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Research Article

Dislocation Synchronization of the Different Complex Value Chaotic Systems Based on Single Adaptive Sliding Mode Controller

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Based on the active control and the adaptive sliding mode controller, a new method of the combination of the active control and the single adaptive sliding mode variable structure control is proposed to realize the dislocation synchronization of the three-dimensional different complex value chaotic systems. The synchronization method is suitable not only for the same complex value chaotic systems, but also for different complex value chaotic systems, so it expands the application of the single sliding mode controller. For the states in the complex space of the driving system and response system, the synchronization for the complex state of the two different chaotic systems is achieved according to the dislocation relationship, not in accordance with the corresponding relationship. The complexity of complex value chaotic system and the diversity of dislocation synchronization increase the security of the chaotic secure communication. This single adaptive sliding mode variable structure controller is simple, and it can enhance the robustness of the system. Theoretical analysis and numerical simulation prove the feasibility and effectiveness of the controller designed.

1. Introduction

Chaos is random and the chaotic communication has the advantages of strong confidentiality, anti-interference, low power, low cost, and so forth. Therefore, chaotic communication has great application in secure communication. Since Pecora and Carroll put forward and achieve the synchronization of the driving and response system, more and more people began to research the chaotic synchronization. Otherwise chaotic synchronization is the key technology of application of chaotic system in secure communication [1, 2]. Various new methods are put forward to realize chaotic synchronization, such as complete synchronization [3], lag synchronization [4], generalized synchronization [5, 6], projective synchronization [7], and sliding mode synchronization [8]. Recently, a new synchronization method is proposed, namely, the dislocation synchronization [9]. The dislocation synchronization methods attract much attention. For example, Min and Wang [10] and Shao et al. [11] realize the dislocation synchronization of the two chaotic systems and the synchronization method is applied to secure communication. Fang and Jiang [12] realize the dislocated modified chaotic function projective synchronization and apply it in secure communication. The synchronization for the complex state of the two different chaotic systems is achieved according to the dislocation relationship, not in accordance with the corresponding relationship. Therefore, synchronization scheme for two different chaotic systems will increase exponentially as the system state variables increase, which improves the ability to resist crack and opens up a new path for the application of chaotic encryption.

Since Fowler et al. [13] proposed the complex Lorenz equation, complex system plays an important role in the field of physics. At present, the complex system has been widely applied to various physical phenomena, such as the detuning of the laser system, the amplitude of the electromagnetic field, and the thermal convection of liquid [14–16]. So we generalize the chaotic system to the complex value space. The complex values not only increase the information transmission, but also improve the security of secure communication [17].

Therefore complex values chaotic system synchronization is widely applied. In recent years, its synchronization and characteristics of the complex value chaotic system have attracted a lot of attention, such as the research for the characteristics and synchronization of time-delay complex Lorenz chaotic system [18], the complex scaling factor modified projective synchronization [19] between the real chaotic system and the complex chaotic system, control and synchronization of hyperchaotic complex Lorenz system [20], modified function projective synchronization of different complex chaotic system [21], and chaos in the fractional-order complex Lorenz system and its synchronization [22]. P. Liu and S. T. Liu [23] realize the adaptive hybrid synchronization of the complex chaotic systems with unknown parameters and external disturbance. In [24] the complex value chaotic system is applied in the secure communication. Otherwise many other synchronization methods have been applied to complex chaotic system.

