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Research Article

Global Exponential Stability for DCNNs with Impulses on Time Scales

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A class of delayed cellular neural networks (DCNNs) with impulses on time scales is considered. By using the topological degree theory, and the time scale calculus theory some sufficient conditions are derived to ensure the existence, uniqueness, and global exponential stability of equilibria for this class of neural networks. Finally, a numerical example illustrates the feasibility of our results and also shows that the continuous-time neural network and the discrete-time analogue have the same dynamical behaviors. The results of this paper are completely new and complementary to the previously known results.

1. Introduction

Chua and Yang [1] proposed a novel class of information-processing systems called cellular neural networks (CNNs) in 1988. The CNNs can be applied in signal processing and can also be used to solve some image processing and pattern recognition problems [2]. Since time delays are unavoidable due to finite switching speeds of the amplifiers, delayed cellular neural networks (DCNNs) have been widely studied and successfully applied to pattern recognition, associative memories, and signal processing and optimization, especially in image processing. The dynamic behavior of the networks plays an important role in such applications [3–8]. Therefore, there are many works on the stability of equilibrium point of delayed cellular neural networks (DCNNs) [5–13].

Most neural networks can be classified into two types: continuous or discrete. However, many real-world systems and natural processes cannot be categorized into one of them. They display characteristics of both continuous and discrete styles. For instance, some biological neural networks in biology, bursting rhythm models in pathology, and optimal control models in economics are characterized by abrupt changes of state. These are the familiar impulsive phenomena. Other examples can also be found in information science, electronics, automatic control systems, computer networking, artificial intelligence, robotics, telecommunications, and so forth. Such a kind of phenomena, in which sudden and sharp changes often occur in a continuous process, cannot be well described by pure continuous or pure discrete models. Therefore, it is important and, in effect, necessary to study a new type of neural networks—impulsive neural networks—as an appropriate description of these phenomena of abrupt qualitative dynamical changes of essentially continuous systems. The fundamental theory of impulsive differential equations has been developed in [14]. Since delays and impulses can affect the dynamical behaviors of the system, it is necessary to investigate both delay and impulsive effects on the stability of neural networks. For more details, one can refer to [10, 13, 15–23].

The theory of time scale was initiated by Hilger in 1988, which has recently received a lot of attention [24–26]. The field of dynamic equations on time scale contains links and extends the classical theory of differential and difference equations. It is well known that both continuous and discrete systems are very important in implementation and applications (see [27–30]). But it is troublesome to study the stability for continuous and discrete systems, respectively.

Therefore, it is significant to study that on time scales which can unify the continuous and discrete situations [21, 31–40].

Motivated by above, in this paper, we are concerned with the following impulsive DCNN on time scales:

$$\begin{gathered} x_i^{\Delta}(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) \\ + \sum_{j=1}^{n} c_{ij} f_j\left(x_j\left(\zeta_{ij}\left(t, x_j(t)\right)\right)\right) + I_i, \\ t \in \mathbb{T}_0^{+},\ t \neq t_k, \\ x_i\left(t_k^{+}\right) = x_i\left(t_k^{-}\right) + P_i\left(x_i\left(t_k^{-}\right)\right), \\ k = 1, 2, \ldots,\ i = 1, 2, \ldots, n, \end{gathered} \tag{1}$$

where n corresponds to the numbers of units in a neural network; $x_i(t)$ corresponds to the state of the ith unit at time t; $f_j(x_j(t))$ denotes the output of the jth unit at time t. $\mathbb{T}_0^{+}$ is the $\mathbb{T}$-interval $\{t \in \mathbb{T}, t \geq 0\}$, and $\mathbb{T}$ denotes a time scale, which is an arbitrary nonempty closed subset of the real number $\mathbb{R}$ and with bounded graininess μ. For the simplicity, we assume that $0 \in \mathbb{T}$ and $\mathbb{T}$ is unbounded above; that is, $\sup \mathbb{T} = +\infty$. Further, b_{ij}, c_{ij}, a_i, and I_i are constants. b_{ij}, c_{ij} denote the strength of the jth unit at time t and $\zeta_{ij}(t, x_j(t))$, respectively. I_i denotes the external bias on the ith unit and a_i represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs. t_k, $k = 1, 2, \ldots$ are the moments of impulsive perturbations and satisfy $0 = t_0 < t_1 < t_2 < \cdots$ and $\lim_{k \to \infty} = \infty$, $\mu(t_k) = 0$ (see Definition 3). $P_i(x_i(t_k))$ represents the abrupt change of the state $x_i(t)$ at the impulsive moment t_k. To the best of our knowledge, this is first paper to study DCNNs with impulses on time scales.

Throughout this paper, we assume that $x_i(t_k) \equiv x_i(t_k^{-})$ and

(H1) functions ζ_{ij} satisfy $\zeta_{ij} : \mathbb{T} \times \mathbb{R} \to \mathbb{T}$ for all $t \in \mathbb{T}$, $i, j = 1, 2, \ldots, n$;

(H2) $f_j \in C(\mathbb{R}, \mathbb{R})$ $(i = 1, 2, \ldots, n)$ and there exists a positive number F_i such that $|f_i(x) - f_i(y)| \leq F_i |x - y|$ for all $x, y \in \mathbb{R}$, $i = 1, 2, \ldots, n$.

Remark 1. The neural network (1) is a system of differential equations with state-dependent deviating arguments and from (H1), one can see that deviating arguments in (1) may be delayed type, advanced type, or mixed type.

Our main purpose of this paper is to study the existence and global exponential stability of the equilibria of (1) by using the topological degree theory and the time scale calculus theory. The results of this paper are completely new and complementary to the previously known results.

The organization of this paper is as follows. In the next section, some notations, definitions, and lemmas are presented. Section 3 addresses the existence and uniqueness of equilibria of system (1) by using the method of topological degree theory. In Section 4, we give the criteria of global exponential stability of the equilibrium point of system (1). In Section 5, an example is also provided to illustrate the effectiveness of the main results in Sections 3 and 4.

