Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 217253, 10 pages
http://dx.doi.org/10.1155/2015/217253

Research Article

Estimation of Nonlinear Functions of State Vector for Linear Systems with Time-Delays and Uncertainties

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Received 4 June 2014; Revised 1 August 2014; Accepted 2 August 2014

Academic Editor: Yuxin Zhao

This paper focuses on estimation of a nonlinear function of state vector (NFS) in discrete-time linear systems with time-delays and model uncertainties. The NFS represents a multivariate nonlinear function of state variables, which can indicate useful information of a target system for control. The optimal nonlinear estimator of an NFS (in mean square sense) represents a function of the receding horizon estimate and its error covariance. The proposed receding horizon filter represents the standard Kalman filter with time-delays and special initial horizon conditions described by the Lyapunov-like equations. In general case to calculate an optimal estimator of an NFS we propose using the unscented transformation. Important class of polynomial NFS is considered in detail. In the case of polynomial NFS an optimal estimator has a closed-form computational procedure. The subsequent application of the proposed receding horizon filter and nonlinear estimator to a linear stochastic system with time-delays and uncertainties demonstrates their effectiveness.

1. Introduction

The Kalman filtering for dynamic systems with time-delays and its variations are well-known state estimation techniques in wide use in a variety of applications such as navigation, target tracking, vehicle state estimation, communications engineering, air traffic control, biomedical and chemical processing, and many other areas [1–6]. Ignorance of the effect of time- delays could cause unpredictable and unsatisfactory system performance with traditional Kalman filters [3, 6, 7]. These applications have motivated researchers to study the control and filtering problem for systems with time-delays.

However, some applications require the estimation of not only a state vector but also a nonlinear function of the state (NFS), which expresses practical and worthwhile information for control systems. For instance, in a mechanical application, such measurands include displacement (linear and angular), which can be interpreted as a quadratic form of a state vector, mass (weight, load, and density), and force (absolute, relative, static, torque, and pressure) [8]. In particular, prediction of future measurands depicted by an NFS can be helpful in several applications, such as system control and target tracking. It is well known that, for dynamical systems in which the stability issue depends explicitly on the time-delay and model uncertainty, in order to overcome this practical issue, it is necessary to study the estimation of NFS in dynamical systems with time-delay and model uncertainty. Most authors have not focused on estimation of NFS but have considered state estimation (filtering) only. To the best of our knowledge, there are no effective methods for estimation of an NFS in a dynamic system with time-delay and model uncertainty in the literature.

The Kalman filters are very difficult to be applied to dynamic system with uncertainty. The standard Kalman filter estimates the state of system if it fully fits the system model; that is, there are no model uncertainties. Otherwise, its algorithm may become unstable [9] and the estimate diverges [10] due to the limited knowledge of the system's

dynamic model and unestimated measurement noise. Thus, it is not robust against modeling uncertainty and disturbances [1]. For estimation problems where we attempt to achieve robustness against temporary uncertainty, numerous strategies have been suggested, rigorously investigated, and implemented over the past few decades. For this reason, in terms of structural adaptation, there are a number of methods currently available for such systems [11, 12]. The structure adaptation filters are based on the Bayesian approach in which the unknown parameter is assumed to be random with a prior known probability [13–15]. Next approach relative to the robust filtering problem with respect to various filtering performance criteria, such as H-infinity filtering and energy-to-peak for linear continuous and discrete-time-delay systems [2, 6, 16, 17].

In this paper, we are concerned with usage of the receding horizon strategy for designing robust receding horizon (finite-memory) filter/estimator for time-delay stochastic systems with model uncertainties. In this way the information obtained over the most recent time interval is only used [15, 18, 19]. As a result, the receding horizon filters are more robust against model uncertainties and numerical errors than standard Kalman filters, which utilize all measurements [20, 21]. Based on aforementioned literature, and to the best of the authors' knowledge, there are no existing results for the receding horizon estimation of NFS for linear systems with time-delays.

Motivated by the above problems, in this paper, we focus on estimating an arbitrary nonlinear function of the state in a linear system with time-delay and model uncertainty. The main contribution of the paper is derivation of receding horizon estimators based on the crucial Lyapunov-like equations for receding horizon initial conditions.

This paper is organized as follows. Section 2 presents a statement of the estimation problem for NFS within the linear system with time-delays. The optimal estimator for NFS is proposed. In Section 3, we present the receding horizon filter/estimator for discrete-time linear system with time-delays. Here, the exact recursive equations for determining receding horizon initial conditions (mean and covariance) are derived and discussed. In Section 4, we present several practical examples of NFS. In Section 5, we derive effective closed-form computational procedure for polynomial NFS. In Section 6, the unscented transformation is introduced, and its application for estimation of NFS is proposed. In Section 7, the efficiency of the proposed receding horizon estimators is studied on real dynamical model with time-delays. Finally, a brief conclusion is given in Section 8.

2. Problem Statement

We first consider a discrete-time linear system described by stochastic recursive equation with time-delays:

$$x(k+1) = \sum_{h=0}^{M} F_h(k-h)\, x(k-h) + w(k), \qquad k = 0, 1, 2, \ldots, \tag{1}$$

where $x(k) \in \mathfrak{R}^n$ is an unknown state, $F_h(k)$, $h = 0, 1, \ldots, M$, are $n \times n$ time-varying matrices, $x(-s) \sim \mathbf{N}(\overline{x}_0, P_0)$, $s = 0, 1, \ldots, M$, are initial conditions, $w(k) \in \mathfrak{R}^n$ is a zero-mean white Gaussian noise with covariance $\mathrm{Cov}\{w(k), w(s)\} = Q(k)\delta_{ks}$, and δ_{ks} is the Kronecker function.

The discrete measurement $y(k) \in \mathfrak{R}^m$ is also described by recursive equation with time- delays:

$$y(k) = \sum_{d=0}^{L} H_d(k-d)\, x(k-d) + v(k), \tag{2}$$

where $H_d(k) \in \mathfrak{R}^{m\times n}$ is the measurement matrix, $d = 0, 1, \ldots, L$, and $v(k) \in \mathfrak{R}^m$ is a zero-mean white Gaussian noise with covariance $\mathrm{Cov}\{v(k), v(s)\} = R(k)\delta_{ks}$.