Sliding mode control has many attractive advantages, such as fast response and good dynamic performance [25–28]. Recently Huang and Qi [29] realize the synchronization of the different dimensions chaotic systems based on the sliding mode variable structure, but the controller needs to design three sliding modes, so the controller is too complicated and has high cost of control. Yu [30], Cao and Zhang [31], and Aghababa [32] realize the synchronization for a class of three-dimensional chaotic system with a single adaptive controller, but the method is applied only to the same chaotic system. In this paper, for the three-dimensional complex value chaotic system, combined with the active control and single adaptive sliding mode variable structure control, a new method is put forward to realize the dislocation synchronization of a class of three-dimensional different chaotic value systems in the complex space. The controller design is divided into two parts: one is the active control and the other contains only a single driving variable adaptive sliding mode variable structure controller. The design of the single adaptive sliding mode variable structure controller not only reduces the complexity of the controller, but also enhances the robustness of the control system. It is significant for the application of chaos synchronization. Finally, take the three-dimensional complex Chen chaotic system and Lü chaotic system as the example and simulate the dislocation synchronization under this controller. The simulation results show the effectiveness and the feasibility of the proposed method.

2. Dislocation Synchronization of Different Complex Value Chaotic Systems Based on Adaptive Sliding Mode Control

2.1. The Synchronization Problem. The driving system is a three-dimensional complex chaotic system as follows:

$$\dot{x} = f(x), \tag{1}$$

where x_n $(n = 1, 2, 3)$ is the state variable in complex space, and the complex state variables x_n can be written as $x_n = x_n^R + jx_n^I$, where R is the real part of complex, I is the imaginary part, and $f(x)$ is a 3×1 matrix of complex nonlinear function.

The response system is a three-dimensional complex chaotic system:

$$\dot{y} = g(y) + u, \tag{2}$$

where y_n $(n = 1, 2, 3)$ is the state variable in complex space, and the complex state variables y_n can be written as $y_n = y_n^R + jy_n^I$. $g(y)$ is a 3×1 matrix of complex nonlinear function, and u is a controller designed, where $u_n = u_n^R + ju_n^I$.

According to the definition of the dislocation synchronization, there are $3! - 1$ dislocation synchronization methods for two different three-dimensional chaotic systems. Moreover, for two different chaotic systems, synchronization scheme will increase exponentially as the numbers of the state variables. Because the diversity of dislocation synchronization and the selection of the synchronization scheme were not known before, the dislocation synchronization enhances the ability to resist crack, which opens up a new path for the application of chaotic encryption.

The definition of the dislocation synchronization error is

$$e_n(t) = y_m(t) - x_n(t) \quad (m, n = 1, 2, 3), \tag{3}$$

where at least one pair meets $m \neq n$, $e_n(t) = e_n^R(t) + je_n^I(t)$, and in this paper the purpose is to design the controller of u, when $t \to \infty$, $e_n(t) \to 0$; that is,

$$\begin{aligned} \lim_{t\to\infty} \left|e_n^R(t)\right| &= \lim_{t\to\infty} \left|y_m^R\left(t, y_0^R\right) - x_n^R\left(t, x_0^R\right)\right| = 0, \\ \lim_{t\to\infty} \left|e_n^I(t)\right| &= \lim_{t\to\infty} \left|y_m^I\left(t, y_0^I\right) - x_n^I\left(t, x_0^I\right)\right| = 0. \end{aligned} \tag{4}$$

If (4) holds,

$$\lim_{t\to\infty} \left|e_n(t)\right| = \lim_{t\to\infty} \sqrt{\left|e_n^R(t)\right|^2 + \left|e_n^I(t)\right|^2} = 0, \tag{5}$$

where y_0^R, x_0^R are the real parts and y_0^I, x_0^I are the imaginary parts of complex initial values of state variable for the systems (1) and (2), respectively.

2.2. Controller Design. According to the definition of the dislocation synchronization error, the error dynamic system is

$$\begin{aligned} \dot{e}_n &= \dot{y}_m - \dot{x}_n = g_m(y) - f_n(x) + u_m \\ &= h_n(e) + F_n(x, y) + u_m \quad m, n = 1, 2, 3, \end{aligned} \tag{6}$$

where at least one pair meets $m \neq n$, $h(e)$ is the error function matrix, and $F(x, y)$ is the function of the complex variables x, y. The controller u is divided into two parts: u_{I} and u_{II}. Then let the active controller $u_{m\mathrm{I}} = -F_n(x, y)$, and the synchronization error dynamic system is

$$\dot{e} = h(e) + u_{\mathrm{II}}. \tag{7}$$

The definition of complex variable integral sliding mode surface is

$$s_n = s_n^R + js_n^I = e_n + d\int_0^t e_n(\tau)\, d\tau \quad (n = 1, 2, 3), \tag{8}$$

where d is a positive real number.

