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*Research Article*

# Derivative-Based Trapezoid Rule for the Riemann-Stieltjes Integral

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The derivative-based trapezoid rule for the Riemann-Stieltjes integral is presented which uses 2 derivative values at the endpoints. This kind of quadrature rule obtains an increase of two orders of precision over the trapezoid rule for the Riemann-Stieltjes integral and the error term is investigated. At last, the rationality of the generalization of derivative-based trapezoid rule for Riemann-Stieltjes integral is demonstrated.

## 1. Introduction

In mathematics, the Riemann-Stieltjes integral is a kind of generalization of the Riemann integral, named after Bernhard Riemann and Thomas Stieltjes. It is Stieltjes that first gave the definition of this integral [1] in 1894. It serves as an instructive and useful precursor of the Lebesgue integral. It is known that the Riemann-Stieltjes integral has wide applications in the field of probability theory [2, 3], stochastic process [4], and functional analysis [5], especially in original formulation of F. Riesz's theorem and the spectral theorem for self-adjoint operators in a Hilbert space.

Definite integration is one of the most important and basic concepts in mathematics. And it has numerous applications in fields such as physics and engineering. In several practical problems, we need to calculate integrals. As is known to all, as for $I = \int_a^b f(x)dx$, once the primitive function $F(x)$ of integrand $f(x)$ is known, the definite integral of $f(x)$ over the interval $[a,b]$ is given by Newton-Leibniz formula, that is,

$$\int_a^b f(x)\,dx = F(b) - F(a). \tag{1}$$

However, the explicit primitive function $F(x)$ is not available or its primitive function is not easy to obtain, such as $e^{\pm x^2}$, $\sin x^2$, and $(\sin x)/x$. Moreover, the integrand $f(x)$ is only available at certain points $x_i$, $i = 1, 2, \ldots, n$. How to get high-precision numerical integration formulas becomes one of the challenges in fields of mathematics [6].

The methods of quadrature are usually based on the interpolation polynomials and can be written in the following form:

$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n} w_i f(x_i), \tag{2}$$

where there are $n+1$ distinct integration points at $x_0, x_1, \ldots, x_n$ within the interval $[a,b]$ and $n+1$ weights $w_i$. If the integration points are uniformly distributed over the interval, so $x_i = x_0 + ih$ in which $h = (b-a)/n$.

These $w_i$ can be derived in several different ways [7–9]. One is interpolate $f(x)$ at the $n+1$ points $x_0, x_1, \ldots, x_n$, using the Lagrange polynomials and then integrating the foresaid polynomials to obtain (2).

The other is based on the precision of a quadrature formula. Select the $w_i$, $i = 0, 1, \ldots, n$, so that the error,

$$R_n(f) = \int_a^b f(x)\,dx - \sum_{i=0}^{n} w_i f(x_i), \tag{3}$$

is exactly zero for $f(x) = x^j$, $j = 0, 1, \ldots, n$. Using the method of undetermined coefficients, this approach generates a system of $n+1$ linear equations for weights $w_i$.

Since the monomials $1, x, \ldots, x^n$ are linearly independent, the linear system of equations has a unique solution.

The trapezoidal rule is the most well-known numerical integration rule of this type. Trapezoidal rule for classical Riemann integral is

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\left(f(a)+f(b)\right) - \frac{(b-a)^3}{12} f''(\xi), \quad (4)$$

where $\xi \in (a, b)$.

In spite of the many accurate and efficient methods for numerical integration being available in [7–9], recently Mercer has obtained trapezoid rule for Riemann-Stieltjes integral which engender a generalization of Hadamard's integral inequality [10]. Then he develops Midpoint and Simpson's rules for Riemann-Stieltjes integral [11] by using the concept of relative convexity. Burg has proposed derivative-based closed Newton-Cotes numerical quadrature [12] which uses both the function value and the derivative value on uniformly spaced intervals. Zhao and Li have proposed midpoint derivative-based closed Newton-Cotes quadrature [13] and numerical superiority has been shown. Recently, Simos and his partners have made a contribution to the Newton-Cotes formula for the Riemann integral and its applications [14–21], especially the connection between closed Newton-Cotes, trigonometrically fitted differential methods, symplectic integrators, and efficient solution of the Schrodinger equation [17–21].