2. Notations and Preliminaries

In this section, we will first recall some basic definitions and lemmas which will be useful for the proof of our main results.

Definition 2 (see [33, 34]). A time scale $\mathbb{T}$ is arbitrary nonempty closed subset of the real set $\mathbb{R}$ with the topology and ordering inherited from $\mathbb{R}$.

Definition 3 (see [33, 34]). On any time scale $\mathbb{T}$, we define the forward and backward jump operators by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) := \sup\{s \in \mathbb{T} : s < t\}. \tag{2}$$

A point t is said to be left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, left-scattered if $\rho(t) < t$, and right-scattered if $\sigma(t) > t$. The graininess function μ for a time scale $\mathbb{T}$ is defined by $\mu(t) := \sigma(t) - t$. If $\mathbb{T}$ has a left-scattered maximum m, then we defined $\mathbb{T}^k$ to be $\mathbb{T} \setminus \{m\}$. Otherwise, $\mathbb{T}^k = \mathbb{T}$.

Definition 4 (see [33, 34]). For a function $f : \mathbb{T} \to \mathbb{R}$ (the range $\mathbb{R}$ of f may be actually replaced by Banach space), the (delta) derivative is defined by

$$f^{\Delta} = \frac{f(\sigma(t)) - f(t)}{\sigma(t) - t} \tag{3}$$

if f is continuous at t and t is right-scattered. If t is not right-scattered, then the derivative is defined by

$$f^{\Delta} = \lim_{s \to t, s \in \mathbb{T}} \frac{f(\sigma(t)) - f(s)}{\sigma(t) - s} = \lim_{s \to t, s \in \mathbb{T}} \frac{f(t) - f(s)}{t - s}, \tag{4}$$

provided this limit exists.

Lemma 5 (see [33, 34]). *If f, g are differential at $t \in \mathbb{T}$, one has*

(1) $f(\sigma(t)) = f(t) + \mu(t) f^{\Delta}(t)$;

(2) $(fg)^{\Delta}(t) = f^{\Delta}(t) g(t) + f(\sigma(t)) g^{\Delta}(t)$.

Definition 6 (see [33, 34]). A function $F : \mathbb{T}^k \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta} = f$ holds for all $t \in \mathbb{T}^k$. In this case, we define the integral of f by

$$\int_a^t f(s) \Delta s = F(t) - F(a) \quad \text{for } t \in \mathbb{T}, \tag{5}$$

and we have the following formula:

$$\int_t^{\sigma(t)} f(s) \Delta s = \mu(t) f(t) \quad \text{for } t \in \mathbb{T}^k. \tag{6}$$

Definition 7 (see [33, 34]). A function $f : \mathbb{T} \to \mathbb{R}$ is called right-dense continuous (rd-continuous) provided it is continuous at right-dense points of $\mathbb{T}$ and the left-sided limit exists (finite) at left-dense point of $\mathbb{T}$. The set of all right-dense continuous functions on $\mathbb{T}$ is defined by $C_{\mathrm{rd}} = C_{\mathrm{rd}}(\mathbb{T}) = C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$. If f is continuous at each right-dense point and each left-dense point, then f is said to be continuous function on $\mathbb{T}$. We define $C(J, \mathbb{R}) = \{f(t) \text{ is continuous on } J\}$.

Lemma 8 (see [33, 34]). *If $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C(\mathbb{T}, \mathbb{R})$, then one has*

(1) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$;

(2) *if* $f(t) \geq 0$ *for all* $a \leq t < b$, *then* $\int_a^b f(t)\Delta t \geq 0$;

(3) *if* $|f(t)| \leq g(t)$ *on* $[a, b) := \{t \in \mathbb{T} : a \leq t < b\}$, *then* $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$.

Definition 9 (see [33, 34]). A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive if $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}$. If p is regressive function, then the generalized exponential function e_p is defined by

$$e_p(t, s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\,\Delta\tau\right\}, \quad \text{for } s, t \in \mathbb{T}, \tag{7}$$

with the cylinder transformation

$$\xi_h(z) = \begin{cases} \dfrac{\log(1 + hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases} \tag{8}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions; we define

$$\begin{gathered} p \oplus q := p + q + \mu pq, \qquad \ominus p := -\frac{p}{1 + \mu p}, \\ p \ominus q := p \oplus (\ominus q). \end{gathered} \tag{9}$$

Then, the generalized exponential function has the following properties.

Lemma 10 (see [33, 34]). *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions; then*

(1) $e_0(t, s) \equiv 1$ *and* $e_p(t, t) \equiv 1$;

(2) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;

(3) $e_p(t, \sigma(s)) = e_p(t, s)/(1 + \mu(s)p(s))$;

(4) $1/e_p(t, s) = e_{\ominus p}(t, s)$;

(5) $e_p(t, s) = 1/e_p(s, t) = e_{\ominus p}(s, t)$;

(6) $e_p(t, s)e_p(s, r) = e_p(t, r)$;

(7) $e_p(t, s)e_q(t, s) = e_{p\oplus q}(t, s)$;

(8) $e_p(t, s)/e_q(t, s) = e_{p\ominus q}(t, s)$.

Definition 11. A point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T \in \mathbb{R}^n$ is called an equilibrium point of model (1) if $x(t) = x^*$ is a solution of (1).

Throughout this paper, we always assume that the impulsive jump vector P satisfies

$$P(x^*) = (P_1(x_1^*), P_2(x_2^*), \ldots, P_n(x_n^*))^T = 0. \tag{10}$$

That is, if x^* is an equilibrium point of the following non-impulsive system:

$$\begin{aligned} x_i^\Delta(t) = {} & -a_i x_i(t) + \sum_{j=1}^n b_{ij} f_j(x_j(t)) \\ & + \sum_{j=1}^n c_{ij} f_j(x_j(\zeta_{ij}(t, x_j(t)))) + I_i, \\ & i = 1, 2, \ldots, n, \end{aligned} \tag{11}$$

then it is also the equilibrium point of impulsive system (1).