We also assume that the initial states $x(-s)$, $s = 0, 1, \ldots, M$, system noise $w(k)$, and measurement error $v(k)$ are mutually uncorrelated; that is,

$$\begin{aligned} \mathrm{Cov}\{x(-s), w(k)\} &= 0, \\ \mathrm{Cov}\{x(-s), v(k)\} &= 0, \\ \mathrm{Cov}\{w(k), v(k)\} &= 0, \\ s &= 0, 1, \ldots, M. \end{aligned} \tag{3}$$

A problem associated with such system (1), (2) is that of estimation of the nonlinear function of state (NFS) variables

$$z(k) = f[x(k)] : \mathfrak{R}^n \longrightarrow \mathfrak{R} \tag{4}$$

from the overall noisy sensor measurements

$$Y^k = \{y(s) : s = 1, \ldots, k\}. \tag{5}$$

Typical examples of such an NFS may be an arbitrary quadratic form $f[x(k)] = x(k)^T \Omega x(k)$, representing an energy-like function of an object or square distance $f[x(k)] = d^2(x(k), x^0(k))$ between the current $x(k)$ and nominal $x^0(k)$ states, respectively.

We propose an optimal estimation algorithm for NFS and its performance in the subsequent sections.

The optimal mean square estimate of the NFS $z(k) = f[x(k)]$ based on the overall sensor measurements (5) represents a conditional mean:

$$\widehat{z}(k) = \mathbf{E}\left[z(k) \mid Y^k\right] = \int f[x(k)]\, p\left(x(k) \mid Y^k\right) dx(k), \tag{6}$$

where $p(x(k) \mid Y^k)$ is a conditional probability density function (pdf).

To calculate a conditional pdf $p(x(k) \mid Y^k)$ we need to solve the filtering problem consisting in calculation of good estimates $\widehat{x}(k \mid k)$ of the state of a stochastic dynamical system based on noisy partial measurements (5).

Next we propose the new receding horizon filtering algorithm for discrete-time linear systems with time-delays and uncertainties (1), (2) determining the conditional pdf $p(x(k) \mid Y^k)$ in (6).

3. Receding Horizon Filter and Nonlinear Estimator

At first assume that the system model (1), (2) does not contain any uncertainties.

3.1. Kalman Filtering with Time-Delays for Model without Uncertainties. According to [22, 23] the optimal mean square estimate of state $\hat{x}(k \mid k)$ based on all current measurements (5) represents conditional mean $\hat{x}(k \mid k) = \mathbf{E}[x(k) \mid Y^k]$ which is described by the Kalman filter equations with time-delays (KFTD):

$$\begin{aligned}
&\hat{x}(k-m \mid k) \\
&\quad = \hat{x}(k-m \mid k-1) \\
&\qquad + G_m(k)\left[y(k) - \sum_{d=0}^{L} H_d(k-d)\,\hat{x}(k-d \mid k-1)\right], \\
&\qquad\qquad m = 1, 2, \ldots, \overline{M}, \quad \overline{M} = \max\{M, L\}, \\
&\hat{x}(k \mid k-1) = \sum_{h=0}^{M} F_h(k-h-1)\,\hat{x}(k-h-1 \mid k-1), \\
&\hat{x}(k \mid k) \\
&\quad = \hat{x}(k \mid k-1) \\
&\qquad + G_0(k)\left[y(k) - \sum_{d=0}^{L} H_d(k-d)\,\hat{x}(k-d \mid k-1)\right],
\end{aligned} \tag{7}$$

where Kalman gains $G_m(k)$, $m = 0, 1, \ldots, \overline{M}$, and error covariances

$$\begin{aligned}
P(k_1, k_2 \mid k) &= \operatorname{Cov}\{e(k_1 \mid k), e(k_2 \mid k)\}, \\
e(k_1 \mid k) &= x(k) - \hat{x}(k_1 \mid k), \\
&\qquad k_1, k_2 \le k
\end{aligned} \tag{8}$$

are given by

$$\begin{aligned}
&G_m(k) \\
&\quad = \sum_{d=0}^{L} P(k-m, k-d \mid k-1)\, H_d^T(k-d) \\
&\qquad \times \left[R(k) + \sum_{d_1, d_2=0}^{L} H_{d_1}(k-d_1)\, P(k-d_1, k-d_2 \mid k-1)\right. \\
&\qquad\qquad \left. \times H_{d_2}^T(k-d_2)\right]^{-1}, \\
&P(k-h_1, k-h_2 \mid k) \\
&\quad = P(k-h_1, k-h_2 \mid k-1) \\
&\qquad - G_{h_1}(k) \sum_{d=0}^{L} H_d(k-d)\, P(k-d, k-h_2 \mid k-1), \\
&P(k+1, k+1 \mid k) \\
&\quad = \sum_{h_1, h_2=0}^{M} F_{h_1}(k-h_1)\, P(k-h_1, k-h_2 \mid k)\, F_{h_2}^T(k-h_2) \\
&\qquad + Q(k).
\end{aligned} \tag{9}$$

To run KFTD equations (7)–(9) we need to know the initial conditions for estimates and error covariances, which represent a priori mean and covariance; that is,

$$\begin{aligned}
\hat{x}(-h \mid 0) &= \overline{x}_0, \\
P(-h, -j) = P(-h, -j \mid 0) &= P_0, \\
h, j &= 0, 1, 2, \ldots, \overline{M}.
\end{aligned} \tag{10}$$

In practice any system model contains uncertainties; therefore, to achieve robustness against temporary uncertainty, numerous strategies have been proposed. In the paper we use the effective receding horizon (finite-memory) strategy using only the measurements $y(k)$ obtained over the most recent time interval (receding horizon) [10, 20, 21].

3.2. Receding Horizon Filtering with Time-Delays for Model with Uncertainties. According to the receding horizon strategy the optimal (in mean square sense) estimate of the unknown state $x(k)$ based on the overall receding horizon sensor measurements,

$$Y_{k-\Delta}^{k} = \{y(s) : s = k-\Delta, k-\Delta+1, \ldots, k\}, \tag{11}$$

with horizon time interval Δ, represents the conditional mean,

$$\hat{x}(k \mid k) = \mathbf{E}\left[x(k) \mid Y_{k-\Delta}^{k}\right]. \tag{12}$$

Using KFTD's equations (7)–(9) we propose their receding horizon version for estimation of the state $x(k)$ using receding horizon measurements $Y_{k-\Delta}^{k}$ on the interval $s \in [k-\Delta, k]$. We obtain

$$\begin{aligned}
&\hat{x}(s-m \mid s) \\
&\quad = \hat{x}(s-m \mid s-1) \\
&\qquad + G_m(s)\left[y(s) - \sum_{d=0}^{L} H_d(s-d)\,\hat{x}(s-d \mid s-1)\right], \\
&\qquad\qquad s = k-\Delta, k-\Delta+1, \ldots, k; \\
&\qquad\qquad m = 1, 2, \ldots, \overline{M}, \quad \overline{M} = \max\{M, L\}, \\
&\hat{x}(s \mid s-1) = \sum_{h=0}^{M} F_h(s-h-1)\,\hat{x}(s-h-1 \mid s-1),
\end{aligned}$$