Motivation for the research presented here lies in construction of derivative-based trapezoid rule for the Riemann-Stieltjes integral, which is generalization of the results in [10–12]. In this paper, the derivative-based trapezoid rule for the Riemann-Stieltjes integral is presented. This new scheme is investigated in Section 2. In Section 3, the error term is presented. Finally, conclusions are drawn in Section 4.

## 2. Derivative-Based Trapezoid Rule for the Riemann-Stieltjes Integral

In this section, by adding the derivatives at the endpoints, derivative-based trapezoid rule for the Riemann-Stieltjes integral is presented.

**Theorem 1.** *Suppose that $f'$ and $g$ are continuous on $[a, b]$ and $g$ is increasing there. The derivative-based trapezoid rule for the Riemann-Stieltjes integral is*

$$\begin{aligned}
&\int_a^b f(t)\,dg \\
&\approx T = \left(\frac{6}{(b-a)^2}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{12}{(b-a)^3}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt - g(a)\right) f(a) \\
&+ \left(g(b) - \frac{6}{(b-a)^2}\int_a^b\int_a^t g(x)\,dx\,dt + \frac{12}{(b-a)^3}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt\right) f(b) \\
&+ \left(\frac{2}{b-a}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{6}{(b-a)^2}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt\right) f'(a) \\
&+ \left(\frac{4}{b-a}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{6}{(b-a)^2}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt - \int_a^b g(t)\,dt\right) f'(b).
\end{aligned} \quad (5)$$

*Proof.* Looking for the derivative-based trapezoid rule for the Riemann-Stieltjes integral, we seek numbers $a_0, a_1, b_0, b_1$ such that

$$\int_a^b f(t)\,dg \approx a_0\, f(a) + a_1 f(b) + b_0 f'(a) + b_1\, f'(b) \quad (6)$$

is equality for $f(t) = 1, t, t^2, t^3$. That is,

$$\begin{aligned}
&\int_a^b 1\,dg = a_0 + a_1; \\
&\int_a^b t\,dg = a_0 a + a_1 b + b_0 + b_1; \\
&\int_a^b t^2\,dg = a_0 a^2 + a_1 b^2 + 2b_0 a + 2b_1 b; \\
&\int_a^b t^3\,dg = a_0 a^3 + a_1 b^3 + 3b_0 a^2 + 3b_1 b^2.
\end{aligned} \quad (7)$$

Therefore,

$$\begin{aligned}
&a_0 + a_1 = g(b) - g(a); \\
&a_0 a + a_1 b + b_0 + b_1 = bg(b) - ag(a) - \int_a^b g(t)\,dt; \\
&a_0 a^2 + a_1 b^2 + 2b_0 a + 2b_1 b \\
&\quad = b^2 g(b) - a^2 g(a) - 2b\int_a^b g(t)\,dt \\
&\quad + 2\int_a^b\int_a^t g(x)\,dx\,dt;
\end{aligned}$$

$$
\begin{aligned}
a_0 a^3 &+ a_1 b^3 + 3b_0 a^2 + 3b_1 b^2 \\
&= b^3 g(b) - a^3 g(a) - 3b^2 \int_a^b g(t)\,dt \\
&\quad + 6b \int_a^b \int_a^t g(x)\,dx\,dt \\
&\quad - 6 \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt.
\end{aligned} \tag{8}
$$

Solving simultaneous (8) for $a_0, a_1, b_0, b_1$, we obtain

$$
\begin{aligned}
a_0 &= \frac{6}{(b-a)^2} \int_a^b \int_a^t g(x)\,dx\,dt \\
&\quad - \frac{12}{(b-a)^2} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt - g(a); \\
a_1 &= g(b) - \frac{6}{(b-a)^2} \int_a^b \int_a^t g(x)\,dx\,dt \\
&\quad + \frac{12}{(b-a)^2} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt; \\
b_0 &= \frac{2}{b-a} \int_a^b \int_a^t g(x)\,dx\,dt \\
&\quad - \frac{6}{(b-a)^2} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt; \\
b_1 &= \frac{4}{b-a} \int_a^b \int_a^t g(x)\,dx\,dt \\
&\quad - \frac{6}{(b-a)^2} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt - \int_a^b g(t)\,dt.
\end{aligned} \tag{9}
$$

So we have the derivative-based trapezoid rule for the Riemann-Stieltjes integral as desired. □

**Corollary 2.** *The precision of the derivative-based trapezoid rule for the Riemann-Stieltjes integral is 3.*

*Proof.* From the construction of $a_0$, $a_1$, $b_0$, $b_1$, we obtain that the derivative-based trapezoidal rule for the Riemann-Stieltjes integral has degree of precision not less than 3.