Definition 12 (see [41]). A real matrix $D = (d_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $d_{ij} \leq 0$ $(i, j = 1, 2, \ldots, n,\ i \neq j)$, and all successive principal minors of D are positive.

Lemma 13 (see [41]). *Let $D = (d_{ij})_{n\times n}$ with $d_{ij} \leq 0$ $(i, j = 1, 2, \ldots, n,\ i \neq j)$; then D is a nonsingular M-matrix if and only if the diagonal elements of D are all positive and there exists a positive vector d such that $Dd > 0$ or $D^T d > 0$.*

3. Existence and Uniqueness of Equilibrium Point

In this section, we will discuss the existence and uniqueness of equilibria of the DCNN with impulses on time scales and give their proofs.

Theorem 14. *Under assumptions (H1) and (H2), if the following condition is satisfied*

(H) $a_i - \sum_{j=1}^n F_j(|b_{ij}| + |c_{ij}|) > 0$, $i = 1, 2, \ldots, n$,

then there is exactly one equilibrium point of model (1).

Remark 15. From Lemma 13, we can easily prove that (H) holds implying that the following condition is true:

(H0) there exists a vector $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T > 0$ such that

$$a_i\xi_i - F_i \sum_{j=1}^n (|b_{ji}| + |c_{ji}|)\,\xi_j > 0, \quad i = 1, 2, \ldots, n. \tag{12}$$

For convenience, we set $A = \operatorname{diag}(a_1, a_2, \ldots, a_n)$, $B = (|b_{ij}|)_{n\times n}$, $C = (|c_{ij}|)_{n\times n}$, and $F = \operatorname{diag}(F_1, F_2, \ldots, F_n)$. Let $\eta = (\underbrace{1, 1, \ldots, 1}_{n})^T$, $D = A - (B + C)F$. From assumption (H), we have

$$\begin{aligned} D\eta &= [A - (B + C)F]\,\eta \\ &= \begin{pmatrix} a_1 - \sum_{j=1}^n F_j(|b_{1j}| + |c_{1j}|) \\ a_2 - \sum_{j=1}^n F_j(|b_{2j}| + |c_{2j}|) \\ \vdots \\ a_n - \sum_{j=1}^n F_j(|b_{nj}| + |c_{nj}|) \end{pmatrix} > 0, \end{aligned} \tag{13}$$

which implies that D is a nonsingular M-matrix. So we know that D^T is a nonsingular M-matrix. Hence, there exists a vector $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T > 0$ such that

$$D^T\xi = \left[A^T - F^T\left(B^T + C^T\right)\right]\xi = \begin{pmatrix} a_1\xi_1 - F_1\sum_{j=1}^{n}\left(\left|b_{j1}\right| + \left|c_{j1}\right|\right)\xi_j \\ a_2\xi_2 - F_2\sum_{j=1}^{n}\left(\left|b_{j2}\right| + \left|c_{j2}\right|\right)\xi_j \\ \vdots \\ a_n\xi_n - F_n\sum_{j=1}^{n}\left(\left|b_{jn}\right| + \left|c_{jn}\right|\right)\xi_j \end{pmatrix} > 0. \tag{14}$$

It follows that (H0) holds.

Now, we prove our theorem.

Proof. Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ be an equilibrium point of system (1); then, we have

$$-a_i x_i^* + \sum_{j=1}^{n} b_{ij} f_j\left(x_j^*\right) + \sum_{j=1}^{n} c_{ij} f_j\left(x_j^*\right) + I_i = 0, \quad i = 1, 2, \ldots, n. \tag{15}$$

We denote $h(x_1, x_2, \ldots, x_n) = (h_1, h_2, \ldots, h_n)^T$, where

$$h_i = a_i x_i - \sum_{j=1}^{n} b_{ij} f_j\left(x_j\right) - \sum_{j=1}^{n} c_{ij} f_j\left(x_j\right) - I_i, \quad i = 1, 2, \ldots, n. \tag{16}$$

Obviously, the equilibrium points of model (1) are solutions of system

$$h_i = 0, \quad i = 1, 2, \ldots, n. \tag{17}$$

Define the following homotopic mapping:

$$H\left(x_1, x_2, \ldots, x_n\right) = \lambda h\left(x_1, x_2, \ldots, x_n\right) + (1 - \lambda)\left(x_1, x_2, \ldots, x_n\right)^T, \tag{18}$$

where $\lambda \in [0, 1]$. Let H_k $(k = 1, 2, \ldots, n)$ denote the kth component of $H(x_1, x_2, \ldots, x_n)$; then, we can get

$$\begin{aligned} \left|H_i\right| &= \left|(1-\lambda)x_i + \lambda a_i x_i - \lambda\sum_{j=1}^{n} b_{ij} f_j\left(x_j\right) - \lambda\sum_{j=1}^{n} c_{ij} f_j\left(x_j\right) - \lambda I_i\right| \\ &\geq (1-\lambda)\left|x_i\right| + \lambda a_i\left|x_i\right| - \lambda\sum_{j=1}^{n}\left|b_{ij}\right|\left|f_j\left(x_j\right)\right| - \lambda\sum_{j=1}^{n}\left|c_{ij}\right|\left|f_j\left(x_j\right)\right| - \lambda\left|I_i\right| \\ &\geq \lambda a_i\left|x_i\right| - \lambda\sum_{j=1}^{n}\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|f_j\left(x_j\right) - f_j(0)\right| \\ &\quad - \lambda\sum_{j=1}^{n}\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|f_j(0)\right| - \lambda\left|I_i\right| \\ &\geq \lambda a_i\left|x_i\right| - \lambda\sum_{j=1}^{n} F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|x_j\right| \\ &\quad - \lambda\sum_{j=1}^{n}\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|f_j(0)\right| - \lambda\left|I_i\right|. \end{aligned} \tag{19}$$

