Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 231749, 11 pages
http://dx.doi.org/10.1155/2015/231749

Research Article

Observer-Based Robust Fault Detection Filter Design and Optimization for Networked Control Systems

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Received 4 April 2014; Accepted 17 August 2014

Academic Editor: Minrui Fei

The problem of robust fault detection filter (FDF) design and optimization is investigated for a class of networked control systems (NCSs) with random delays. The NCSs are modeled as Markovian jump systems (MJSs) by assuming that the random delays obey a Markov chain. Based on the model, an observer-based residual generator is constructed and the corresponding fault detection problem is formulated as an H_∞ filtering problem by which the error between the residual signal and the fault is made as small as possible. A sufficient condition for the existence of the desired FDF is derived in terms of linear matrix inequalities (LMIs). Furthermore, to improve the performance of the robust fault detection systems, a time domain optimization approach is proposed. The solution of the optimization problem is given in the form of Moore-Penrose inverse of matrix. A numerical example is provided to illustrate the effectiveness and potential of the proposed approach.

1. Introduction

Networked control systems (NCSs) are feedback control systems in which sensors, controllers, actuators, and other system components are connected with real-time networks [1, 2]. The new structure has many advantages over conventional control systems, such as low cost, simple installation and maintenance, reliability, and enhanced resource utilization, which make NCSs a promising structure for control systems [3]. However, this structure also brings challenges on NCSs analysis and design, for instance, network induced delay and packet dropout [4–8], which inevitably increase the complexity of system design and degrade the system performance [9]. As an important essential to improve the performance, safety, and reliability of dynamic systems, fault detection technique for NCSs has recently attracted considerable attention [10, 11].

Network induced delay is an active field of NCSs research. So far, there are fruitful results in fault detection for NCSs with various network induced delays [12–16]. In [12], the influence caused by unknown network induced delays is transformed into time-varying polytopic uncertainty. Assisted by parameter-dependent Lyapunov function matrix, an optimal fault detection filter (FDF) is designed to detect faults. In many cases, network induced delays are random and can be modeled as Markov chains [17–20]. In the literature [17], by employing the multirate sampling method together with the augmented state matrix method, NCSs with long random delays are modeled as Markovian jump systems (MJSs). Then based on the model, an H_∞ filter is designed for detecting faults. Since all or part of the elements in the desired transition probabilities matrix are hard or costly to obtain, a robust FDF for discrete-time MJSs with partially known transition probabilities is designed in the literature [21]. Moreover, in order to improve the performance of fault detection systems, time domain optimization approaches are proposed for observed-based fault detection systems [22–24].

To the best of authors' knowledge, the problem of robust FDF design and optimization for a class of NCSs, which can be modeled as MJSs, has not been fully investigated yet. This motivates us to study this interesting

and challenging problem, which has great potential in practical applications.

This paper addresses the problem of robust FDF design and optimization for a class of NCSs with random delays and the main achievement is composed of the following four steps. Firstly, the NCSs are modeled as MJSs and the partially known transition probabilities of the Markov process are taken into account. Secondly, an observer-based residual generator is constructed and the robust fault detection problem is formulated as an H_∞ filtering problem. A sufficient condition for the existence of the desired FDF is derived in terms of linear matrix inequalities (LMIs). Thirdly, a time domain optimization approach for detecting smaller faults is proposed for the robust fault detection systems. The optimal solution of the problem is given in terms of Moore-Penrose inverse of matrix. Lastly, a numerical example is provided to illustrate the effectiveness and potential of the proposed approach.

2. Problem Formulation

Consider the following continuous-time, state-space model of the linear time-invariant plant dynamics:

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) + B_{cd} d(t) + B_{cf} f(t) \\ y(t) &= Cx(t) + D_d d(t) + D_f f(t), \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^l$, and $f \in R^q$ denote the state, the control input, the output, and the latent fault, respectively, $d \in R^p$ is the external disturbance belonging to $l_2[0,\infty)$, and the real matrices $A_c, B_c, B_{cd}, B_{cf}, C, D_d$, and D_f are of appropriate dimensions.

Consider the NCSs as [17]; we introduce the following assumptions.

Assumption 1 (see [1, 14]). The sampling period of the NCSs is T. The sensors are clock-driven, and the controllers and actuators are time-division-driven with the same time-division. There are no packet dropout and packet disordering. The control law is fixed. The sensor-to-controller delay is τ_k^{sc} and the controller-to-actuator delay is τ_k^{ca}. $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ is introduced to denote the network induced delay at time instant k and supposed to be smaller than the sampling period T in this paper.

Assumption 2 (see [17]). The sampling interval $[kT, (k+1)T]$ is divided into N pieces.

From Assumption 1, system (1) can be transformed into the following discrete time model:

$$\begin{aligned} x(k+1) &= Ax(k) + \sum_{i=0}^{1} B_i(\tau_k) u(k-i) \\ &\quad + B_d d(k) + B_f f(k), \\ y(k) &= Cx(k) + D_d d(k) + D_f f(k), \end{aligned} \tag{2}$$

where $A = e^{A_c T}$, $B_0(\tau_k) = \int_0^{T-\tau_k} e^{A_c t} B_c dt$, $B_1(\tau_k) = \int_{T-\tau_k}^{T} e^{A_c t} B_c dt$, $B_d = \int_0^T e^{A_c(T-t)} B_{cd} dt$, and $B_f = \int_0^T e^{A_c(T-t)} B_{cf} dt$.

Remark 3. In this paper, we only consider the network induced delay $\tau_k < T$. In more general case, the network induced delay $\tau_k < d_1 T$ and $\tau_k^{sc} < d_2 T$ with d_1 and d_2 being nonnegative integers. System (1) can be written as

$$\begin{aligned} x(k+1) &= Ax(k) + \sum_{i=0}^{d_1} B_i(\tau_k) u(k-i) \\ &\quad + B_d d(k) + B_f f(k) \\ y(k) &= Cx\left(k - \operatorname{floor}\left(\frac{\tau_k^{sc}}{T}\right)\right) + D_d d(k) + D_f f(k), \end{aligned} \tag{3}$$

where floor$(\cdot)$ stands for the function rounding towards minus infinity.

