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Research Article

Nonlinear Vibration Analysis of Moving Strip with Inertial Boundary Condition

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According to the movement mechanism of strip and rollers in tandem mill, the strip between two stands was simplified to axially moving Euler beam and the rollers were simplified to the inertial component on the fixed axis rotation, namely, inertial boundary. Nonlinear vibration mechanical model of Euler beam with inertial boundary conditions was established. The transverse and longitudinal motion equations were derived based on Hamilton's principle. Kantorovich averaging method was employed to discretize the motion equations and the inertial boundary equations, and the solutions were obtained using the modified iteration method. Depending on numerical calculation, the amplitude-frequency responses of Euler beam were determined. The axial velocity, tension, and rotational inertia have strong influences on the vibration characteristics. The results would provide an important theoretical reference to control and analyze the vertical vibration of moving strip in continuous rolling process.

1. Introduction

Vibration phenomenon of moving strip is extremely complicated in continuous rolling process. It may lead to strip quality decline, equipment trouble, and production efficiency reduction [1, 2]. For rolling mill vibrations, there are more studies on vertical, axial, torsional, horizontal, cross, and swing vibration modes of roller [3]. However, the strip vibration exists as well in practical production and scholars provided a little focus for this aspect. Therefore, it is significant to study the vibration of moving strip between two stands during rolling.

The moving strip during continuous rolling process can be similar to an axially moving beam if the vertical vibration of mill can be neglected [4]. Using the theory of moving beam, the vertical vibration of strip can be equivalent to transverse vibration of axially moving beam. Some studies have been already conducted. Sun et al. [5, 6] established two-dimensional dynamic model and three-dimensional dynamic model of moving strip between two mills with time-dependent tension, and the stability of principle parametric resonances was analyzed. Huang and Chen [7, 8] investigated nonlinear vibration of the axially moving beam with coupled transverse and longitudinal motions, and the incremental harmonic balance method was used for analysis. Świątoniowski and Bar [9, 10] analyzed chatter vibration phenomena in tandem rolling mill with use of mathematical models of parametrical self-excited vibration and the nonlinear mathematical model of oscillated system was given: the continuous group of the rolling stands coupled by transferring strip. Miranker [11] put forward differential equations of transverse vibration on moving strings firstly, and the corresponding moving string was analyzed. Hu and Zhang [12] investigated an axially accelerating rectangular thin plate subjected to parametric excitations resulting from the axial time-varying tension and axial time-varying speed in the magnetic field. The parametric resonance of plates with axis tension during zinc coasting process was studied by Kim et al. [13]. Stability of transverse vibration in axial variable motion system was studied by Mote Jr. [14]. The mathematical model of a strip-roller-flexible-supporting hybrid system in coating section of a continuous hot-dip galvanizing line was

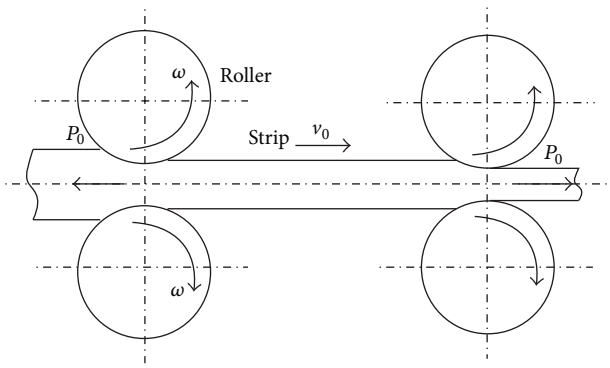


Figure 1: The schematic diagram of moving strip between two stands.

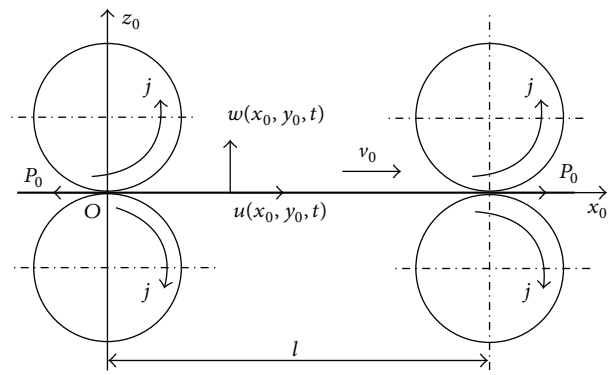


Figure 2: Mechanical model of Euler beam with inertial boundary conditions.

established by Wang et al. [15]. The above studies considered some aspects of moving strip but did not give the influences of rotational inertia and other factors on the strip. It will not be able to analyze the vibration characteristics of strip accurately.

Based on the movement mechanism of rolling and theory of axially moving beam, the vertical vibration of moving strip can be similar to a transverse vibration of axially moving Euler beam with tension [16–18]. The author established a dynamic model of interaction between inertial component and Euler beam. When the rotational inertia is considered, the nonlinear vibration mechanical model of the Euler beam with inertial boundary conditions is established. The modified iteration method is employed to solve the motion equations and used Matlab for simulation and analysis [19, 20]. Hereby, the relationships of axial velocity, tension, and rotational inertia are investigated, and the results may provide an important theoretical reference to control and analyze the vertical vibration of strip during continuous rolling process.