It follows that

$$\begin{aligned} \sum_{i=1}^{n}\left|H_i\right| &\geq \sum_{i=1}^{n}\left[\lambda a_i\left|x_i\right| - \lambda\sum_{j=1}^{n} F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|x_j\right| \right. \\ &\quad \left. - \lambda\sum_{j=1}^{n}\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|f_j(0)\right| - \lambda\left|I_i\right|\right] \\ &\geq \lambda\sum_{i=1}^{n}\left[a_i - \sum_{j=1}^{n} F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\right]\left|x_j\right| \\ &\quad - \lambda\sum_{i=1}^{n}\left[\sum_{j=1}^{n}\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|f_j(0)\right| + \left|I_i\right|\right]. \end{aligned} \tag{20}$$

Let

$$\begin{aligned} \theta &= \min_{1\leq i\leq n}\left\{a_i - \sum_{j=1}^{n} F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\right\}, \\ \gamma &= \max_{1\leq i\leq n}\left\{\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|f_j(0)\right|\right\}, \\ \|x\|_1 &= \sum_{i=1}^{n}\left|x_i\right|. \end{aligned} \tag{21}$$

From the assumption of the theorem, we can easily see that $\theta > 0$. Let

$$\Omega = \left\{x \mid \|x\|_1 \leq \frac{n(\gamma + 1)}{\theta}\right\}. \tag{22}$$

Then, for any $x \in \partial\Omega$, we have

$$\begin{aligned} \sum_{i=1}^{n}\left|H_i\right| &\geq \lambda\theta\sum_{i=1}^{n}\left|x_i\right| - \lambda n\gamma \\ &= \lambda\theta\frac{n(\gamma + 1)}{\theta} - \lambda n\gamma \\ &> 0, \quad \lambda \in (0, 1]. \end{aligned} \tag{23}$$

As $\lambda = 0$, we have

$$H\left(x_1, x_2, \ldots, x_n\right) = \left(x_1, x_2, \ldots, x_n\right)^T \neq 0, \quad x \in \partial\Omega. \tag{24}$$

Hence, all the above conclusions mean that

$$H\left(x_1, x_2, \ldots, x_n\right) \neq 0, \quad \text{for any } x \in \partial\Omega,\ \lambda \in [0,1]. \tag{25}$$

From the homotopy invariance theorem, we obtain

$$\deg(h, \Omega, 0) = \deg(H, \Omega, 0) = \deg(I, \Omega, 0) = 1, \tag{26}$$

where I is the identity operator. By topological degree theory, we can easily know that system (11) has at least one solution in Ω. That means model (1) has at least an equilibrium point.

In order to prove the uniqueness of the equilibrium point, let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ and $y^* = (y_1^*, y_2^*, \ldots, y_n^*)^T$ be two equilibrium points of system (1). So, we have

$$\begin{aligned} &-a_i x_i^* + \sum_{j=1}^n b_{ij} f_j\left(x_j^*\right) + \sum_{j=1}^n c_{ij} f_j\left(x_j^*\right) + I_i = 0, \\ &\qquad i = 1, 2, \ldots, n, \\ &-a_i y_i^* + \sum_{j=1}^n b_{ij} f_j\left(y_j^*\right) + \sum_{j=1}^n c_{ij} f_j\left(y_j^*\right) + I_i = 0, \\ &\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{27}$$

Then,

$$a_i\left(x_i^* - y_i^*\right) = \sum_{j=1}^n \left(b_{ij} + c_{ij}\right)\left[f_j\left(x_j^*\right) - f_j\left(y_j^*\right)\right], \quad i = 1, 2, \ldots, n. \tag{28}$$

By using assumption (H2), we get

$$a_i\left|x_i^* - y_i^*\right| \le \sum_{j=1}^n F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|x_j^* - y_j^*\right|, \quad i = 1, 2, \ldots, n. \tag{29}$$

It follows that

$$a_i \xi_i\left|x_i^* - y_i^*\right| \le \xi_i \sum_{j=1}^n F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|x_j^* - y_j^*\right|, \quad i = 1, 2, \ldots, n. \tag{30}$$

Hence,

$$\begin{aligned} \sum_{i=1}^n a_i \xi_i\left|x_i^* - y_i^*\right| &\le \sum_{i=1}^n \xi_i \sum_{j=1}^n F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|x_j^* - y_j^*\right| \\ &= \sum_{i=1}^n \sum_{j=1}^n \xi_i F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)\left|x_j^* - y_j^*\right| \\ &= \sum_{i=1}^n \sum_{j=1}^n \xi_j F_i\left(\left|b_{ji}\right| + \left|c_{ji}\right|\right)\left|x_i^* - y_i^*\right| \\ &= \sum_{i=1}^n \left[F_i \sum_{j=1}^n \left(\left|b_{ji}\right| + \left|c_{ji}\right|\right)\xi_j\right]\left|x_i^* - y_i^*\right|. \end{aligned} \tag{31}$$

So, we get

$$\sum_{i=1}^n \left(a_i \xi_i - F_i \sum_{j=1}^n \left(\left|b_{ji}\right| + \left|c_{ji}\right|\right)\xi_j\right)\left|x_i^* - y_i^*\right| \le 0. \tag{32}$$

From the assumption (H0), we get $x_i^* = y_i^*, i = 1, 2, \ldots, n$. Therefore, system (1) has one unique equilibrium point. The proof is complete. □