From Assumption 2 and similar to [10, 17], we can obtain that the random network induced delay $\tau_k = (h(k) - 1)T/N$, where the sequence $\{h(k)\}$ can be considered as a discrete-time homogeneous Markov chain taking values in the following finite state space $\Gamma = \{1, 2, \ldots, N\}$ and $\pi = [\pi_{ij}]$ is the stationary transition probability matrix with its elements defined as $\pi_{ij} = \text{Prob}\{h(k+1) = j \mid h(k) = i\} > 0$ and $\sum_{j\in\Gamma} \pi_{ij} = 1$. In addition, for all or part of the elements in π are hard or costly to obtain, we assume that the stationary transition probabilities of the Markov chain in this paper are partially known. For notation clarity [21], $\forall i \in \Gamma$, we denote $\Gamma_k^i = \{j : \pi_{ij} \text{ is known}\}$, $\Gamma_{uk}^i = \{j : \pi_{ij} \text{ is unknown}\}$. Also, we denote that $\pi_k^i = \sum_{j\in\Gamma_k^i} \pi_{ij}$ throughout the paper.

Considering the control input signal $u(k) = -Kx(k)$, then (2) can be equivalently written as the following MJSs:

$$\begin{aligned} z(k+1) &= A_{i,k} z(k) + \overline{B}_d d(k) + \overline{B}_f f(k), \\ y(k) &= \overline{C} z(k) + D_d d(k) + D_f f(k), \end{aligned} \tag{4}$$

where one has $z(k) = \begin{bmatrix} x^T(k) & x^T(k-1) \end{bmatrix}^T$, $A_{i,k} = \begin{bmatrix} A - B_0(\tau_k)K & -B_1(\tau_k)K \\ I & 0 \end{bmatrix}$, $\overline{B}_d = \begin{bmatrix} B_d^T & 0 \end{bmatrix}^T$, $\overline{B}_f = \begin{bmatrix} B_f^T & 0 \end{bmatrix}^T$, $\overline{C} = \begin{bmatrix} C & 0 \end{bmatrix}$, and the subscript i, k of $A_{i,k}$ denotes $h(k) = i \in \Gamma$ at time instant k.

An observer-based FDF is adopted to generate residual signal:

$$\begin{aligned} \hat{z}(k+1) &= A_{i,k} \hat{z}(k) + L_{i,k}\left(y(k) - \overline{C}\hat{z}(k)\right), \\ r(k) &= y(k) - \overline{C}\hat{z}(k), \end{aligned} \tag{5}$$

where $\hat{z} \in R^{2n}$ is the state estimation vector of $z(k)$, $r \in R^l$ is the generated residual signal, and $L_{i,k}$ is the filter's gain matrix to be designed.

Set the filter error $e(k) = z(k) - \hat{z}(k)$, and then the overall fault detection system is given by

$$\begin{aligned} e(k+1) &= \overline{A}_{i,k} e(k) + \overline{B}_{i,k} w(k) \\ r(k) &= \overline{C} e(k) + \overline{D} w(k), \end{aligned} \tag{6}$$

where $\overline{A}_{i,k} = A_{i,k} - L_{i,k}\overline{C}$, $\overline{B}_{i,k} = \left[B_{Ld}^{i,k} \;\; B_{Lf}^{i,k}\right]$, $B_{Ld}^{i,k} = \overline{B}_d - L_{i,k}D_d$, $B_{Lf}^{i,k} = \overline{B}_f - L_{i,k}D_f$, $\overline{D} = [D_d \;\; D_f]$, and $w(k) = \left[d^T(k) \;\; f^T(k)\right]^T$.

Remark 4. In (5) and (6), the subscripts or superscripts i, k have the same meanings as the subscript of $A_{i,k}$ in system (4).

After the above manipulations, the original robust FDF problem for system (1) can be further converted to find a series of filter gain matrices $L_{i,k}$ such that the MJSs (6) are stochastically stable and under zero initial condition, and the H_∞ performance index γ is made as small as possible in the feasibility of [21]

$$\sup_{w(k)\neq 0} E\left\{\frac{\|r(k)\|^2}{\|w(k)\|^2}\right\} < \gamma^2, \quad \gamma > 0. \tag{7}$$

For improving the performance of the fault detection system (6), we use a time domain optimization approach to optimize the fault detection system (6). Let $\xi(k) = V_k(z)r(k) = (V_{s,k} + V_{s-1,k}z^{-1} + \cdots + V_{0,k}z^{-s})r(k)$ denote the modified residual signal [25], where matrix $V_k(z)$ is called the postfilter [22, 25]. Then the residual evaluation function can be selected as

$$J(k) = \|\xi(k)\|_e = \left[\frac{1}{\beta+1}\sum_{i=k-\beta}^{k} \xi^T(i)\,\xi(i)\right]^{1/2}, \tag{8}$$

where β denotes the detection window.

Then the fault can be observed by comparing $J(k)$ with a threshold J_{th} according to the following logic:

$$\begin{aligned} J(k) > J_{\text{th}} &\Longrightarrow \text{alarm for fault} \\ J(k) \le J_{\text{th}} &\Longrightarrow \text{no fault.} \end{aligned} \tag{9}$$

Remark 5. Note that the threshold J_{th} in (9) is the minimum threshold that prevents false alarms and it is also an adaptive threshold which will be shown in the following section.

3. Main Results

In this section, we will discuss the robust FDF design problem of system (4) with partially known transition probabilities and the time domain optimization of fault detection systems (6).

3.1. Filter Gain Design. To finish the filter design based on the MJSs model (6) with partially known transition probabilities, we first introduce the following lemma which will help us to derive the gain of the FDF (5).

Lemma 6 (see [21]). *Consider system* (6) *with partially known transition probabilities and let $\gamma > 0$ be a given scalar. If there exist matrices $P_i > 0, G_i > 0$, $\forall i \in \Gamma$, such that*

$$\begin{bmatrix} -P_i & 0 & \overline{A}_{i,k}^T G_i^T & \overline{C}^T \\ * & -\gamma^2 I & \overline{B}_{i,k}^T G_i^T & \overline{D}^T \\ * & * & -G_i - G_i^T + \overline{P}_j & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{10}$$

where asterisk ($*$) is used to represent a term that is induced by symmetry in symmetric block matrices, and

$$\begin{aligned} \overline{P}_j &= \sum_{j\in\Gamma_k^i} \frac{\pi_{ij}P_j}{\pi_k^i} \quad j \in \Gamma_k^i \\ \overline{P}_j &= P_j \quad j \in \Gamma_{uk}^i \end{aligned} \tag{11}$$

then the system (6) *is stochastically stable with an H_∞ performance index γ.*

As an application of Lemma 6, the following theorem provides a sufficient condition for the existence of an admissible H_∞ FDF in the form of (5).