2. Mechanical and Mathematical Models

The strip moves between two stands during rolling process; the schematic diagram is shown in Figure 1. The moving strip can be simplified to moving Euler beam which is supported by both rollers. The rolling direction can be considered as axial motion direction, and it is assumed that the motion is uniform movement, v_0 is axial velocity, and P_0 is tension. Rollers can be equivalent to symmetric and rigid inertial components and regarded as the disc of fixed axis rotation along the width of beam; j is rotational inertia. Therefore, Figure 2 shows an axially moving Euler beam at velocity v_0 and acted tension P_0 with inertial boundary conditions. The origin of coordinate locates as a crossing point of the center line of the first stand and the passing line. The transverse displacement and the longitudinal displacement of Euler beam are $w(x_0, y_0, t)$ and $u(x_0, y_0, t)$, respectively. l is the distance of Euler beam between two stands.

Based on Hamilton's principle, the mathematic model is established. The following equation can be obtained:

$$\delta \int_{t_2}^{t_1} (T_1 + T_2 - U_1 - U_2)\, dt = 0. \tag{1}$$

Kinetic energy T_1 of moving Euler beam can be written as

$$T_1 = \frac{1}{2} \int_0^l \rho A \left[\left(v_0 + u_{,t} + v_0 u_{,x_0} \right)^2 + \left(w_{,t} + v_0 w_{,x_0} \right)^2 \right] dx_0, \tag{2}$$

where ρ is density of strip, A is cross-sectional area of Euler beam, and $(\cdot)_{,t} = \partial(\cdot)/\partial t$ and $(\cdot)_{,x_0} = \partial(\cdot)/\partial x_0$ denote the partial derivatives in time t and spatial coordinate x_0, respectively.

Potential energy U_1 of Euler beam can be written as

$$U_1 = \frac{1}{2} \int_0^l \left[EA \left(u_{,x_0} + \frac{1}{2} w_{,x_0}^2 \right)^2 + EI w_{,x_0 x_0}^2 \right] dx_0, \tag{3}$$

where E is Young's modulus and I is moment of inertia.

Kinetic energy T_2 of roller can be written as

$$T_2 = \frac{1}{2} j \left(\frac{du}{dt} \frac{1}{r} \right)^2 = \frac{j}{2r^2} \left(u_{,t} + v_0 u_{,x_0} \right)^2, \tag{4}$$

where r is radius of roller.

Potential energy U_2 of tension can be written as

$$U_2 = \int_0^l P_0 \left(u_{,x_0} + \frac{1}{2} w_{,x_0}^2 \right) dx_0. \tag{5}$$

Substitution of (2)~(5) into (1), the motion equations can be expressed as

$$\begin{aligned} &\rho A u_{,tt} + \left(\rho A {v_0}^2 - EA \right) u_{,x_0 x_0} + 2\rho A v_0 u_{,x_0 t} \\ &\quad - EA w_{,x_0} w_{,x_0 x_0} = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} &\rho A w_{,tt} + EI w_{,x_0 x_0 x_0 x_0} + \left(\rho A {v_0}^2 - P_0 \right) w_{,x_0 x_0} \\ &\quad + 2\rho A v_0 w_{,x_0 t} \\ &\quad - EA \left(w_{,x_0 x_0} u_{,x_0} + u_{,x_0 x_0} w_{,x_0} + \frac{3}{2} w_{,x_0}^2 w_{,x_0 x_0} \right) = 0. \end{aligned} \tag{7}$$

Boundary conditions can be written as

$$\begin{aligned}
&EAru_{,x_0} - P_0 r \\
&\quad = -j\left(u_{,tt} + 2v_0 u_{,x_0 t} + {v_0}^2 u_{,x_0 x_0} + v_{,t} u_{,x_0}\right)\frac{1}{r} \\
&\qquad\qquad \text{when } x_0 = 0, \\
&P_0 r + EAru_{,x_0} \\
&\quad = -j\left(u_{,tt} + 2v_0 u_{,x_0 t} + {v_0}^2 u_{,x_0 x_0} + v_{,t} u_{,x_0}\right)\frac{1}{r} \\
&\qquad\qquad \text{when } x_0 = l, \\
&w(0) = w(l) = w_{,x_0 x_0}(0) = w_{,x_0 x_0}(l) = 0, \\
&w_{\max} = \varphi_m,
\end{aligned} \tag{8}$$

where $w_{\max}$ is the maximum transverse displacement.

In general case, the kinetic energy of axial movement caused by transverse vibration is relatively small. So, let $u_{,t} = u_{,tt} = 0$; (6) can be simplified as

$$\left(\rho A {\overline{v}_0}^2 - EA\right) u_{,x_0 x_0} - EAw_{,x_0} w_{,x_0 x_0} = 0. \tag{9}$$

By substitution of $w = \overline{\varphi}(x_0)\cos\overline{\omega}t$ into (9), the following equation can be obtained:

$$u = \overline{\phi}(x_0)\cos^2\overline{\omega}t, \tag{10}$$

where $\overline{\omega}$ is vibration frequency.

According to Kantorovich averaging method on an interval $[0, 2\pi/\overline{\omega}]$, the time variables can be eliminated. The motion equations and the inertial boundary equations after being discreted are

$$\left(\rho A {v_0}^2 - EA\right)\overline{\phi}_{,x_0 x_0} - EA\overline{\varphi}_{,x_0}\overline{\varphi}_{,x_0 x_0} = 0, \tag{11}$$

$$\begin{aligned}
&EI\overline{\varphi}_{,x_0 x_0 x_0 x_0} + \left(\rho A {v_0}^2 - P_0\right)\overline{\varphi}_{,x_0 x_0} - {\omega_0}^2 \rho A\overline{\varphi} \\
&\quad - \frac{3}{4}EA\left(\overline{\varphi}_{,x_0 x_0}\overline{\phi}_{,x_0} + \overline{\varphi}_{,x_0}\overline{\phi}_{,x_0 x_0} + \frac{3}{2}{\overline{\varphi}_{,x_0}}^2\overline{\varphi}_{,x_0 x_0}\right) \\
&\quad = 0,
\end{aligned} \tag{12}$$