4. Global Exponential Stability of the Equilibrium Point

In this section, we consider the following DCNN system with impulses of the type

$$\begin{aligned} x_i^\Delta(t) &= -a_i x_i(t) + \sum_{j=1}^n b_{ij} f_j\left(x_j(t)\right) \\ &\quad + \sum_{j=1}^n c_{ij} f_j\left(x_j\left(t - \tau_{ij}\right)\right) + I_i, \quad t \in \mathbb{T}_0^+,\ t \neq t_k, \\ \Delta x_i\left(t_k\right) &= x_i\left(t_k^+\right) - x_i\left(t_k^-\right) = P_i\left(x_i\left(t_k\right)\right) \\ &= -\gamma_{ik}\left(x_i\left(t_k\right) - x_i^*\right), \\ &\quad k = 1, 2, \ldots,\ i = 1, 2, \ldots, n, \end{aligned} \tag{33}$$

where a_i, b_{ij}, c_{ij}, I_i, and f_j $(i, j = 1, 2, \ldots, n)$ are defined as those in (1) and τ_{ij} $(i, j = 1, 2, \ldots, n)$ are positive constants which satisfy $t - \tau_{ij} \in \mathbb{T}$ for all $t \in \mathbb{T}$, $i, j = 1, 2, \ldots, n$. Let $\tau = \max_{1 \le i, j \le n}(\tau_{ij})$. Then, the initial conditions associated with (33) are of the form

$$x_i(s) = \phi_i(s), \quad s \in [-\tau, 0] \cap \mathbb{T}, \tag{34}$$

where $\phi_i \in C_{\mathrm{rd}}([-\tau, 0] \cap \mathbb{T}, \mathbb{R})$, $i = 1, 2, \ldots, n$ are rd-continuous.

Definition 16. Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ be an equilibrium point of (33) with initial value $\phi^* = (\varphi_1^*, \varphi_2^*, \ldots, \varphi_n^*)^T$. If there exists a positive constant λ with $-\lambda \in \mathscr{R}^+$ such that for $t_0 \in [-\tau, 0]_{\mathbb{T}}$, there exists $M > 1$ such that for an arbitrary solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of (33) with initial value $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T$ satisfies

$$\left|x(t) - x^*\right|_1 \le M\left\|\phi - \phi^*\right\| e_{-\lambda}\left(t, t_0\right), \quad t \in [-\tau, \infty)_{\mathbb{T}},\ t \ge t_0, \tag{35}$$

where $|x(t) - x^*|_1 = \max_{1 \le i \le n}\{|x_i(t) - x_i^*|\}$, $\|\phi - \phi^*\| = \max_{1 \le i \le n} \sup_{s \in [-\tau, 0]_{\mathbb{T}}}\{|\varphi_i(s) - \varphi_i^*|\}$. Then the equilibrium point x^* is said to be exponentially stable.

Now, we study the global exponential stability of the unique equilibrium to (33) on time scales by using Lyapunov method. We have the following.

Theorem 17. *Let (H2) and (H) hold. Suppose further that*

(H_3) $0 < \gamma_{ik} < 2$, $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots$.

Then, the equilibrium $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ *of system* (33) *is globally exponentially stable.*

Remark 18. We denote the $\mathbb{T}$-interval $[a, b]_{\mathbb{T}}$ as $[a, b]_{\mathbb{T}} := \{t \in \mathbb{T} \mid a \le t \le b\}$.

Now, we prove Theorem 17.

Proof. Let $y_i(t) = x_i(t) - x_i^*$ $(i = 1, 2, \ldots, n)$. Then, we can rewrite (33) as

$$\begin{aligned} y_i^{\Delta}(t) &= -a_i y_i(t) + \sum_{j=1}^{n} b_{ij}\left[f_j\left(y_j(t) + x_j^*\right) - f_j\left(x_j^*\right)\right] \\ &\quad + \sum_{j=1}^{n} c_{ij}\left[f_j\left(y_j\left(t - \tau_{ij}\right) + x_j^*\right) - f_j\left(x_j^*\right)\right], \\ &\qquad t \ne t_k,\ t \in \mathbb{T}_0^+, \\ y_i\left(t_k^+\right) &= y_i\left(t_k\right) - \gamma_{ik} y_i\left(t_k\right), \quad i = 1, 2, \ldots, n,\ k = 1, 2, \ldots. \end{aligned} \tag{36}$$

Multiplying both sides of the first equation of (4.2) by $e_{-a_i}(t, \sigma(s))$ and integrating on $[t_0, t]_{\mathbb{T}}$, where $t_0 \in [-\tau, 0]_{\mathbb{T}}$, we get

$$\begin{aligned} y_i(t) &= y_i(t_0) e_{-a_i}(t, t_0) \\ &\quad + \int_{t_0}^{t} e_{-a_i}(t, \sigma(s)) \\ &\quad \times \left\{ \sum_{j=1}^{n} b_{ij}\left[f_j\left(y_j(s) + x_j^*\right) - f_j\left(x_j^*\right)\right] \right. \\ &\quad + \sum_{j=1}^{n} c_{ij}\left[f_j\left(y_j\left(s - \tau_{ij}\right) + x_j^*\right) \right. \\ &\quad \left.\left. - f_j\left(x_j^*\right)\right]\right\} \Delta s, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{37}$$

For positive constant $\alpha < \min_{1 \le i \le n} a_i$ with $-\alpha \in \mathscr{R}^+$, we have $e_{\ominus\alpha}(t, t_0) > 1$, where $t \in [-\tau, t_0]_{\mathbb{T}}$. Take

$$M > \max_{1 \le i \le n} \left\{ \frac{a_i}{a_i - \sum_{j=1}^{m} F_j\left(\left|b_{ij}\right| + \left|c_{ij}\right|\right)} \right\}. \tag{38}$$

In view of (H), we have $M > 1$. Hence, it is obvious that

$$\left|y(t)\right|_1 \le M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad \forall t \in [-\tau, t_0]_{\mathbb{T}}. \tag{39}$$

We claim that

$$\left|y(t)\right|_1 \le M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad \forall t \in \left(t_0, t_1\right]_{\mathbb{T}}. \tag{40}$$

To prove this claim, we show that for any $p > 1$, the following inequality holds

$$\left|y(t)\right|_1 < p M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad \forall t \in \left(t_0, t_1\right]_{\mathbb{T}}. \tag{41}$$