Theorem 7. *Consider system* (6) *with partially known transition probabilities and let $\gamma > 0$ be a given scalar. If there exist matrices $P_i > 0$, $G_i > 0$, and $\forall i \in \Gamma$ and matrices K_i, $\forall i \in \Gamma$, such that*

$$\begin{bmatrix} -P_i & 0 & 0 & A_{i,k}^T G_i^T - \overline{C}^T K_i^T & \overline{C}^T \\ * & -\gamma^2 I & 0 & \overline{B}_d^T G_i^T - D_d^T K_i^T & D_d^T \\ * & * & -\gamma^2 I & \overline{B}_f^T G_i^T - D_f^T K_i^T & D_f^T \\ * & * & * & -G_i - G_i^T + \overline{P}_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{12}$$

where $A_{i,k}$, $\overline{C}$, and $\overline{P}_j$ are defined in (4) *and* (11), *then the system* (6) *is stochastically stable with an H_∞ performance index γ. Moreover, the filter gains of an admissible H_∞ FDF in the form of* (5) *are given by $L_{i,k} = G_i^{-1}K_i$, $\forall i \in \Gamma$.*

Proof. From (6), we can replace $\overline{A}_{i,k}, \overline{B}_{i,k}, \overline{D}$ in (10) by $\overline{A}_{i,k} = A_{i,k} - L_{i,k}\overline{C}$, $\overline{B}_{i,k} = \left[\overline{B}_d - L_{i,k}D_d \;\; \overline{B}_f - L_{i,k}D_f\right]$, and

$\overline{D} = [D_d \;\; D_f]$, respectively. Then, (10) in Lemma 6 can be written as

$$\begin{bmatrix} -P_i & 0 & 0 & A_{i,k}^T G_i^T - \overline{C}^T L_{i,k}^T G_i^T & \overline{C}^T \\ * & -\gamma^2 I & 0 & \overline{B}_d^T G_i^T - D_d^T L_{i,k}^T G_i^T & D_d^T \\ * & * & -\gamma^2 I & \overline{B}_f^T G_i^T - D_f^T L_{i,k}^T G_i^T & D_f^T \\ * & * & * & -G_i - G_i^T + \overline{P}_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \quad (13)$$

Define $K_i = G_i L_{i,k}$; (13) can be transformed into (12). This completes the proof. □

Remark 8. The optimal H_∞ performance index γ^* and the corresponding filter gains can be obtained by setting $\delta = \gamma^2$ and solving the following optimization problem:

$$\begin{aligned} &\text{Minimize} : \delta, \\ &\text{s.t.} \, (12). \end{aligned} \quad (14)$$

3.2. Determination of Threshold. From Section 3.1, we can obtain the residual signal $r(k)$, and then the modified residual signal $\xi(k)$ can be generated by using $r(k)$ as the input of the postfilter. According to the system (6) and the definition of $\xi(k)$, we can rewrite $\xi(k)$ in the following compact form [22]:

$$\xi(k) = V_k \left(H_{d,k} d_s(k) + H_{f,k} f_s(k) \right), \quad (15)$$

where

$$H_{f,k} = \begin{bmatrix} D_f & 0 & \cdots & 0 \\ g_f(k-s+1, k-s) & D_f & \cdots & 0 \\ & \vdots & & \\ g_f(k, k-s) & \cdots & g_f(k, k-1) & D_f \end{bmatrix},$$

$$f_s(k) = \begin{bmatrix} f(k-s) \\ f(k-s+1) \\ \vdots \\ f(k) \end{bmatrix},$$

$$H_{d,k} = \begin{bmatrix} g_e(k-s, k-s) & D_d & 0 & \cdots & 0 \\ g_e(k-s+1, k-s) & g_d(k-s+1, k-s) & D_d & \cdots & 0 \\ & & \vdots & & \\ g_e(k, k-s) & g_d(k, k-s) & \cdots & g_d(k, k-1) & D_d \end{bmatrix}, \quad (16)$$

$$d_s(k) = \begin{bmatrix} e(k-s) \\ d(k-s) \\ \vdots \\ d(k) \end{bmatrix},$$

$$g_f(k, j) = \overline{C}\Phi(k, j+1) B_{Lf}^{i,j}, \qquad g_e(k, j) = \overline{C}\Phi(k, j), \qquad g_d(k, j) = \overline{C}\Phi(k, j+1) B_{Ld}^{i,j},$$

$$\Phi(k, j) = \prod_{a=j}^{k-1} \overline{A}_{i,a}, \quad \Phi(k, k) = \mathrm{I}, \quad k-s \le j \le k-1,$$

$$V_k = [V_{0,k} \;\; V_{1,k} \;\; \cdots \;\; V_{s,k}].$$

Remark 9. From (16), we know that the matrices $H_{f,k}$ and $H_{d,k}$ are time-varying matrices since they are constructed by $B_{Lf}^{i,j}$, $B_{Ld}^{i,j}$, and $\overline{A}_{i,j}$ ($i \in \Gamma, k-s \le j \le k-1$) which are influenced by the mode of system (6). We can obtain the mode and these matrices online at each time instant. It also should be noted that the selection of index s which is the order of the postfilter $V_k(z)$ is arbitrary in principle, but, in this paper, considering the computational complexity of online implementation, we set it equal to $2n$.

Once the modified residual signal has been generated and the residual evaluation function has been selected, we

can determine the threshold J_{th}. From (8) and (15), we have

$$J_{\text{th}} = \sup_{d,f=0} J(k) = \sup_{d,f=0} \|\xi(k)\|_e = \sup_{d,f=0}\left[\frac{1}{\beta+1}\sum_{i=k-\beta}^{k}\xi^T(i)\,\xi(i)\right]^{1/2}. \tag{17}$$

It should be pointed out that the threshold defined in (17) is the minimum threshold that prevents false alarms. It follows from (15) and (17) that

$$\begin{aligned} J_{\text{th}} &= \sup_{d}\left[\frac{1}{\beta+1}\sum_{j=k-\beta}^{k}\left(V_kH_{d,k}d_s(j)\right)^TV_kH_{d,k}d_s(j)\right]^{1/2} \\ &\le \overline{\sigma}\left(V_kH_{d,k}\right)\sup_{d}\left[\frac{1}{\beta+1}\sum_{j=k-\beta}^{k}d_s^T(j)\,d_s(j)\right]^{1/2}, \end{aligned} \tag{18}$$

where $\overline{\sigma}(\cdot)$ denotes the maximum singular value.

