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Research Article

Fusion Control of Flexible Logic Control and Neural Network

Lihua Fu and Dan Wang

College of Computer Science, Beijing University of Technology, Beijing 100124, China

Correspondence should be addressed to Lihua Fu; fulihuapaper@sohu.com

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Based on the basic physical meaning of error E and error variety EC, this paper analyzes the logical relationship between them and uses *Universal Combinatorial Operation Model* in *Universal Logic* to describe it. Accordingly, a flexible logic control method is put forward to realize effective control on multivariable nonlinear system. In order to implement fusion control with artificial neural network, this paper proposes a new neuron model of *Zero-level Universal Combinatorial Operation* in *Universal Logic*. And the artificial neural network of flexible logic control model is implemented based on the proposed neuron model. Finally, stability control, anti-interference control of double inverted-pendulum system, and free walking of cart pendulum system on a level track are realized, showing experimentally the feasibility and validity of this method.

1. Introduction

In recent years, fuzzy control has made a rapid development, and it has found a considerable number of successful industrial applications [1–3]. But fuzzy control has two shortcomings in the process of controlling some practical complex systems. One is that the number of control rules increases exponentially with the increase of the number of inputs, and the other one is that the precision of control system is low [4].

To reduce the dimension of control model, hierarchical fuzzy logic control divides the collection of control rules into several collections based on different functions [5, 6]. Compound control combines fuzzy control and other relatively mature control methods to realize the effective control [7], such as Fuzzy-PID Compound Control [8], fuzzy predication control [9], adaptive fuzzy H_∞ control [10], and so forth. The basic idea of adaptive fuzzy control based on variable universe [11, 12] is to keep the form of rules and varies universe of discourse according to the control error. Though a great deal of research has been done to improve the performance of fuzzy control, most of these methods are based on the basic idea that fuzzy controller is a piecewise approximator. However, to date, there has been relatively little research conducted on the internal relations among input variables of fuzzy controllers.

Based on analysis of the logical relationship between the system's error E and error variety EC, this paper indicates that the relationship is just universal combinatorial relation in *Universal Logic* [13], and the simple *Universal Combinatorial Operation* can be used instead of complex fuzzy rule-based reasoning process. As a result, a flexible logic control method is proposed to realize effective control on multivariable nonlinear system.

Artificial neural network is widely used in modelling and controlling thanks to its properties of self-learning, self-organizing, and self-adapting [14]. In order to realize fusion control with artificial neural network, this paper attempts to study the neuron model of *Zero-level Universal Combinatorial Operation* in *Universal Logic* and propose a new neural model. Based on this neuron model, the artificial neural network of flexible logic control model is implemented. Finally, stability control, anti-interference control of double inverted-pendulum system, and free walking of cart pendulum system on a level track are realized to prove the feasibility and validity of this method.

The rest of the paper is organized as follows. Section 2 introduces necessary background on *Universal Combinatorial Operation Model* and flexible logic control method and gives and proves some important theorems of *Universal Combinatorial Operation Model* in the interval $[a, b]$. Section 3 puts forward a new neuron model of *Zero-level Universal*

Combinatorial Operation. Based on this neuron model, the artificial neural network of flexible logic control model is implemented in Section 4. The designed flexible logic control model is applied to treat the double inverted-pendulum system in Section 5. Finally, concluding remarks are given in Section 6.

2. Universal Combinatorial Operation Model

2.1. Universal Combinatorial Operation Model. In order to deal reasonably with the complex relation between factors in complex system, T-norm, S-norm or *Mean* operators are taken as Aggregation Operators.

However, T-norm result is not bigger than the minimum value, and S-norm result is not less than the maximum value. As a result, T-norm, or S-norm can only handle mutually conflictive relation. In contrast, *Mean* operators can vary only between the minimum and maximum values based on its "tradeoff" concept, so it can only handle mutually consistent relation [13].

Universal Logic [13], proposed by Professor He et al., is a kind of flexible logic. It considers the continuous change of not only the truth value of propositions, which is called truth value flexibility, but also the relation between propositions, which is called relation flexibility. Based on fuzzy logic, it puts forward two important coefficients: *generalized correlation coefficient* "h" and *generalized self-correlation coefficient* "k". The flexible change of universal logic operations is based on "h" and "k". So *Universal Logic* provides a new theoretical foundation to realize more effective control for complex systems.

Universal Combinatorial Operation Model is the combinatorial connective of *Universal Logic*. In this paper, we will only consider *generalized correlation coefficient h*. So *Zero-level Universal Combinatorial Operation Model* is defined as follows.

Definition 1 (see [13]). Set mapping $C^e : [0,1] \times [0,1] \to [0,1]$ and

$$\begin{aligned} & C^e(x, y, h) \\ & = \text{ite}\left\{ \Gamma^e \left[\left(x^m + y^m - e^m\right)^{1/m} \right] \mid x + y < 2e; \right. \\ & \qquad 1 - \left(\Gamma^{1-e} \left[(1-x)^m + (1-y)^m - (1-e)^m \right] \right)^{1/m} \mid x \\ & \qquad \left. + \ y > 2e; e \right\}. \end{aligned} \tag{1}$$

So C^e is *Zero-level Universal Combinatorial Operation Model*, denoted by C^e_h, where $m = (3-4h)/(4h(1-h))$, $h \in [0,1]$, $m \in R$, $e \in [0,1]$.

Note 1. Conditional expression $\text{ite}\{\beta \mid \alpha; \gamma\}$ means if α is true, then β; otherwise γ. $\text{ite}\{\beta_1 \mid \alpha_1; \beta_2 \mid \alpha_2; \gamma\} = \text{ite}\{\beta_1 \mid \alpha_1; \text{ite}\{\beta_2 \mid \alpha_2; \gamma\}\}$. Amplitude limiting function $\Gamma^1[x] = \text{ite}\{1 \mid x > 1; 0 \mid x < 0 \text{ or imaginary number}; x\}$.

Universal Combinatorial Operation Model is a cluster of combinatorial operators, which is determined by *general correlation coefficient h* between propositions. In practical application, according to general correlation between propositions, we can take the corresponding one from the cluster. As *generalized correlation coefficient h* is equal to some special values, the corresponding combinatorial operators are given as follows.

(1) When $h = 1$, it means two propositions attract each other to the maximum extent. And $C^e(x, y, 1) = \text{ite}\{\min(x, y) \mid x + y < 2e; \max(x, y) \mid x + y > 2e; e\}$ is *Zadeh* combination $\mathbf{C}^e_3$.

(2) When $h = 0.75$, it means two propositions are independently correlated. And $C^e(x, y, 0.75) = \text{ite}\{xy/e \mid x + y < 2e; (x + y - xy - e)/(1 - e) \mid x + y > 2e; e\}$ is probability combination $\mathbf{C}^e_2$.

(3) When $h = 0.5$, it means two propositions reject each other to the maximum extent or restrain each other to the minimum extent. And $C^e(x, y, 0.5) = \Gamma^1[x + y - e]$ is bounded combination $\mathbf{C}^e_1$.

(4) When $h = 0$, it means two propositions restrain each other to the maximum extent. And $C^e(x, y, 0) = \text{ite}\{0 \mid x, y < e; 1 \mid x, y > e; e\}$ is drastic combination $\mathbf{C}^e_0$.

2.2. Universal Combinatorial Operation Model in Any Interval $[a, b]$. In practical control application, fuzzy domain of fuzzy variables, E and EC, is mostly symmetrical, such as $[-6, 6]$. However, the conventional *Universal Combinatorial Operation Model* has been limited in the interval $[0, 1]$. To this end, Chen, based on the basic idea of *Universal Logic*, sets up *Fractal Logic* in his doctoral dissertation [15], which can make inference in any interval $[a, b]$.

The combinatorial operation model in *Fractal Logic* is described below.

Definition 2 (see [15]). Set mapping $GN : [a, b] \to [a, b]$ and

$$GN(x) = b + a - x. \tag{2}$$

Then GN is normal universal *Not* operation model in any interval $[a, b]$, denoted by GN.

For the above definition, normal universal *Not* operation model has the following characters.

(1) *Closure:*

$$GN(x) \in [a, b]. \tag{3}$$

(2) *Two polar law:*

$$GN(a) = b, \qquad GN(b) = a. \tag{4}$$

(3) *Symmetric involution:*

$$GN(GN(x)) = x. \tag{5}$$

Definition 3 (see [15]). Set mapping $GC^e : [a,b] \times [a,b] \to [a,b]$ and

$$\begin{aligned} GC^e(x,y,h) &= \text{ite}\Big\{ \min\Big(e, (b-a) \\ &\quad \times \big[\max\big(0, \big((x-a)^m + (y-a)^m \\ &\quad - (e-a)^m\big) \\ &\quad \times \big((b-a)^m\big)^{-1}\big)\big]^{1/m} + a\Big) \mid \\ &\quad x+y<2e; b+a \\ &\quad - \min\Big(e', (b-a) \\ &\quad \times \big[\max\big(0, \big((b-x)^m + (b-y)^m \\ &\quad -(b-e)^m\big) \\ &\quad \times \big((b-a)^m\big)^{-1}\big)\big]^{1/m} + a\Big) \mid \\ &\quad x+y>2e; e\Big\}. \end{aligned} \tag{6}$$

Then GC^e is *Zero-level Universal Combinatorial Operation Model* in any interval $[a,b]$, denoted by GC_h^e, where $m = (3-4h)/(4h(1-h))$, $h \in [0,1]$, $m \in R$, $e, e' \in [a,b]$, and $e' = GN(e)$.