By way of contradiction, assume that (41) does not hold. Then, there exist $\rho \in (t_0, t_1]_{\mathbb{T}}$ and $i_0 \in \{1, 2, \ldots, n\}$ such that

$$\begin{gathered} \left|y_{i_0}(\rho)\right| \ge p M e_{\ominus\alpha}(\rho, t_0)\left\|\phi - \phi^*\right\|, \\ \left|y_{i_0}(t)\right| < p M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad t \in \left(t_0, \rho\right)_{\mathbb{T}}, \\ \left|y_l(t)\right| < p M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad \text{for } l \ne i_0, \\ t \in \left(t_0, \rho\right]_{\mathbb{T}},\ l = 1, 2, \ldots, n. \end{gathered} \tag{42}$$

Therefore, there must be a constant $\delta_1 \ge 1$ such that

$$\begin{gathered} \left|y_{i_0}(\rho)\right| = \delta_1 p M e_{\ominus\alpha}(\rho, t_0)\left\|\phi - \phi^*\right\|, \\ \left|u_{i_0}(t)\right| < \delta_1 p M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad t \in \left(t_0, \rho\right)_{\mathbb{T}}, \\ \left|u_l(t)\right| < \delta_1 p M e_{\ominus\alpha}(t, t_0)\left\|\phi - \phi^*\right\|, \quad \text{for } l \ne i_0, \\ t \in \left(t_0, \rho\right]_{\mathbb{T}},\ l = 1, 2, \ldots, n. \end{gathered} \tag{43}$$

Note that, in view of (37), we have

$$\begin{aligned} \left|y_{i_0}(\rho)\right| &= \left| y_{i_0}(t_0) e_{-a_{i_0}}(\rho, t_0) \right. \\ &\quad + \int_{t_0}^{\rho} e_{-a_{i_0}}(\rho, \sigma(s)) \\ &\quad \times \left\{ \sum_{j=1}^{n} b_{i_0 j}\left[f_j\left(y_j(s) + x_j^*\right) - f_j\left(x_j^*\right)\right] \right. \\ &\quad + \sum_{j=1}^{n} c_{i_0 j}\left[f_j\left(y_j\left(s - \tau_{ij}\right) + x_j^*\right) \right. \\ &\quad \left.\left.\left. - f_j\left(x_j^*\right)\right]\right\} \Delta s \right| \\ &\le e_{-a_{i_0}}(\rho, t_0)\left\|\phi - \phi^*\right\| \\ &\quad + \int_{t_0}^{\rho} e_{-a_{i_0}}(\rho, \sigma(s)) \\ &\quad \times \left(\sum_{j=1}^{m} \left|b_{i_0 j}\right| F_j \left|y_j(s)\right| \right. \\ &\quad \left. + \sum_{j=1}^{n} \left|c_{i_0 j}\right| F_j \left|y_j\left(s - \tau_{i_0 j}\right)\right| \right) \Delta s \\ &\le e_{-a_{i_0}}(\rho, t_0)\left\|\phi - \phi^*\right\| \\ &\quad + \int_{t_0}^{\rho} e_{-a_{i_0}}(\rho, \sigma(s)) \\ &\quad \times \left(\sum_{j=1}^{m} \left|b_{i_0 j}\right| F_j \delta_1 p M e_{\ominus\alpha}(s, t_0) \right. \\ &\quad \left. + \sum_{j=1}^{n} \left|c_{i_0 j}\right| F_j \delta_1 p M e_{\ominus\alpha}\left(s - \tau_{i_0 j}, t_0\right) \right) \Delta s \end{aligned}$$

$$
\begin{aligned}
&= \|\phi-\phi^*\| e_{\ominus\alpha}(\rho,t_0)\, e_{-a_{i_0}\oplus\alpha}(\rho,t_0) \\
&\quad + \delta_1 pMe_{\ominus\alpha}(\rho,t_0)\|\phi-\phi^*\| \\
&\quad \times \int_{t_0}^{\rho} e_{-a_{i_0}\oplus\alpha}(\rho,\sigma(s)) \\
&\qquad \times \left(\sum_{j=1}^{m}\left|b_{i_0 j}\right| F_j e_{\ominus\alpha}(\rho,\sigma(s)) \right. \\
&\qquad\quad \left. + \sum_{j=1}^{n}\left|c_{i_0 j}\right| F_j e_{\ominus\alpha}\left(\sigma(s), s-\tau_{i_0 j}\right)\right)\Delta s \\
&\le \|\phi-\phi^*\| e_{\ominus\alpha}(\rho,t_0)\, e_{-a_{i_0}+\alpha}(\rho,t_0) \\
&\quad + \delta_1 pMe_{\ominus\alpha}(\rho,t_0)\|\phi-\phi^*\| \\
&\quad \times \int_{t_0}^{\rho} e_{-a_{i_0}}(\rho,\sigma(s)) \\
&\qquad \times \left(\sum_{j=1}^{m}\left|b_{i_0 j}\right| F_j + \sum_{j=1}^{n}\left|c_{i_0 j}\right| F_j\right)\Delta s \\
&\le \|\phi-\phi^*\| e_{\ominus\alpha}(\rho,t_0) \\
&\quad + \delta_1 pMe_{\ominus\alpha}(\rho,t_0)\|\phi-\phi^*\| \frac{1}{-a_{i_0}} \\
&\quad \times \int_{t_0}^{\rho}\left(-a_{i_0}\right) e_{-a_{i_0}}(\rho,\sigma(s))\,\Delta s \\
&\quad \times \left(\sum_{j=1}^{m}\left|b_{i_0 j}\right| F_j + \sum_{j=1}^{n}\left|c_{i_0 j}\right| F_j\right) \\
&= \|\phi-\phi^*\| e_{\ominus\alpha}(\rho,t_0) \\
&\quad - \frac{1}{a_{i_0}}\delta_1 pMe_{\ominus(-\alpha)}(\rho,t_0)\|\phi-\phi^*\| \\
&\quad \times \left(e_{-a_{i_0}}(\rho,t_0)-1\right)\left(\sum_{j=1}^{m}\left|b_{i_0 j}\right| F_j + \sum_{j=1}^{n}\left|c_{i_0 j}\right| F_j\right) \\
&< \delta_1 pM\|\phi-\phi^*\| e_{\ominus\alpha}(\rho,t_0) \\
&\quad \times \left(\frac{1}{\delta_1 pM} + \frac{1}{a_{i_0}}\sum_{j=1}^{m} F_j\left(\left|b_{i_0 j}\right| + \left|c_{i_0 j}\right|\right)\right) \\
&< \delta_1 pM\|\phi-\phi^*\| e_{\ominus\alpha}(\rho,t_0) \\
&\quad \times \left(\frac{1}{M} + \frac{1}{a_{i_0}}\sum_{j=1}^{m} F_j\left(\left|b_{i_0 j}\right| + \left|c_{i_0 j}\right|\right)\right) \\
&< \delta_1 pMe_{\ominus\alpha}(\rho,t_0)\|\phi-\phi^*\|.
\end{aligned}
\tag{44}
$$