In Section 3 Theorem 3, we can easily see that the quadrature is not equality for $f(t) = t^4$. So the precision of this method is 3. □

## 3. The Error Term for Riemann-Stieltjes Derivative-Based Trapezoid Rule

In this section, the error term of the derivative-based trapezoid rule for the Riemann-Stieltjes is investigated. The error term can be found in mainly 3 different ways [8, 9].

Here, we use the concept of precision to calculate the error term, where the error term is related to the difference between the quadrature formula for the monomial $x^{p+1}/(p+1)!$ and the exact value $(1/(p+1)!) \int_a^b x^{p+1} dx$, where $p$ is the precision of the quadrature formula.

**Theorem 3.** *Suppose that $f^{(4)}$ and $g'$ are continuous on $[a, b]$ and $g$ is increasing there. The derivative-based trapezoid rule for the Riemann-Stieltjes integral with the error term is*

$$
\begin{aligned}
\int_a^b & f(t)\,dg \\
&= \left( \frac{6}{(b-a)^2} \int_a^b \int_a^t g(x)\,dx\,dt \right. \\
&\qquad - \frac{12}{(b-a)^3} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt \\
&\qquad \left. - g(a) \right) f(a) \\
&\quad + \left( g(b) - \frac{6}{(b-a)^2} \int_a^b \int_a^t g(x)\,dx\,dt \right. \\
&\qquad \left. + \frac{12}{(b-a)^3} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt \right) f(b) \\
&\quad + \left( \frac{2}{b-a} \int_a^b \int_a^t g(x)\,dx\,dt \right. \\
&\qquad \left. - \frac{6}{(b-a)^2} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt \right) f'(a) \\
&\quad + \left( \frac{4}{b-a} \int_a^b \int_a^t g(x)\,dx\,dt \right. \\
&\qquad - \frac{6}{(b-a)^2} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt \\
&\qquad \left. - \int_a^b g(t)\,dt \right) f'(b) \\
&\quad + \left( \frac{(b-a)^2}{12} \int_a^b \int_a^t g(x)\,dx\,dt \right. \\
&\qquad - \frac{b-a}{12} \int_a^b \int_a^t \int_a^y g(x)\,dx\,dy\,dt \\
&\qquad \left. + \int_a^b \int_a^t \int_a^z \int_a^y g(x)\,dx\,dy\,dz\,dt \right) \\
&\quad \times f^{(4)}(\xi)\, g'(\eta),
\end{aligned} \tag{10}
$$

*where $\xi, \eta \in (a,b)$. And the error term $R[f]$ of this method is*

$$\left(\frac{(b-a)^2}{12}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{b-a}{12}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt + \int_a^b\int_a^t\int_a^z\int_a^y g(x)\,dx\,dy\,dz\,dt\right) f^{(4)}(\xi)\, g'(\eta). \tag{11}$$

*Proof.* Let $f(t) = t^4/4!$.

So

$$\begin{aligned}\frac{1}{4!}\int_a^b t^4 dg &= \frac{1}{24}\left(b^4 g(b) - a^4 g(a)\right) - \frac{b^3}{6}\int_a^b g(t)\,dt + \frac{b^2}{2}\int_a^b\int_a^t g(x)\,dx \\ &\quad - b\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt + \int_a^b\int_a^t\int_a^z\int_a^y g(x)\,dx\,dy\,dz\,dt.\end{aligned} \tag{12}$$

By Theorem 1, we have

$$\begin{aligned}T &= \left(\frac{6}{(b-a)^2}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{12}{(b-a)^3}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt - g(a)\right)\frac{a^4}{24} \\ &\quad + \left(g(b) - \frac{6}{(b-a)^2}\int_a^b\int_a^t g(x)\,dx\,dt + \frac{12}{(b-a)^3}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt\right)\frac{b^4}{24} \\ &\quad + \left(\frac{2}{b-a}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{6}{(b-a)^2}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt\right)\frac{a^3}{6} \\ &\quad + \left(\frac{4}{b-a}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{6}{(b-a)^2}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt - \int_a^b g(t)\,dt\right)\frac{b^3}{6}.\end{aligned} \tag{13}$$