$$\begin{aligned}
&j{v_0}^2\overline{\phi}_{,x_0 x_0} + EAr^2\overline{\phi}_{,x_0} - \frac{4}{3}j{\omega_0}^2\overline{\phi} - \frac{4}{3}P_0 r^2 = 0 \\
&\qquad\qquad \text{when } x_0 = 0,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&j{v_0}^2\overline{\phi}_{,x_0 x_0} + EAr^2\overline{\phi}_{,x_0} - \frac{4}{3}j{\omega_0}^2\overline{\phi} + \frac{4}{3}P_0 r^2 = 0 \\
&\qquad\qquad \text{when } x_0 = l,
\end{aligned} \tag{14}$$

$$\overline{\varphi}(0) = \overline{\varphi}(l) = \overline{\varphi}_{,x_0 x_0}(0) = \overline{\varphi}_{,x_0 x_0}(l) = 0, \tag{15}$$

$$\overline{\varphi}\left(\frac{1}{2}\right) = \overline{\varphi}_m. \tag{16}$$

The dimensionless quantities are given by

$$\begin{aligned}
x &= \frac{x_0}{l}, \\
\phi &= \frac{\overline{\phi}}{l}, \\
\varphi &= \frac{\overline{\varphi}}{l}, \\
v &= v_0\sqrt{\frac{\rho}{E}}, \\
\omega &= \omega_0\sqrt{\frac{\rho A l^4}{EI}}, \\
S &= \frac{Al^2}{I}, \\
J &= \frac{j}{\rho A r^2 l}, \\
P &= \frac{P_0 l^2}{EI}.
\end{aligned} \tag{17}$$

Then, the dimensionless form of (11)~(16) can be obtained, respectively:

$$\left(v^2 - 1\right)\phi_{,xx} - \varphi_{,x}\varphi_{,xx} = 0, \tag{18}$$

$$\begin{aligned}
&\varphi_{,xxxx} + \left(Sv^2 - P\right)\varphi_{,xx} - \omega^2\varphi \\
&\quad - \frac{3}{4}S\left(\varphi_{,xx}\phi_{,x} + \varphi_{,x}\phi_{,xx} + \frac{3}{2}{\varphi_{,x}}^2\varphi_{,xx}\right) = 0,
\end{aligned} \tag{19}$$

$$Jv^2\phi_{,xx} + \phi_{,x} - \frac{4J}{3S}\omega^2\phi - \frac{4P}{3S} = 0 \quad \text{when } x = 0, \tag{20}$$

$$Jv^2\phi_{,xx} + \phi_{,x} - \frac{4J}{3S}\omega^2\phi + \frac{4P}{3S} = 0 \quad \text{when } x = 1, \tag{21}$$

$$\varphi(0) = \varphi(1) = \varphi_{,xx}(0) = \varphi_{,xx}(1) = 0, \tag{22}$$

$$\varphi\left(\frac{1}{2}\right) = \varphi_m. \tag{23}$$

3. Solution Based on Modified Iteration Method

Due to more complicated solving process of (18)~(23), the modified iteration method will be employed to solve the equations in this section.

3.1. The First-Order Approximate Solution. Firstly, all the nonlinear terms will be omitted in (19), and (19) can be transformed into

$$\varphi_{1,xxxx} - \omega^2\varphi_1 = 0. \tag{24}$$

The series solution of (24) is

$$\varphi_1 = a_0 M_0(x) + a_1 N_0(x) + a_2 I_0(x) + a_3 K_0(x), \tag{25}$$

where

$$M_0 = \sum_{n=0}^{\infty} \frac{\left(\omega^2\right)^n}{(4n)!} x^{4n},$$
$$N_0 = \sum_{n=0}^{\infty} \frac{\left(\omega^2\right)^n}{(4n+1)!} x^{4n+1},$$
$$I_0 = \sum_{n=0}^{\infty} \frac{\left(\omega^2\right)^n}{(4n+2)!} x^{4n+2},$$
$$K_0 = \sum_{n=0}^{\infty} \frac{\left(\omega^2\right)^n}{(4n+3)!} x^{4n+3}. \tag{26}$$

By substitution of (25) into (22) and (23), one has $\omega_1 = 14.709$. Then, the coefficients of (25) have

$$a_0 = 0,$$
$$a_1 = \mu_1 \varphi_m,$$
$$a_2 = 0,$$
$$a_3 = \mu_2 \varphi_m, \tag{27}$$

where

$$\mu_1 = \frac{-K_0(1)}{N_0(1) K_0(1/2) - N_0(1/2) K_0(1)},$$
$$\mu_2 = \frac{N_0(1)}{N_0(1) K_0(1/2) - N_0(1/2) K_0(1)}. \tag{28}$$

Then,

$$\begin{aligned}\varphi_1(x) &= \varphi_m \left[\mu_1 N_0(x) + \mu_2 K_0(x)\right] \\ &= \varphi_m \left[\mu_1 \sum_{n=0}^{\infty} \frac{{\omega_1}^{2n} x^{4n+1}}{(4n+1)!} + \mu_2 \sum_{n=0}^{\infty} \frac{{\omega_1}^{2n} x^{4n+3}}{(4n+3)!}\right].\end{aligned} \tag{29}$$

