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Research Article

Product Reliability Oriented Design Scheme of Control Chart Based on the Convergent CEV for Censored Characteristics

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Since the censored characteristics are unmonitored effectively in the manufacturing process, the produced product tends to have an unexpected high infant failure rate. Thus, this paper analyzes the associated relationship between product reliability and censored quality characteristics in manufacturing process firstly, and the dynamic and universal control demands for censored characteristics are presented. In view that traditional CEV-based control charts are usually confined to some specific types when dealing with censored characteristics, which greatly restrict the wide application of censored control charts in the high-quality manufacturing process, the convergent CEV technique based on the principle of asymptotic reduction is proposed to be better applied in the analysis of censored characteristics. And novel design procedures and simple but practical performance indicator in the form of the most dangerous alarm distance for censored characteristics of Weibull distribution are put forward. Finally, the validity of the proposed method is verified by a case study of monitoring a censored characteristic relative to lifetime of some aeronautical bearings, and the result proves that the sensitivity of the proposed control chart increased 79.96% compared to traditional CEV chart and is proved to be applicable to monitoring mixed censored characteristics affecting product reliability in manufacturing process.

1. Introduction

To meet the demand of high reliability and long life of the product, targeted analysis, assurance, and optimization for reliability are required to be carried out in the lifecycle of design, manufacture, and usage. But, for a long time, traditional reliability studies only focused on the design and usage stages, and the reliability technologies applicable to the manufacturing process have always been ignored, and they have not drawn the attention it deserves [1–4]. The manufacturing process is the bridge to transfer the reliability from design to usage, and it is the key process to achieve lossless evolution of product reliability. Those uncertainties, mainly reflected as quality variations of key quality characteristics in manufacturing process, are the root causes of high infant failure rate of product [5]. Infant mortality failures are generally the result of manufacturing errors that are not caught in inspection prior to burn-in or placing into service [6].

Fierce global competition has made the requirements of manufacturing quality and the complexity of product improved increasingly. Due to the detection technology and time and cost constraints, censored characteristics that contain incomplete information are widely present in the manufacturing process, which constitutes the overall product quality data together with the normal (uncensored) characteristics [7, 8]. Deviation of both types of characteristics transfers and restricts the produced quality and reliability in the manufacturing process. Censored characteristics are ambiguous and dubious, which has a natural connection with product reliability, and reliability oriented censored control chart studies are a considerable potential research area for statistical process monitoring [9].

Despite the broad application prospect of the censored charts to the product reliability assurance, research on the product reliability oriented control chart is few. Most current studies are focused on the conventional control charts based on the principle of CEV: based on the conditional expectation

method, Steiner et al. [7, 10, 11] adopted traditional template of Shewhart charts and developed conditional expected value (CEV) weight control charts with the constructing steps for single-side control chart of normal censored characteristics presented. Based on the principle of probability limit, Liu [12] discussed the nonnormal statistical model for high-quality assurance and determined the upper and lower limits of quality control charts with skewed distribution; Erto et al. [13] put forward DT (Data Technology) approach to transform the process nonstatistical data into statistical data for joint use in process monitoring, affirming the meaning and status of censored characteristics' monitoring in the manufacturing process to some extent; Montgomery [14] stressed that there were many situations where the proportion of censored observations was between 50% and 95% and these conventional control chart design approaches seemed to be invalid, and the conditional expected value (CEV) weight control charts were a possible approach to resolving this problem. Based on the method of probability plotting, Steiner and Jones [15] proposed a new EWMA control chart for monitoring the survival data mixed with competitive risk factors; Asadzadeh and Aghaie [16] studied the adjusted regression model based CUSUM control charts for the mean reduction in the detection process; Guo and Wang [17] presented an order statistic of uniform distribution for type II censored characteristics and gave one-sided and two-sided control limit with degrees of freedom $(2r - 1)$ of χ^2 distribution.

From the above literature review, we can see that, compared with researches of normal control charts, the number of charts for monitoring censored characteristics is very small. The only few control charts for the study of censored characteristics are usually confined to the specific distributions and chart types, which is inconvenient to be widely conducted in the manufacturing process. Though the concept of reliability monitoring based on the control charts has been brought forth for years by Professor Xie et al. [18], the control chart suited to control demands for the censored characteristics in the manufacturing process is very rare. The few studies are about the statistical characteristics of failure data like times between failures and so forth in production [19, 20], but still a large number of censored key quality characteristics remain unmonitored. The bottleneck of the product reliability oriented censored control chart study is the associated relationship between product reliability and censored quality characteristics in manufacturing process. Therefore, in order to resolve the dilemma, a product reliability oriented design scheme of control chart based on the convergent CEV for censored characteristics is proposed in this paper. Specifically, the distortion problems of censored characteristics by the aid of the mathematical principle of successive approximation in approximation problem [21, 22] are expounded fully. And thus the convergent CEV based on the idea of asymptotic reduction mathematically is proposed to process censored characteristics under Weibull distribution for its unique flexibility and diversity in dealing with censored data [20]. With more restored censored data, statistical control becomes more reasonable and feasible. Naturally, the two outstanding problems in statistical control of censored characteristics, namely, the optimal value of censored data and the customized effective control chart for censored data, are being boosted for further study.

The rest of the paper is organized as follows: Section 2 presents the principle of monitoring censored characteristics in the manufacturing process for assurance of product reliability; Section 3 proposes the asymptotic reduction method for statistical processing of censored data; Section 4 presents design scheme based on the convergent CEV. An example of censored characteristics monitoring in a bearing manufacturing process is given in Section 5. Finally, Section 6 concludes the paper and presents the future perspectives.

2. Principle of Monitoring Censored Characteristics in the Manufacturing Process for Product Reliability Assurance

2.1. Monitoring Demands of Censored Characteristics in Manufacturing. Reliability is quality over time, and reliability becomes a dynamic indicator of product quality on the time axis. Classical reliability theory deems that product reliability refers to the ability to accomplish the required function in specific time and under specific conditions, which lays more emphasis on the field reliability of product. While taking into account the whole life cycle activities of products, we can find that product reliability is determined in the design phase and is formed in the manufacturing process, and, after the transportation and storage to sales network, it is finally handed over to customers to be used and perceived, forming the evolution process from design reliability, manufacturing reliability, and reliability at sale to field reliability [5, 23]. When analyzing the reliability level simply from the sense of field reliability (but let alone and leave out the link of manufacturing reliability), it is easy to abandon the invisible deviation originated in the manufacturing process, even though some dominant deviation factors like the size tolerance and geometric deformation are extracted through the detection of failed quality. From producers' perspective, manufacturing is the primary downstream link to transfer design reliability with a certain product design scheme to users. The stableness of manufacturing quality should play a decisive role in the reliability level for the produced products. Instead, the enforcement of monitoring on the key quality characteristics could directly determine the loss severity of design reliability, which resulted from the quality variations in the manufacturing process. As shown in Figure 1, variations from man, machine, material, method, measurement, and environment (5M1E) in production are the original factors that cause the degradation of design reliability. These variations could be transmitted from manufacturing system to the process quality and ultimately affect the product reliability [24, 25], which should cause the degradation of the produced product reliability. Obviously, the variations of censored and normal KQCs (Key Quality Characteristics) [26] have a negative impact on the reliability of manufactured products.