2.3. Demonstration of Attributes of Universal Combinatorial Operation Model in Any Interval. According to the definition of *Universal Combinatorial Operation Model* in any interval, the following characters [15] are attained.

(1) $GC^e(x,y,h)$ conforms to the combination axiom:

(i) *Boundary condition GC1*:
If $x, y < e$, then $GC^e(x,y,h) \le \min(x,y)$.
If $x, y > e$, then $GC^e(x,y,h) \ge \max(x,y)$.
If $x + y = 2e$, then $GC^e(x,y,h) = e$.
Otherwise, $\min(x,y) \le GC^e(x,y,h) \le \max(x,y)$.

(ii) *Monotonicity GC2*:
$GC^e(x,y,h)$ increases monotonously along with x and y.

(iii) *Continuity GC3*:
When $h \in (0,1)$, $GC^e(x,y,h)$ is continuous for all x and y.

(iv) *Commutative law GC4*:

$$GC^e(x,y,h) = GC^e(y,x,h). \tag{7}$$

(v) *Law of identical element GC5*:

$$GC^e(x,e,h) = x. \tag{8}$$

(2) *Closure*:

$$GC^e(x,y,h) \in [a,b]. \tag{9}$$

(3) *Inverse law*:

$$GC^e(x, 2e-x, h) = e. \tag{10}$$

(4) *Renunciation law*

$$GC^e(e,e,h) = e. \tag{11}$$

Theorem 4. $GN(GC^{GN(e)}(GN(x), GN(y), h)) = GC^e(x,y,h)$, $x, y \in [a,b]$, $e \in [a,b]$.

Proof. $x, y \in [a,b]$, $e \in [a,b]$, according to the closure of normal universal *Not* operation and universal combinatorial operation: $GN(x), GN(y) \in [a,b]$, $GN(e) \in [a,b]$ and $GC^{GN(e)}(GN(x), GN(y), h) \in [a,b]$ and according to the definition of universal combinatorial operation:

(1) when $x + y < 2e$

$$\begin{aligned} GN(x) + GN(y) &= (b+a-x) + (b+a-y) \\ &= 2(b+a) - (x+y) \\ &> 2(b+a-e) = 2GN(e). \end{aligned} \tag{12}$$

Then, according to the definition of $GC^e(x,y,h)$:

$$\begin{aligned} &GC^{GN(e)}(GN(x), GN(y), h) \\ &= b+a \\ &\quad - \min\Big(ab + a - GN(e), (b-a) \\ &\quad \times \big[\max\big(0, \big((b-GN(x))^m \\ &\quad + (b-GN(y))^m - (b-GN(e))^m\big) \\ &\quad \times \big((b-a)^m\big)^{-1}\big)\big]^{1/m} + a\Big). \end{aligned} \tag{13}$$

According to the definition of central generalized negation operation:

$$GN(x) = b + a - x, \tag{14}$$

$$GN(y) = b + a - y, \tag{15}$$

$$GN(e) = b + a - e. \tag{16}$$

Substituting (14), (15), and (16) separately into (13):

$$\begin{aligned}
&GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)\\
&=b+a\\
&\quad-\min\Big(e,(b-a)\\
&\qquad\times\Big[\max\Big(0,\big((x-a)^m+(y-a)^m\\
&\qquad\qquad-(e-a)^m\big)\\
&\qquad\qquad\times((b-a)^m)^{-1}\Big)\Big]^{1/m}+a\Big).
\end{aligned}\tag{17}$$

And then

$$\begin{aligned}
&GN\left(GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)\right)\\
&=b+a-\Big(b+a\\
&\qquad-\min\Big(e,(b-a)\\
&\qquad\quad\times\Big[\max\Big(0,\big((x-a)^m+(y-a)^m\\
&\qquad\qquad-(e-a)^m\big)\\
&\qquad\qquad\times\ ((b-a)^m)^{-1}\Big)\Big]^{1/m}+a\Big)\Big)\\
&=\min\Big(e,(b-a)\\
&\qquad\times\Big[\max\Big(0,\big((x-a)^m+(y-a)^m\\
&\qquad\qquad-(e-a)^m\big)\times((b-a)^m)^{-1}\Big)\Big]^{1/m}+a\Big)\\
&=GC^e\left(x,y,h\right)
\end{aligned}\tag{18}$$

(2) when $x+y>2e$

$$\begin{aligned}
&GN\left(x\right)+GN\left(y\right)\\
&\quad=\left(b+a-x\right)+\left(b+a-y\right)\\
&\quad=2\left(b+a\right)-\left(x+y\right)\\
&\quad<2\left(b+a-e\right)=2GN\left(e\right).
\end{aligned}\tag{19}$$

So, according to the definition of $GC^e(x,y,h)$:

$$\begin{aligned}
&GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)\\
&=\min\Big(GN\left(e\right),(b-a)\\
&\qquad\times\Big[\max\Big(0,\big((GN\left(x\right)-a)^m+(GN\left(y\right)-a)^m\\
&\qquad\qquad-(GN\left(e\right)-a)^m\big)\\
&\qquad\qquad\times((b-a)^m)^{-1}\Big)\Big]^{1/m}+a\Big).
\end{aligned}\tag{20}$$

Substituting (14), (15), and (16) separately into (20):

$$\begin{aligned}
&GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)\\
&=\min\Big(b+a-e,(b-a)\\
&\qquad\times\Big[\max\Big(0,\big((b-x)^m+(b-y)^m\\
&\qquad\qquad-(b-e)^m\big)\times((b-a)^m)^{-1}\Big)\Big]^{1/m}+a\Big).
\end{aligned}\tag{21}$$

And then

$$\begin{aligned}
&GN\left(GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)\right)\\
&=b+a\\
&\quad-\min\Big(b+a-e,(b-a)\\
&\qquad\times\Big[\max\Big(0,\big((b-x)^m+(b-y)^m\\
&\qquad\qquad-(b-e)^m\big)\times((b-a)^m)^{-1}\Big)\Big]^{1/m}+a\Big)\\
&=GC^e\left(x,y,h\right)
\end{aligned}\tag{22}$$

(3) when $x+y=2e$

$$\begin{aligned}
&GN\left(x\right)+GN\left(y\right)\\
&\quad=\left(b+a-x\right)+\left(b+a-y\right)\\
&\quad=2\left(b+a\right)-\left(x+y\right)\\
&\quad=2\left(b+a-e\right)\\
&\quad=2GN\left(e\right).
\end{aligned}\tag{23}$$

According to the definition of $GC^e(x,y,h)$:

$$GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)=GN\left(e\right).\tag{24}$$

Then:

$$GN\left(GC^{GN(e)}\left(GN\left(x\right),GN\left(y\right),h\right)\right)=GN\left(GN\left(e\right)\right)=e.\tag{25}$$

From the above, the theorem is true. □

Lemma 5. $GC^{GN(e)}(GN(x),GN(y),h)=GN(GC^e(x,y,h))$.

Table 1: Fuzzy rules defining the fuzzy composed variable GE_θ.

Error of pendulum rod	Angle speed of pendulum rod						
	NB	NM	NS	ZE	PS	PM	PB
Angle of pendulum rod							
NB	**NB**	**NB**	**NB**	NM	**NM**	**NS**	**ZE**
NM	**NB**	**NB**	**NM**	NM	**NS**	**ZE**	**PS**
NS	**NB**	**NM**	**NM**	NS	**ZE**	**PS**	**PM**
ZE	NM	NM	NS	ZE	PS	PM	PM
PS	**NM**	**NS**	**ZE**	PS	**PM**	**PM**	**PB**
PM	**NS**	**ZE**	**PS**	PM	**PM**	**PB**	**PB**
PB	**ZE**	**PS**	**PM**	PM	**PB**	**PB**	**PB**

Proof. According to Theorem 4 and involution law of normal universal *Not* operation in any interval $[a,b]$, the theorem can be proved simply. □

Lemma 6. $C^e(x,y,h) = 1 - C^{1-e}(1-x, 1-y, h)$.

Proof. Setting the interval $[a,b]$ of x, y as $[0,1]$, the lemma can be proved simply. □

Lemma 7. *If the interval $[a,b]$ is symmetrical about e, then $GC^e(x^*, y^*, h) = (GC^e(x,y,h))^*$, where x^* and x are symmetrical about e, namely, $x^* = 2e - x$, y^*, e^*, and $(GC^e(x,y,h))^*$ are similar, $e \in [a,b]$, $h \in [0,1]$.*

Proof. Since the interval $[a,b]$ is symmetrical about e, then $a + b = 2e$ and $GN(x) = a + b - x = 2e - x$; thus

$$GN(x) = x^*. \tag{26}$$

Similarly

$$\begin{gathered} GN(y) = y^*, \\ GN(e) = a + b - e = 2e - e = e. \end{gathered} \tag{27}$$

From (26) and (27) the following could be obtained:

$$\begin{gathered} GC^{GN(e)}(GN(x), GN(y), h) = GC^e(x^*, y^*, h), \\ GN(GC^e(x,y,h)) = (GC^e(x,y,h))^*. \end{gathered} \tag{28}$$

And then from Lemma 5

$$GC^e(x^*, y^*, h) = (GC^e(x,y,h))^*. \tag{29}$$

So the theorem is true. □

Lemma 8. $C^{0.5}(1-x, 1-y, h) = 1 - C^{0.5}(x,y,h)$, *where* $h \in [0,1]$.