$$\begin{aligned}\hat{x}(s \mid s) &= \hat{x}(s \mid s-1) \\ &\quad + G_0(s)\left[y(s) - \sum_{d=0}^{L} H_d(s-d)\,\hat{x}(s-d \mid s-1)\right],\end{aligned} \tag{13}$$

where the receding horizon filter gains $G_m(k)$, $m = 0, 1, \ldots, \overline{M}$, and error autocovariances

$$\begin{aligned}P(s_1, s_2 \mid s) &= \operatorname{Cov}\{e(s_1 \mid s), e(s_2 \mid s)\}, \\ e(s_1 \mid s) &= x(s) - \hat{x}(s_1 \mid s), \\ & s_1, s_2 \le s\end{aligned} \tag{14}$$

are described by

$$\begin{aligned}G_m(s) &= \sum_{d=0}^{L} P(s-m, s-d \mid s-1)\, H_d^T(s-d) \\ &\quad \times \left[R(s) + \sum_{d_1,d_2=0}^{L} H_{d_1}(s-d_1)\, P(s-d_1, s-d_2 \mid s-1) \right. \\ &\qquad \left. \times H_{d_2}^T(s-d_2)\right]^{-1}, \\ P(s-h_1, s-h_2 \mid s) &= P(s-h_1, s-h_2 \mid s-1) \\ &\quad - G_{h_1}(s) \sum_{d=0}^{L} H_d(s-d)\, P(s-d, s-h_2 \mid s-1), \\ P(s+1, s+1 \mid s) &= \sum_{h_1,h_2=0}^{M} F_{h_1}(s-h_1)\, P(s-h_1, s-h_2 \mid s)\, F_{h_2}^T(s-h_2) \\ &\quad + Q(s).\end{aligned} \tag{15}$$

In contrast to KFTD (7)–(9) the receding horizon filter (RHF) (13)–(15) needs to initialize $(M+1)$ horizon initial conditions at $s = k - \Delta$ which represent unconditional means and covariances of the state $x(k)$; that is,

$$\begin{aligned}m\left(k-\Delta-\overline{M}+1\right) &\equiv \hat{x}\left(k-\Delta-\overline{M}+1 \mid k-\Delta\right) \\ &= \mathbf{E}\left\{x\left(k-\Delta-\overline{M}+1\right)\right\}, \\ m\left(k-\Delta-\overline{M}+2\right) &\equiv \hat{x}\left(k-\Delta-\overline{M}+2 \mid k-\Delta\right) \\ &= \mathbf{E}\left\{x\left(k-\Delta-\overline{M}+2\right)\right\}, \\ &\vdots \\ m(k-\Delta+1) &\equiv \hat{x}(k-\Delta+1 \mid k-\Delta) \\ &= \mathbf{E}\{x(k-\Delta+1)\},\end{aligned} \tag{16}$$

$$\begin{aligned}P(h_1, h_2) &\equiv P(h_1, h_2 \mid k-\Delta) = \operatorname{Cov}\{x(h_1), x(h_2)\}, \\ & h_1, h_2 = k-\Delta-\overline{M}+1, \ldots, k-\Delta+1.\end{aligned} \tag{17}$$

We have the following.

Theorem 1. *The horizon initial means* (16) *are described by*

$$\begin{aligned}m(t+1) &= \sum_{h=0}^{M} F_h(t-h)\, m(t-h), \\ & t = 0, 1, \ldots, k-\Delta+1, \\ m(0) = m(-1) &= \cdots = m(-M) = \overline{x}_0.\end{aligned} \tag{18}$$

Theorem 2. *The horizon initial covariances* (17) *satisfy Lyapunov-like recursive equations*

$$\begin{aligned}P(t+1, t+1) &= \sum_{h_1,h_2=0}^{M} F_{h_1}(t-h_1)\, P(t-h_1, t-h_2)\, F_{h_2}^T(t-h_2) \\ &\quad + Q(t), \quad t = 0, 1, \ldots, k-\Delta+1,\end{aligned} \tag{19a}$$

$$\begin{aligned}P(t-h_1+1, t-h_2+1) &= \sum_{\ell_1=0}^{M} F_{\ell_1}(t-h_1-\ell_1)\, P(t-h_1-\ell_1) \\ &\quad + Q(t-h_1)\,\delta_{t-h_1,t-h_2}, \quad k-h_1 \ge k-h_2, \\ P(t-h_1, t-h_2) &= P(t-h_2, t-h_1)^T, \quad t-h_1 < t-h_2, \\ P(-s_1, s_2) &= P_0, \quad s_1, s_2 = 1, 2, \ldots, M.\end{aligned} \tag{19b}$$

Derivation of Lyapunov-like equations for mean and covariances (18), (19a), and (19b) is given in the appendix.

Remark 3. Original initial conditions (10) for the KFTD (7)–(9) at different time instants $s = 0, -1, \ldots, -\overline{M}$ are identical in contrast to new horizon initial conditions (18), (19a), and (19b) which are more realistic for practice.

Remark 4 (real-time implementation of RHF). The RHF equations (13)–(19b) can be divided into two parts as follows.

Part 1 (offline equations). We may note that the RHF gains $G_m(s)$ and error autocovariances $P(s_1, s_2 \mid s)$ can be *precomputed*, since they do not depend on the receding horizon measurements $Y_{k-\Delta}^k$, but only on the system matrices $F_h(k-h)$, $H_d(k-d)$, noise statistics $Q(k)$, $R(k)$, and horizon initial conditions (18), (19a), and (19b) which are the part of system model (1) and (2).

Part 2 (online equations). Thus, once the measurement schedule has been settled, the real-time implementation of

the RHF requires only the computation of the RHF estimates $\hat{x}(s-m \mid s)$, $s = k-\Delta, k-\Delta+1, \ldots, k$, using only current measurements $y(k)$.

3.3. Conditional PDF and Optimal Receding Horizon Estimator. From (6) the optimal mean square estimate of the NCF $z(k) = f[x(k)]$ based on the receding horizon sensor measurements (11) also represents a conditional mean:

$$\hat{z}(k) = \mathbf{E}\left[z(k) \mid Y_{k-\Delta}^{k}\right] = \int f[x(k)]\, p\left(x(k) \mid Y_{k-\Delta}^{k}\right) dx(k), \tag{20}$$

where $p(x(k) \mid Y_{k-\Delta}^{k}) = \mathbf{N}(\hat{x}(k \mid k), P(k \mid k))$ is a Gaussian probability density function with conditional mean $\hat{x}(k \mid k) = \mathbf{E}[x(k) \mid Y_{k-\Delta}^{k}]$ and covariance $P(k \mid k)$ determined by RHF equations (13) and (15).

Thus, the estimate in (20) represents the *optimal receding horizon estimator* (RHE):

$$\hat{z}(k) = F(\hat{x}(k \mid k), P(k \mid k)), \tag{21}$$

which depends on the receding horizon estimate $\hat{x}(k \mid k)$ and its error covariance $P(k \mid k)$.