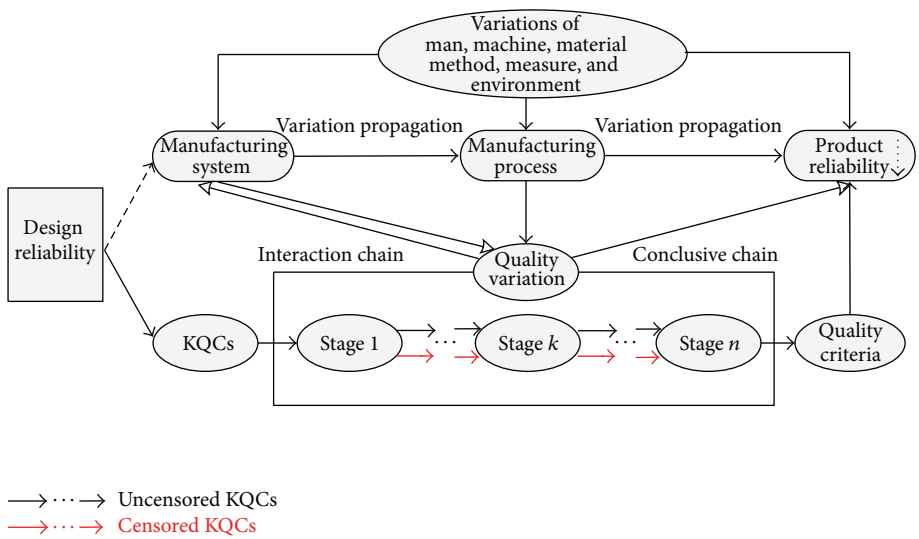

Figure 1: The impact of censored KQCs on the product reliability.

As shown in Figure 1, on the one hand, KQCs are the important quantitative indicators of product quality and the main carriers of process quality information. They are classified into normal KQCs and censored KQCs when transferring deviations among the n-position manufacturing process. Coupling effect of deviations brings about errors of manufacturing system-level, contributing to the decline in the quality for the manufacturing process, and, accompanied with quality deviations accumulated, it yields the degradation of product reliability. Therefore, in order to prevent reliability degradation in the manufacturing, it is vital to conduct overall detection and monitoring of all kinds of KQCs.

In genetic engineering [27], hereditary information is passed through a combination of genes being dominant and recessive. Because of certain external conditions environmentally or internal factors biologically, errors of DNA occur occasionally during the replication process, resulting in human cancer and other serious diseases. In order to prevent cancer and other life threatening diseases, it is of great significance to recognize the principle roles of dominant and recessive genes with gene sequencing techniques.

Prevention of manufactured products from serious quality problems in the course of usage is similar to the identification and control of key genes in genetic engineering. During the manufacturing process, both censored and normal quality characteristics are responsible for the transmission of the design reliability index like the reliability genes in the manufacturing process. As shown in Figure 1, joint deviation transmission of censored KQCs and normal ones, just similar to the recessive genes and dominant genes, has a comprehensive impact on the manufactured reliability (lifetime). Thus, when analyzing and guaranteeing product reliability during the manufacturing process, it is indispensable to cover both types of KQCs. Any truncation or omission of process quality data evolution chain should cause a transmission loss of the intended design reliability.

Furthermore, censoring means loss of information, where the negative effects of the censored KQCs are greater than the normal KQCs in the manufacturing process, and thus the censored KQCs should be the key factors to monitor the product reliability in manufacturing.

2.2. Quantitative Relationship between Product Reliability and Censored Quality Characteristics. In general, products are manufactured with reference to the designed quality indexes via a plurality of processing stations as assumed. And, affected by quality variations of the manufacturing process, the manufactured products are facing reliability degradation in different degrees. Practices of engineering have shown that quality information usually tends to be characterized by adopting some key quality characteristics of products. Therefore, deviations of process quality can be quantified as the fluctuations of those key quality characteristics. Based on theory of stream of variations [28], the author [25] proposed a model named RPPD (the relationship of reliability of products and the process dimensions) to explore the relationship model between dimensional deviations of product KQCs and the related product reliability. Namely, the functional relationship of product reliability R and deviations of some relative key quality characteristics is as follows:

$$R = F(X) + v. \tag{1}$$

Specifically, integrated key quality characteristics X include components of dominant normal characteristics X_n and recessive censored ones X_c. And v stands for the noise factor. Accordingly, design reliability that fits requirements set in the design phase and product reliability of the manufacturing process can be modeled, respectively, as below:

$$\begin{aligned} R_d &= F_{X\sim R}(X_d) + v_d, \\ R_m &= F_{X\sim R}(X_m) + v_m. \end{aligned} \tag{2}$$

Comparing these two formulas, degradation of the product reliability in the manufacturing process $\Delta R(\geq 0)$ can be obtained, denoted by

$$\begin{aligned}\Delta R &= R_d - R_m = F_{x\sim\Delta R}(x) + v_\Delta \\ &= \sum_{i=1}^{r}\frac{\partial F}{\partial (x_i)_N}\Delta(x_i)_N + \sum_{j=r+1}^{n}\frac{\partial F}{\partial (x_i)_C}\Delta(x_i)_C.\end{aligned} \tag{3}$$

Here, $\sum_{i=1}^{r}(\partial F/\partial(x_i)_N)\Delta(x_i)_N$ corresponds to dominant deviation effect of normal KQCs, and $\sum_{j=r+1}^{n}(\partial F/\partial(x_i)_C)\Delta(x_i)_C$ corresponds to the recessive one of censored KQCs.

Deviations of reliability could be decomposed by

$$\Delta R = \Delta R_1 + \Delta R_2. \tag{4}$$

According to formula (3), ΔR_1 of the normal data part and ΔR_2 of the censored one are displayed, respectively:

$$\begin{aligned}\Delta R_1 &= \sum_{i=1}^{r}\frac{\partial F}{\partial (x_i)_N}\Delta(x_i)_N \\ &= \frac{\partial F}{\partial x_1}\Delta x_1 + \frac{\partial F}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial F}{\partial x_r}\Delta x_r,\end{aligned} \tag{5}$$

$$\begin{aligned}\Delta R_2 &= \sum_{j=r+1}^{n}\frac{\partial F}{\partial (x_i)_C}\Delta(x_i)_C \\ &= \frac{\partial F}{\partial x_{n-r}}\Delta x_{n-r} + \cdots + \frac{\partial F}{\partial x_n}\Delta x_n.\end{aligned} \tag{6}$$