From (16), we can obtain

$$\begin{aligned} &\sum_{j=k-\beta}^{k}d_s^T(j)\,d_s(j) \\ &\quad = \sum_{j=k-\beta}^{k}\left[e^T(j-s)\,e(j-s)+\sum_{n=0}^{s}d^T(j-n)\,d(j-n)\right]. \end{aligned} \tag{19}$$

According to (6), we have [22]

$$\begin{aligned} &\left[\sum_{j=k-\beta}^{k}e^T(j-s)\,e(j-s)\right]^{1/2} \\ &\quad \le (\beta+1)^{1/2}\left\|\left(e^{j\omega}I-\overline{A}_{i,k}\right)^{-1}B_{Ld}^{i,k}\right\|_{\infty}\|d(k)\|_e \\ &\quad \le (\beta+1)^{1/2}\sup_{i\in\Gamma,\omega}\overline{\sigma}\left(\left(e^{j\omega}I-\overline{A}_{i,k}\right)^{-1}B_{Ld}^{i,k}\right)\Delta d, \end{aligned} \tag{20}$$

where $\Delta d \ge \|d(k)\|_e$, and note that

$$\left[\sum_{j=k-\beta}^{k}\sum_{n=0}^{s}d^T(j-n)\,d(j-n)\right]^{1/2} \le \left[(\beta+1)(s+1)\right]^{1/2}\Delta d. \tag{21}$$

So according to (18)–(21), the threshold can be defined as

$$J_{\text{th}} = \overline{\sigma}\left(V_kH_{d,k}\right)\left(s+1+\lambda_d\right)^{1/2}\Delta d, \tag{22}$$

where $\lambda_d = (\sup_{i\in\Gamma,\omega}\overline{\sigma}((e^{j\omega}I-\overline{A}_{i,k})^{-1}B_{Ld}^{i,k}))^2$.

Note that V_k, $H_{d,k}$ vary with the mode of system (6), so J_{th} is an adaptive threshold which can be obtained online.

3.3. Optimization of Fault Detection Systems. The objective of optimizing the fault detection system (6) is to seek a performance index in order to detect faults as small as possible. For describing the performance index, we first give the following definitions [22].

Definition 10. The set of detectable faults which are denoted by S_f can be expressed by

$$S_f = \left\{f \mid \inf_d\|\xi\|_e \ge J_{\text{th}}\right\} = \left\{f \mid \left\|V_kH_{f,k}f_s(k)\right\|_e \ge 2J_{\text{th}}\right\}. \tag{23}$$

Definition 11. Minimum detectable faults, denoted by $f_{\min}$, are faults which belong to S_f and minimize $\inf_d\|\xi\|_e$. So an $f_{\min}$ can be obtained by solving the following extreme problem:

$$\inf_{f\in S_f,d}\|\xi\|_e = J_{\text{th}}. \tag{24}$$

Definition 12. Maximal minimum detectable faults, denoted by f_{mmin}, are defined by

$$\begin{aligned} \|f_{\text{mmin}}\|_{2,[0,\beta]} &= \left[\sum_{a=-\beta}^{0}f_{\text{mmin}}^T(k+a)f_{\text{mmin}}(k+a)\right]^{1/2} \\ &= \max_{f_{\min}}\left[\sum_{a=-\beta}^{0}f_{\min}^T(k+a)f_{\min}(k+a)\right]^{1/2}. \end{aligned} \tag{25}$$

Note that the smaller f_{mmin} becomes, the more faults can be detected. So our objective of optimizing can be formulated as

$$\min_{V_k}J = \min_{V_k}\|f_{\text{mmin}}\|_{2,[0,\beta]}. \tag{26}$$

Followed from (23) and (24), the minimum detectable faults $f_{\min}$ ensure that

$$\begin{aligned} &\left[\frac{1}{\beta+1}\sum_{a=-\beta}^{0}\left(V_kH_{f,k}f_{s,\min}(k+a)\right)^TV_kH_{f,k}f_{s,\min}(k+a)\right]^{1/2} \\ &\quad = 2J_{\text{th}}, \end{aligned} \tag{27}$$

where one has $f_{s,\min}(k+a) = \left[f_{\min}^T(k+a-s)\ \ f_{\min}^T(k+a-s+1)\ \cdots\ f_{\min}^T(k+a)\right]^T$.

Since

$$\begin{aligned}
&\left[\frac{1}{\beta+1}\sum_{a=-\beta}^{0}\left(V_kH_{f,k}f_{s,\min}(k+a)\right)^TV_kH_{f,k}f_{s,\min}(k+a)\right]^{1/2}\\
&\geq \underline{\sigma}\left(V_kH_{f,k}\right)\\
&\quad\times\left[\frac{1}{\beta+1}\sum_{a=-\beta}^{0}\left(f_{s,\min}(k+a)\right)^Tf_{s,\min}(k+a)\right]^{1/2}\\
&=\underline{\sigma}\left(V_kH_{f,k}\right)\\
&\quad\times\left[\frac{1}{\beta+1}\sum_{a=-\beta}^{0}\sum_{b=0}^{s}f_{\min}^T(k+a-b)f_{\min}(k+a-b)\right]^{1/2}\\
&=\underline{\sigma}\left(V_kH_{f,k}\right)\\
&\quad\times\left[\frac{1}{\beta+1}\sum_{b=0}^{s}\sum_{a=-\beta}^{0}f_{\min}^T(k+a-b)f_{\min}(k+a-b)\right]^{1/2},
\end{aligned}\tag{28}$$

where $\underline{\sigma}(\cdot)$ denotes the minimum singular value, then we have

$$\begin{aligned}
&\left[\frac{1}{\beta+1}\sum_{b=0}^{s}\sum_{a=-\beta}^{0}f_{\min}^T(k+a-b)f_{\min}(k+a-b)\right]^{1/2}\\
&\qquad\leq\frac{2J_{\text{th}}}{\underline{\sigma}\left(V_kH_{f,k}\right)}.
\end{aligned}\tag{29}$$