Lemma 9. *If the interval $[a,b]$ is symmetrical about the original point, then $GC^0(-x,-y,h) = -GC^0(x,y,h)$, $h \in [0,1]$.*

This lemma indicates that, when the interval $[a,b]$ is symmetrical about the origin and identity element e is 0, *Universal Combinatorial Operation* $GC^e(x,y,h)$ is also symmetrical about the origin.

As pointed out in the literature [4], the logical relationship between input variables, error E and error variety EC of normal two-dimensional fuzzy controller, is universal combinatorial relation in *Universal Logic*. Consequently, the complex reasoning process based on fuzzy rule can be replaced by the simple universal combinatorial operation, and a flexible logic control model is presented accordingly. In fuzzy control, the domains of input variables and output variable are generally symmetric to the origin, such as $[-5, 5]$. Obviously, it is the prerequisite of control model that the operation model be symmetric to the origin. Therefore, Lemma 9 provides a basis for the *Universal Combinatorial Operation*'s application in control.

2.4. Flexible Logic Control Method. Xiao et al. have put forward a concept of fuzzy composed variable to reduce effectively fuzzy control rules in multivariable nonlinear system [16]. According to the characteristics of controlled system and the internal relationship between input variables, the core is to construct a fuzzy composed variable by the fuzzy logic system to synthetically reflect the deviation between reference and the process output.

Four output variables in single inverted pendulum are considered, which are the displacement and speed of cart, x and x', and the angle and angle speed between pendulum bar and vertical line, θ and θ'. Four input variables are involved for the fuzzy controller. In the input variables of control system, the angle and angle speed, θ and θ', directly reflected the motion of pendulum. Therefore, according to the angle and angle speed of pendulum and the language rules shown in Table 1, a fuzzy composed variable, the error GE_θ of pendulum, can be defined to synthetically describe the motion of pendulum with the fuzzy logic system. Similarly, a fuzzy composed variable, the error GE_x of cart, can be defined according to the displacement and speed of cart, x and x', so as to synthetically describe the motion of cart. For multivariable system, it need not define, respectively, fuzzy logic system for every fuzzy composed variable. We can use a uniform fuzzy rule table, such as Table 1, and just select different quantization factors to obtain different fuzzy composed variables.

Note 2. In the paper, the fuzzy controller discussed has inputs, such as error e and error variety ec, and output variable u is the control signal. The variables, e,ec, and u, are crisp values from the practical process. The fuzzy language variables, E, EC, and U, are the corresponding fuzzy ones, and the fuzzy domains are unified as $[-1, 1]$ with fuzzy subsets, such as negative big (NB), negative middle (NM), negative small (NS), zero (ZE), positive small (PS), positive middle (PM), and positive big (PB).

Obviously, the language rules shown in Table 1 actually reflect the essential relationship between error E and error variety EC. The contents in Table 1 can be approximately divided into four parts, which separately give the language rules used to define composed variables under four conditions, such as error E and error variety EC are both negative, error E is negative but error variety EC is positive, error E is positive but error variety EC is negative, and error E and error variety EC are both positive.

Both error E and error variety EC reflect the deviation between the reference and the output; and then a composed variable E' can be defined to synthetically describe the control deviation of system, based on the essential relationship between them.

According to the physical meanings of error E and error variety EC, we can get the following conclusions.

(1) When error E and error variety EC are both positive, the control deviation of system is positive and tends to further increase positively. So the value of composed variable E' should not be smaller than their maximum value and the direction is positive. The combination rules are shown by the lower right bold corner of Table 1.

(2) When error E is positive but error variety EC is negative, the control deviation of system is positive but tends to decrease. So the value of composed variable E' should be between E and EC in this case. The combination rules are shown by the lower left bold corner of Table 1.

(3) Similarly, the value of composed variable E' under two other conditions can also be obtained.

Through above analysis, it is easy to know that the essential relationship among error E, error variety EC and the composed variable E' is just a kind of universal combinatorial one in *Universal Logic*. As a result, we have

$$E' = GC^e(E, EC, h). \tag{30}$$

Obviously, now the identity element e is zero.

Moreover, according to the concept of negative feedback control, there is only a difference of a single sign between composed variable E' describing control deviation and the output variable U of controller, namely;

$$U = -E'. \tag{31}$$

Therefore, the relationship among error E, error variety EC, and the output variable U is

$$U = -GC^e(E, EC, h), \tag{32}$$

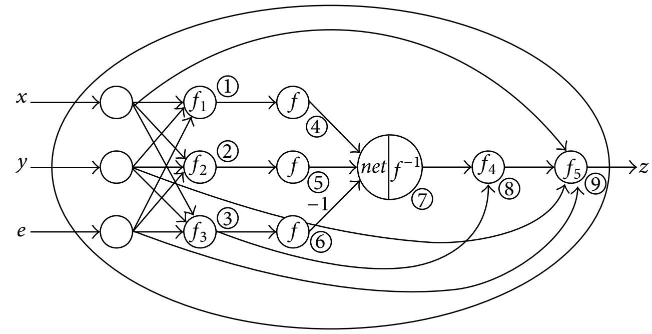


Figure 1: The neuron model of zero-level universal combinatorial operation.

where $E, EC, U \in [-1, 1]$, $e = 0$, and $h \in [0, 1]$. The control method is called *Flexible Logic Control Method* [4].

At the same time, a weighted factor $\alpha, \alpha \in [0, 1]$ is introduced to meet the requirements of different controlled objects. By adjusting the value of α, it is possible to change the weighting degrees for error E and error variety EC. When the *general correlation coefficient h* is 0.5, there is

$$\begin{aligned} U &= -GC^e(\alpha E, (1-\alpha) EC, 0.5) \\ &= \Gamma_{-1}^{1}[\alpha E + (1-\alpha) EC - e] \\ &= \alpha E + (1-\alpha) EC. \end{aligned} \tag{33}$$

And the formula (33) is just the fuzzy control method proposed by Long and Wang [17]. He used a linear equation, such as (33), to describe fuzzy control rules. But the relationship among E, EC, and U is not only linear. So (32) is a cluster of operators determined by *general correlation coefficient h*, and (33) is only a special operator in the cluster as h is equal to 0.5. As a result, flexible logic control method can realize the effective control for complex system.

3. Neuron Model of Zero-Level Universal Combinatorial Operation

The uniform neuron model of generalized logic operators in any intervals $[a, b]$ is established in the literature [18]. Seven neuron models of logic operation are given separately, such as *Not*, *And*, *Or*, *Implication*, *Equation*, *Average*, and *Combination*.

But the neuron model of combination logic operation is too complicated for practical applications.

In this section, a new neuron model of zero-level universal combinatorial operation is constructed based on Lemma 6 in Section 2.3.

It is learnt from Lemma 6 in Section 2.3 that

$$C^e(x, y, h) = 1 - C^{1-e}(1-x, 1-y, h). \tag{34}$$

Therefore, the model of zero-level universal combinatorial operation can be represented by the neuron shown in Figure 1.

The neuron model is composed of several subneurons which are interconnected. In the model, there are three input

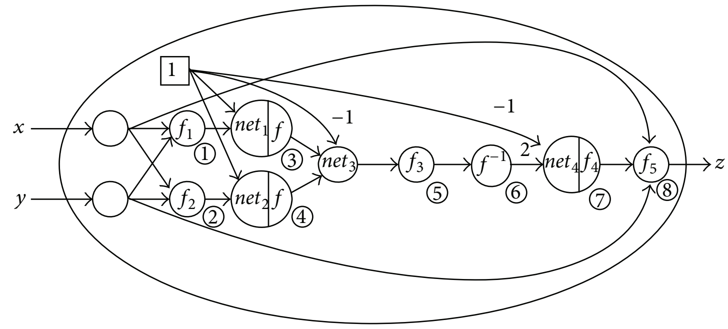


Figure 2: The neuron model of zero-level universal combinatorial operation in the interval $[-1, 1]$.

parameters: x, y, and e. And the output z, *net* denotes the weighted sum of all inputs, and all unmarked weights are 1. The transfer functions of the subneurons marked ④, ⑤, ⑥ are $f(x) = x^m$ with the parameter m and $m = (3-4h)/(4h(1-h))$. And the transfer function of the sub-neuron marked ⑦ is $f^{-1}(x)$. The sub-neurons marked ①, ②, ③, ⑧, and ⑨ have the following transfer functions, respectively:

$$f_1(u, v, e) = \begin{cases} u & u + v < 2e \\ 1 - u & u + v > 2e \\ e & \text{otherwise,} \end{cases}$$

$$f_2(u, v, e) = \begin{cases} v & u + v < 2e \\ 1 - v & u + v > 2e \\ e & \text{otherwise,} \end{cases}$$

$$f_3(u, v, e) = \begin{cases} e & u + v < 2e \\ 1 - e & u + v > 2e \\ e & \text{otherwise,} \end{cases} \tag{35}$$

$$f_4(u, v) = \Gamma^{v}[u],$$

$$f_5(u, v, w, e) = \begin{cases} w & u + v < 2e \\ 1 - w & u + v > 2e \\ w & \text{otherwise.} \end{cases}$$

We will discuss if the neuron model shown in Figure 1 has realized zero-level universal combinatorial operation.