And substituting (29) into (18), one can get

$$\phi_1(x) = \frac{1}{2\left(v^2-1\right)} \int \varphi_{1,x}^2 \, dx + c_1 x + c_2. \tag{30}$$

The coefficients c_1 and c_2 in (30) can be determined by (20) and (21), and the results are as follows:

$$\begin{aligned}c_1 &= \frac{1}{2\left(v^2-1\right)} \left(\frac{3Sv^2}{2{\omega_1}^2} \varphi_{1,x} \varphi_{1,xx} + \frac{3S}{4J{\omega_1}^2} \varphi_{1,x}^2 \right. \\ &\quad \left. - \int \varphi_{1,x}^2 \, \mathrm{d}x\right)_{x=1} + \frac{2P}{J{\omega_1}^2} \\ c_2 &= \frac{3S}{8J{\omega_1}^2\left(v^2-1\right)} \left(\frac{3Sv^2}{2{\omega_1}^2} \varphi_{1,x} \varphi_{1,xx} + \frac{3S}{4J{\omega_1}^2} \varphi_{1,x}^2 \right. \\ &\quad \left. - \int \varphi_{1,x}^2 \, \mathrm{d}x\right)_{x=1} + \left(\frac{3S}{2J{\omega_1}^2} - 1\right) \frac{P}{J{\omega_1}^2}.\end{aligned} \tag{31}$$

3.2. The Second-Order Modified-Iterative Solution. In this subsection, the second-order modified-iterative solution can be given.

By substitution of (29) and (30) into (19), one has

$$\varphi_{2,xxxx} - \omega^2 \varphi_2 = \alpha \varphi_{1,xx} + \beta \varphi_{1,x}^2 \varphi_{1,xx}, \tag{32}$$

where

$$\begin{aligned}\alpha &= P + \frac{3}{4} S c_1 - S v^2, \\ \beta &= \frac{9Sv^2}{8\left(v^2-1\right)}, \\ \varphi_{1,x}^2(x) &= {\varphi_m}^2 \left(\sum_{n=0}^{\infty} A_n^{(1)} x^{4n} + \sum_{n=0}^{\infty} B_n^{(1)} x^{4n+2}\right), \\ \varphi_{1,xx}(x) &= \varphi_m \left(\sum_{n=1}^{\infty} C_n^{(1)} x^{4n-1} + \sum_{n=0}^{\infty} D_n^{(1)} x^{4n+1}\right),\end{aligned} \tag{33}$$

where

$$\begin{aligned}A_0^{(1)} &= {\mu_1}^2, \\ A_n^{(1)} &= \sum_{n_1=0}^{n} \frac{{\mu_1}^2 {\omega_1}^{2n}}{(4n_1)!\,(4n-4n_1)!} \\ &\quad + \sum_{n_1=0}^{n-1} \frac{{\mu_2}^2 {\omega_1}^{2n-2}}{(4n_1+2)!\,(4n-4n_1-2)!} \\ &\qquad (n = 1, 2, \ldots), \\ B_n^{(1)} &= \sum_{n_1=0}^{n} \frac{2\mu_1 \mu_2 {\omega_1}^{2n}}{(4n_1)!\,(4n-4n_1+2)!} \quad (n = 0, 1, \ldots), \\ C_n^{(1)} &= \frac{\mu_1 \omega_1^{2n}}{(4n-1)!}, \quad (n = 1, 2, \ldots), \\ D_n^{(1)} &= \frac{\mu_2 \omega_1^{2n}}{(4n+1)!}, \quad (n = 0, 1, \ldots).\end{aligned} \tag{34}$$

Due to the property of series solution, the solution of (32) can be written as

$$\begin{aligned}&\varphi_2(x) \\ &= \varphi_m \left[\zeta_1 \sum_{n=0}^{\infty} \frac{\omega^{2n} x^{4n+1}}{(4n+1)!} + \zeta_2 \sum_{n=0}^{\infty} \frac{\omega^{2n} x^{4n+3}}{(4n+3)!}\right] \\ &\quad + \varphi_m \left(\sum_{n=1}^{\infty} A_n x^{4n-1} + \sum_{n=0}^{\infty} B_n x^{4n+1}\right) \\ &\quad + {\varphi_m}^3 \left(\sum_{n=1}^{\infty} C_n x^{4n-1} + \sum_{n=0}^{\infty} D_n x^{4n+1} + \sum_{n=0}^{\infty} E_n x^{4n+3}\right),\end{aligned} \tag{35}$$

where ζ_1 and ζ_2 are undetermined coefficients. Consider

$$\begin{aligned} A_1 &= B_0 = C_1 = D_0 = E_0 = 0, \\ A_{n+1} &= \frac{\omega^2 A_n + \alpha C_n^{(1)}}{(4n+3)(4n+2)(4n+1)4n}, \\ B_{n+1} &= \frac{\omega^2 B_n + \alpha D_n^{(1)}}{(4n+5)(4n+4)(4n+3)(4n+2)}, \\ C_{n+1} &= \frac{\omega^2 C_n + \beta A_n^{(2)}}{(4n+3)(4n+2)(4n+1)4n}, \\ D_{n+1} &= \frac{\omega^2 D_n + \beta B_n^{(2)}}{(4n+5)(4n+4)(4n+3)(4n+2)}, \\ E_{n+1} &= \frac{\omega^2 E_n + \beta C_n^{(2)}}{(4n+7)(4n+6)(4n+5)(4n+4)}, \end{aligned} \tag{36}$$

where

$$\begin{aligned} A_n^{(2)} &= \sum_{m=0}^{n} A_n^{(1)} C_{n-m}^{(1)}, \quad (n = 1, 2, \ldots), \\ C_n^{(2)} &= \sum_{m=0}^{n} B_n^{(1)} D_{n-m}^{(1)}, \quad (n = 0, 1, \ldots), \\ B_0^{(2)} &= \mu_1^2 \mu_2, \\ B_n^{(2)} &= \sum_{m=0}^{n} \left(A_n^{(1)} D_{n-m}^{(1)} + B_n^{(1)} C_{n-m}^{(1)} \right), \quad (n = 1, 2, \ldots). \end{aligned} \tag{37}$$