Further, we consider several examples of application of the general nonlinear estimator (20).

4. Examples of Optimal Nonlinear Estimator

4.1. Estimation of Distance. Consider an NFS representing a distance between an unknown scalar position $x(k)$ and given point $a(k)$. In this case an NFS becomes $z(k) = |x(k) - a(k)|$. Using (20) the best estimate of the distance between an unknown position $x(k)$ and moving point $a(k)$, $k = 0, 1, \ldots$, is

$$\begin{aligned}\hat{z}(k) &= \int_{-\infty}^{\infty} |x(k) - a(k)|\, \mathbf{N}(\hat{x}(k \mid k), P(k \mid k))\, dx(k) \\ &= |\hat{x}(k \mid k) - a(k)|\, \Phi\left(\frac{|\hat{x}(k \mid k) - a(k)|}{\sqrt{P(k \mid k)}}\right) \\ &\quad + \sqrt{2P(k \mid k)} \exp\left[-\frac{(\hat{x}(k \mid k) - a(k))^2}{2P(k \mid k)}\right], \\ \Phi(u) &= \frac{1}{\sqrt{2\pi}} \int_0^u e^{-t^2/2} dt,\end{aligned} \tag{22}$$

where $\hat{x}(k \mid k)$ and $P(k \mid k)$ are determined by (13) and (15) (or (7) and (9)), respectively, and $\Phi(u)$ is the error function.

In the particular case with $a(k) = 0$, the estimate of an unknown modulus $z(k) = |x(k)|$ takes the form

$$\begin{aligned}\hat{z}(k) &= \int_{-\infty}^{\infty} |x(k)|\, \mathbf{N}(\hat{x}(k \mid k), P(k \mid k))\, dx(k) \\ &= |\hat{x}(k \mid k)|\, \Phi\left(\frac{|\hat{x}(k \mid k)|}{\sqrt{P(k \mid k)}}\right) \\ &\quad + \sqrt{2P(k \mid k)} \exp\left[-\frac{\hat{x}(k \mid k)^2}{2P(k \mid k)}\right].\end{aligned} \tag{23}$$

4.2. Quadratic Function of State Vector. Consider an arbitrary quadratic function of a state vector $x(k) \in \mathfrak{R}^n$,

$$z(k) = f[x(k)] = x(k)^T \Omega x(k) + a(k)^T x(k), \quad \Omega^T = \Omega. \tag{24}$$

Then optimal nonlinear estimate (20) can be explicitly calculated in terms of a state estimate $\hat{x}(k \mid k)$ and its error covariance $P(k \mid k)$. Using the formula [24]

$$\begin{gathered}\mathbf{E}\left(x^T \Omega x\right) = \operatorname{tr}\left[\Omega\left(P + mm^T\right)\right], \\ m = \mathbf{E}(x), \; P = \operatorname{cov}(x, x),\end{gathered} \tag{25}$$

we obtain an optimal estimate for the quadratic function

$$\begin{aligned}\hat{z}(k) = &\operatorname{tr}\left[\Omega\left(P(k \mid k) + \hat{x}(k \mid k)\hat{x}(k \mid k)^T\right)\right] \\ &+ a(k)^T \hat{x}(k \mid k).\end{aligned} \tag{26}$$

In (25) and (26), $\operatorname{tr}(A)$ is the trace of a matrix A.

In the particular case a quadratic function represents a power of a signal $z(k) = x(k)^2$. Using (26) at $\Omega = 1$, $a(k) = 0$, we obtain

$$\hat{z}(k \mid k) = P(k \mid k) + \hat{x}(k)^2. \tag{27}$$

4.3. Estimation of Sine and Cosine Functions. In this case an NCF becomes

$$\begin{gathered}z^{(s)}(k) = \sin(x(k)), \\ z^{(c)}(k) = \cos(x(k)),\end{gathered} \tag{28}$$

where $x(k)$ is an unknown angle.

Then the best estimates of the trigonometrical functions of an unknown angle are

$$\begin{aligned}\hat{z}^{(s)}(k \mid k) &= \int_{-\infty}^{\infty} \sin(x(k))\, \mathbf{N}(\hat{x}(k \mid k), P(k \mid k))\, dx(k) \\ &= e^{-P(k|k)/2} \sin(\hat{x}(k \mid k)), \\ \hat{z}^{(c)}(k \mid k) &= \int_{-\infty}^{\infty} \cos(x(k))\, \mathbf{N}(\hat{x}(k \mid k), P(k \mid k))\, dx(k) \\ &= e^{-P(k|k)/2} \cos(\hat{x}(k \mid k)).\end{aligned} \tag{29}$$

5. Multivariate Polynomial Nonlinear Function

Here we consider a special NFS representing an arbitrary multivariate polynomial function of the form

$$\begin{aligned}&z(k) \\ &\quad = f[x(k)] \\ &\quad = \sum_{0 \le \ell_1 + \cdots + \ell_n \le n} A_{\ell_1 \ell_2, \ldots, \ell_n} x_1(k)^{\ell_1} x_2(k)^{\ell_2}, \ldots, x_n(k)^{\ell_n}, \\ &\qquad\qquad \ell_1, \ldots, \ell_n \ge 0.\end{aligned} \tag{30}$$

In this case, the algorithm for calculation of the optimal estimator $\hat{z}(k) = \mathbf{E}[f(x(k)) \mid Y_{k-\Delta}^{k}]$ has a closed form because conditional expectation

$$\begin{aligned}\hat{z}(k) &= \mathbf{E}\left[f(x(k)) \mid Y_{k-\Delta}^{k}\right] \\ &= \sum_{0\le \ell_1+\cdots+\ell_n\le n} A_{\ell_1\ell_2,\ldots,\ell_n}\mathbf{E}\left(x_1^{\ell_1}x_2^{\ell_2},\ldots,x_n^{\ell_n} \mid Y_{k-\Delta}^{k}\right)\end{aligned} \tag{31}$$

depends on high-order moments $m_{\ell_1\ell_2,\ldots,\ell_n} \equiv \mathbf{E}(x_1^{\ell_1}x_2^{\ell_2},\ldots,x_n^{\ell_n} \mid Y_{k-\Delta}^{k})$ of a multivariate Gaussian distribution, which can be explicitly calculated in terms of first- and second-order conditional moments $(\hat{x}(k \mid k), P(k \mid k))$ [24–26]. For example,

$$\begin{gathered} z(k) = A_0 + \sum_{i=1}^{N} A_i x(k)^i, \quad x(k) \in \mathfrak{R}, \\ \hat{z}(k) = A_0 + \sum_{i=1}^{N} A_i m_i(k), \quad m_i(k) = \mathbf{E}\left[x(k)^i \mid Y_{k-\Delta}^{k}\right], \end{gathered} \tag{32}$$