In fact, the parallel transmission of deviations for both dominant and recessive effects serves as a bridge linking the required design reliability and the manufactured product reliability, which constitutes the vital factor to be monitored to prevent degradation of product reliability in manufacturing, wherein actual deviations of $\Delta x_{n-r}, \ldots, \Delta x_n$ representing the censored part are all nonnegative, with ΔR_2 being a nonnegative value. That is, deviation in the censored parts authentically plays a certain role in the variation of reliability ΔR. Considering that degrees of uncertainty of the characteristics deviation determine the influence on the reliability strongly or weakly, what can be ensured is that the higher the uncertainty is, the tougher it is to guarantee the reliability. Moreover, with censored characteristics containing less information due to censoring itself, they own a higher extent of uncertainty than that of normal ones. That is to say, deviations of censored KQCs act more prominently on the reliability degradation in actual manufacturing process. However, in practical process monitoring, dominant deviations of normal KQCs often become the monitoring focuses for quality control. Even though censored characteristics exist objectively and are important factors caused by deviations of the process quality, as a rule, they are on the margin of monitoring and are apt to be ignored. Accordingly, this paper focuses on the more influential censored deviations and conducts targeted research emphatically. For one thing, inaccuracy of censored data due to censoring mechanism is fully acknowledged; for another thing, to achieve preventive guaranteeing for reliability degradation in the manufacturing process, building censored deviations oriented control charts from the point of data restoring is imperative and challenging.

3. Asymptotic Reduction Method For Censored Characteristics

3.1. Convergent CEV Based on Asymptotic Reduction. Censored characteristics differ from normal ones in their numerical value, along with censoring contrast to completeness and accuracy. Different censoring mechanism and classification basis determine the sorting of censored characteristics. At one time, censored characteristics can be divided into time censored ones and number censored ones. Also, they can be left-censored, right-censored, interval-censored, and so forth. Practically, engineering applications have ascertained that right-censored characteristics abound excessively and should be extensively focused [7, 8]. Thus, this paper lays more emphasis on right-censored characteristics and conducts related technique research and application.

Set $X = (x_1, x_2, \ldots, x_r, C_{n-r}, \ldots, C_n)$ as a sample of right-censored observations, with sample size of n and censoring level at C. Introduce the variable γ of 0-1 and stipulate that $\gamma_i = 0$ when the value of KQCs obeys $X_i \leq C$, corresponding to the real observation of X_i; $\gamma_i = 1$ when $X_i > C$, with X_i being a censored observation, and in the meantime record its numerical value as the censoring level C. Therefore, the mixed couple (X_i, γ_i) can be used to describe the right-censored characteristics with the number of censored samples being $m = \sum_{i=1}^{n}\gamma_i$ in total and noncensored ones being $r = \sum_{i=1}^{n}(1-\gamma_i)$.

Mathematical idea of asymptotic reduction successively tries to determine the approximate scope of solution with given conditions, and the solution space is gradually reduced till the occurrence of the optimal solution with the impossible cases ruled out through the continuous improvement method. Based on the principle of asymptotic reduction, the convergent CEV for a simulated restored observation handles the conditional expected value $v_c = E_c(X \mid X \geq C)$ with a basic limit processing by the censored expected value $v_{cc} = \lim_{\theta\to\hat{\theta}} E_{cc}\{(X,\theta)|(X,\theta) \geq C)\}$, which guarantees a more accurate sample value. Since the initial censored observation no longer roughly takes the threshold C but refers to a higher estimator as v_{cc}, a more realistic state for censored samples is restored and recurred. And it is obvious that the average value of the censored samples approximates more to the practical process average value, which greatly reduces the estimation deviation due to a skewed distribution caused by the data censoring.

With the orientation of function limit, procedures of convergent CEV based on the combination of maximum likelihood estimation and the censored expected value for censored characteristics are presented as below.

Step 1. Establish the density distribution function of right-censored characteristics $f(x_i)^{1-\gamma_i}\Pr(X_i > C)^{\gamma_i}$, namely, $f(x_i)^{1-\gamma_i}S(x)^{\gamma_i}$, and then, considering the connotation of the

maximum likelihood function, the logarithm maximum likelihood function subjected to the right censoring mechanism is deduced as follows.

First, the likelihood function $L(\theta)$ of the right-censored samples is expressed by

$$L(\theta) = \prod_{i=1}^{N} f(x_i)^{1-\gamma_i} S(x)^{\gamma_i}. \tag{7}$$

Then, after taking the logarithm for formula (7), the logarithm maximum likelihood function is thus obtained by

$$\log L(\theta) = (N-r) S(C;\theta) + \sum_{i\in D} f(x_i;\theta). \tag{8}$$

Here, θ represents one or more distribution parameters of the quality characteristics; x_i is the detected sample value of the quality characteristic; C is the censoring level; f is the failure rate function; S is the survival function; r is the number of noncensored characteristics; D is the set of noncensored characteristics.

Step 2. Clarify the relevance between the censored expected value v_c and the potential distribution of the censored characteristics and it can be manifested by

$$v_c = E_c\{(X,\theta) \mid (X,\theta) \geq C\} = \frac{\int_C^{\infty} x f(x;\theta)\,dx}{\int_C^{\infty} f(x;\theta)\,dx} = h(\theta). \tag{9}$$

Step 3. Restore and update the sample information with the definition for (X_i, γ_i) by

$$v_i = \begin{cases} x & \text{if } x \leq C \\ v_c & \text{if } x > C. \end{cases} \tag{10}$$

Here, the censored sample with threshold C is replaced with the censored expected value and the noncensored sample gets the original value.

Step 4. Determine the estimator for θ based on the restored sample information and the maximum likelihood function by

$$\widehat{\theta} = g(v_i). \tag{11}$$

More specifically, when it comes to specific calculation, it is required to assign θ an appropriate initial value and then calculate $\widehat{\theta}$ according to formulas (9)–(11). Afterwards, the renewable parameter values are obtained and substituted back to formulas (9)–(11) with several iterations until the estimated parameter values converge to the desired accuracy. In turn, the censored expected value under convergent CEV v_{cc} will be as

$$v_{cc} = \lim_{\theta\to\widehat{\theta}} E_{cc}\{(X,\theta) \mid (X,\theta) \geq C\}. \tag{12}$$

Figure 2 shows the convergent curves of the estimated parameters whose potential distribution is $N(5, 20^2)$ with