It is evident that the equality in (29) holds true only if vectors $f_{\min}(k+a-b)$, $b=0,\ldots,s$, satisfy

$$f_{\min}(k+a)=f_{\min}(k+a-1)=\cdots=f_{\min}(k+a-s)\tag{30}$$

and are equal to the eigenvector of matrix $(V_kH_{f,k})^TV_kH_{f,k}$ corresponding to $\underline{\sigma}^2(V_kH_{f,k})$. According to the definition of f_{mmin}, we finally have

$$\begin{aligned}
&\max_{f_{\min}}\left[\sum_{a=-\beta}^{0}f_{\min}^T(k+a)f_{\min}(k+a)\right]^{1/2}\\
&\quad=\frac{2J_{\text{th}}}{\underline{\sigma}\left(V_kH_{f,k}\right)}\left(\frac{\beta+1}{s+1}\right)^{1/2}\\
&\quad=2\Delta d(\beta+1)^{1/2}\left(1+\frac{\lambda_d}{s+1}\right)^{1/2}\frac{\overline{\sigma}\left(V_kH_{d,k}\right)}{\underline{\sigma}\left(V_kH_{f,k}\right)}.
\end{aligned}\tag{31}$$

Thus, we can know that the objective of optimizing system (6) is reduced to finding matrices V_k at each time instant that solve the following optimization problem:

$$\begin{aligned}
\min_{V_k}J&=\min_{V_k}\left\|f_{\text{mmin}}\right\|_{2,[0,\beta]}\\
&=2\Delta d(\beta+1)^{1/2}\left(1+\frac{\lambda_d}{s+1}\right)^{1/2}\min_{V_k}\left[\frac{\overline{\sigma}\left(V_kH_{d,k}\right)}{\underline{\sigma}\left(V_kH_{f,k}\right)}\right].
\end{aligned}\tag{32}$$

Next, we give the following lemma that plays a key role in deriving the solution of optimization problem (32).

Lemma 13 (see [22]). *Given matrices H,P of appropriate dimensions, then the optimal solution X for optimization problem $\min_X\overline{\sigma}(P+XH)$ is given by*

$$X=-PH^+,\tag{33}$$

and furthermore

$$\min_X\overline{\sigma}(P+XH)=\overline{\sigma}\left(P-XH^+H\right),\tag{34}$$

where H^+ denotes the pseudoinverse or Moore-Penrose inverse of matrix H.

Based on Lemma 13, we have the following theorem to determine the optimal solution for problem (32).

Theorem 14. *Given time-varying matrices $H_{d,k}$, $H_{f,k}$ with $\operatorname{rank}(H_{f,k})=q(s+1)=\alpha$ which are defined as (16) at each time instant, then the optimal solution V_k^* for (32) is given by*

$$\begin{aligned}
V_k^*&=H_{f,k}^- - H_{f,k}^-H_{d,k}\left(H_{fn,k}H_{d,k}\right)^+H_{fn,k}\\
&\quad\text{with } H_{f,k}^-H_{f,k}=I_{\alpha\times\alpha},\quad H_{fn,k}H_{f,k}=0.
\end{aligned}\tag{35}$$

Furthermore,

$$\begin{aligned}
\min_{V_k}J&=\min_{V_k}\left\|f_{\text{mmin}}\right\|_{2,[0,\beta]}\\
&=2\Delta d(\beta+1)^{1/2}\left(1+\frac{\lambda_d}{s+1}\right)^{1/2}\\
&\quad\times\overline{\sigma}\left(H_{f,k}^-H_{d,k}\left(I-\left(H_{fn,k}H_{d,k}\right)^+H_{fn,k}H_{d,k}\right)\right),
\end{aligned}\tag{36}$$

where $(\cdot)^+$ denotes the Moore-Penrose inverse.

Proof. From Theorem 7 and (22), we know that λ_d is a constant which can be calculated offline. So the original optimization problem (32) is equivalent to the following time-varying optimization problem:

$$\min_{V_k}\left[\frac{\overline{\sigma}\left(V_kH_{d,k}\right)}{\underline{\sigma}\left(V_kH_{f,k}\right)}\right].\tag{37}$$

For deriving the solution of problem (37), at each time instant, we set

$$V_k^* = X_{1,k}H_{f,k}^- + X_k H_{fn,k}, \tag{38}$$

and substitute it into (37)

$$\begin{aligned}\frac{\overline{\sigma}\left(V_k^* H_{d,k}\right)}{\underline{\sigma}\left(V_k^* H_{f,k}\right)} &= \frac{\overline{\sigma}\left(\left(X_{1,k}H_{f,k}^- + X_k H_{fn,k}\right)H_{d,k}\right)}{\underline{\sigma}\left(\left(X_{1,k}H_{f,k}^- + X_k H_{fn,k}\right)H_{f,k}\right)} \\ &= \frac{\overline{\sigma}\left(\left(X_{1,k}H_{f,k}^- + X_k H_{fn,k}\right)H_{d,k}\right)}{\underline{\sigma}\left(X_{1,k}\right)},\end{aligned} \tag{39}$$

where $X_{1,k} \in R^{\alpha\times\alpha}$ and $\operatorname{rank}(X_{1,k}) = \alpha$, X_k is arbitrarily selectable. Note that

$$\begin{aligned}&\frac{\overline{\sigma}\left(\left(X_{1,k}H_{f,k}^- + X_k H_{fn,k}\right)H_{d,k}\right)}{\underline{\sigma}\left(X_{1,k}\right)} \\ &\quad\geq \frac{\underline{\sigma}\left(X_{1,k}\right)\overline{\sigma}\left(H_{f,k}^- H_{d,k} + X_{1,k}^{-1}X_k H_{fn,k}H_{d,k}\right)}{\underline{\sigma}\left(X_{1,k}\right)} \\ &\quad= \overline{\sigma}\left(H_{f,k}^- H_{d,k} + X_{1,k}^{-1}X_k H_{fn,k}H_{d,k}\right),\end{aligned} \tag{40}$$

and the equality holds true if and only if

$$\overline{\sigma}\left(X_{1,k}\right) = \underline{\sigma}\left(X_{1,k}\right) \Longleftrightarrow X_{1,k} = I_{\alpha\times\alpha}. \tag{41}$$