Thus, we get a contradiction. Hence, (41) holds. Let $p \to 1$; then (40) holds. From (40), we have that $|y_i(t_1)| \le Me_{\ominus\alpha}(t,t_0)\|\phi-\phi^*\|$, $i = 1,2,\ldots,n$. Since $|y_i(t_1^+)| = |1-\gamma_{i1}||y_i(t_1)| < |y_i(t_1)|$, $i = 1,2,\ldots,n$, it follows that

$$
\begin{gathered}
\left|y_i\left(t_1^+\right)\right| \le Me_{\ominus\alpha}(t,t_0)\|\phi-\phi^*\|, \\
\forall t \in [-\tau,t_1]_{\mathbb{T}},\ i = 1,2,\ldots,n.
\end{gathered}
\tag{45}
$$

Thus, for $t \in [t_1,t_2]_{\mathbb{T}}$, we may repeat the above procedure and obtain

$$
\begin{gathered}
\left|y_i(t)\right| \le Me_{\ominus\alpha}(t,t_0)\|\phi-\phi^*\|, \\
\forall t \in [t_1,t_2]_{\mathbb{T}},\ i = 1,2,\ldots,n.
\end{gathered}
\tag{46}
$$

Similarly, we have

$$
\begin{gathered}
\left|y_i(t)\right| \le Me_{\ominus\alpha}(t,t_0)\|\phi-\phi^*\|, \\
\forall t \in [-\tau,\infty)_{\mathbb{T}},\ i = 1,2,\ldots,n.
\end{gathered}
\tag{47}
$$

Take $-\lambda = \ominus\alpha$; then $\lambda > 0$ and $-\lambda \in \mathscr{R}^+$. Hence, we have that

$$
\begin{gathered}
\left|y(t)\right|_1 \le M\|\phi-\phi^*\| e_{-\lambda}(t,t_0), \\
t \in [-\tau,\infty)_{\mathbb{T}},\ t \ge t_0,
\end{gathered}
\tag{48}
$$

which means that the equilibrium point x^* of (33) is exponentially stable. This completes the proof. □

Remark 19. If the time scale $\mathbb{T} = \mathbb{R}$, then $\mu(t) = 0$ and system (33) becomes the following model:

$$
\begin{aligned}
\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) \\
&\quad + \sum_{j=1}^{n} c_{ij} f_j\left(x_j\left(t-\tau_{ij}\right)\right) + I_i, \quad t \ne t_k, \\
x_i\left(t_k^+\right) &= x_i(t_k) - \gamma_{ik}\left(x_i(t_k) - x_i^*\right), \\
& k = 1,2,\ldots,\ i = 1,2,\ldots,n.
\end{aligned}
\tag{49}
$$

From Theorem 17, we can immediately derive the following result.

Corollary 20. *Suppose that system* (49) *satisfies condition (H2) and (H), and the following assumptions hold:*

(1) $-a_i + \sum_{j=1}^{n} F_j(|b_{ij}| + |c_{ij}|) < 0$;

(2) $0 < \gamma_{ik} < 2$.

Then, the equilibrium of system (49) *is globally exponentially stable.*

Remark 21. In [42], by utilizing the time scale calculus theory, topological degree theory, and Hölder's inequality on time scales, authors studied the existence and the global exponential stability of equilibrium point to a class of impulsive BAM neural networks with distributed delays on time scales. But, results obtained in [42] cannot be applied to (1). Also, for establishing the global exponential stability of equilibrium point to (1), our method used in this paper is totally different from that used in [42].

5. An Example

In this section, an example is given to show the effectiveness of the result obtained in the previous section. Because the condition (4.2) is not dependent on the impulses, we just need to check it with the nonimpulsive system.

Consider the following simple DCNN on time scale $\mathbb{T}$:

$$\begin{aligned} x_i^{\Delta}(t) &= -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) \\ &\quad + \sum_{j=1}^{n} c_{ij} f_j\left(x_j\left(t-\tau_{ij}\right)\right) + I_i, \quad t \in \mathbb{T}_0^{+}, \end{aligned} \tag{50}$$

where $(a_1, a_2)^T = (1,1)^T$, $I_i = 2$, $\tau_{ij} = 2$ $(i, j = 1, 2)$,

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{pmatrix}, \qquad \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 0.02 & 0.03 \\ 0.03 & 0.02 \end{pmatrix}. \tag{51}$$

Taking $f_1(x) = f_2(x) = (1/2)(|x+1| + |x-1|)$, we can easily see that $F_1 = F_2 = 1$.

Let $\mathbb{T} = \bigcup_{k=0}^{\infty} [2k, 2k+1]$, then for $t \in \mathbb{T}$, $t \neq 2k+1$ $(k = 0, 1, 2, \ldots)$, we have $\mu(t) = 0$, and for $t = 2k+1$ $(k = 0, 1, 2, \ldots)$, we have $\mu(t) = 1$.