where

$$\begin{aligned} m_1(k) &= \hat{x}(k \mid k), \\ m_2(k) &= \hat{x}(k \mid k)^2 + P(k \mid k), \\ m_3(k) &= \hat{x}(k \mid k)^3 + 3\hat{x}(k \mid k)P(k \mid k), \\ m_4(k) &= \hat{x}(k \mid k)^4 + 6\hat{x}(k \mid k)^2 P(k \mid k) + 3P(k \mid k)^2, \\ m_5(k) &= \hat{x}(k \mid k)^5 + 10\hat{x}(k \mid k)^3 P(k \mid k) \\ &\quad + 15\hat{x}(k \mid k)P(k \mid k)^2, \\ m_6(k) &= \hat{x}(k \mid k)^6 + 15\hat{x}(k \mid k)^4 P(k \mid k) \\ &\quad + 45\hat{x}(k \mid k)^2 P(k \mid k)^2 + 15P(k \mid k)^3, \\ &\vdots \end{aligned} \tag{33}$$

Here $\hat{x}(k \mid k)$ and $P(k \mid k)$ are determined by the RHF equations.

For general NFS the unscented transformation serves as a useful tool for their calculations.

Remark 5. For typical NFS $z(k) = f[x(k)]$ the integral (20) for normal distribution is known; that is, it can be explicitly expressed in terms of a receding horizon state estimate and its error covariance (see examples in Sections 4 and 5). But for general NFS the integral (20) can be calculated only approximately, for example, using the unscented transformation.

6. General Nonlinear Function and Unscented Transformation

The unscented transformation (UT) was first proposed by Julier and Uhlmann in light of the intuition that it is significantly easier to approximate the statistics of a transformed random variable, for example, the mean and covariance [27, 28]. During the last decade, the UT has become a powerful approach for designing new filtering and control algorithms for nonlinear dynamic models [29, 30]. Following this approach, the procedure to calculate the best estimate of an NFS (conditional mean)

$$\hat{z}(k) = \mathbf{E}\left[f(x(k)) \mid Y_{k-\Delta}^{k}\right] \tag{34}$$

using the UT can be summarized as follows.

Generate the sigma points $\{X_s(k)\}_{s=0}^{2n}$ with corresponding weights $\{W_s\}_{s=0}^{2n}$:

$$\begin{aligned} X_0(k) &= \hat{x}(k \mid k), \qquad W_0 = \frac{\ell}{n+\ell}. \\ X_s(k) &= \hat{x}(k \mid k) + \left[\sqrt{(n+l)P(k \mid k)}\right]_s, \\ W_s &= \frac{1}{2(n+\ell)}, \\ X_{s+n}(k) &= \hat{x}(k \mid k) - \left[\sqrt{(n+l)P(k \mid k)}\right]_s, \\ W_{s+n} &= \frac{1}{2(n+\ell)}, \\ s &= 1, 2, \ldots, n, \end{aligned} \tag{35}$$

where $[\sqrt{P(k \mid k)}]_s$ is the sth column of the matrix square root of $P(k \mid k)$ and ℓ is the scaling parameter influencing the spreading of the points in the state space and thus the accuracy of the approximation [31]. Propagate each of these sigma points through a nonlinear function as

$$\xi_s(k) = f\left(X_s(k)\right), \quad s = 0, 1, \ldots, 2n \tag{36}$$

and the resulting best estimate of the NFS is given as

$$\hat{z}(k) = \sum_{s=0}^{2n} W_s \xi_s(k). \tag{37}$$

Thus, the estimate (37) is approximately represented by known function of the receding horizon estimate $\hat{x}(k \mid k)$ and error covariance $P(k \mid k)$.

Application of the obtained results is illustrated by example of wind tunnel system with model uncertainties.

7. Numerical Example: The Wind Tunnel System

A comparative experimental analysis of the proposed RHF (11)–(19b) and KFTD (7)–(10) is considered on example of nonlinear estimation of a kinetic energy of the wind tunnel system with two model time-invariant uncertainties δ_1 and δ_2. In this case, we numerically evaluate and compare three mean square errors (MSEs) of the RHF, KFTD, and nonlinear estimator for kinetic energy.

A discretization of the state differential equations and the design based on a discrete-time model has been applied to the high-speed closed-air unit wind tunnel model by Armstrong

and Tripp [32]. The state vector $x(k) \in \Re^3$ consists of the state variables $x_1(k)$, $x_2(k)$, and $x_3(k)$, representing derivatives from a chosen equilibrium point of the following quantities: x_1 = Mach number, x_2 = actuator position guide vane angle in a driving fan, and x_3 = actuator rate. Then the discretized system model with a sampling period of $T = 0.5$ (sec) is given by

$$x(k+1) = \begin{bmatrix} 0.9032+\delta_1 & 0 & 0 \\ 0 & 0.6245 & 0.0701 \\ 0 & -2.5247 & -0.0488 \end{bmatrix} x(k) + \begin{bmatrix} 0 & -0.0011+\delta_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(k-1) + w(k), \tag{38}$$

where $\delta_1(k) = \delta_1$ and $\delta_2(k) = \delta_2$ are unknown time-invariant model parameters and a wind tunnel system with the initial mean and covariance is $\overline{x}(0) = \mathbf{E}[x(0)] = [3 \;\; 28 \;\; 10]^T$ and $P(0) = \mathrm{diag}[1 \;\; 1 \;\; 0 \;\; 0]$. The covariance of the system white Gaussian noise $w(k) \in \Re^3$ is subjected to $Q(k) = \mathrm{diag}\{0.01^2 \;\; 0.01^2 \;\; 0.01^2\}$.

The first coordinate $x_1(k)$, related to Mach number of the wind tunnel system, is observable through a measurement model. We have

$$y(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + v(k), \tag{39}$$

where $v(k) \in \Re^2$ is zero-mean white Gaussian measurement noise with covariance $R(k) = \mathrm{diag}[0.5 \;\; 0.5]$. We assume that the true values of the model uncertainties δ_1 and δ_2 on the given *uncertainty interval* (UI) $T_{\mathrm{UI}} = [80, 200]$ are

$$\begin{aligned} \delta_1(k) &= \delta_1 = 0.5, \\ \delta_2(k) &= \delta_2 = 0.1, \\ k &\in T_{\mathrm{UI}} = [80, 200] \end{aligned} \tag{40}$$

and without it, that is,

$$\begin{aligned} \delta_1(k) &= \delta_2(k) = 0, \\ k &\in T_{\mathrm{UI}} = [80, 200]. \end{aligned} \tag{41}$$

The horizon length Δ of the RHF (13)–(19b) and RHE (20) takes three values as $\Delta = 3, 5$, and 10.

