$$\dot{z}_2(t) = A_s z_2(t) + \psi(z_1(t)), \quad (27)$$

$$x_1(t) = C_s z_2(t). \quad (28)$$

Consider a quadratic Lyapunov function candidate:

$$V(t) = \frac{1}{2} z_2^T(t) P_s z_2(t), \quad (29)$$

$$\begin{aligned} \dot{V}(t) &= z_2^T(t) P_s (A_s z_2(t) + \psi(z_1(t))) \\ &= -\frac{1}{2} z_2^T(t) Q_s z_2(t) + z_2^T(t) P_s \psi(z_1(t)) \\ &\leq z_2^T(t) P_s \psi(z_1(t)) < 0, \end{aligned} \quad (30)$$

$$P_s A_s + A_s^T P_s = -Q_s, \quad (31)$$

where  $Q_s$  and  $P_s$  are symmetric positive definite matrices defined by (27). If  $A_s$  is a stable matrix, we can get a unique  $P_s$  from (31) when  $Q_s$  is given. Therefore,  $z_2(t)$  is bounded.

## 5. Simulation Results

This simulation, which proposes static state feedback AQM controller for the time-delay system, verifies the performance.

TABLE 1: Sender 1-2 node and receiver node parameters.

Parameter	Value
Application	ftp
Transport layer protocol	TCP
TCP agent SACK bit	False
Packet size	1448 (byte)
Acknowledgment size	40 (byte)
Flow number (TCP session number) (disturbance)	8 (+1) (flow)

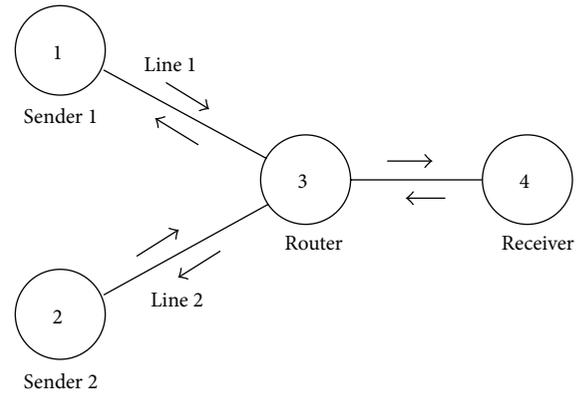


FIGURE 3: The network topology.

The network topology is simple and it is shown in Figure 3. But this small-scale network is developed by using four computers, and some experiments are carried out.

This network consists of two senders with four data flows (this means four sessions and  $N = 8$ ), one bottleneck router, and one receiver, which receives data from senders through the bottleneck router (see Table 1).

In the responses (Figures 4, 5, and 6) of the congestion system with time-delay, the output errors converge to zero. So the effectiveness of this method is verified.

## 6. Conclusions

In this paper, a new design method for the congestion controller of the TCP/AQM networks is introduced. The developed approach can theoretically guarantee the system performances, including the disturbance rejection and the implied stability of the closed-loop system. This property is useful for congestion controller design. This paper studies a control system with time-delay using a model following method which is one of the effective means of solving time-delay problems in a control system. The method can efficaciously control time-delay under disturbances and has excellent practicability.

By using this model, the nonlinear input time-delay system, which describes a TCP/AQM network, is transformed into an equivalent nonlinear system, and it is possible to design controllers based on nonlinear control theories. For a congestion control problem, a round packet trip time is

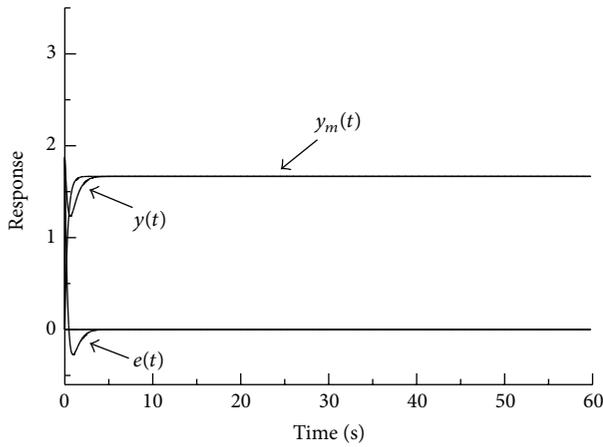


FIGURE 4: Responses of the congestion system with time-delay (the reference queue size is  $\bar{q}_0 = 1.7$  (packets); time-delay is 0.12 (ms)).

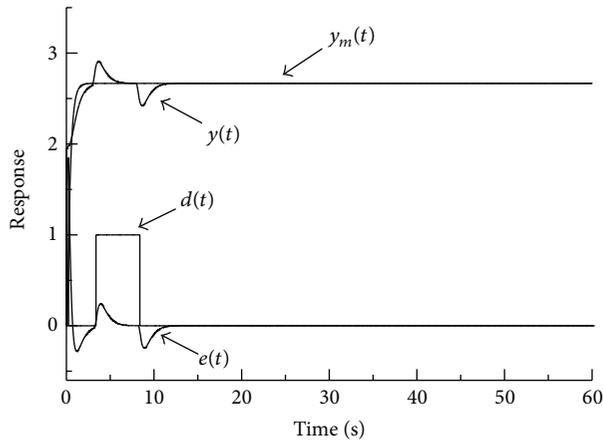


FIGURE 5: Responses of the congestion system with time-delay (the reference queue size is  $\bar{q}_0 = 2.7$  (packets), time-delay is 12 (ms), and  $d(t)$  is white noises).

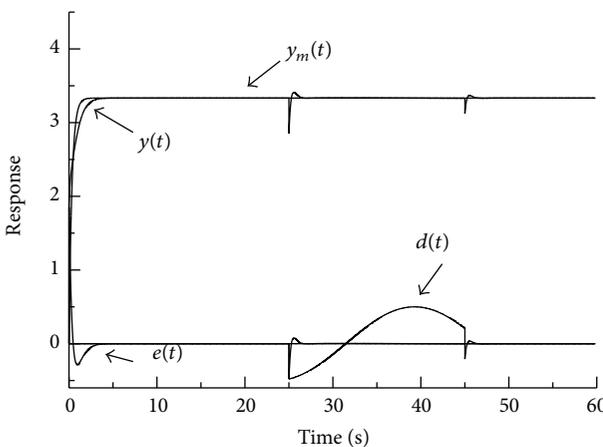


FIGURE 6: Responses of the congestion system with time-delay (the reference queue size is  $\bar{q}_0 = 3.3$  (packets), time-delay is 0.1 (ms), and  $d(t)$  is white noises).

not stationary and depends on the actual traffic. Finally, it is pointed out that the effectiveness of the proposed approach is only verified via simulations.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgments

This work was financially supported by the Shanghai University of Engineering Science Development Fund to cultivate high-level special project (2012gp02). The authors would like to thank the editor and the reviewers for their constructive comments and suggestions which improved the quality of the paper.

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