(1) $x + y < 2e$. According to the definition of transfer functions f_1, f_2, and f_3, it is easy to find that the outputs of the sub-neurons marked ①, ②, ③ separately are x, y, and e. According to the definition of transfer function f, it is found that the outputs of the sub-neuron marked ④, ⑤, ⑥ separately are x^m, y^m, and e^m. Therefore, the output of the sub-neuron marked ⑦ is $(x^m + y^m - e^m)^{1/m}$. According to the definition of transfer function f_4, the output of the sub-neuron marked ⑧ is $\Gamma^e[(x^m + y^m - e^m)^{1/m}]$. According to the definition of transfer function f_5, the output of the sub-neuron marked ⑨ is $\Gamma^e[(x^m + y^m - e^m)^{1/m}]$. When $x + y < 2e$, the output of the neuron is

$$\Gamma^e\left[\left(x^m + y^m - e^m\right)^{1/m}\right] \tag{36}$$

(2) $x + y > 2e$. It is similarly learnt that the output of the sub-neuron marked ⑦ is $((1-x)^m + (1-y)^m - (1-e)^m)^{1/m}$, the output of the sub-neuron marked ⑧ is $\Gamma^{1-e}[((1-x)^m + (1-y)^m - (1-e)^m)^{1/m}]$, and the output of the sub-neuron marked ⑨ is $1 - \Gamma^{1-e}[((1-x)^m + (1-y)^m - (1-e)^m)^{1/m}]$. When $x + y > 2e$, the output of the neuron is

$$1 - \Gamma^{1-e}\left[\left((1-x)^m + (1-y)^m - (1-e)^m\right)^{1/m}\right]. \tag{37}$$

(3) $x + y = 2e$. It is similarly learnt that the output of the sub-neuron marked ⑦ is e, the output of the sub-neuron marked ⑧ is e, and the output of the sub-neuron marked ⑨ is e. When $x + y = 2e$, the output of neuron is e.

In summary, the neuron model shown in Figure 1 has fully realized the model of zero-level universal combinatorial operation.

In practical control application, the fuzzy domains of system variables are generally expressed as $[-n, n]$, for example, $[-6, 6]$. To simplify the neuron model, we can unify the fuzzy domains of system variables as $[-1, 1]$. Obviously, the identity element e is 0. Therefore, the model of zero-level universal combinatorial operation in the interval $[-1, 1]$ can be obtained by the definition of universal combinatorial operation model in any intervals $[a, b]$ in Section 2.2,

$$\begin{aligned} &GC^0(x, y, h) \\ &= \text{ite}\left\{\min\left(0, 2\left[\max\left(0, \frac{(x+1)^m + (y+1)^m - 1}{2^m}\right)\right]^{1/m}\right.\right. \\ &\qquad \left. -1\right) \mid x + y < 0; \\ &\qquad -\min\left(0, 2\left[\max\left(0, \frac{(1-x)^m + (1-y)^m - 1}{2^m}\right)\right]^{1/m}\right. \\ &\qquad \left.\left. -1\right) \mid x + y > 0; 0\right\}. \end{aligned} \tag{38}$$

Therefore, it is easy to obtain the neuron model of zero-level universal combinatorial operation in the interval $[-1, 1]$

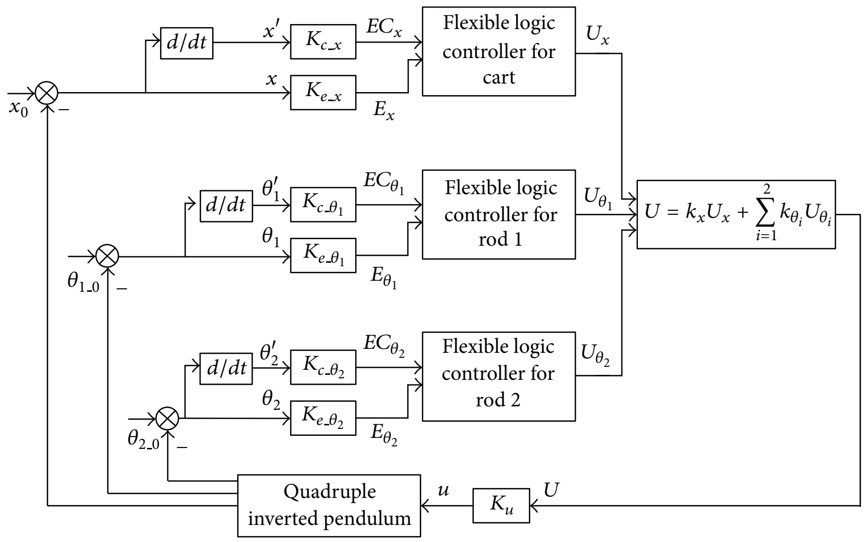


Figure 3: The flexible logic control model of double inverted-pendulum.

according to the neuron model given in Figure 1, as shown in Figure 2.

The neuron model is composed of interconnected multiple sub-neurons, and its internal structure is a small artificial neural network. In the model, x and y are the inputs, z is the output, net_1, net_2, net_3, and net_4 are the weighted sum of all inputs, the values of connecting weights without specific marking are 1, and ①, ②,. . ., ⑧ are marks of some sub-neurons for convenient discussion. The sub-neurons marked ③ and ④ have the transfer function $f(x) = x^m$ with the parameter m. And the sub-neuron marked ⑤ also has the transfer function $f_3(x) = \max(0, x/(2^m))$ with the parameter m, where $m = (3 - 4h)/(4h(1 - h))$. The sub-neuron marked ⑥ also has the transfer function $f^{-1}(x)$ with the parameter m, and the sub-neurons marked ①, ②, ⑦, and ⑧ have the following transfer functions, respectively:

$$f_1(u,v) = \begin{cases} u & u+v<0 \\ -u & u+v>0 \\ 0 & \text{otherwise,} \end{cases}$$

$$f_2(u,v) = \begin{cases} v & u+v<0 \\ -v & u+v>0 \\ 0 & \text{otherwise,} \end{cases} \tag{39}$$

$$f_4(u) = \min(0,u),$$

$$f_5(u,v,w) = \begin{cases} w & u+v<0 \\ -w & u+v>0 \\ w & \text{otherwise.} \end{cases}$$

4. Realization of Artificial Neural Network of Flexible Logic Control Model

In the practical complex system, the control objects are always not less than one, so multiple system variables are selected for feedback control. The basic idea of flexible logic control model presented in Section 2.4 is to design sub-goal controller based on the control object, and each subgoal controller is designed with the flexible logic control method. Then the output variable of control system is the weighted sum of the outputs of these sub-goal controllers.

Obviously, most of the operators involved in this model are universal combinatorial operation. Therefore, it is easy to implement the artificial neural network of the flexible logic control model by using the neuron model of zero-level universal combinatorial operation in Section 3. In this section, the artificial neural network structure of flexible logic control model is presented using the double inverted-pendulum system as the control object.

The objective is to maintain the rods in an upright position and the cart in an appointed position in the rail. There are six output variables and one input variable in double inverted-pendulum, which are x, x', θ_1, $\theta_1{}'$, θ_2, $\theta_2{}'$, and u. The variables, E_x, EC_x, E_{θ_1}, EC_{θ_1}, E_{θ_2}, EC_{θ_2}, and U, are the corresponding fuzzy ones, and the fuzzy domains are unified as $[-1, 1]$.

We can design three subcontrollers with the flexible logic control method. One is to maintain the cart in an appointed position with two input variables, E_x and EC_x. The other ones are to maintain, respectively, the rods in an upright position with two input variables E_{θ_i} and EC_{θ_i} $(i = 1, 2)$. And we lead into weighted factors, such as α_x, α_{θ_1}, and α_{θ_2}. The three sub-controllers are designed as follows:

$$U_x = -GC^0\left(\alpha_x E_x, (1-\alpha_x) EC_x, h_x\right), \tag{40}$$

$$\alpha_x = \left(\alpha_{s\_x} - \alpha_{0\_x}\right)\left|E_x\right| + \alpha_{0\_x}, \tag{41}$$

$$U_{\theta_i} = -GC^0\left(\alpha_{\theta_i} E_{\theta_i}, \left(1-\alpha_{\theta_i}\right) EC_{\theta_i}, h_{\theta_i}\right), \tag{42}$$

$$\alpha_{\theta_i} = \left(\alpha_{s\_\theta_i} - \alpha_{0\_\theta_i}\right)\left|E_{\theta_i}\right| + \alpha_{0\_\theta_i}. \tag{43}$$

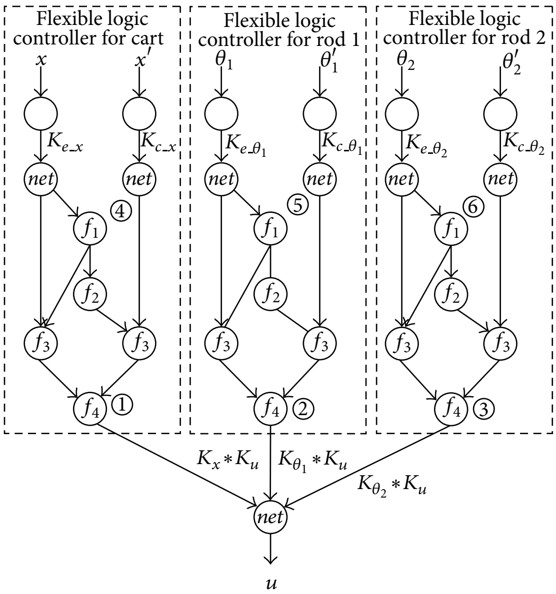


FIGURE 4: The neural network structure of flexible logic control model.