By substitution of (35) into (22) and (23), the solvability condition is

$$D\xi = 0, \tag{38}$$

where

$$\begin{aligned} \zeta &= \left[\zeta_1 \quad \zeta_2 \quad 1\right]^T, \\ D &= \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}, \end{aligned} \tag{39}$$

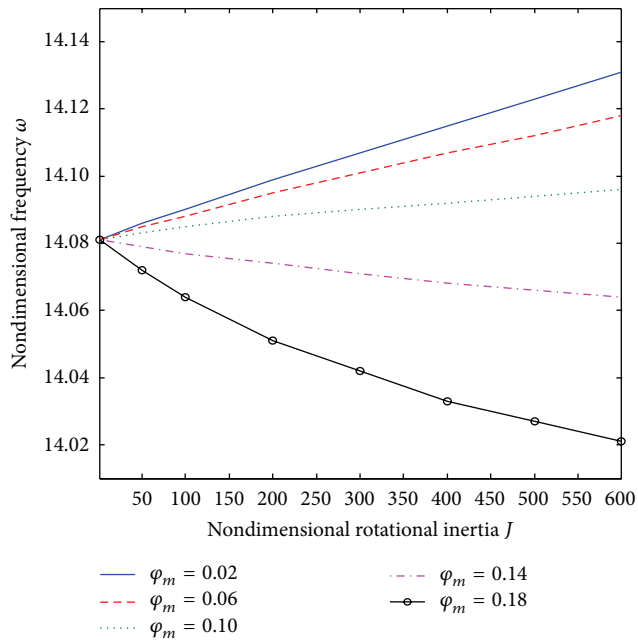
where

$$\begin{aligned} d_{11} &= \sum_{n=0}^{\infty} \frac{\omega^{2n}}{(4n+1)!}, \\ d_{12} &= \sum_{n=0}^{\infty} \frac{\omega^{2n}}{(4n+3)!}, \\ d_{13} &= \left(\sum_{n=1}^{\infty} A_n + \sum_{n=0}^{\infty} B_n \right) + {\varphi_m}^2 \left(\sum_{n=1}^{\infty} C_n + \sum_{n=0}^{\infty} D_n \right. \\ &\quad \left. + \sum_{n=0}^{\infty} E_n \right), \\ d_{21} &= \sum_{n=0}^{\infty} \frac{\omega^{2n}}{(4n+1)!} \left(\frac{1}{2}\right)^{4n+1}, \\ d_{22} &= \sum_{n=0}^{\infty} \frac{\omega^{2n}}{(4n+3)!} \left(\frac{1}{2}\right)^{4n+3}, \\ d_{23} &= \left(\sum_{n=1}^{\infty} A_n \left(\frac{1}{2}\right)^{4n-1} + \sum_{n=0}^{\infty} B_n \left(\frac{1}{2}\right)^{4n+1} \right) \\ &\quad + {\varphi_m}^2 \left[\sum_{n=1}^{\infty} C_n \left(\frac{1}{2}\right)^{4n-1} \right. \\ &\quad \left. + \sum_{n=0}^{\infty} D_n \left(\frac{1}{2}\right)^{4n+1} + \sum_{n=0}^{\infty} E_n \left(\frac{1}{2}\right)^{4n+3} \right] - 1 \\ d_{31} &= \sum_{n=1}^{\infty} \frac{\omega^{2n}}{(4n-1)!}, \\ d_{32} &= \sum_{n=0}^{\infty} \frac{\omega^{2n}}{(4n+1)!}, \\ d_{33} &= \left(\sum_{n=1}^{\infty} (4n-1)(4n-2) A_n + \sum_{n=0}^{\infty} (4n+1) 4n B_n \right) \\ &\quad + {\varphi_m}^2 \left[\sum_{n=1}^{\infty} (4n-1)(4n-2) C_n \right. \\ &\quad \left. + \sum_{n=0}^{\infty} (4n+1) 4n D_n + \sum_{n=0}^{\infty} (4n+3)(4n+2) E_n \right]. \end{aligned} \tag{40}$$

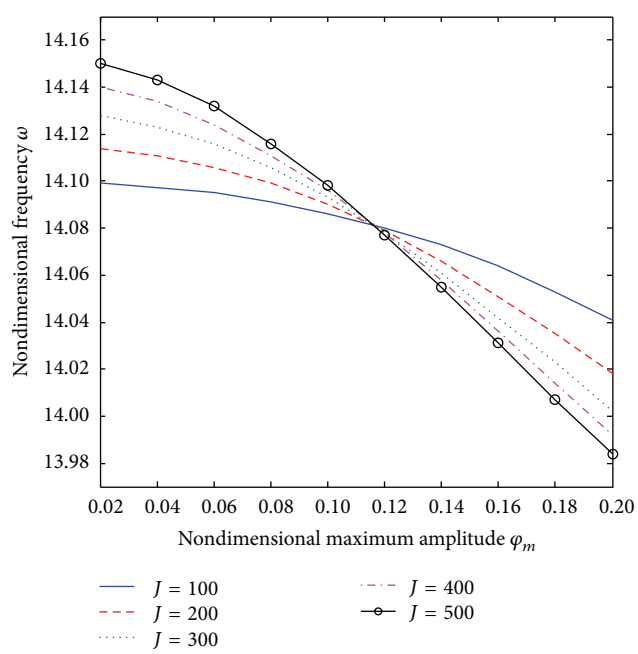
The frequency ω_2 can be obtained by $\det D = 0$. The coefficients ζ_1 and ζ_2 can be determined by (38); then the second-order modified-iterative solution can be obtained.