The adaptive sliding mode variable structure controller $u_{i\mathrm{II}}$ for the complex chaotic system is as shown in

$$u_{n\mathrm{II}} = u_{n\mathrm{II}}^R + ju_{n\mathrm{II}}^I = -k_n\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\operatorname{sign}(s_n), \quad (9)$$

where $\dot{k}_n = \rho(\sum_{i=1}^{3}(|e_i^R| + |e_i^I|))|s_n|$ (ρ is a constant and greater than zero).

In order to make the complex chaotic systems (1) and (2) achieve a single adaptive sliding mode variable structure control, we can make any one of the sliding mode controllers work and the other two sliding mode controllers zero. Then let

$$\begin{aligned}\dot{e}_1 &= h_1(e_1, e_2, e_3)\\ \dot{e}_2 &= h_2(e_1, e_2, e_3) - k_2\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\operatorname{sign}(s_2)\\ \dot{e}_3 &= h_3(e_1, e_2, e_3).\end{aligned} \quad (10)$$

So the two hypotheses below must be satisfied.

Hypothesis 1. The complex functions $h_1(e_1, e_2, e_3)$ and $h_3(e_1, e_2, e_3)$ in system (10) are smooth and continuous in the $e_2 = 0$ field. For all e_1, e_3 about $e_1 = 0$, $e_3 = 0$ in the subsystem $h_1(e_1, 0, e_3)$ and $h_3(e_1, 0, e_3)$ are uniform exponential stability.

Hypothesis 2. The system (10) is the error system, so the positive large enough M exists:

$$\begin{aligned}h_2^R(e_1, e_2, e_3) &\le \lambda_1|e_1| + \lambda_2|e_2| + \lambda_3|e_3|\\ &\le M\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right),\\ h_2^I(e_1, e_2, e_3) &\le \lambda_4|e_1| + \lambda_5|e_2| + \lambda_6|e_3|\\ &\le M\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right).\end{aligned} \quad (11)$$

So the sliding mode surface is designed as $s_2 = s_2^R + js_2^I = e_2 + \int_0^t de_2(\tau)d\tau$; when the system moves on the sliding surface, the following conditions need to be satisfied:

$$\begin{aligned}s_2 &= e_2 + \int_0^t de_2(\tau)\,d\tau = 0,\\ \dot{s}_2 &= \dot{e}_2 + ke_2 = 0.\end{aligned} \quad (12)$$

From (12)

$$\dot{e}_2 = -de_2. \quad (13)$$

Since $d > 0$, (13) is asymptotically stable; namely,

$$\lim_{t\to\infty} e_2 \longrightarrow 0. \quad (14)$$

From Hypothesis 1, $\lim_{t\to\infty} e_3 \to 0$ and $\lim_{t\to\infty} e_1 \to 0$; namely,

$$\lim_{t\to\infty} e_n \longrightarrow 0 \quad (n = 1, 2, 3). \quad (15)$$

Theorem 1. *Under the control of the active controller u_{I} and a single adaptive sliding mode controller u_{II}, the error system* (10) *which starts from the arbitrary initial value can be $\lim_{t\to\infty} e_n \to 0$ $(n = 1, 2, 3)$. So the dislocation synchronization of different complex value chaotic systems will be achieved.*