When the control signal U is combined, we lead into three weighted factors, such as k_x, k_{θ_1}, and k_{θ_2}, for the three sub-controllers. According to (40) and (42), we can get the output of the controller as follows:

$$\begin{aligned} U &= k_x U_x + k_{\theta_1} U_{\theta_1} + k_{\theta_2} U_{\theta_2} \\ &= k_x \left(-GC^0 \left(\alpha_x E_x, (1-\alpha_x) EC_x, h_x\right)\right) \\ &\quad + \sum_{i=1}^{2} k_{\theta_i} \left(-GC^0 \left(\alpha_{\theta_i} E_{\theta_i}, \left(1-\alpha_{\theta_i}\right) EC_{\theta_i}, h_{\theta_i}\right)\right), \end{aligned} \tag{44}$$

where E_x, EC_x, E_{θ_i}, EC_{θ_i}, and $U \in [-1,1]$, $GC^0(x,y,h)$ is (38), h_x, h_{θ_1}, and h_{θ_2} are general correlation coefficients, h_x, h_{θ_1}, and $h_{\theta_2} \in [0,1]$, α_x, α_{θ_1}, and $\alpha_{\theta_2} \in [0,1]$, k_x, k_{θ_1}, and $k_{\theta_2} \in [-1,1]$, $0 \le \alpha_{0\_x} \le \alpha_{s\_x} \le 1$, $0 \le \alpha_{0\_\theta_1} \le \alpha_{s\_\theta_1} \le 1$, and $0 \le \alpha_{0\_\theta_2} \le \alpha_{s\_\theta_2} \le 1$.

By the above analysis, we can obtain the control model of double inverted-pendulum, as shown in Figure 3. Based on the neuron model of zero-level universal combinatorial operation, the artificial neural network structure of the control model could be obtained, as shown in Figure 4.

The displacement and speed of cart, x and x', the angle and angle speed between the lower pendulum bar and vertical line, θ_1 and $\theta_1{}'$, and the angle and angle speed between the upper pendulum bar and vertical line, θ_2 and $\theta_2{}'$, are the input variables of the artificial neural network. The output variable is the control signal. ①, ②,. . ., ⑥ are marks of some sub-neurons for convenient discussion. *Net* is the weighted sum of these inputs of the corresponding sub-neurons. The connection weight values of the unmarked sub-neurons are 1. The sub-neurons marked ①, ②, and ③ are the neuron model of zero-level universal combinatorial operation in the interval $[-1,1]$, as given in Section 3. The other sub-neurons have the following transfer functions, respectively:

$$\begin{gathered} f_1(u) = \left(\alpha_s - \alpha_0\right)|u| + \alpha_0, \\ f_2(u) = 1-u, \qquad f_3(u,v) = uv, \end{gathered} \tag{45}$$

where the transfer function f_1 has the parameters such as α_0 and α_s, and the transfer functions of the sub-neurons marked ④, ⑤, and ⑥ separately have the parameters α_{0_x}, α_{s_x}, $\alpha_{0\_\theta_1}$, $\alpha_{s\_\theta_1}$, $\alpha_{0\_\theta_2}$, and $\alpha_{s\_\theta_2}$.

From the Figure 4, we can see that the artificial neural network structure of the control model is a feed forward neural network. *Genetic algorithms* (*GAs*) are a robust and efficient optimization technique based on the mechanism of natural selection and natural genetics [19]. One of the important features of *GAs* is that they are a population-based search technique. Instead of moving from one single point to another like traditional mathematical programming techniques, *GAs* always maintain and manipulate a solution set. Therefore, *GAs* are used to train the artificial neural network.

TABLE 2: The physical parameters of the double inverted-pendulum.

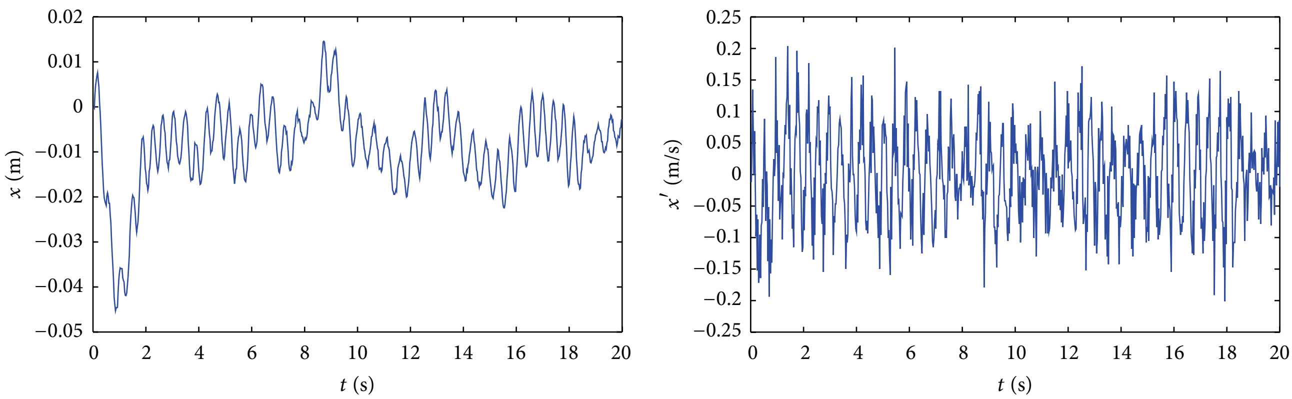
Symbol	Value
m_0	0.924 kg
m_1	0.185 kg
m_2	0.2 kg
l_1	0.283 m
l_2	0.245 m
L_1	0.483 m
f_0	0.1 N·s/m
J_1	0.00547 kg·m^2
J_2	0.00549 kg·m^2

5. Results of Experiments

This section takes a double inverted-pendulum physical system, for example, to show the feasibility and validity of the flexible logic control method based on artificial neural network.

Owing to the rapidity and the absolute instability of the inverted-pendulum system, it requires a high real-time processing frequency. Therefore, the sampling interval of this system is set as 5 ms. When the system is running for 20 s, the control effect comparison could be conducted. The physical parameters of the system are given in Table 2.

The flexible logic control method based on artificial neural network is applied into the above double inverted-pendulum physical system. The network parameters are optimized by genetic algorithms. The definition of the fitness



(a) The cart displacement curve

(b) The cart speed curve

Figure 5: The car displacement and speed curve when the system is steady.

Table 3: The control parameters of the control model.

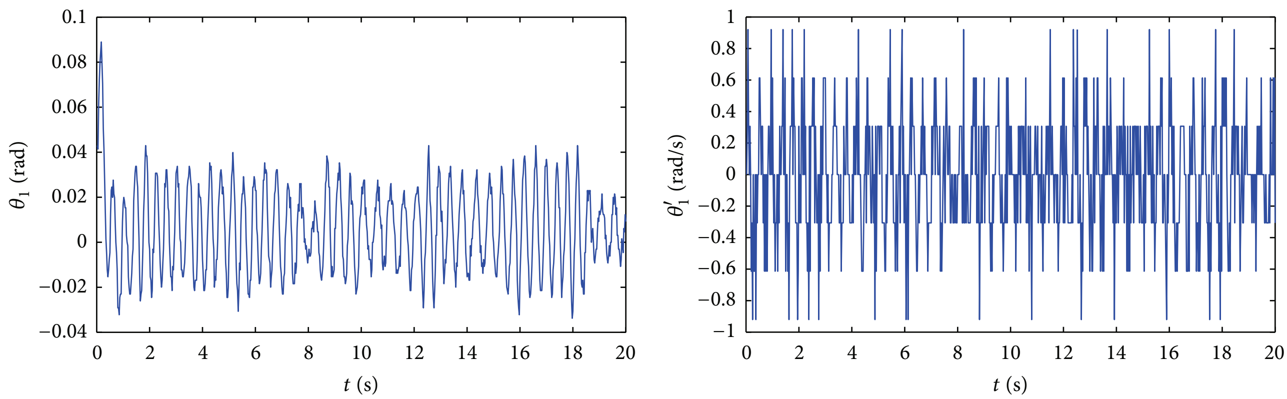
Symbol	Value	Description
K_{e_x}	29.4118	Quantification factor for E_x
K_{c_x}	16.3529	Quantification factor for EC_x
h_x	0.2902	General correlation coefficient between E_x and EC_x
a_{0_x}	0.2157	Minimum value of a_x
a_{s_x}	0.5258	Maximum value of a_x
$K_{e\_\theta 1}$	73.3333	Quantification factor for $E_{\theta 1}$
$K_{c\_\theta 1}$	8.0000	Quantification factor for $EC_{\theta 1}$
$h_{\theta 1}$	0.8275	General correlation coefficient between $E_{\theta 1}$ and $EC_{\theta 1}$
$a_{0\_\theta 1}$	0.7569	Minimum value of $a_{\theta 1}$
$a_{s\_\theta 1}$	0.7701	Maximum value of $a_{\theta 1}$
$K_{e\_\theta 2}$	47.4510	Quantification factor for $E_{\theta 1}$
$K_{c\_\theta 2}$	14.9412	Quantification factor for $EC_{\theta 1}$
$h_{\theta 2}$	0.5674	General correlation coefficient between $E_{\theta 2}$ and $EC_{\theta 2}$
$a_{0\_\theta 2}$	0.6745	Minimum value of $a_{\theta 2}$
$a_{s\_\theta 2}$	0.7530	Maximum value of $a_{\theta 2}$
K_x	0.4902	Weighted factor of the subcontroller for cart
$K_{\theta 1}$	−0.3645	Weighted factor of the subcontroller for rod1
$K_{\theta 2}$	0.9686	Weighted factor of the subcontroller for rod2
K_u	8.7843	Proportion factor for U

function is shown as (46) and the control parameters of the system are shown in Table 3. Consider

$$\mathrm{Dis} = \sum_{i=0}^{N} \left(\left(\left(\frac{x^2(i)}{25} \right) + \left(\frac{x'^2(i)}{50} \right) + \left(\frac{\theta_1^2(i)}{5} \right) + \left(\frac{\theta_1'^2(i)}{10} \right) + \theta_2^2(i) + \theta_2'^2(i) \right) \times (N)^{-1} \right),$$
$$\text{fitness} = \frac{1}{(\mathrm{Dis}/10)}. \tag{46}$$