By Equations (12)-(13), we obtain

$$\begin{aligned}\frac{1}{4!}\int_a^b t^4 dg - T &= \frac{(b-a)^2}{12}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{b-a}{12}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt \\ &\quad + \int_a^b\int_a^t\int_a^z\int_a^y g(x)\,dx\,dy\,dz\,dt.\end{aligned} \tag{14}$$

This implies that

$$\begin{aligned}R[f] &= \left(\frac{(b-a)^2}{12}\int_a^b\int_a^t g(x)\,dx\,dt - \frac{b-a}{12}\int_a^b\int_a^t\int_a^y g(x)\,dx\,dy\,dt + \int_a^b\int_a^t\int_a^z\int_a^y g(x)\,dx\,dy\,dz\,dt\right) \\ &\quad \times f^{(4)}(\xi)\, g'(\eta).\end{aligned} \tag{15}$$

□

**Corollary 4.** *Conditions are the same as for Theorem 3. When $g(t) = t$, (10) reduces to the derivative-based trapezoid rule (see [12]) for the classical Riemann integral.*

*Proof.* It is easy to obtain

$$\begin{aligned}&\int_a^b\int_a^t x\,dx\,dt = \frac{1}{6}b^3 - \frac{1}{2}a^2 b + \frac{1}{3}a^3, \\ &\int_a^b\int_a^t\int_a^y x\,dx\,dy\,dt = \frac{1}{24}b^4 - \frac{1}{4}a^2b^2 + \frac{1}{3}a^3 b - \frac{1}{8}a^4, \\ &\int_a^b\int_a^t\int_a^z\int_a^y x\,dx\,dy\,dz\,dt = \frac{1}{120}b^5 - \frac{1}{12}a^2b^3 + \frac{1}{6}a^3b^2 - \frac{1}{8}a^3 b + \frac{1}{30}a^5.\end{aligned} \tag{16}$$

By Theorem 3,

$$\begin{aligned}\int_a^b f(t)\,dg = \int_a^b f(t)\,dt &= \left(\frac{6}{(b-a)^2}\int_a^b\int_a^t x\,dx\,dt - \frac{12}{(b-a)^3}\int_a^b\int_a^t\int_a^y x\,dx\,dy\,dt - a\right) f(a)\end{aligned}$$

$$
\begin{aligned}
&+\left(b-\frac{6}{(b-a)^2}\int_a^b\int_a^t x\,dx\,dt\right.\\
&\qquad\left.+\frac{12}{(b-a)^3}\int_a^b\int_a^t\int_a^y x\,dx\,dy\,dt\right)f(b)\\
&+\left(\frac{2}{b-a}\int_a^b\int_a^t x\,dx\,dt\right.\\
&\qquad\left.-\frac{6}{(b-a)^2}\int_a^b\int_a^t\int_a^y x\,dx\,dy\,dt\right)f'(a)\\
&+\left(\frac{4}{b-a}\int_a^b\int_a^t x\,dx\,dt\right.\\
&\qquad-\frac{6}{(b-a)^2}\int_a^b\int_a^t\int_a^y x\,dx\,dy\,dt\\
&\qquad\left.-\int_a^b t\,dt\right)f'(b)\\
&+\left(\frac{(b-a)^2}{12}\int_a^b\int_a^t x\,dx\,dt\right.\\
&\qquad-\frac{b-a}{12}\int_a^b\int_a^t\int_a^y x\,dx\,dy\,dt\\
&\qquad\left.+\int_a^b\int_a^t\int_a^z\int_a^y x\,dx\,dy\,dz\,dt\right)f^{(4)}(\xi)\\
&=\frac{b-a}{2}\left(f(a)+f(b)\right)\\
&\quad+\frac{(b-a)^2}{12}\left(f'(a)-f'(b)\right)\\
&\quad+\frac{(b-a)^5}{720}f^{(4)}(\xi).
\end{aligned}
\tag{17}
$$

□

*Remark 5.* From Corollary 4, we know that the results in Theorem 3 possess the reducibility. When $g(t) = t$, formula (10) reduces to the derivative-based trapezoid rule for the classical Riemann integral. So Theorem 3 is a reasonable generalization of the results in [12].

## 4. Conclusions

We briefly summarize our main conclusions in this paper as follows.

(1) The derivative-based trapezoid rule for the Riemann-Stieltjes integral is presented which uses 2 derivative values at the endpoints.