Proof. Select the Lyapunov function $V = (1/2)\sum_{n=1}^{3}[((s_n^R)^2 + (s_n^I)^2) + (1/\rho)((k_n^R - k_*)^2 + (k_n^I - k_*)^2)]$, where $k_* > M + d$. Then the time derivative of the $V(t)$ is

$$\begin{aligned}\dot{V}(t) &= \sum_{n=1}^{3}\left(s_n^R\dot{s}_n^R + s_n^I\dot{s}_n^I\right)\\ &\quad + \frac{1}{\rho}\sum_{n=1}^{3}\left(\left(k_n^R - k_*\right)\dot{k}_n^R + \left(k_n^I - k_*\right)\dot{k}_n^I\right)\\ &= s_2^R\left(\dot{e}_2^R + de_2^R\right) + s_2^I\left(\dot{e}_2^I + de_2^I\right)\\ &\quad + \left(k_2^R - k_*\right)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^R\right|\\ &\quad + \left(k_2^I - k_*\right)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^I\right|\\ &= s_2^R\Bigg(h_2^R(e_1, e_2, e_3)\\ &\qquad - k_2^R\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\operatorname{sign}\left(s_2^R\right) + de_2^R\Bigg)\\ &\quad + s_2^I\Bigg(h_2^I(e_1, e_2, e_3)\\ &\qquad - k_2^I\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\operatorname{sign}\left(s_2^I\right) + de_2^I\Bigg)\\ &\quad + \left(k_2^R - k_*\right)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^R\right|\\ &\quad + \left(k_2^I - k_*\right)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^I\right|\\ &\le M\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^R\right| + d\left|e_2^R\right|\left|s_2^R\right|\\ &\quad - k_*\left(\sum_{i=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^R\right|\\ &\quad + M\left(\sum_{n=1}^{3}\left(\left|e_i^R\right| + \left|e_i^I\right|\right)\right)\left|s_2^I\right| + d\left|e_2^I\right|\left|s_2^I\right|\end{aligned}$$

$$
\begin{aligned}
&-k_*\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^I\right| \\
&\leq -(k_*-M)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^R\right| \\
&\quad -(k_*-M)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^I\right| \\
&\quad +d\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^R\right| \\
&\quad +d\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^I\right| \\
&= -(k_*-M-d)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^R\right| \\
&\quad -(k_*-M-d)\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\left|s_2^I\right| \\
&\leq 0.
\end{aligned}
\tag{16}
$$

Based on the Lyapunov theory, the error in the system (10) starts from any initial condition, which satisfies the reaching condition of sliding mode, and (15) holds on the sliding surface. Therefore, the error system (10) can stabilize to equilibrium point ultimately based on Lyapunov stability theory. So, under the control of the active controller and a single adaptive sliding mode variable structure controller, the dislocation synchronization of the different complex chaotic systems is achieved. □

For the error system of complex chaotic systems (1) and (2), since $|e_n(t)| = \sqrt{|e_n^R(t)|^2 + |e_n^I(t)|^2}$, the simulation results are shown in Figure 3.

3. Numerical Simulation Analysis

The complex value chaotic system in secure communication not only increases the content of the transmission information, but also improves the communication complexity. So it is important to the security of the secret communication. In order to verify effectiveness of the designed controller, two different complex chaotic systems are selected to be simulated.

The driving system is a complex Chen chaotic system [33]:

$$
\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1), \\
\dot{x}_2 &= (c-a)x_1 - x_1 x_3 + c x_2, \\
\dot{x}_3 &= \frac{1}{2(\overline{x}_1 x_2 + x_1 \overline{x}_2)} - b x_3,
\end{aligned}
\tag{17}
$$

where $a = 35$, $b = 3$, and $c = 28$. And $x_1 = x_1^R + jx_1^I$ and $x_2 = x_2^R + jx_2^I$ are complex variable, and x_3 is the real variable. The upper score is the conjugate variable.