Three experiments as the stability control, anti-interference control, and the free movement in a level track of cart pendulum system for the above double inverted-pendulum physical system are carried out by using the controller made of these control parameters.

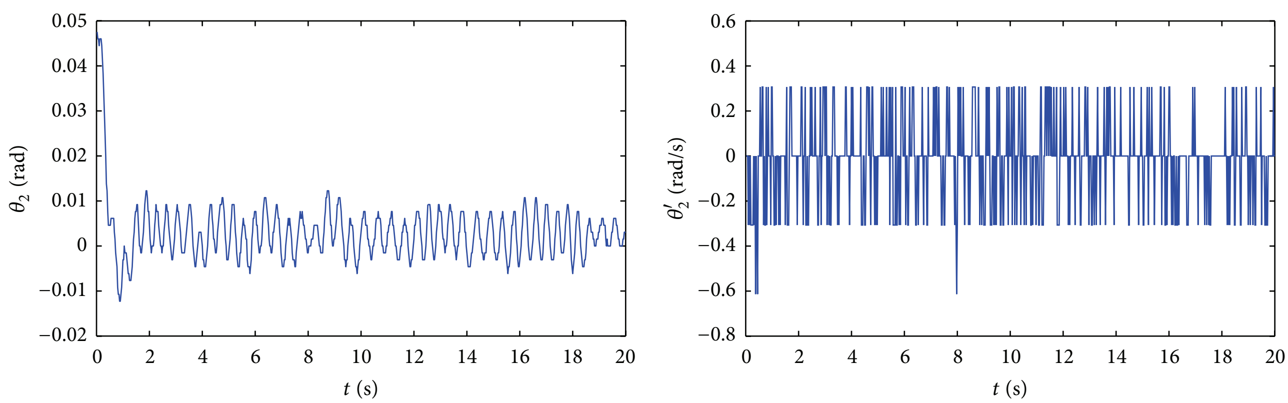
5.1. Stability Control Experiment. Under the same initial state such as $x(0) = 0$ m, $\theta_1(0) = 0.05$ rad and $\theta_2(0) = 0.05$ rad, the stability control of the double inverted-pendulum physical system has been realized. The experimental results are shown in Figures 5, 6, 7, and 8 (running time: 20 s).



(a) The lower rod angle curve

(b) The lower rod angular velocity curve

FIGURE 6: The lower rod angle curve and angular velocity curve when the system is steady.



(a) The upper rod angle curve

(b) The upper rod angular velocity curve

FIGURE 7: The upper rod angle curve and angular velocity curve when the system is steady.

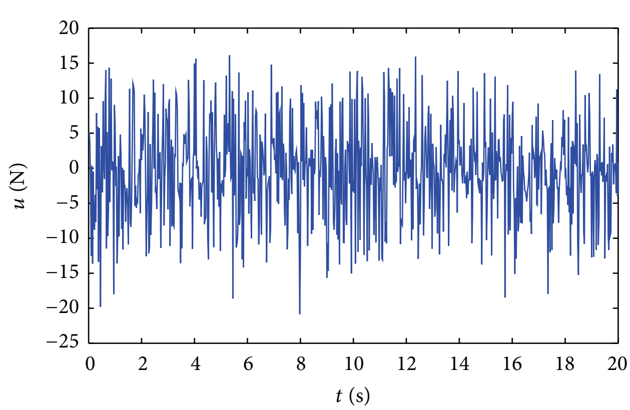


FIGURE 8: The controlled quantity curve when the system is steady.

From the experimental results, we can find when the system is in a stable state, the deviation of the cart displacement could be about 0.01 m with a good stability control effect.

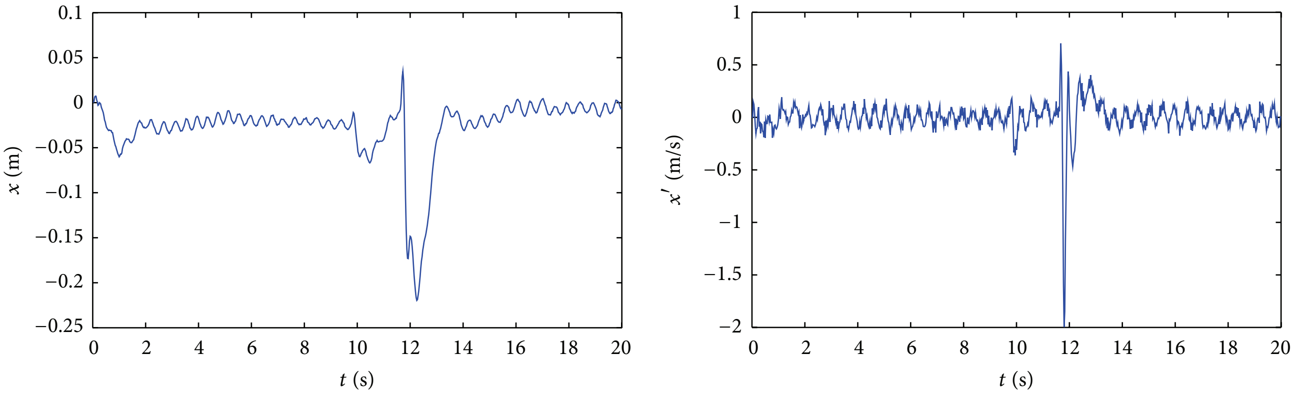
5.2. Anti-Interference Control Experiment. After knocking the upper pendulum twice when the system reaches a steady state, it shows strong anti-interference ability. The experimental results are shown in Figures 9, 10, 11, and 12.

From the experimental results, we can find the system becomes stable again after 1.9 s.

5.3. Free Movement Control Experiment of Cart Pendulum System. When the system reaches a steady state, the target position x_d is changed on-line, such as from 0 m to −0.2 m. So the free movement control of cart pendulum on a level track could be realized. The experimental results are shown in Figures 13, 14, 15, and 16.

From the experimental results, we can see the cart moves to the new target location after 2.7 s and the system remains stable.

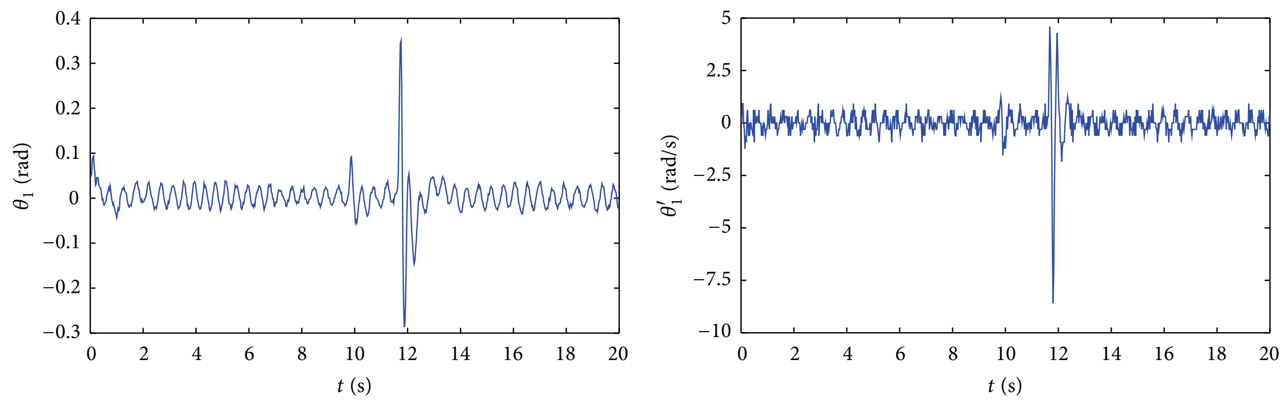
5.4. Self-Adaptive Experiment of Control Model. Some physical parameters have been changed in another double inverted-pendulum physical system. The physical parameters of the system are shown in Table 4. The control model