The total kinetic energy of an actuator representing the NFS, $z(k) = f[x(k)]$, can be expressed as the sum of the translational kinetic energy of the center of mass (E^t) and the rotational kinetic energy about the center of mass (E^r). The kinetic energies can be expressed in the following quadratic form (see Section 4.2):

$$\begin{aligned} E^r &= \frac{I\omega^2}{2} = \frac{I\dot{x}_2^2}{2}, \qquad E^t = \frac{mv^2}{2} = \frac{mx_3^2}{2}, \\ \text{Total Energy} &= z(k) = E^r + E^t = \frac{I\dot{x}_2^2 + mx_3^2}{2}, \end{aligned} \tag{42}$$

where I is rotational inertia, ω is angular velocity, m is mass, and v is linear velocity. Concrete numerical values for $I = 0.136\,\mathrm{kgm}^2$, $\omega = 55$ rad/s, $m = 7.39$ kg, and $v = 15$ m/s.

7.1. Simulation Results with Parametric Uncertainty. We assume that time-invariant uncertain model parameters $\delta_i(k) \equiv \delta_i$ take the form (40). We specially focus on comparing the MSEs of the Mach number of the wind tunnel system $x_1(k)$ that directly contain the uncertainties δ_i in (38) and the total kinetic energy $z(k)$ in (42).

Our point of interest is the behavior of the nonlinear RHE $\hat{z}(k)$ using proposed RHF $\hat{x}(k \mid k)$ around the uncertainty interval $T_{\mathrm{UI}} = [80, 200]$. Here we describe the results of simulations of two filters (KFTD, RHF) and RHE $\hat{z}(k)$ with three different receding horizon lengths Δ in terms of MSEs such that

$$\begin{aligned} P_1^{\mathrm{KFTD}}(k \mid k) &= \mathbf{E}\left[x_1(k) - \hat{x}_1^{\mathrm{KFTD}}(k \mid k)\right]^2, \\ P_{1,\Delta}^{\mathrm{RHF}}(k \mid k) &= \mathbf{E}\left[x_1(k) - \hat{x}_{1,\Delta}^{\mathrm{RHF}}(k \mid k)\right]^2, \\ P_z^{\mathrm{KFTD}}(k \mid k) &= \mathbf{E}\left[z(k) - \hat{z}^{\mathrm{KFTD}}(k \mid k)\right]^2, \\ P_{z,\Delta}^{\mathrm{RHF}}(k \mid k) &= \mathbf{E}\left[z(k) - \hat{z}_{\Delta}^{\mathrm{RHE}}(k \mid k)\right]^2, \\ &\qquad \text{for } \Delta = 3, 5, 10. \end{aligned} \tag{43}$$

In Figures 1 and 2, we observe that, within the uncertainty interval $T_{\mathrm{UI}} = [80, 200]$, the time histories of the MSEs for the KFTD are notably larger than the other RHFs. However, the KFTD performs slightly worse than the RHF with the biggest horizon length of $\Delta = 10$. Also, the RHF/RHE with the small horizon length $\Delta = 3$ is more accurate than the RHF/RHE with the big horizon lengths $\Delta = 5$ and $\Delta = 10$. Therefore,

$$\begin{aligned} &P_{1,\Delta=3}^{\mathrm{RHF}}(k \mid k) < P_{1,\Delta=5}^{\mathrm{RHF}}(k \mid k) < P_{1,\Delta=10}^{\mathrm{RHF}}(k \mid k) \\ &\qquad < P_1^{\mathrm{KFTD}}(k \mid k), \\ &P_{z,\Delta=3}^{\mathrm{RHF}}(k \mid k) < P_{z,\Delta=5}^{\mathrm{RHF}}(k \mid k) < P_{z,\Delta=10}^{\mathrm{RHF}}(k \mid k) \\ &\qquad < P_z^{\mathrm{KFTD}}(k \mid k) \\ &\qquad\qquad \text{for } k \in T_{UI}. \end{aligned} \tag{44}$$

The reason for the presence of such a robustness property (44) is to compensate for the given uncertainties $\delta_i(k)$, as the horizon length Δ for sensor (memory of RHF) should be minimal. In this case, it is equal, as $\Delta = 3$.

The comparison shows us that performance of the RHF/RHE is quite good and it can be widely used in different areas of applications: industrial, military, space, inertial navigation, and others [33].

8. Conclusion

In this paper we propose a new RHF for discrete-time linear system with time-delays and model uncertainties with application to estimation of nonlinear functions of state vector. We have derived the Lyapunov-like recursive equations for horizon initial mean and covariance of a system state with an arbitrary number of time-delays and verified the effectiveness

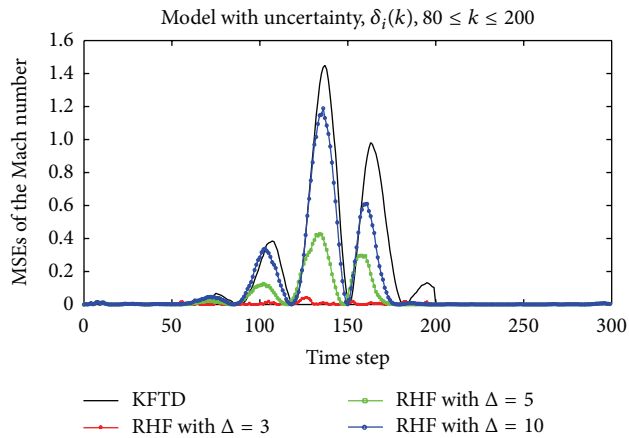

FIGURE 1: MSEs of the Mach number x_1 comparison among KFTD and three RHFs with different horizon lengths: $\Delta = 3$ (red line); $\Delta = 5$ (green line); $\Delta = 10$ (blue line); KFTD (black line). The x-axis represents time and y-axis represents MSEs.

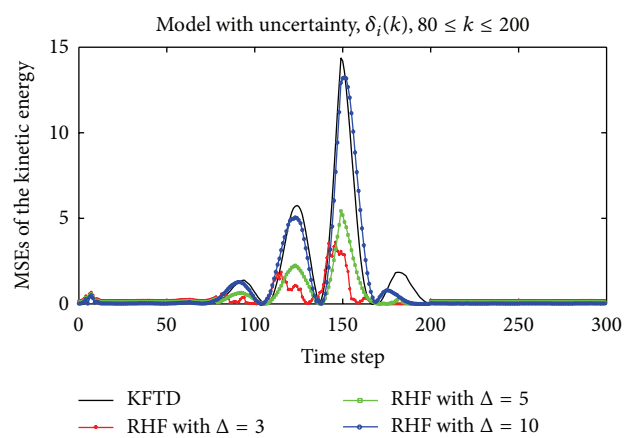

FIGURE 2: Comparison of MSEs of the total kinetic energy z using KFTD and three RHFs with horizon lengths: $\Delta = 3$ (red line); $\Delta = 5$ (green line); $\Delta = 10$ (blue line); KFTD (black line). The x-axis represents time and y-axis represents MSEs.

of the proposed filter RHF through numerical simulation and comparison with KFTD.