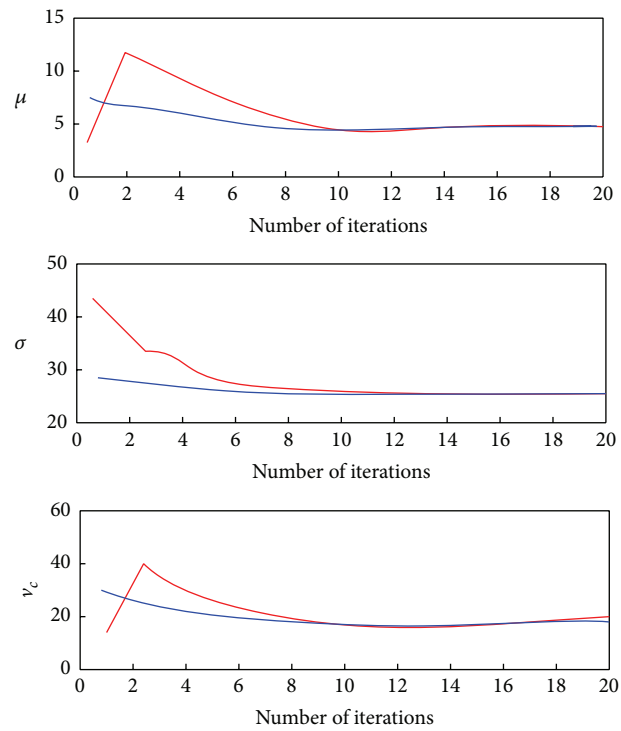

Figure 2: Convergent curves of the estimated parameters.

initial values of $\mu = 15, \sigma = 1$ and $\mu = 2, \sigma = 25$ contrastively. Here, the former corresponds to the red curve and the latter to the blue curve.

Through 100 times' simulations, it is shown that whatever the initial values for the potential parameters may be, μ, σ, and v_c become convergent around 10 times after the iterations. Although the estimated convergence values may not be equal to the real values of the potential distribution, it is assured that the deviation between them is successively reduced along with a decreasing censoring proportion.

By the above method, the initial group of mixed right-censored characteristics $X = (x_1, x_2, \ldots, x_r, C_{n-r}, \ldots, C_n)$ is replaced with $(x_1, x_2, \ldots, x_r, v_{cc}, v_{cc}, \ldots, v_{cc})$, which fulfills the restoring process of the censored characteristics.

3.2. Sensitivity Analysis. Generally, the test statistic of samples in traditional process control is measured as the arithmetic mean of the observed quality data in each group as below:

$$T = \sum \frac{X_i}{n}. \tag{13}$$

When there is censoring of quality characteristics denoted by (X_i, γ_i), use the censoring point C as the current measured value:

$$T = \frac{\sum_{i\in D} X_i + mC}{n}. \tag{14}$$

Here, X_i is the noncensored quality data and D, m, n have the same meaning as before.

Meanwhile, under convergent CEV method, the censored expected value v_{cc} is used to substitute censored observations that exceed the threshold C. Thus, the test statistic has the expression as

$$T = \frac{\sum_{i\in D} X_i + mv_{cc}}{n}. \tag{15}$$

Along with the process mean shifting to the left, quality data of the samples offset to the left, too. At the same time, the number of censored data reduces from m to m', with the offset of the test statistic as Δ_1 by traditional process control and Δ_2 by the convergent CEV as below:

$$\Delta_1 = T - T' = \frac{\sum_{i\in D} X_i + mC}{n} - \frac{\sum_{i\in D'} X_i + m'C}{n} \tag{16}$$

$$\Delta_2 = T - T' = \frac{\sum_{i\in D} X_i + mv_{cc}}{n} - \frac{\sum_{i\in D'} X_i + m'v_{cc}}{n}. \tag{17}$$

Formula (17) is subtracted from formula (16):

$$\Delta_2 - \Delta_1 = (v_c - C)\cdot\frac{m-m'}{n} > 0. \tag{18}$$

In formula (18), with $v_{cc} > C$ and $m > m'$, it can be judged that formula (18) is consistently greater than 0, meaning that test statistic under convergent CEV method observed a bigger offset than traditional test statistic. Theoretically, application of the convergent CEV method for censored characteristics could make the test statistic more sensitive to the deviation of process parameters. From the perspective of the data itself, the convergence CEV based on the asymptotic reduction updates the censored data at the extreme, which reflects the real state of the obtained observations more accurately and further provides an effective input for monitoring process parameters.

4. Design Scheme for Censored Control Chart under Weibull Distribution

4.1. Calculation of the Censored Expected Value v_{cc}. Weibull distribution involves forms of two parameters and three parameters, of which the two-parameter one is mainly used for describing lifetimes of rolling bearings and material fatigue suffering high stress and the three-parameter one is mainly used for characterizing lifetimes of materials or some special parts under low stress. Here, this paper primarily focuses on two-parameter Weibull distribution and conducts relevant study of censored control chart.

It is well known that the density function and the survival function of two-parameter Weibull distribution are given as, respectively,

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha^\beta}x^{\beta-1}\exp\left\{-\left[\frac{x}{\alpha}\right]^\beta\right\}, \tag{19}$$

$$S(x;\alpha,\beta) = \exp\left\{-\left[\frac{x}{\alpha}\right]^\beta\right\}. \tag{20}$$

Given the above formulas, the mean μ and the variance σ^2 of the random variable x are

$$\begin{aligned}\mu &= \alpha\Gamma\left(1+\frac{1}{\beta}\right),\\ \sigma^2 &= \alpha^2\left\{\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right\}.\end{aligned} \tag{21}$$

And α is the scale parameter; β is the shape parameter; $\Gamma(x)$ is the Gamma function:

$$\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx, \quad a > 0. \tag{22}$$

For key quality characteristics of right-censored obeying two-parameter Weibull distribution, (X_i, γ_i) has the density function as below:

$$\begin{aligned}&f(x_i;\alpha,\beta)^{1-\gamma_i}S(x_i;\alpha,\beta)^{\gamma_i}\\ &\quad= \left\{\frac{\beta}{\alpha^\beta}x^{\beta-1}\exp\left\{-\left[\frac{x}{\alpha}\right]^\beta\right\}\right\}^{\wedge(1-\gamma_i)}\\ &\quad\cdot\left\{\exp\left\{-\left[\frac{x}{\alpha}\right]^\beta\right\}\right\}^{\wedge\gamma_i}.\end{aligned} \tag{23}$$

Correspondingly, the logarithmic maximum likelihood function of (X_i, γ_i) is

$$\begin{aligned}\log L_w(\alpha,\beta) &= (N-r)\log S(C;\alpha,\beta)\\ &+ \sum_{i\in D}\log f(x_i;\alpha,\beta) = (N-r)\left[-\left(\frac{C}{\alpha}\right)^\beta\right]\\ &+ \sum_{i\in D}\left\{\log(\beta) - \log(\alpha)\right.\\ &\left.+ (\beta-1)\left[\log(v_i) - \log(\alpha)\right] - \left(\frac{v_i}{\alpha}\right)^\beta\right\}.\end{aligned} \tag{24}$$

With reference to formula (9), bring the related functions of Weibull distribution into calculation and obtain the result of v_c:

$$\begin{aligned}v_c = E_c(w) \mid E_c(w\ge C) &= \frac{\int_C^\infty wf(w;\alpha,\beta)\,dw}{\int_C^\infty f(w;\alpha,\beta)\,dw}\\ &= \frac{(\beta/\alpha^\beta)\int_C^\infty w^\beta\exp\left\{-[w/\alpha]^\beta\right\}dw}{(\beta/\alpha^\beta)\int_C^\infty w^{\beta-1}\exp\left\{-[w/\alpha]^\beta\right\}dw}.\end{aligned} \tag{25}$$