(a) The cart displacement curve

(b) The cart speed curve

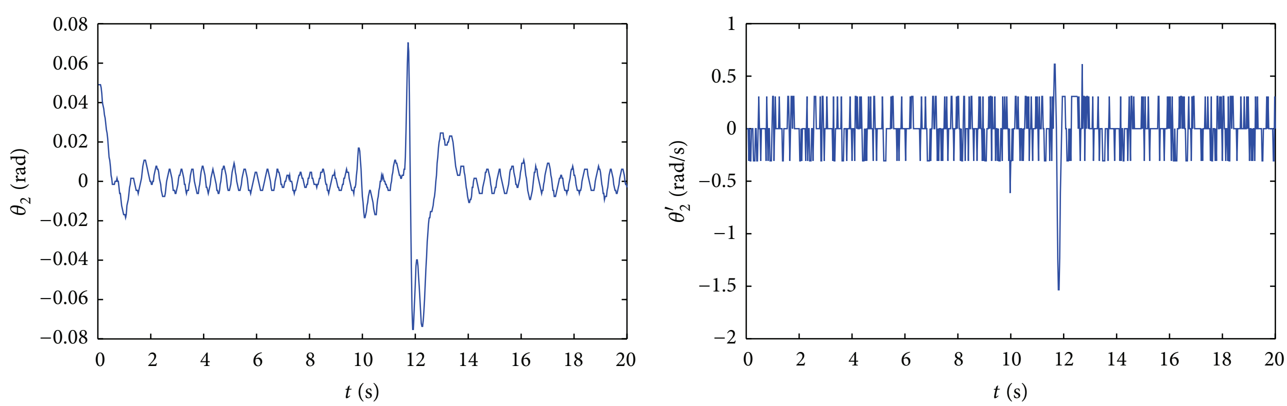
Figure 9: The car displacement and speed curve when the system is interfered.



(a) The lower rod angle curve

(b) The lower rod angular velocity curve

Figure 10: The lower rod angle curve and angular velocity curve when the system is interfered.



(a) The upper rod angle curve

(b) The upper rod angular velocity curve

Figure 11: The upper rod angle curve and angular velocity curve when the system is interfered.

also can realize the stability control of the new double inverted-pendulum after changing some parameters. The experimental results are shown in Figures 17, 18, 19, and 20. The experimental results show that the control model based on artificial neural network has excellent self-adaptability and portability.

5.5. Comparison of the Experimental Results. Cheng et al. [20] put forward a parameter fuzzy control method in 1996. The core idea of this method is to find the synthetical relationship among state variables by using modern control theory and form composed error and composed variety of error so as to construct a fuzzy controller.

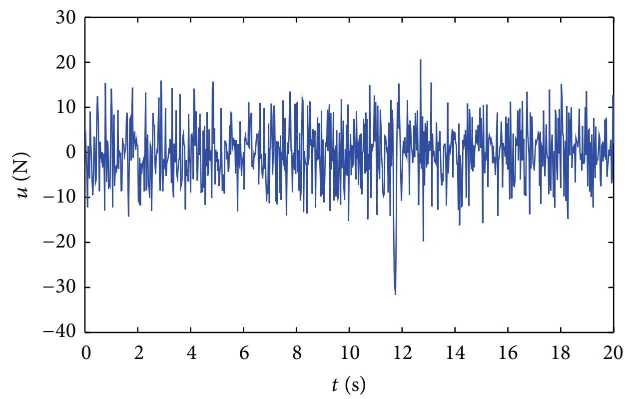
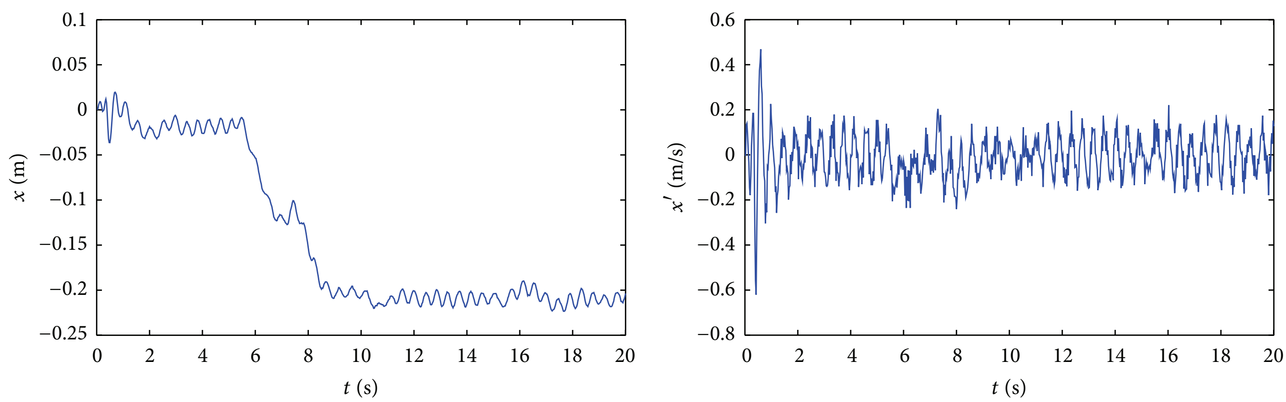


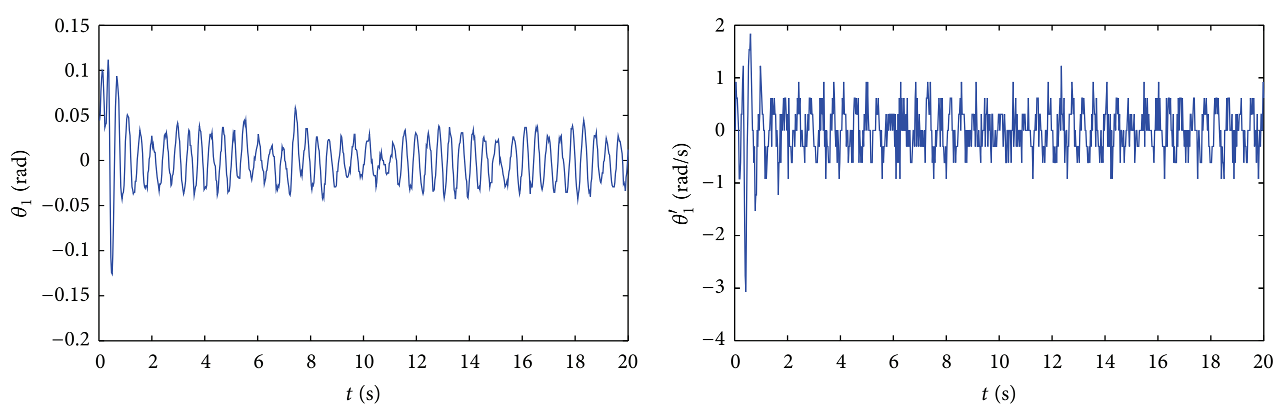
Figure 12: The controlled quantity curve when the system is interfered.



(a) The cart displacement curve

(b) The cart speed curve

Figure 13: The car displacement and speed curve when the cart is moving in a level track.



(a) The lower rod angle curve

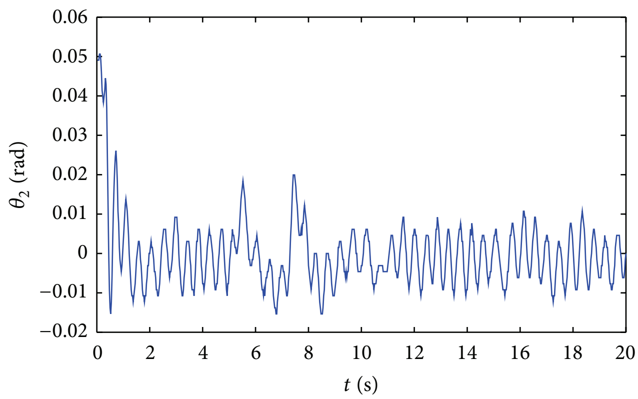
(b) The lower rod angular velocity curve

Figure 14: The lower rod angle curve and angular velocity curve when the cart is moving in a level track.

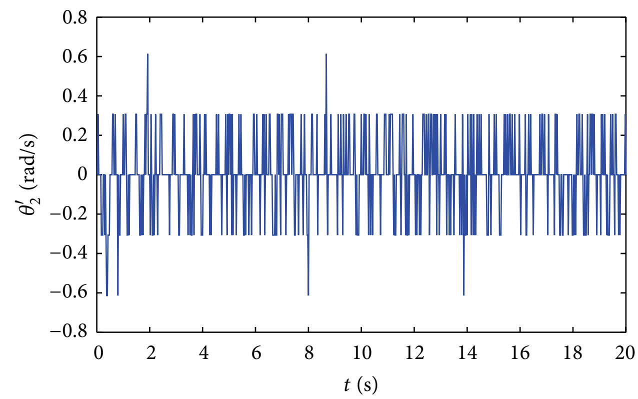
The parameter fuzzy control method is used to stabilize the above double inverted-pendulum physical system. The control parameters of this system are shown in Table 5.

Real-time control is carried out in the above double inverted-pendulum physical system. When the initial state of the system is $x(0) = 0\,\text{m}$, $\theta_1(0) = 0.05\,\text{rad}$, and $\theta_2(0) = 0.05\,\text{rad}$, the running time is 20 s. The comparison of the experimental results for two control methods is shown in Table 6.