4. Numerical Simulation

To verify the proposed method, some numerical simulations will be given. The main parameters of a tandem rolling mill are listed as below: the distance between stand F2 and stand F3 is 3000 mm, the thickness of strip is 20 mm, rolling velocity of mill is 3 m/s, the nondimensional velocity is 6×10^{-4}, and the nondimensional tension is 5×10^4.



(a) Different values of maximum amplitude



(b) Different values of rotational inertia

Figure 3: Influence of rotational inertia on frequency.

Figure 3 illustrates the influence of nondimensional rotational inertia J on nondimensional frequency ω under the condition of constant velocity and constant tension. It can be found that the rotational inertia of roller has great effect on amplitude-frequency response characteristics, and the vibration performance will be changed. From Figure 3(a), one can see that the vibration performance is changed from sclerotic type to intenerate type gradually and the nondimensional maximum amplitude about φ_m is 0.02, 0.06, 0.10, 0.14, and 0.18, respectively. When $\varphi_m \leq 0.10$, the nondimensional frequencies increase with increasing of rotational inertia, but the hardening degree decreases with increasing of φ_m. When $\varphi_m \geq 0.14$, with increasing of rotational inertia, the nondimensional frequencies decrease, and the softening degree increases with increasing of φ_m. It is obvious that Figure 3(b) shows the same vibration performance. With increasing of φ_m, the frequencies decrease, and vibration performance present uniformly changes. When $\varphi_m = 0.12$, the point is limit between sclerotic type and intenerate type. In addition, when rotational inertia increases to infinity, the boundary condition tends to be fixed. And the bigger the rotational inertia, the greater the impact of amplitude on the frequency.

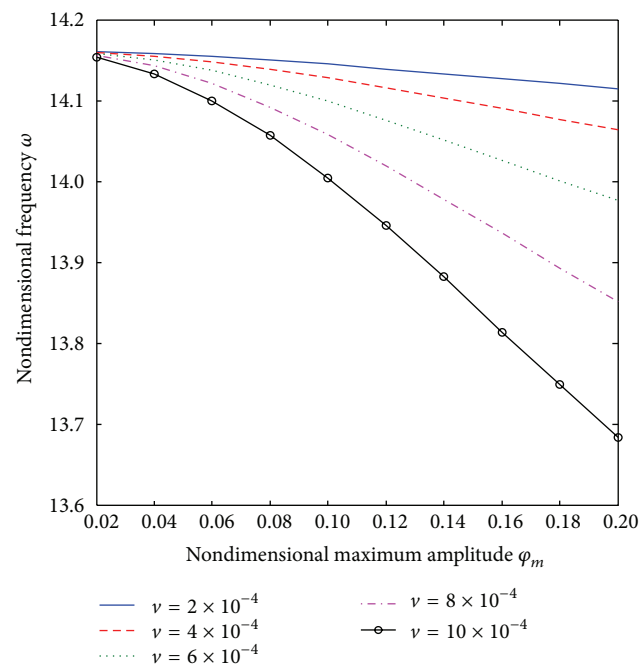


Figure 4: Amplitude-frequency responses under the condition of different velocities.

Figure 4 shows the amplitude-frequency responses under the condition of different nondimensional velocities, which are 2×10^{-4}, 4×10^{-4}, 6×10^{-4}, 8×10^{-4}, and 10×10^{-4}, respectively. The nondimensional rotational inertia $J = 500$; the nondimensional tension $P = 5 \times 10^4$. From Figure 4, one can see that the nondimensional frequencies decrease with the increasing of amplitude. With the decrease of velocity, the influence of amplitude acts the less on frequency, and the downtrend of amplitude-frequency responses becomes more gradual. That is, the larger velocity results in the greater influence of the amplitude on frequency.

Figure 5 depicts the amplitude-frequency responses with different nondimensional tensions: 1×10^4, 2×10^4, 3×10^4, 4×10^4, and 5×10^4. The nondimensional rotational inertia $J = 500$; the nondimensional velocity $v = 6 \times 10^{-4}$. As can be seen from Figure 5, with increasing of amplitude, the nondimensional frequencies decrease. And the smaller is the tension, the faster declines the amplitude-frequency response. When $\varphi_m \leq 0.11$, the larger tension leads to less

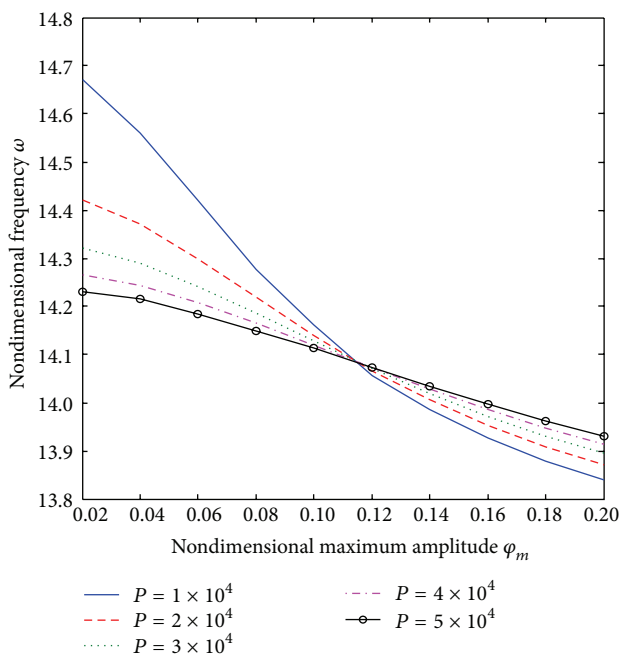


FIGURE 5: Amplitude-frequency responses under the condition of different tensions.