And thus the censored expected value under convergent CEV becomes

$$\begin{aligned}v_{cc} &= \lim_{\theta\to\hat{\theta}} E_{cc}\{(w,\theta) \mid (w,\theta)\ge C\}\\ &= \frac{\alpha\Gamma * \left[(C/\alpha)^\beta, 1+1/\beta\right]}{\exp\left[-[C/\alpha]^\beta\right]}.\end{aligned} \tag{26}$$

Here, $\Gamma * (x, a) = \int_{y=0}^{x} y^{a-1} \exp(-y) dy$ is the incomplete Gamma function. And the restored censored characteristics have the values as

$$v_i = \begin{cases} x & \text{if } x \le C \text{ (not censored)} \\ \dfrac{\widehat{\alpha}\Gamma * \left[(C/\widehat{\alpha})\,\widehat{\beta}, 1+\widehat{\beta}\right]}{\exp\left[-(C/\widehat{\alpha})\,\widehat{\beta}\right]} & \text{if } x > C \text{ (censored)}. \end{cases} \tag{27}$$

4.2. Design Procedures for Censored Control Chart. Control chart performs as a statistical tool to determine whether the process is in control or not via the analysis of some specific key quality characteristics. And the key is to construct the corresponding control chart which lies in the selected test statistic and the proper control limits based on the features of the specific key quality characteristics.

Different from traditional variable characteristic and attribute characteristic, censored characteristic is featured by its mixture of the two mentioned characteristics. Namely, not only the metrological value but also the number of unqualified sample is recorded simultaneously for characteristic that is censored. Thus, it turns out that whichever types of control charts simply for variable data or for attribute data are helpless for the mixed censored data since the information inside cannot be fully used, which greatly reduces the monitoring effect with a soaring quality loss. Faced with this dilemma, it becomes so urgent to build censored control charts to make full use of the limited but mixed censored quality characteristics.

For censored characteristics, the censoring proportion influences greatly the form of control chart as $\overline{X}$ chart or S chart. Steiner and MacKay [7] have proved that, for censoring proportion that is rather high (bigger than 0.5), censored $\overline{X}$ chart is also applicable to detect the change of the process standard deviation. Assuming an increased process variance, censored characteristics that located left of the process mean shift to lower values while censored characteristics at the right of the process mean give way to the unified censored expected value v_{cc} which holds back the increasing trend of the characteristics values, exhibiting a decreased process mean value. Since, for censored characteristics of high censoring proportion, an increase in process variance has the similar performance with the decrease in process mean value, censored $\overline{X}$ control chart is recommended and discussed here.

Affected by the censoring mechanism, test statistic for censored control chart is different from that of simply taking the arithmetic average of the quality characteristics but probably has a more complicated form which contains censored data or their deformation. And then, it is difficult to derive an exact formula to determine the control limits in accordance with the traditional principle of 3σ [11]. Given all this, this paper uses Monte Carlo simulation to obtain the control limits for different sample size with different censoring ratios. The specific implementation steps are as follows.

Step 1. Choose the estimated value $\widehat{\theta} = g(v_i)$ by formula (11) as an input and generate N random variables obeying the Weibull distribution. Subsequently, based on the practical censoring level C, carry out the replacement for data greater than C just by C.

Step 2. Sort the random data generated from Step 1 into groups sized by n and calculate the test statistic for each group by formula (28) as follows, namely, the monitoring parameter, which constitutes the statistic matrix $TS = [T_1, T_2, \ldots, T_{N/n}]^T$:

$$T_j = \frac{\sum_{i \in D} X_i + mC}{n}, \quad \left(j = 1, 2, \ldots, \frac{N}{n}\right). \tag{28}$$

Step 3. Ascend $TS = [T_1, T_2, \ldots, T_{N/n}]^T$ and obtain the order statistic matrix $TS' = [T_1', T_2', \ldots, T_{N/n}']^T$, which satisfies the condition $T_1' \le T_2' \le \cdots \le T_{N/n}'$. Define the location scalar $p = [p_1, p_2] = [(N/n) * R_f, (N/n) * (1 - R_f)]$, and with the expected false alarm rate R_f, the estimated control limits are gotten as

$$\begin{aligned} \text{Lower Control Limit: LCL} &= \frac{1}{2}\left(T_{p_1}' + T_{p_1+1}'\right), \\ \text{Upper Control Limit: UCL} &= \frac{1}{2}\left(T_{p_2}' + T_{p_2+1}'\right). \end{aligned} \tag{29}$$

Here, the two control limits by formula (29) are estimated mainly based on the principle of the type I error signified by the expected false alarm rate R_f with the ascending ordered statistic matrix originated from the initial samples. Concretely, the lower control limit depicts the error risk boundary of the small observations, while the upper control limit shows that of the large observations naturally.

Step 4. Carry out Steps 1–3 repeatedly for the arithmetic mean of the control limits derived from each calculation and then select the arithmetic mean as the ultimate control limits. Together with the updated censored samples depicted in the control chart, construction of the censored control chart is thus completed.

According to the number of control limits contained in the control chart, control charts can be divided into one-sided control charts and two-sided ones. Obviously, for censored control charts, they possess the unilateral structure owing to the censoring mechanisms naturally. When it happens to the right-censoring, quality characteristics should be the type of the Larger the Better. That is, upward shift of the process mean will bring about augment of the censoring proportion, resulting in less information contained in the sample data and more difficulty in judging whether the process mean shifts or not. On the other hand, engineering applications often focus on the value decline of quality characteristics being the Larger the Better without having to worry about exceeding the target values. There is no need to worry about values of the characteristics being higher than that of the target. So, it is reasonable and practical to establish a unilateral lower limit $\text{LCL} = (1/2)(T_{p_1}' + T_{p_1+1}')$ for right-censored characteristics in this paper.