From Table 6, the adjusting time and the steady-state errors of the displacement and the angle are compared with



(a) The upper rod angle curve



(b) The upper rod angular velocity curve

FIGURE 15: The upper rod angle curve and angular velocity curve when the cart is moving in a level track.

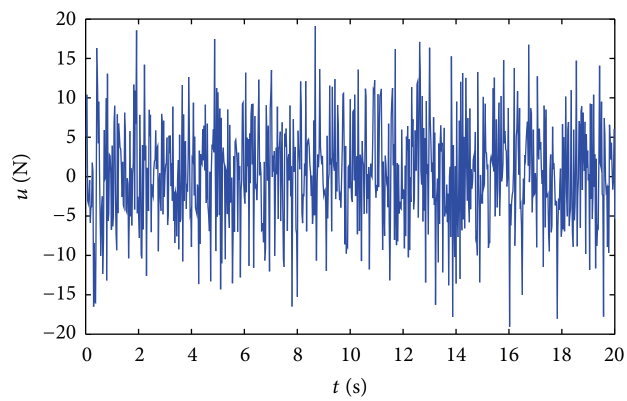


FIGURE 16: The controlled quantity curve when the cart is moving in a level track.

TABLE 4: The physical parameters of another double inverted-pendulum.

Symbol	Value
m_0	0.595 kg
m_1	0.158 kg
m_2	0.142 kg
l_1	0.1237 m
l_2	0.2265 m
L_1	0.151 m
f_0	0.1 N·s/m
J_1	0.0028394 kg·m^2
J_2	0.0024329 kg·m^2

parameter fuzzy control method about the stability control in double inverted-pendulum physical system.

In summary, the flexible logic control model based on artificial neural networks has good control effects. And the controlling system has good stability and anti-interference ability.

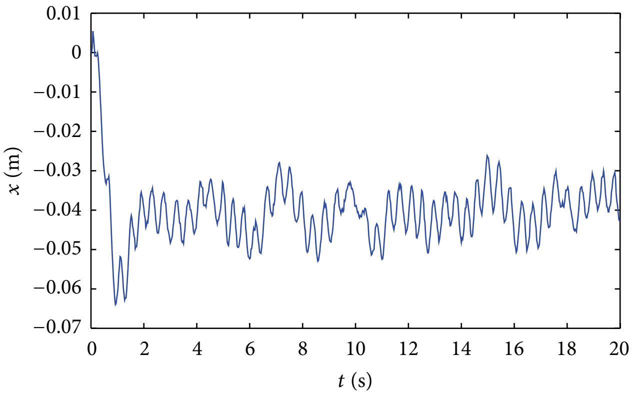
TABLE 5: The control parameters of the parameter fuzzy controller.

Symbol	Value
k_1	8.3089
k_2	33.4311
k_3	−114.1740
k_4	0.0978
k_5	26.8817
k_6	−22.1896
a_0	0.3333
a_s	0.7518

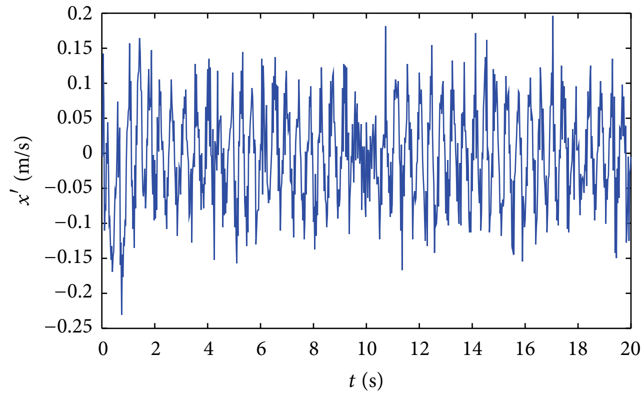
6. Conclusion

Based on the basic physical meaning of error E and error variety EC, this paper analyzes the logical relationship between them and uses universal combinational operation model to describe it. And a flexible logic control method is put forward to realize effective control on multivariable nonlinear system.

In order to implement the fusion control of *Universal Logic* and artificial neural networks, this paper puts forward a new neuron model of zero-level universal combinational

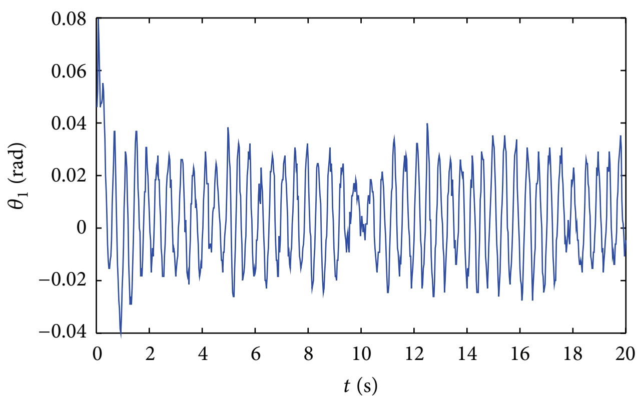


(a) The cart displacement curve

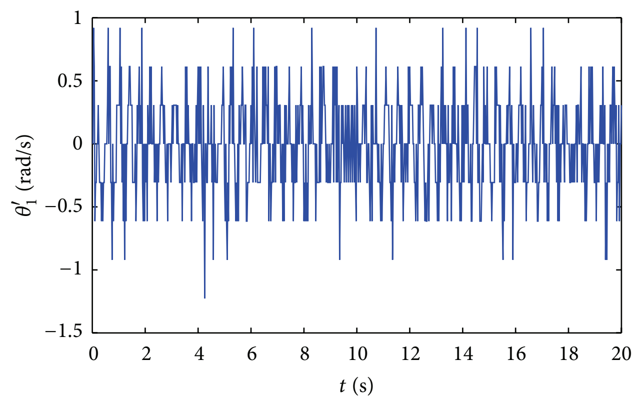


(b) The cart speed curve

FIGURE 17: The car displacement and speed curve when the system is steady.

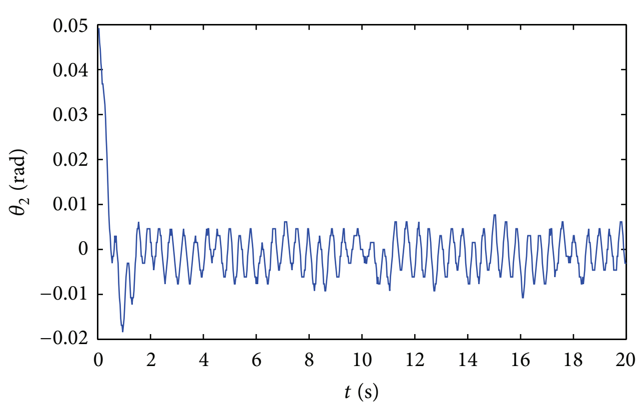


(a) The lower rod angle curve

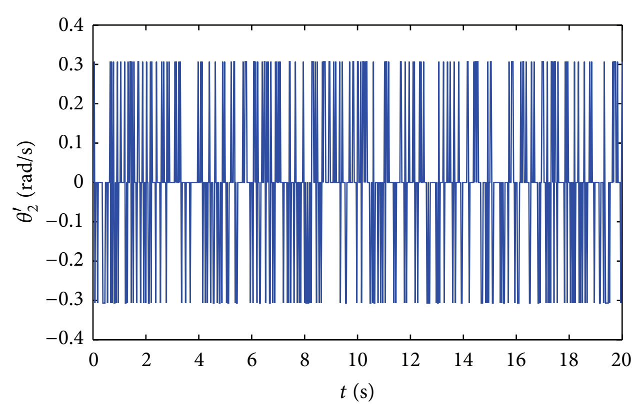


(b) The lower rod angular velocity curve

FIGURE 18: The lower rod angle curve and angular velocity curve when the system is steady.



(a) The upper rod angle curve



(b) The upper rod angular velocity Curve

FIGURE 19: The upper rod angle curve and angular velocity curve when the system is steady.

Table 6: The comparison of the control effects for two control methods.

Control method	Control effects: Evaluation indexes: Steady state deviation: The cart displacement (m)	The angle of the lower rod (rod)	The angle of the upper rod (rod)	Adjusting time (s)
Fuzzy control parameter method	0.125	0.01	0.004	2.37
Flexible logic control method	0.01	0.004	0.002	1.7

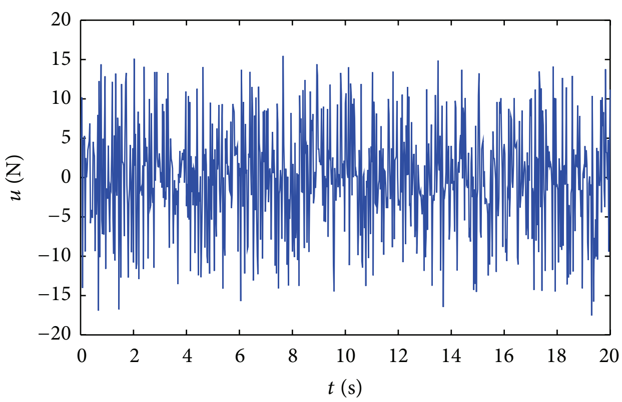


Figure 20: The controlled quantity curve when the system is steady.

operation. Based on this neuron model, artificial neural network is designed for flexible logic control model. Meanwhile, the stability control of the double inverted-pendulum physical system, anti-interference control, and free movement of the cart pendulum have been realized in a level track by using the proposed control model.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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