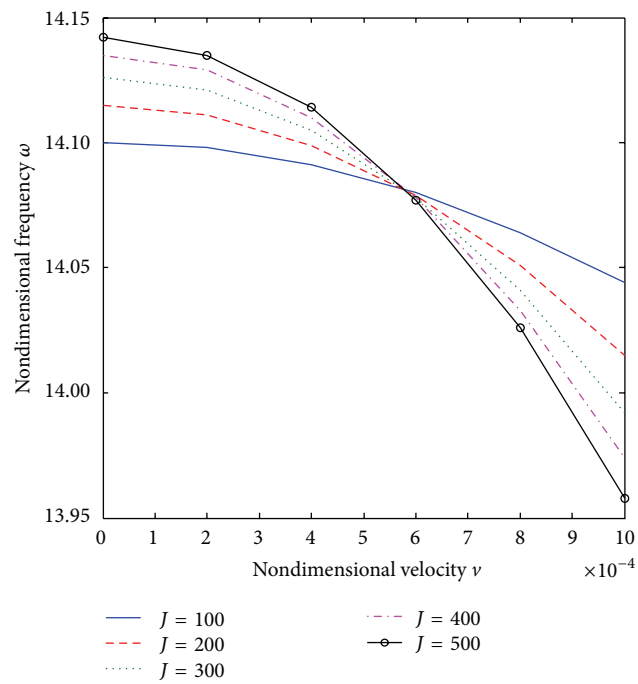


FIGURE 6: Relationship between velocity and frequency.

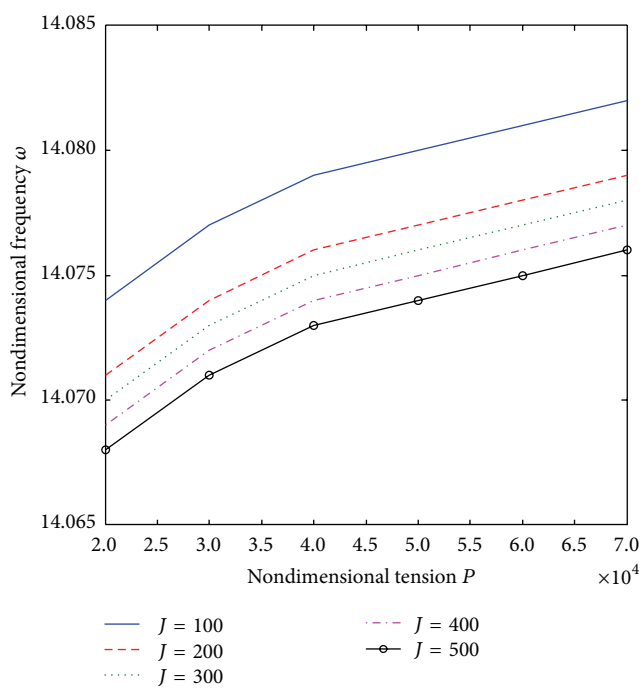


FIGURE 7: Relationship between tension and frequency.

frequency, whereas an adverse result is presented from $\varphi_m > 0.11$. That is to say, the smaller tension has greater effect on vibration frequency of strip.

Figure 6 displays the relationship between nondimensional velocity and nondimensional frequency under the condition of different nondimensional rotational inertia, which is 100, 200, 300, 400, and 500, respectively. The amplitude $\varphi_m = 0.12$; the nondimensional tension $P = 5 \times 10^4$. From Figure 6, one can see that, with increasing of velocity, the nondimensional frequencies decrease. And the bigger is the rotational inertia, the faster declines the trend of velocity-frequency curve. When $v \leq 5.7 \times 10^{-4}$, the bigger is the rotational inertia, the greater is the frequency, whereas when $v > 5.7 \times 10^{-4}$, the result shows the opposite. So, the velocity has strong influence on frequency, while the rotational inertia is bigger than others. It also can be known that the velocity-frequency curves are closer with bigger rotational inertia. In other words, a further proof is given to the rule of boundary condition that tends to be fixed at that time.

Figure 7 demonstrates relationship between nondimensional tension and nondimensional frequency under the condition of different nondimensional rotational inertia, which is 100, 200, 300, 400, and 500, respectively. The nondimensional amplitude φ_m is 0.12; the nondimensional velocity v is 6×10^{-4}. From Figure 7, one can see that, with increasing of tension, the nondimensional frequencies increase, and the rising tendencies keep consistent completely. Also it can be seen that the bigger is the rotational inertia, the less is the frequency and the more approximate is the tension-frequency curves. Moreover, it is shown that the less rotational inertia has much influence on frequency.

In Figures 6 and 7, it can be seen that the rotational inertia of roller has much influence on vertical vibration. When the rotational inertia is bigger, the velocity variables have greater influence on vibration, but the tension variables have less influence. Thus, rotational inertia is not to be ignored in researching of strip vibration. To improve the strip quality and prolong the service life of roller, harmful vibration

phenomenon of strip can be depressed during rolling process by adjusting the above parameters.