The response system is a complex Lü chaotic system [33]:

$$
\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + u_1, \\
\dot{y}_2 &= -y_1 y_3 + \beta y_2 + u_2, \\
\dot{y}_3 &= \frac{1}{2(\overline{y}_1 y_2 + y_1 \overline{y}_2)} - \gamma y_3 + u_3,
\end{aligned}
\tag{18}
$$

where $\alpha = 29$, $\beta = 21$, and $\gamma = 2$. And $y_1 = y_1^R + jy_1^I$ and $y_2 = y_2^R + jy_2^I$ are complex variable, and y_3 is the real variable. The upper score is the conjugate variable.

In this paper, one of the dislocation synchronization schemes is chosen, and the other schemes can be analyzed by similar methods. The definition of dislocation synchronization error is

$$
\begin{aligned}
e_1 &= y_2 - x_1, \\
e_2 &= y_1 - x_2, \\
e_3 &= y_3 - x_3.
\end{aligned}
\tag{19}
$$

Then (17) and (18) can be simplified:

$$
\begin{aligned}
\dot{e}_1 &= -ae_1 + ae_2 + (a+\beta)y_2 - ay_1 - y_1 y_3 + u_2, \\
\dot{e}_2 &= -\alpha e_2 + (c-a)e_1 - (\alpha+c)x_2 \\
&\quad + (a+\alpha-c)y_2 + x_1 x_3 + u_1, \\
\dot{e}_3 &= -\gamma e_3 + \frac{1}{2}\left(y_2\overline{e}_1 + \overline{x}_2 e_2 + y_1\overline{e}_2 + \overline{x}_1 e_1\right) + (b-\gamma)x_3 + u_3.
\end{aligned}
\tag{20}
$$

The active controller is designed as follows:

$$
\begin{aligned}
u_{2\mathrm{I}} &= -(a+\beta)y_2 + ay_1 + y_1 y_3, \\
u_{1\mathrm{I}} &= -(\alpha+c)x_2 - (a+\alpha-c)y_2 - x_1 x_3, \\
u_{3\mathrm{I}} &= -(b-\gamma)x_3.
\end{aligned}
\tag{21}
$$

And the single adaptive sliding mode variable structure controller is

$$
\begin{aligned}
u_{2\mathrm{II}} &= 0, \\
u_{1\mathrm{II}} &= -k_2\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\operatorname{sign}(s_2), \\
u_{3\mathrm{II}} &= 0.
\end{aligned}
\tag{22}
$$

Let the adaptive rate $\dot{k}_2 = \rho(\sum_{i=1}^{3}(|e_i^R| + |e_i^I|))|s_2|$, so the error system is

$$
\begin{aligned}
\dot{e}_1 &= -ae_1 + ae_2, \\
\dot{e}_2 &= -\alpha e_2 + (c-a)e_1 - k_2\left(\sum_{i=1}^{3}\left(\left|e_i^R\right|+\left|e_i^I\right|\right)\right)\operatorname{sign}(s_2), \\
\dot{e}_3 &= -\gamma e_3 + \frac{1}{2}\left(y_2\overline{e}_1 + \overline{x}_2 e_2 + y_1\overline{e}_2 + \overline{x}_1 e_1\right).
\end{aligned}
\tag{23}
$$