Thus, we finally have

$$\begin{aligned}\min_{V_k}\left[\frac{\overline{\sigma}\left(V_k H_{d,k}\right)}{\underline{\sigma}\left(V_k H_{f,k}\right)}\right] &= \min_{X_{1,k},X_k}\frac{\overline{\sigma}\left(\left(X_{1,k}H_{f,k}^- + X_k H_{fn,k}\right)H_{d,k}\right)}{\underline{\sigma}\left(X_{1,k}\right)} \\ &= \min_{X_k}\overline{\sigma}\left(H_{f,k}^- H_{d,k} + X_{1,k}^{-1}X_k H_{fn,k}H_{d,k}\right).\end{aligned} \tag{42}$$

Using Lemma 13, we can obtain

$$X_k = -H_{f,k}^- H_{d,k}\left(H_{fn,k}H_{d,k}\right)^+. \tag{43}$$

Hence, the optimal solution V_k^* for (32) is given by

$$V_k^* = H_{f,k}^- - H_{f,k}^- H_{d,k}\left(H_{fn,k}H_{d,k}\right)^+ H_{fn,k}. \tag{44}$$

Furthermore, substituting V_k^* into (32) leads to (36). This completes the proof. □

Remark 15. From Remark 9 and (35), we can know that the optimal solution V_k^* for (32) is time-varying and can be obtained online at each time instant. As a result, the postfilter $V_k(z)$ is time-varying as well. That is different from the conventional approach in [22], in which the postfilter $V_k(z)$ is time-invariant.

Remark 16. Note that if $H_{f,k}$ is a full rank square matrix which is a special case that is often met, we have $H_{f,k}^- = H_{f,k}^{-1}$, $H_{fn,k} = 0$. Thus, the optimal solution $V_k^* = H_{f,k}^{-1}$.

3.4. Summary. The following Algorithm 17 summarizes the essential parts of this section and the approach proposed above for the FDF system design.

Algorithm 17. Consider the following steps.
Step 1. Solve the optimal H_∞ problem in Theorem 7 and Remark 8 for $L_{i,k}, \forall i \in \Gamma$.
Step 2. Generate residual signal $r(k)$ from FDF (5).
Step 3. From (16), form $H_{f,k}$, $H_{d,k}$, and $H_{y,k}$.
Step 4. Find $H_{f,k}^-$, $H_{fn,k}$ with $H_{f,k}^- H_{f,k} = I_{\alpha\times\alpha}$, $H_{fn,k}H_{f,k} = 0$, and calculate $(H_{fn,k}H_{d,k})^+$.
Step 5. Set the optimal postfilter $V_k^* = H_{f,k}^- - H_{f,k}^- H_{d,k}(H_{fn,k}H_{d,k})^+ H_{fn,k}$.
Step 6. Establish the adaptive threshold $J_{\text{th}} = \overline{\sigma}(V_k H_{d,k})(s + 1 + \lambda_d)^{1/2}\Delta d$.
Step 7. Let $\xi(k) = V_k^* r(k)$ denote the modified residual signal; then the residual evaluation function is

$$J(k) = \|\xi(k)\|_e = \left[\frac{1}{\beta+1}\sum_{i=k-\beta}^{k}\xi^T(i)\,\xi(i)\right]^{1/2}. \tag{45}$$

From Algorithm 17, it can be easily known that these steps are implemented online except Step 1.

4. Numerical Example

In this section, a numerical example is given to show the effectiveness of the proposed method. Consider the following continuous dynamics model:

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}x(t) + \begin{bmatrix}2\\1\end{bmatrix}u(t) + \begin{bmatrix}0.1\\0.05\end{bmatrix}d(t) + \begin{bmatrix}2\\1\end{bmatrix}f(t), \\ y(t) &= \begin{bmatrix}1 & 0\end{bmatrix}x(t) + 0.1d(t) + 0.8f(t).\end{aligned} \tag{46}$$

We choose the sampling period of NCSs as 0.3 s and the division of the sample interval as $N = 3$; then it is easily obtained that the Markov chain $h(k) \in \{1\ \ 2\ \ 3\}$. The initial mode is set to be $\tau_0 = 0$, and the detection window $\beta = 10$. For $k = 0, 1, \ldots, 200$, the external disturbance $d(k)$ is supposed to be a random noise uniformly distributed over $[-0.5, 0.5]$, and the fault signal $f(k)$ is given as

$$f(k) = \begin{cases}0.13, & \text{for } k = 100, 101, \ldots, 200\\ 0 & \text{others.}\end{cases} \tag{47}$$

The discrete control law and the transition probability matrices are given as

$$\begin{aligned}K &= \begin{bmatrix}1.6918\\-0.0623\end{bmatrix}^T, \qquad \pi_1 = \begin{bmatrix}0.5 & 0.4 & 0.1\\0.2 & 0.6 & 0.2\\0.2 & 0.3 & 0.5\end{bmatrix}, \\ \pi_2 &= \begin{bmatrix}0.5 & ? & ?\\ ? & ? & 0.2\\0.2 & 0.3 & 0.5\end{bmatrix}, \qquad \pi_3 = \begin{bmatrix}? & ? & ?\\ ? & ? & ?\\ ? & ? & ?\end{bmatrix},\end{aligned} \tag{48}$$

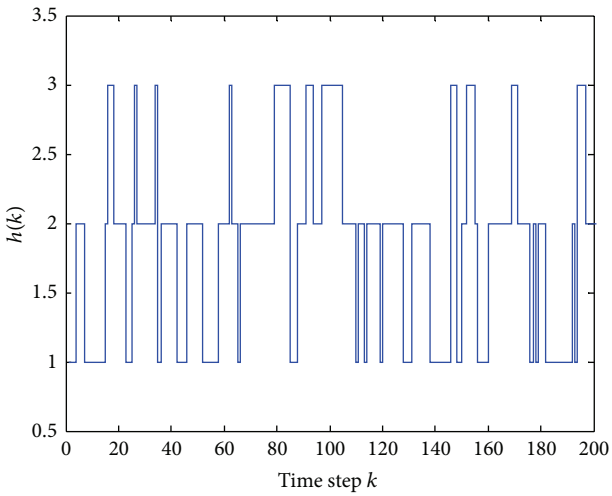

FIGURE 1: Mode evolution.

where "?" denotes the inaccessible elements of the matrices. So π_1, π_2, and π_3 denote the transition probability matrix with completely known transition probabilities (Case 1), partially known transition probabilities (Case 2), and completely unknown transition probabilities (Case 3), respectively.