In order to estimate an arbitrary NCF in uncertain environment, a new RHE is proposed. Two important classes of an NCF (quadratic, polynomial) are considered in detail, and effective computational procedures based on formulas for high-order moments of a Gaussian distribution and UT transformation are proposed.

In verifying we implemented example for real dynamic system with time-delays and parametric model uncertainty $\delta(k)$. Through the implementations, the robustness of the RHF/RHE in terms of MSEs is demonstrated and the proposed receding horizon estimators have a potential to be used in a wide range of practical applications.

During the last decades, there has been extensive interest in the study of a class of physical systems modeled by hybrid system dynamics known as Markovian jump systems including multiple delays [34], mode-dependent delays [35], singular time-delays [36], and time-variant delays [37]. As a generalization of the obtained results for linear stochastic systems with time-delays and uncertainties we would like to point out that it is possible to extend the main results to Markovian jump systems with time-delays and complex dynamical systems with prevalent network induced phenomena such as missing measurements, sensor delays, multiple fading measurements, and signal quantization.

Appendix

Derivation of Equation for Horizon Initial Mean (18). Taking expectation on both sides of (1) and using $\mathbf{E}[w(t)] = 0$ we immediately obtain recursive equation (18) for mean $m(t) = \mathbf{E}[x(t)]$.

Derivation of Equation For Horizon Initial Covariance (19a). Subtracting (18) from (1) we obtain time propagation of the centered state:

$$\tilde{x}(t+1) = \sum_{h=0}^{M} F_h(t-h)\,\tilde{x}(t-h) + w(t), \quad t = 0, 1, 2, \ldots. \tag{A.1}$$

Next we have

$$\begin{aligned}
&\tilde{x}(t+1)\,\tilde{x}(t+1)^T \\
&\quad = \sum_{h_1,h_2=0}^{M} F_{h_1}(t-h_1)\,\tilde{x}(t-h_1)\,\tilde{x}(t-h_2)^T F_{h_2}^T(t-h_2) \\
&\qquad + w(t)\,w(t)^T + \sum_{h_1=0}^{M} F_{h_1}(t-h_1)\,\tilde{x}(t-h_1)\,w(t)^T \\
&\qquad + \sum_{h_2=0}^{M} w(t)\,\tilde{x}(t-h_2)^T F_{h_2}^T(t-h_2).
\end{aligned} \tag{A.2}$$

Taking expectation on both sides of (A.2) and using the fact that current noise $w(t)$ does not depend on current and past states $\tilde{x}(t-h_1)$, $\tilde{x}(t-h_2)$ we obtain recursive equation for covariances (19a) and (19b),

$$\begin{aligned}
&P(t+1, t+1) \\
&\quad = \sum_{h_1,h_2=0}^{M} F_{h_1}(t-h_1)\,P(t-h_1, t-h_2)\,F_{h_2}^T(t-h_1) + Q(t).
\end{aligned} \tag{A.3}$$

Note that (A.3) contains autocovariance:

$$P(t-h_1, t-h_2) = \mathbf{E}\left[\tilde{x}(t-h_1)\,\tilde{x}(t-h_2)^T\right], \tag{A.4}$$

$$\begin{aligned}
\tilde{x}(t-h) &= x(t-h) - m(t-h), \\
h_1, h_2 &= 0, 1, \ldots, M.
\end{aligned} \tag{A.5}$$

Derivation of Equation for Autocovariance (19b). Using "symmetric" property of autocovariance $P(t-h_2, t-h_1) = P(t-h_1, t-h_2)^T$ and without loss of generality we can assume that $k-h_1 \geq k-h_2$. Substituting $t \to t-h_1$ in (A.1) we obtain

$$\tilde{x}(t-h_1+1) = \sum_{\ell_1=0}^{M} F_{l_1}(t-h_1-\ell_1)\,\tilde{x}(t-h_1-\ell_1) + w(t-h_1). \tag{A.6}$$

Multiplying both sides of (A.6) by $\tilde{x}(t-h_2+1)^T$ and using (A.5) we obtain

$$\begin{aligned}\tilde{x}(t-h_1+1)\,\tilde{x}(t-h_2+1)^T &= \sum_{\ell_1=0}^{M} F_{\ell_1}(t-h_1-\ell_1)\,\tilde{x}(t-h_1-\ell_1)\,\tilde{x}(t-h_2+1)^T \\ &\quad + w(t-h_1)\,\tilde{x}(t-h_2+1)^T,\end{aligned} \tag{A.7}$$

$$\begin{aligned}P(t-h_1+1, t-h_2+1) &= \sum_{\ell_1=0}^{M} F_{\ell_1}(t-h_1-\ell_1)\,P(t-h_1-\ell_1, t-h_2+1) \\ &\quad + \mathbf{E}\left[w(t-h_1)\,\tilde{x}(t-h_2+1)^T\right], \\ &\qquad h_1, h_2 = 0, 1, \ldots, M.\end{aligned} \tag{A.8}$$

It remains to calculate expectation in (A.8); that is,

$$\mathbf{E}\left[w(t-h_1)\,\tilde{x}(t-h_2+1)^T\right] \quad \text{for } t-h_1 \geq t-h_2. \tag{A.9}$$

Calculating product $w(t-h_1)\tilde{x}(t-h_2+1)^T$ using (A.6) and after that taking expectation we get

$$\begin{aligned}\mathbf{E}\left[w(t-h_1)\,\tilde{x}(t-h_2+1)^T\right] &= \sum_{\ell_2=0}^{M} \mathbf{E}\left[w(t-h_1)\,\tilde{x}(t-h_2-\ell_2)^T\right] F_{l_2}^T(t-h_2-\ell_2) \\ &\quad + \mathbf{E}\left[w(t-h_1)\,w(t-h_2)^T\right].\end{aligned} \tag{A.10}$$

According to assumption $t-h_1 \geq t-h_2$, "future" noise $w(t-h_1)$ does not depend on current and past states $\tilde{x}(t-h_2-\ell_2)$; therefore $\mathbf{E}[w(t-h_1)\tilde{x}(t-h_2-\ell_2)^T] = 0$. Next using property of white noise we obtain

$$\mathbf{E}\left[w(t-h_1)\,w(t-h_2)^T\right] = Q(t-h_1)\,\delta_{t-h_1,t-h_2}. \tag{A.11}$$

Finally using (A.8), (A.10), and (A.11) we get equation for autocovariance (19b).