We have that

$$\begin{aligned} a_1 - \sum_{j=1}^{2} F_j\left(\left|b_{1j}\right| + \left|c_{1j}\right|\right) = 0.93 > 0, \\ a_2 - \sum_{j=1}^{2} F_j\left(\left|b_{2j}\right| + \left|c_{2j}\right|\right) = 0.93 > 0, \end{aligned} \tag{52}$$

which imply that the assumption (H) of Theorem 14 holds. Thus, it follows from Theorems 14 and 17 that system (50) has a unique equilibrium point which is globally exponentially stable (see Figure 1).

Since $\mu(t) \equiv 0$ for $t \in \mathbb{T} = \mathbb{R}$ and $\mu(t) \equiv 1$ for $t \in \mathbb{T} = \mathbb{Z}$, from the discussion above one can easily see that for $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$, (50) always has a unique equilibrium point which is globally exponentially stable. That is, the following continuous-time system

$$\begin{aligned} x_i'(t) &= -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) \\ &\quad + \sum_{j=1}^{n} c_{ij} f_j\left(x_j\left(t-\tau_{ij}\right)\right) + I_i, \quad t \in \mathbb{R}^{+} \end{aligned} \tag{53}$$

and its discrete-time analogue

$$\begin{aligned} \Delta x_i(t) &= -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) \\ &\quad + \sum_{j=1}^{n} c_{ij} f_j\left(x_j\left(t-\tau_{ij}\right)\right) + I_i, \quad t \in \mathbb{Z}^{+} \end{aligned} \tag{54}$$

have the same dynamical properties, where a_i, I_i, τ_{ij}, and f_j are the same as those in (50) (see Figures 2 and 3).

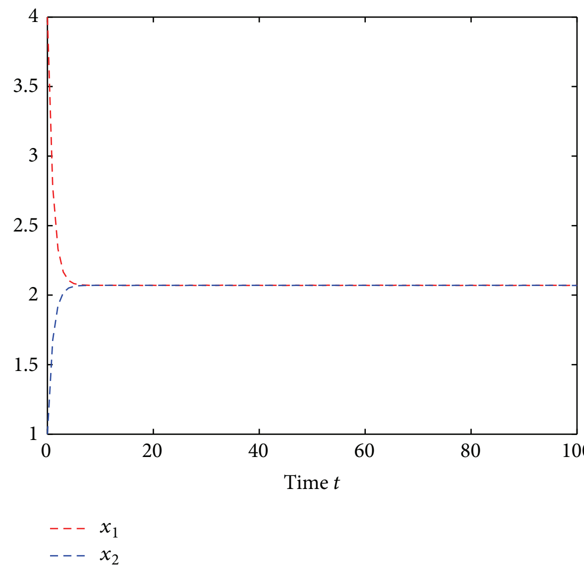

Figure 1: Transient responses of states x_1, x_2 in Example when $\mathbb{T} = \bigcup_{k=1}^{\infty} [2k, 2k+1]$.

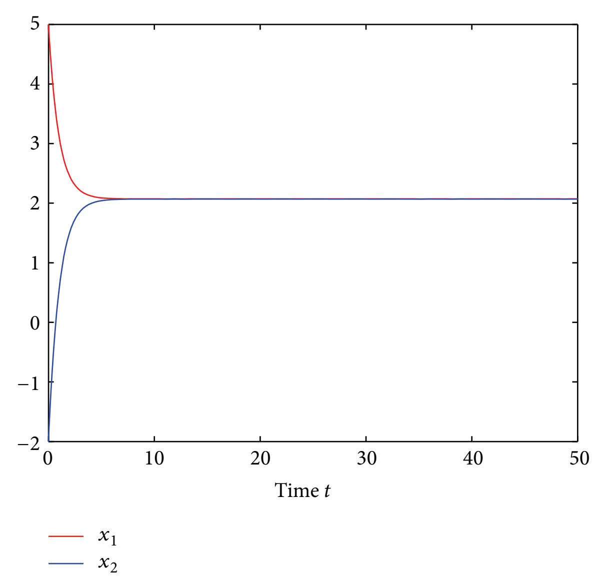

Figure 2: Transient responses of states x_1, x_2 in Example when $\mathbb{T} = \mathbb{R}$.

6. Conclusion

Using the topological degree theory and the time scale calculus theory, some sufficient conditions are obtained to ensure the existence and the global exponential stability of equilibria for DCNNs neural networks with impulses on time scales. This is the first time to apply the time scale calculus theory to unify the study of the stability of the equilibrium for DCNNs with impulses on time scales under the same framework. The results obtained in this paper possess highly important significance and are easily checked in practice. In addition, the method in this paper may be applied to

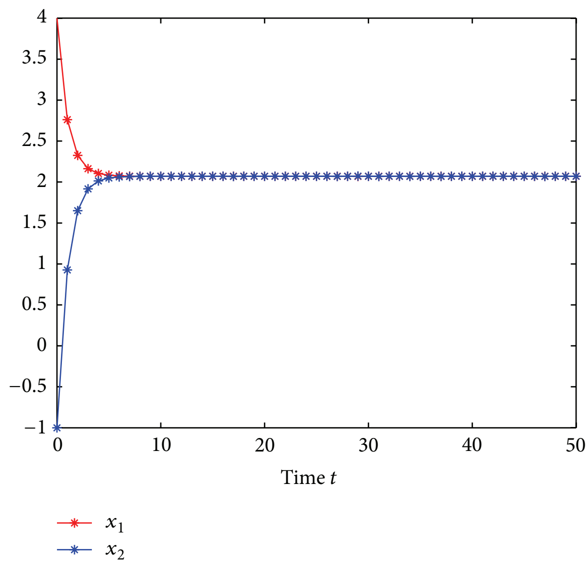

Figure 3: Transient responses of states x_1, x_2 in Example when $\mathbb{T} = \mathbb{Z}$.

some other systems such as the BAM and Cohen-Grossberg systems with impulses and so on.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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