Considering Case 1 as the practical one for the other two cases, we can generate a possible evolution of system modes as shown in Figure 1.

Then according to Theorem 7, the filter's gain matrices for the three cases of the observer (5) are, respectively, given by

$$\begin{aligned}
&\text{Case 1: } L_{1,k} = \begin{bmatrix} 0.7505 \\ -0.0231 \\ 0.0074 \\ 0.0053 \end{bmatrix}, \quad L_{2,k} = \begin{bmatrix} 0.7533 \\ -0.0265 \\ 0.0098 \\ 0.0023 \end{bmatrix}, \\
&\qquad L_{3,k} = \begin{bmatrix} 0.7550 \\ -0.0268 \\ 0.0098 \\ -0.0001 \end{bmatrix}, \\
&\text{Case 2: } L_{1,k} = \begin{bmatrix} 0.7487 \\ -0.0231 \\ 0.0113 \\ 0.0129 \end{bmatrix}, \quad L_{2,k} = \begin{bmatrix} 0.7505 \\ -0.0277 \\ 0.0120 \\ 0.0007 \end{bmatrix}, \\
&\qquad L_{3,k} = \begin{bmatrix} 0.7552 \\ -0.0296 \\ 0.0143 \\ 0.0002 \end{bmatrix}, \\
&\text{Case 3: } L_{1,k} = \begin{bmatrix} 0.7312 \\ -0.0334 \\ 0.0288 \\ 0.0028 \end{bmatrix}, \quad L_{2,k} = \begin{bmatrix} 0.7431 \\ -0.0386 \\ 0.0317 \\ 0.0040 \end{bmatrix}, \\
&\qquad L_{3,k} = \begin{bmatrix} 0.7548 \\ -0.0445 \\ 0.0393 \\ 0.0010 \end{bmatrix}.
\end{aligned} \tag{49}$$

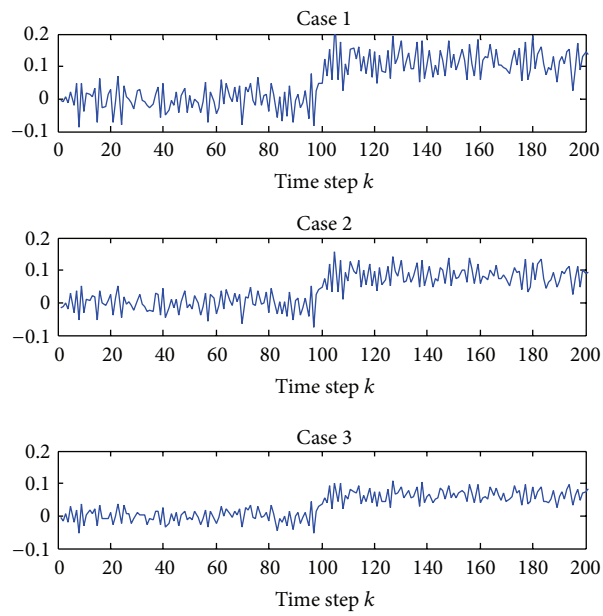

FIGURE 2: Residual signals $r(k)$ for three different cases.

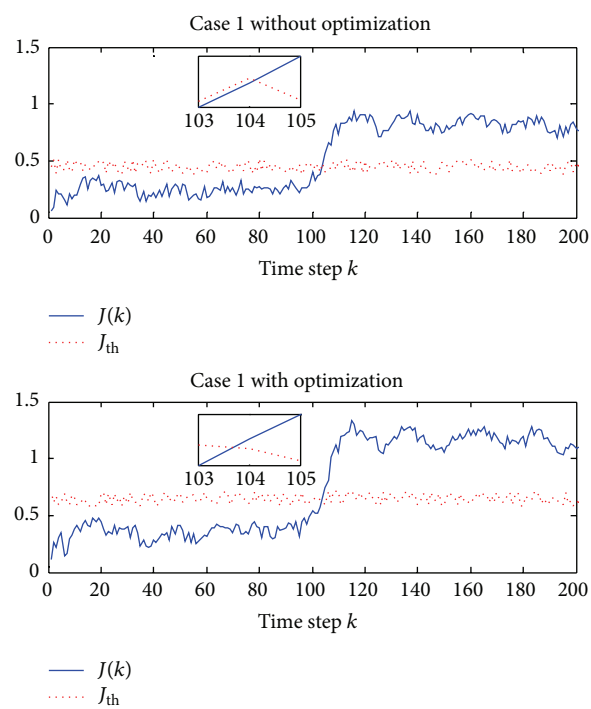

FIGURE 3: Evaluation of $J(k)$ and threshold J_{th} for case 1.

Accordingly, Figure 2 shows the generated residual signals $r(k)$ for three different cases, and Figures 3, 4, and 5 present the evolution of $J(k)$ and the corresponding threshold J_{th}, respectively, for three different transition probability matrices. In order to show the time steps N_d for the fault detection in different case, the corresponding enlarged figures are given in Figures 3–5 as well.

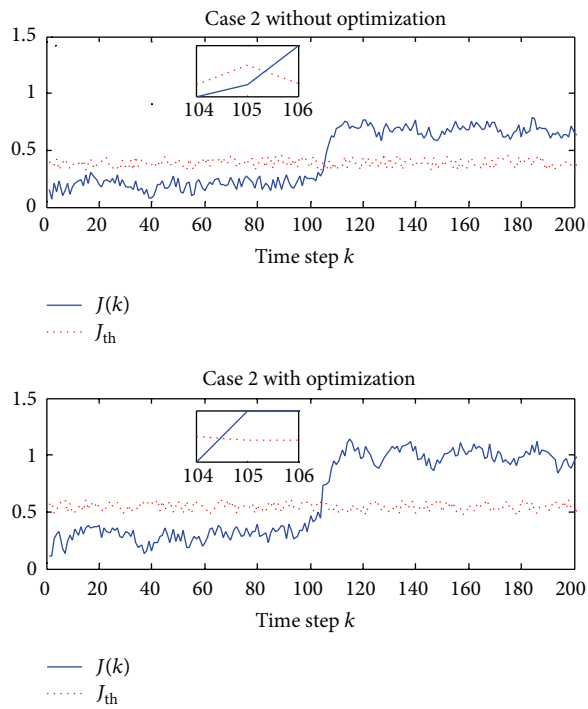

FIGURE 4: Evaluation of $J(k)$ and threshold J_{th} for case 2.