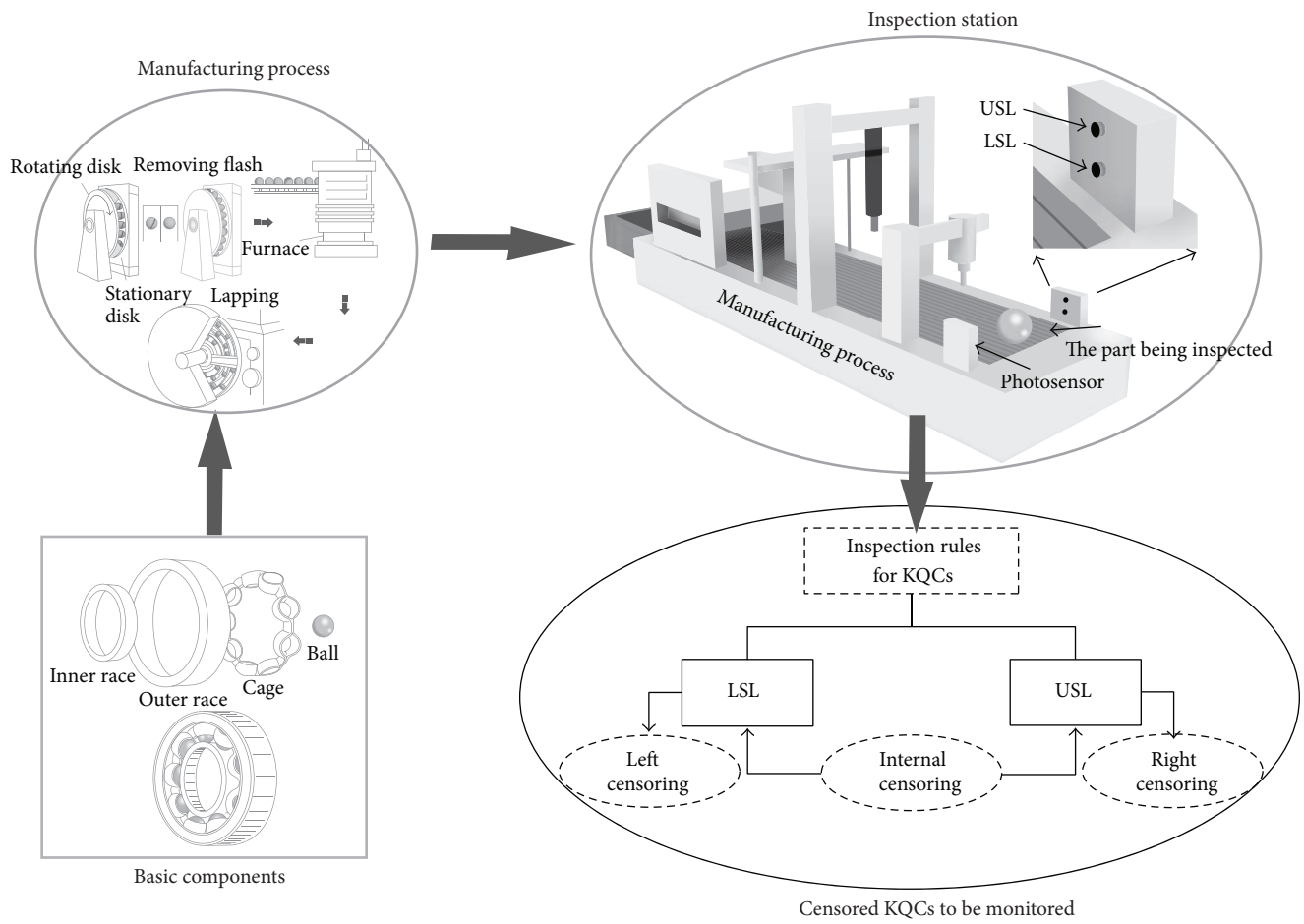

Figure 3: A panoramic view of quality monitoring in manufacturing for bearings.

4.3. Performance Evaluation of the Control Chart. The average run length (ARL) is currently the most commonly used means to evaluate the performance of control charts. It begins with the results and conducts the measurement for the taken samples. When the process is close to the abnormal state but with no distinct signals, ARL becomes no longer appropriate. Thus, this paper intends to adopt a new index R_d different from ARL to carry out the performance evaluation for censored control chart from the preventive control perspective, as shown in the following formula:

$$R_d = \min |\text{Test Statistic} - \text{Control Limit}|, \tag{30}$$

where R_d refers to the most dangerous alarm distance of the nearest sample point to the control limit in a preliminary controlled state, which ascertains the most dangerous alarm sample point. That is to say, even if the process is initially determined as stable, the most dangerous alarm sample point can be diagnosed and analyzed and then make preventative improvements to ensure the reliability of the manufacturing process.

5. Case Study

5.1. Background. In recent years, with the wide applications of bearings in the field of aviation, aerospace, automotive, shipbuilding, and so forth, demand for high reliability and long life bearings is further enhanced, along with more and more attention paid on the quality of the manufacturing process. As bearings make up the most key part of the aircrafts or the automobiles' engine, their manufacturing quality has always been a bottleneck in the engine manufacturing process. In particular, monitoring the variations of the KQCs becomes the most important task to ensure the quality and reliability of the engines. Quality variations of the bearings' surface roughness, geometrical shape, machining dimension, position accuracy, and so forth have a great influence on the assembly accuracy and the reliability of the whole engine. Figure 3 shows the basic four components of bearings including the inner race, outer race, cage, and ball. At the same time, for balls being manufactured, they will go through the inspection station to be detected under specific inspection rules. That is to say, components that are manufactured by previous machines are to enter the inspection station with rule of either LSL (Lower Specification Limit) or USL (Upper Specification Limit), otherwise jointly the rule of both LSL and USL. It is apparent that the setting of inspection rules is just like a concrete kind of censoring we proclaimed previously from the point of data loss. And thus, for KQCs that we studied emphatically, the LSL rule equivalent to

left censoring corresponds to left-censored KQCs; the USL rule equivalent to right censoring corresponds to right-censored KQCs; and both of the LSL rule and the USL rule just equivalent to interval censoring correspond to interval censored KQCs.

In practical multistation manufacturing process, due to the combined effects of various sensitive factors, the reliability of the produced bearings is always less than the design requirements. Variations of man, machine, material, method, measurement, and environment (5W1E) are transferred by each workstation cumulatively, which should affect the quality characteristics that represent the bearings' performance. Since the variations are reflected as the deviation of the process quality, with the cumulative effect of the deviations, ultimately, the bearing reliability formed in the manufacturing process cannot meet the design requirements. Therefore, there is a practical demand to reduce the reliability degradation by carrying out effective control of the bearing quality in the manufacturing process.

Based on the identification result of KQCs of a bearing producer in China, the quality characteristics whose variations are dominant or recessive are being classified into two types of normal KQCs and censored KQCs. As shown in Figure 3, censored KQCs are the key factors of quality monitoring in manufacturing for bearings. When having analyzed the recessive negative impact from the KQCs' deviations, establish the statistical analysis for censored characteristics by means of the convergent CEV processing method proposed in this paper. Afterwards, convergent CEV control chart is constructed to analyze the test data that are censored and identify the key factors whose variations mainly influence the bearings quality, which provides effective assurance to producing high reliability and long life bearing components.

5.2. Numerical Example. Select the test data for some key censored characteristics related to the bearing fatigue life in the field. Table 1 (which lists only the first 20 groups) gives 100 groups of Weibull distribution data sized by 5 in each group (a total of 500 data items), and the potential distribution conforms to both the shape parameter and the size parameter as 1, wherein the censoring proportion p_c is 75% and the censoring level is $C = 0.279$.