5. Conclusions

(1) Based on the movement mechanism of rolling and the theory of axially moving beam, the mechanical model of the Euler beam with inertial boundary conditions is established. The Kantorovich averaging method is employed to separate the time variables and space variables of equations. Using modified iteration method, the solutions of nonlinear vibration equations are obtained.

(2) The simulation results indicate that axial velocity, tension, and rotational inertia of the roller have strong influences on the vibration characteristics. Under the case that tension and velocity are constants, the vibration performances are changed by the maximum amplitude while the rotational inertia is considered. The point $\varphi_m = 0.12$ is limit between sclerotic type and intenerate type. When $\varphi_m \leq 0.12$, with increasing of rotational inertia, the vibration frequency increases. The vibration performance shows sclerotic type and hardening degree is decreased along with amplitude increase gradually. With further increasing the amplitude, that is, $\varphi_m > 0.12$, the curves present downward trend with rotational inertia increases, namely, intenerate type. And with increasing of φ_m, the softening degree increases. In addition, when rotational inertia increases infinitely, the boundary condition tends to be fixed, and the bigger the rotational inertia, the greater the impact of amplitude on the frequency.

(3) Depending on the variation of velocity and tension, distinct cases arise. When the velocity is faster or the tension is smaller, the influences of amplitude on frequency are strong. Moreover, when the rotational inertia is bigger, the velocity variables have greater influence on vibration, but the tension variables have less influence.

(4) The study on vibration characteristics of axially moving Euler beam with inertial boundary conditions accords with rolling actuality. The simulation results can provide an important theoretical reference to control and analyze the vertical vibration of strip during rolling process of tandem mill.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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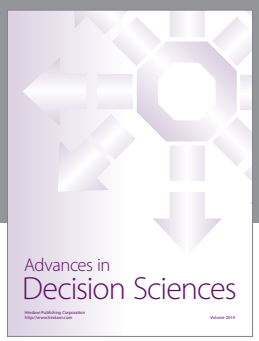

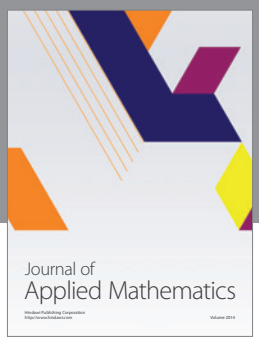

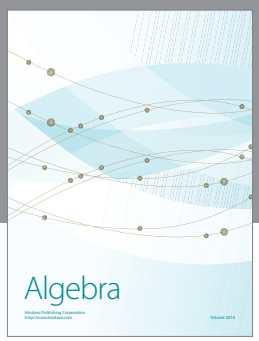

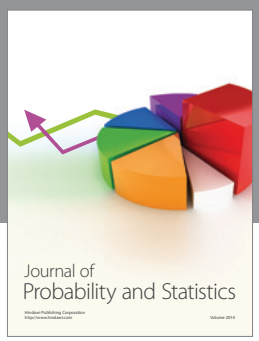

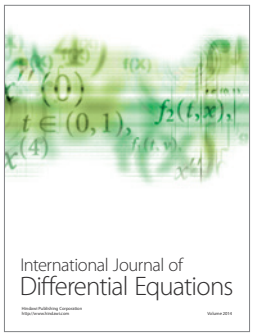

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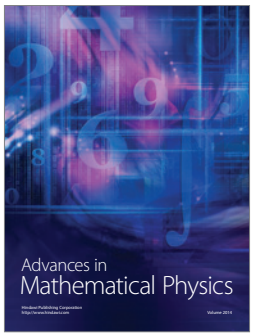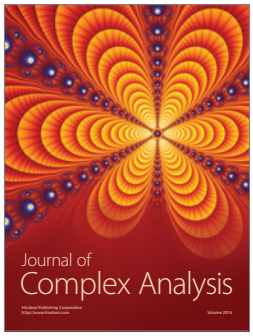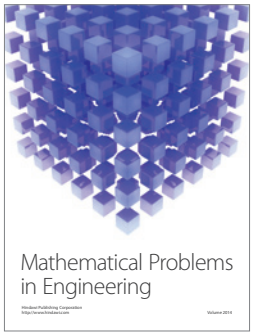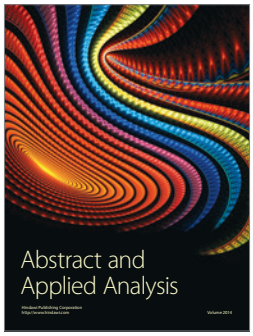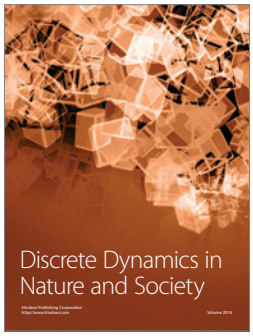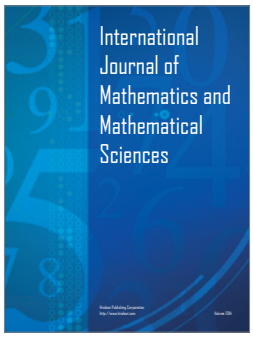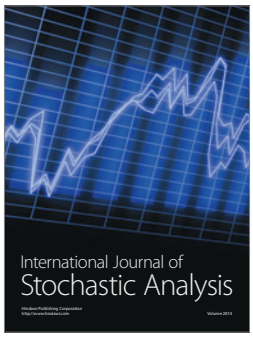
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