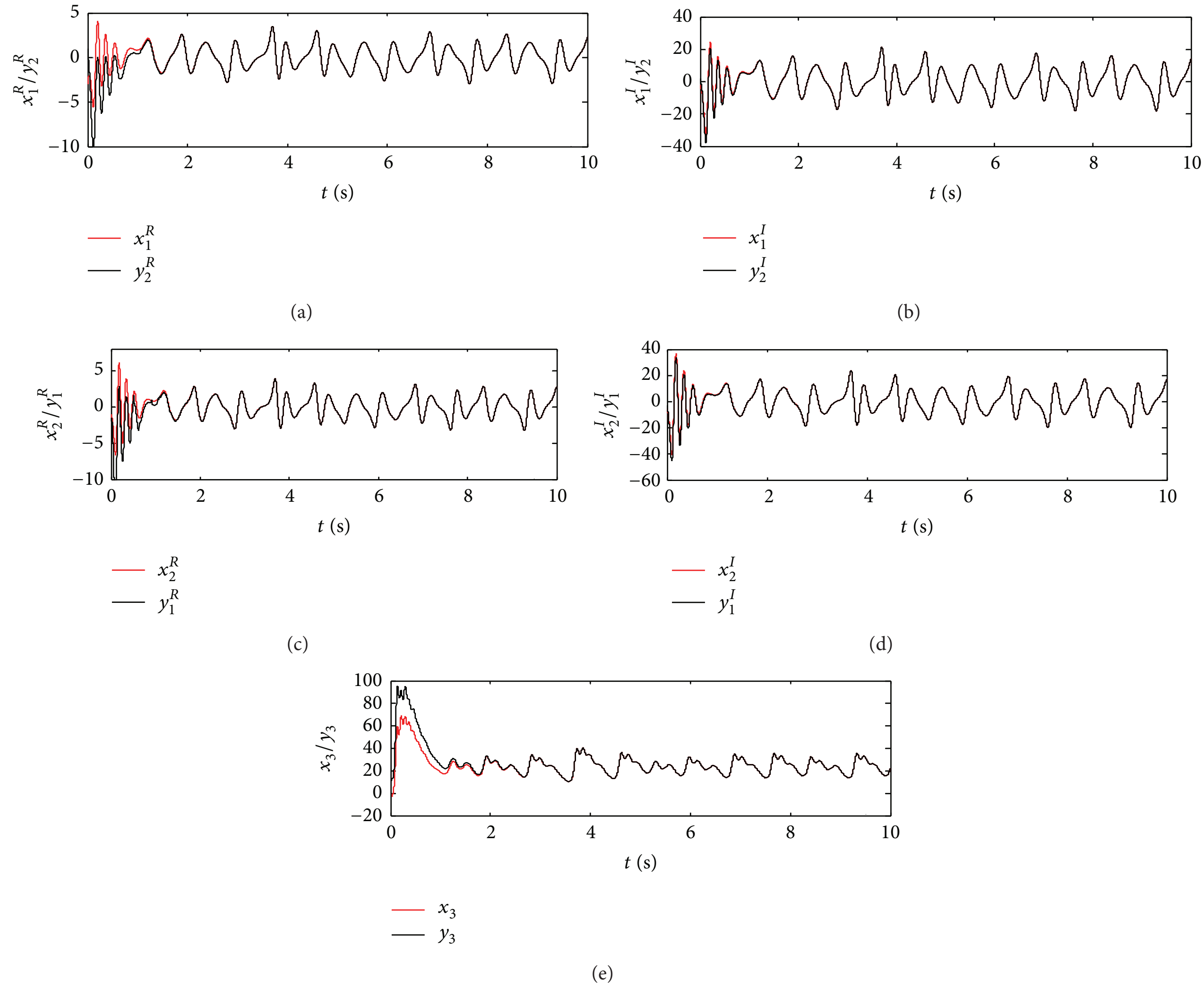

Figure 1: The synchronization curves of the state of the complex value chaotic system. (a) x_1^R, y_2^R; (b) x_1^I, y_2^I; (c) x_2^R, y_1^R; (d) x_2^I, y_1^I; (e) x_3, y_3.

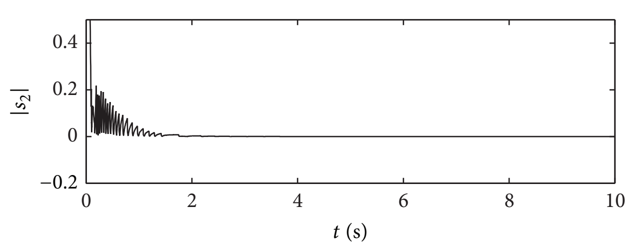

Figure 2: The sliding surface.