This completes the derivation Lyapunov-like equations for receding horizon mean and covariances.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported in part by the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (no. 2012R1A1A2000679).

References

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1979.

[2] E. K. Boukas and Z.-K. Liu, *Deterministic and Stochastic Time-Delay Systems*, Birkhauser, Boston, Mass, USA, 2002.

[3] M. Malek-Zavarei and M. Jamshidi, *Time-Delay Systems: Analysis, Optimization and Application*, North-Holland, Amsterdam, The Netherlands, 1987.

[4] J. Chiasson and J. J. Loiseau, *Applications of Time Delay Systems*, vol. 352 of *Lecture Notes in Control and Information Sciences*, Springer, London, UK, 2007.

[5] R. Sipahi, T. Vyhlidal, S.-I. Niculescu, and P. Pepe, Eds., *Time Delay Systems: Methods, Applications and New Trends*, vol. 423 of *Lecture Notes in Control and Information Sciences*, Springer, London, UK, 2012.

[6] M. S. Mahmoud, *Robust Control and Filtering for Time-Delay Systems*, Marcel Dekker, New York, NY, USA, 2000.

[7] H. Kwakernaak, "Optimal filtering in linear systems with time delays," *IEEE Transactions on Automatic Control*, vol. 12, no. 2, pp. 169–173, 1967.

[8] A. G. O. Mutambara, *Decentralized Estimation and Control for Multisensor Systems*, CRC Press, New York, NY, USA, 1998.

[9] K. Kobayashi, K. C. Cheok, K. Watanabe, and F. Munekata, "Accurate differential global positioning system via fuzzy logic kaiman filter sensor fusion technique," *IEEE Transactions on Industrial Electronics*, vol. 45, no. 3, pp. 510–518, 1998.

[10] W. H. Kwon and S. Han, *Receding Horizon Control: Model Predictive Control for State Models*, Springer, London, UK, 2005.

[11] W. Watanabe, *Adaptive Estimation and Control*, Prentice-Hall, New York, NY, USA, 1991.

[12] F. C. Schweppe, *Uncertain Dynamic Systems*, Prentice-Hall, Upper Saddle River, NJ, USA, 1973.

[13] D. Y. Kim, J. I. Ahn, and V. Shin, "Suboptimal filter for continuous-time linear systems with unknown parameters," *Asian Journal of Control*, vol. 10, no. 5, pp. 525–534, 2008.

[14] V. Shin, J. I. Ahn, and D. Y. Kim, "New algorithm for recursive estimation in linear discrete-time systems with unknown parameters," *International Journal of Control, Automation and Systems*, vol. 4, no. 4, pp. 456–465, 2006.

[15] A. H. Jazwinski, *Stochastic Processes and Filtering Theory*, Academic Press, New York, NY, USA, 1970.

[16] H. Gao, J. Lam, and C. Wang, "Robust energy-to-peak filter design for stochastic time-delay systems," *Systems and Control Letters*, vol. 55, no. 2, pp. 101–111, 2006.

[17] P. Shi, "Robust filtering for uncertain delay systems under sampled measurements," *Signal Processing*, vol. 58, no. 2, pp. 131–151, 1997.

[18] A. Alessandri, M. Baglietto, and G. Battistelli, "Receding-horizon estimation for discrete-time linear systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 3, pp. 473–478, 2003.

[19] A. Alessandri, M. Baglietto, and G. Battistelli, "Receding-horizon estimation for switching discrete-time linear systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1736–1748, 2005.

[20] W. H. Kwon, P. S. Kim, and P. Park, "A receding horizon Kalman FIR filter for discrete time-invariant systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 9, pp. 1787–1791, 1999.

[21] D. Y. Kim and V. Shin, "Optimal receding horizon filter for continuous-time nonlinear stochastic systems," in *Proceedings of the 6th WSEAS International Conference on Signal Processing (WSEAS '07)*, pp. 112–116, Dallas, Tex, USA, 2007.

[22] J. Mishra and V. S. Rajamani, "Least-squares state estimation in time-delay systems with colored observation noise: an innovation approach," *IEEE Transactions on Automatic Control*, vol. 20, no. 1, pp. 140–142, 1975.

[23] R. Priemer and A. G. Vacroux, "Estimation in linear discrete systems with multiple time delays," *IEEE Transactions on Automatic Control*, vol. 14, pp. 384–387, 1969.

[24] V. S. Pugachev and I. N. Sinitsyn, *Stochastic Differential Systems. Analysis and Filtering*, Wiley & Sons, New York, NY, USA, 1987.

[25] R. Kan, "From moments of sum to moments of product," *Journal of Multivariate Analysis*, vol. 99, no. 3, pp. 542–554, 2008.

[26] B. Holmquist, "Expectations of products of quadratic forms in normal variables," *Stochastic Analysis and Applications*, vol. 14, no. 2, pp. 149–164, 1996.

[27] S. Julier, J. Uhlmann, and H. F. Durrant-Whyte, "A new method for the nonlinear transformation of means and covariances in filters and estimators," *IEEE Transactions on Automatic Control*, vol. 45, no. 3, pp. 477–482, 2000.

[28] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proceedings of the IEEE*, vol. 92, no. 3, pp. 401–422, 2004.

[29] D. J. Lee, "Nonlinear estimation and multiple sensor fusion using unscented information filtering," *IEEE Signal Processing Letters*, vol. 15, pp. 861–864, 2008.

[30] F. Gustafsson and G. Hendeby, "Some relations between extended and unscented Kalman filters," *IEEE Transactions on Signal Processing*, vol. 60, no. 2, pp. 545–555, 2012.

[31] L. Chang, B. Hu, A. Li, and F. Qin, "Transformed unscented Kalman filter," *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 252–257, 2013.

[32] E. S. Armstrong and J. S. Tripp, "An application of multivariable design techniques to the control of the National Transonic Facility," NASA Technical Reports 1887, 1981.

[33] R. C. Luo and M. G. Kay, "Multisensor integration and fusion in intelligent systems," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 19, no. 5, pp. 901–931, 1989.

[34] F. Li, P. Shi, L. Wu, and X. Zhang, "Fuzzy-model-based D-stability and non-fragile control for discrete-time descriptor systems with multiple selays," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 1019–1025, 2014.

[35] F. Li, L. Wu, and P. Shi, "Stochastic stability of semi-Markovian jump systems with mode-dependent delays," *International Journal of Robust and Nonlinear Control*, 2014.

[36] L. Wu, X. Su, and P. Shi, "Slidin g mode control with bounded l_2 gain performance of Markovian jump singular time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1929–1933, 2012.

[37] F. Li and X. Zhang, "A delay-dependent bounded real lemma for singular LPV systems with time-variant delay," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 5, pp. 559–574, 2012.