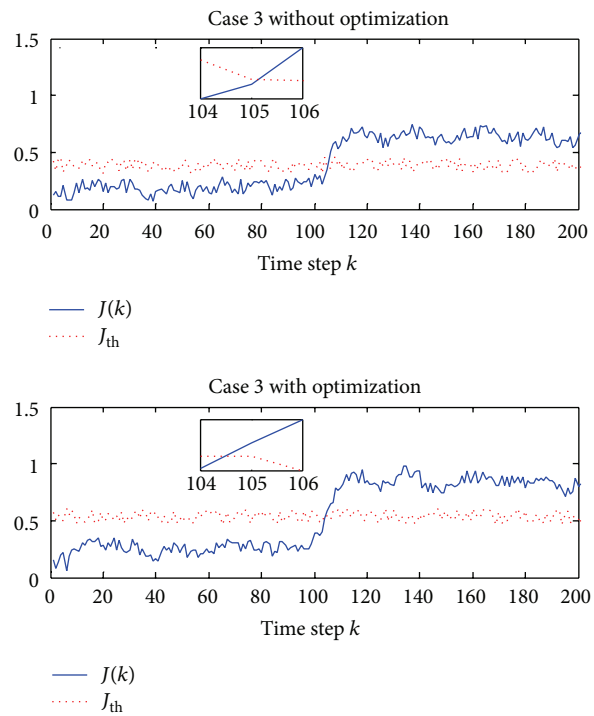

FIGURE 5: Evaluation of $J(k)$ and threshold J_{th} for case 3.

TABLE 1: The optimal H_∞ performance index γ^*, time steps N_d, and the minimum detectable faults f_{min} for three different cases.

Case	γ^*	With optimization		Without optimization	
		N_d	f_{min}	N_d	f_{min}
1	0.8108	4	0.1053	5	0.1204
2	0.8121	5	0.1056	6	0.1211
3	0.8225	5	0.1057	6	0.1215

In order to compare the performance of detection systems in three different cases before and after optimization, the minimum detectable faults f_{min} for the six different conditions are obtained by 500 times simulation. For example, when the fault signal $f(k)$ is given as

$$f(k) = \begin{cases} 0.12, & \text{for } k = 100, 101, \ldots, 200, \\ 0 & \text{others}, \end{cases} \tag{50}$$

the evolution of $J(k)$ and the corresponding threshold J_{th} in Case 3 are shown in Figure 6. It is obvious from Figure 6 that the FDF system with optimization can detect the given fault but the FDF system without optimization cannot.

Based on the path in Figure 1 and the selected threshold J_{th}, the optimal H_∞ performance index γ^* by Theorem 7, the time steps N_d for the fault detection by the evaluation function $J(k)$ and logic (9), and the minimum detectable faults f_{min} can be obtained and given in Table 1.

Obviously, it can be seen from Figures 3–6 and Table 1 that the fault detection systems with optimization can detect the smaller faults and need less time steps than the system without optimization; that is, the fault detection systems

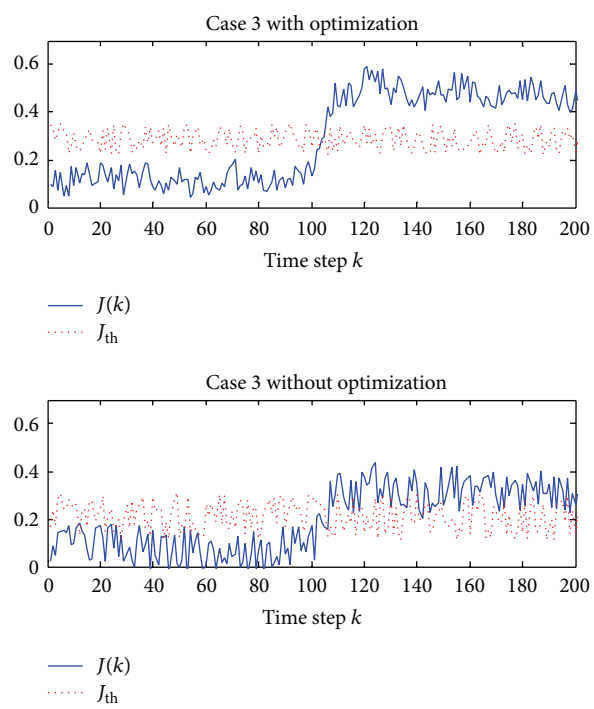

FIGURE 6: Evaluation of $J(k)$ and threshold J_{th} for case 3 when $f(k)$ is given as (50).

with optimization have a better performance than the system without optimization for each case. It also can be depicted from Figures 3–5 and Table 1 that the more transition probability knowledge we have, the better H_∞ performance index can be achieved, the less time is needed, and the smaller faults can be detected. Therefore, our design and

optimization approaches for robust fault detection systems actually build a tradeoff in practice between the complexity to obtain transition probabilities and the performance benefits and efficiency of detection.

5. Conclusion

The problem of observer-based robust FDF design and optimization for NCSs with random delays is investigated in this paper. A MJSs model has been developed by assuming the random delays to obey a Markov chain, and the partially known transition probabilities of the Markov process are taken into account. Based on the developed model, an H_∞ FDF is derived in terms of LMIs. Furthermore, to improve the performance of the FDF, a time domain optimization approach is proposed for the robust fault detection systems. The optimal solution of the problem is given in the form of Moore-Penrose inverse of matrix. Finally, a numerical example has been given to demonstrate the effectiveness and potential of the proposed approach. Some extensions of the present method are under investigation. For example, the problem of FDF design and optimization for NCSs with unknown delays or packet dropout needs to be further studied. Besides, in order to reduce the computation of the fault detection algorithm and enhance the engineering practicability of the time domain optimization approach, a recursive algorithm is worth forthcoming investigation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (60974014 and 61074027).

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