Using methods of maximum likelihood estimation and the convergent censored expected value shown in formulas (9)–(11), the parameters for Weibull distribution are estimated by Matlab programming with iterative operation. For the initial 500 sample data items, initialize $\alpha = 0.5$, $\beta = 0.5$ to be involved in the iterative operation. And then, parameters for Weibull distribution are estimated as $\widehat{\alpha} = 0.9916$, $\widehat{\beta} = 0.7135$, also with the censored expected value acquired as $v_{cc} = 3.1483$. Afterwards, according to formula (10), censored sample data can be updated finally.

So, according to the output parameter of the iterative process $\widehat{\alpha} = 0.9916$ and based on the designed method for determining the lower limit, the Matlab programming can give the summary matrix for the lower limits $T\_\mathrm{LCL} = [\mathrm{LCL}_1, \mathrm{LCL}_2, \ldots, \mathrm{LCL}_t]$ when setting the number of iterations as $t = 10^5$. And the final lower control limit is computed by $\mathrm{LCL} = \sum_{i=1}^{t} \mathrm{LCL}_i/t = 0.6345$, which contrasts to the lower control limit determined by the initial unrestored samples as $\mathrm{LCL}' = 0.3771$. When it is applied to monitor the actual process, the lower control limit under convergent CEV and the initial one are drawn on the control chart at the same time, as shown in Figure 4.

As shown in Figure 4, it is seen that all the points are above the control limit, and the process is preliminarily perceived as stable and controlled. That is to say, the convergent

Table 1: Initial sample data (the first 20 groups).

Number	Sample value				
1	0.2790	0.2448	0.2790	0.2790	0.2790
2	0.2790	0.2790	0.2790	0.2790	0.2790
3	0.0030	0.2790	0.1312	0.0686	0.1992
4	0.2790	0.1705	0.2790	0.2095	0.2790
5	0.2790	0.0809	0.2790	0.2790	0.2790
6	0.2790	0.2601	0.0372	0.2787	0.2790
7	0.2790	0.2790	0.2790	0.2790	0.2790
8	0.2790	0.2790	0.0274	0.0286	0.0345
9	0.2790	0.2790	0.2790	0.0121	0.2790
10	0.2790	0.2790	0.2790	0.1460	0.2790
11	0.2790	0.2790	0.2790	0.2790	0.2790
12	0.2790	0.2790	0.2790	0.2790	0.2790
13	0.2790	0.2790	0.2790	0.2790	0.2790
14	0.1725	0.2790	0.2790	0.2428	0.2790
15	0.2790	0.2790	0.2093	0.1246	0.2790
16	0.2790	0.2790	0.2790	0.0902	0.2790
17	0.2790	0.2790	0.2790	0.2790	0.2790
18	0.0436	0.1868	0.0255	0.2790	0.2790
19	0.2790	0.2654	0.2790	0.2790	0.1788
20	0.0784	0.0678	0.2790	0.1057	0.2790

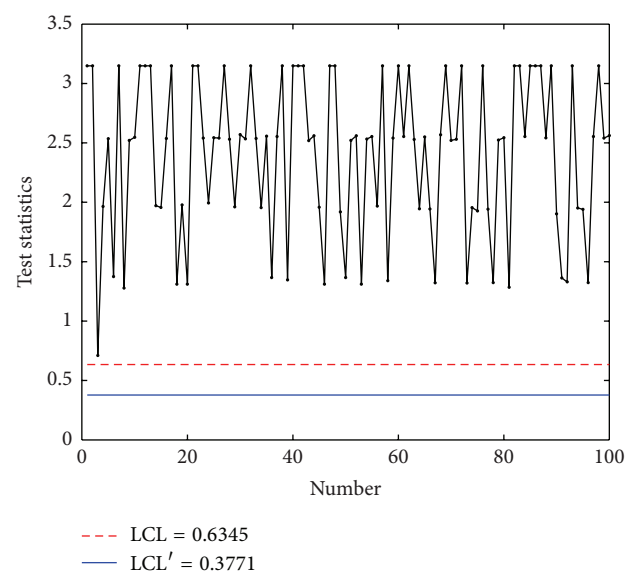

Figure 4: The convergent control chart for censored characteristics related to the bearings lifetime.

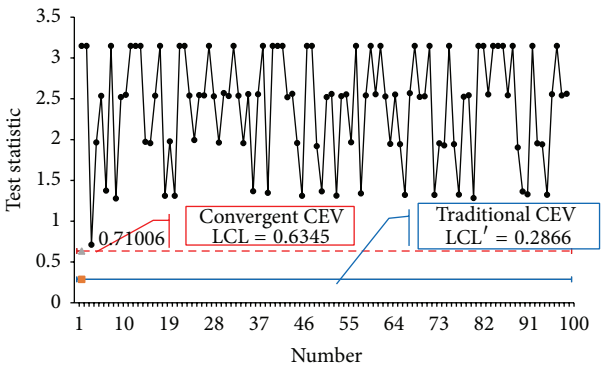

Figure 5: Performance analysis.

control chart for censored characteristics can be used to monitor the process.

5.3. Result Analysis. As traditional CEV with Weibull distribution is built on the Weibull scale, process parameters are thus determined as $\widehat{\alpha}' = 0.9989$, $\widehat{\beta}' = 0.9935$. When it comes to constructing the control chart, standard form of Weibull distribution $f(w; 1, \beta)$ is highly dependent on the shape parameter β, making separation of parameters impossible, and then the stimulated control limit for CEV $\overline{X}$ differs along parameter β without a unified table for reference. However, standard form of exponential distribution shows its independence on scale parameter α and shape parameter β of Weibull distribution, which enables a fixed limit curve under standard exponential distribution. Furthermore, the lower control limit of traditional CEV with Weibull distribution can be ascertained as $\mathrm{LCL} = (\alpha \mathrm{lcl}^X)^{1/\beta}$ [10] by a simple formula change. And the lower control limit of traditional CEV becomes $\mathrm{LCL}' = (0.9989 * 0.289)\hat{}(1/0.9935) = 0.2866$. When it is specified to the actual monitoring process, the convergent lower control limit and the traditional one are presented on the control chart at the same time, as is shown in Figure 5.

From Figure 5, it is seen that all the points are above the control limit, and the process is preliminarily perceived as stable and controlled. That is to say, the proposed control chart for censored characteristics can be used to monitor the process. Furthermore, it can be comparatively concluded that control limit of the convergent CEV (the dashed line) corresponds to $R_{d-d} = \text{Test Statistic}_{(3)} - \mathrm{LCL}$—namely, $R_{d-s} = 0.71006 - 0.6345 = 0.07556$—while the traditional CEV control limit (the solid line) is $R_{d-d} = 0.3771$. Obviously, control limit under the convergent CEV acts more timely to locate the most dangerous alarm point, showing greater sensitivity to detect the abnormality in the process by that of 79.96% $((R_{d-d} - R_{d-s})/R_{d-d} = (0.3771 - 0.07556)/0.3771)$.