(2) This kind of quadrature rule obtains an increase of two orders of precision over the trapezoid rule for the Riemann-Stieltjes integral.

(3) The error term for Riemann-Stieltjes derivative-based trapezoid rule is investigated. And the rationality of the generalization of derivative-based trapezoid rule for Riemann-Stieltjes integral is demonstrated.

The derivative-based midpoint and Simpson's rules for the Riemann-Stieltjes integral will be achieved by further research.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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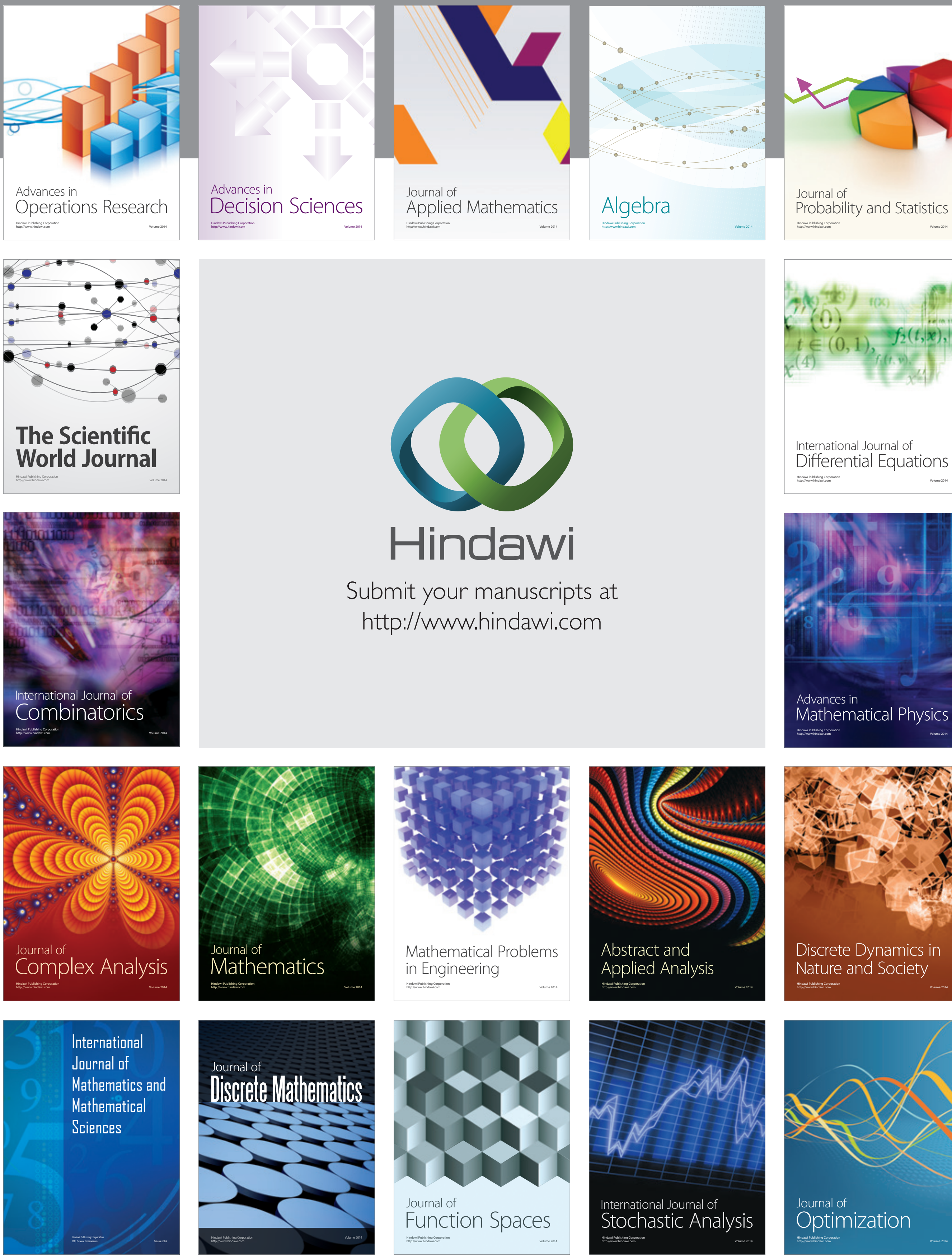
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