The final result shows that the proposed control chart could provide more precise control limit to ensure that the dynamic change process of the studied censored characteristics can be effectively detected. And it provides the chance for capturing the monitoring focus via the designed precautionary reference point in the first place.

6. Conclusions

Considering the increasing monitoring demands of product reliability and censored characteristics in manufacturing process, the convergent CEV method based on the principle of asymptotic reduction is put forward to preprocess the censored characteristics. On this basis, parameter estimation for censored characteristics under Weibull distribution is given, and the construction steps for the censored control chart with the convergent CEV are presented. Ultimately, the effectiveness of the proposed method is validated by the case of monitoring censored characteristics in the manufacturing process for bearings. The result shows that the convergent CEV method is a highly versatile method in processing censored data. And the proposed censored control chart can give a general monitoring parameter, distinct building steps, and a unique simple but practical performance analysis index, which contributes to a control chart with broad applicability prospect. Further researches on product reliability oriented control charts for different distributions of censored characteristics using the principle of convergent CEV method are planned.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] P. C. Teoh and K. Case, "An evaluation of failure modes and effects analysis generation method for conceptual design," *International Journal of Computer Integrated Manufacturing*, vol. 18, no. 4, pp. 279–293, 2005.

[2] W. Wang, P. Scarf, S. Wu, and E. Zio, "Mathematical applications to reliability and maintenance problems in engineering systems," *Mathematical Problems in Engineering*, vol. 2015, Article ID 629497, 2 pages, 2015.

[3] G. Levitin and L. Xing, "Reliability and performance of multi-state systems with propagated failures having selective effect," *Reliability Engineering & System Safety*, vol. 95, no. 6, pp. 655–661, 2010.

[4] V. Volovoi, "Modeling of system reliability Petri nets with aging tokens," *Reliability Engineering & System Safety*, vol. 84, no. 2, pp. 149–161, 2004.

[5] Y. He, L. Wang, Z. He, and M. Xie, "A fuzzy TOPSIS and rough set based approach for mechanism analysis of product infant failure," *Engineering Applications of Artificial Intelligence*, 2015.

[6] M. A. Durivage, *Practical Engineering, Process, and Reliability Statistics*, American Society for Quality, Quality Press, Milwaukee, Wis, USA, 2015.

[7] S. H. Steiner and R. J. Mackay, "Monitoring processes with highly censored data," *Journal of Quality Technology*, vol. 32, no. 3, pp. 199–208, 2000.

[8] H. Wu and Y. Luan, "An efficient estimation of the mean residual life function with length-biased right-censored data," *Mathematical Problems in Engineering*, vol. 2014, Article ID 937397, 5 pages, 2014.

[9] W. H. Woodall and D. C. Montgomery, "Some current directions in the theory and application of statistical process monitoring," *Journal of Quality Technology*, vol. 46, no. 1, pp. 78–94, 2014.

[10] S. H. Steiner and R. J. MacKay, "Detecting changes in the mean from censored lifetime data," in *Frontiers in Statistical Quality Control*, vol. 6 of *Frontiers in Statistical Quality Control*, pp. 275–289, Physica, Heidelberg, Germany, 2001.

[11] S. H. Steiner and R. J. MacKay, "Monitoring processes with data censored owing to competing risks by using exponentially weighted moving average control charts," *Journal of the Royal Statistical Society. Series C. Applied Statistics*, vol. 50, no. 3, pp. 293–302, 2001.

[12] S. T. Liu, "Springer handbook of engineering statistics," *Technometrics*, vol. 49, no. 4, pp. 494–494, 2007.

[13] P. Erto, G. Pallotta, and S. H. Park, "An example of data technology product: a control chart for Weibull processes," *International Statistical Review*, vol. 76, no. 2, pp. 157–166, 2008.

[14] D. C. Montgomery, *Introduction to Statistical Process Control*, John Wiley & Sons, Hoboken, NJ, USA, 6th edition, 2009.

[15] S. H. Steiner and M. Jones, "Risk-adjusted survival time monitoring with an updating exponentially weighted moving average (EWMA) control chart," *Statistics in Medicine*, vol. 29, no. 4, pp. 444–454, 2010.

[16] S. Asadzadeh and A. Aghaie, "Improving the product reliability in multistage manufacturing and service operations," *Quality and Reliability Engineering International*, vol. 28, no. 4, pp. 397–407, 2012.

[17] B. Guo and B. X. Wang, "Control charts for monitoring the weibull shape parameter based on type-II censored sample," *Quality and Reliability Engineering International*, vol. 30, no. 1, pp. 13–24, 2014.

[18] M. Xie, T. N. Goh, and P. Ranjan, "Some effective control chart procedures for reliability monitoring," *Reliability Engineering & System Safety*, vol. 77, no. 2, pp. 143–150, 2002.

[19] B. Sürücü and H. S. Sazak, "Monitoring reliability for a three-parameter Weibull distribution," *Reliability Engineering & System Safety*, vol. 94, no. 2, pp. 503–508, 2009.

[20] D. A. Olteanu, *Cumulative sum control charts for censored reliability data [Ph.D. thesis]*, Virginia Polytechnic Institute and State University, Blacksburg, Va, USA, 2010.

[21] L. Lu, "Weighted progressive iteration approximation and convergence analysis," *Computer Aided Geometric Design*, vol. 27, no. 2, pp. 129–137, 2010.

[22] S.-O. Caballero-Morales, "Building cost function 3D benchmarks to improve the economic statistical design of $\overline{X}$ control charts," *Mathematical Problems in Engineering*, vol. 2014, Article ID 879456, 20 pages, 2014.

[23] P. Murthy, "New research in reliability, warranty and maintenance," in *Proceedings of the 4th Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling (APARM '10)*, pp. 504–515, McGraw-Hill International Enterprises, Wellington, New Zealand, December 2010.

[24] J. V. Abellan-Nebot, J. Liu, F. R. Subirn, and J. Shi, "State space modeling of variation propagation in multistation machining processes considering machining-induced variations," *Journal of Manufacturing Science and Engineering*, vol. 134, no. 2, Article ID 021002, 2012.

[25] K. Mi, Y.-H. He, and C.-H. Wu, "The relationship of product reliability and its process dimensions: a study on product reliability assurance in manufacturing," in *Proceedings of the IEEE 18th International Conference on Industrial Engineering and Engineering Management (IE & EM '11)*, pp. 1177–1180, September 2011.

[26] Y. He, X. Tang, and W. Chang, "Technical decomposition approach of critical to quality characteristics for product design for six sigma," *Quality and Reliability Engineering International*, vol. 26, no. 4, pp. 325–339, 2010.

[27] I. A. Kurnaz, *Techniques in Genetic Engineering*, CRC Press, Taylor & Francis Group, New York, NY, USA, 2015.

[28] S. J. Hu and Y. Koren, "Stream-of-variation theory for automotive body assembly," *CIRP Annals—Manufacturing Technology*, vol. 46, no. 1, pp. 1–6, 1997.