If $e_2 = 0$, the error of the system (23) will be

$$\begin{aligned} \dot{e}_1 &= -ae_1, \\ \dot{e}_3 &= -\gamma e_3 + \frac{1}{2}\left(y_2\bar{e}_1 + \bar{x}_1 e_1\right). \end{aligned} \tag{24}$$

Because the eigenvalues of the coefficient matrix for the system (24) are all negative, the errors e_1, e_3 will both satisfy Hypothesis 1. Then, when $e_2 = 0$, e_1 and e_3 are uniform exponential stability, respectively.

Let the initial state of the system (1) be $(x_1(0), x_2(0), x_3(0)) = (-3-2j, -1-5j, -4)$, let the initial state of the system (2) be $(y_1(0), y_2(0), y_3(0)) = (-2-3j, 1+2j, 12)$, the parameter $d = 3$, and $\rho = 2$. Then the synchronization results between the systems (17) and (18) are shown in Figure 1.

For the sliding surface designed for the complex chaotic system, based on $|s_2| = \sqrt{|s_2^R|^2 + |s_2^I|^2}$, we can get the simulation result under the control of the sliding mode controller. The results are shown in Figure 2.

From Figures 1, 2, and 3, the dislocation synchronization errors of the complex Chen chaotic system and complex Lü chaotic system under the controller proposed tend to zero asymptotically; namely, the dislocation synchronization of two different complex value chaotic systems is realized. It shows the effectiveness of the synchronization controller based on the active control and the single adaptive sliding mode variable. Compare Figures 1 and 3 to Figures 3 and

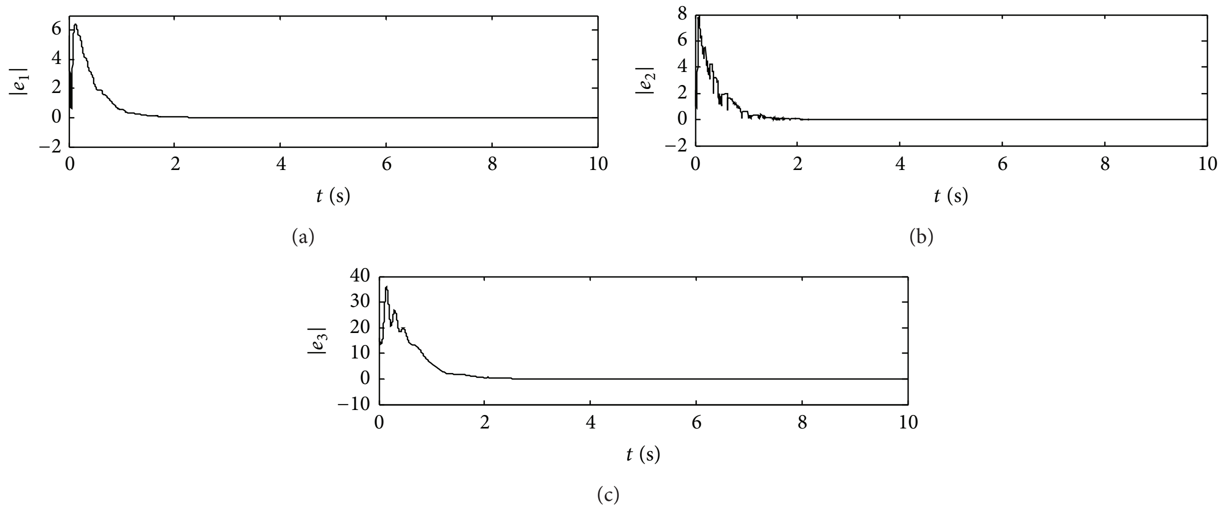

Figure 3: The synchronization error curves of the complex chaotic system. (a) $|e_1(t)|$; (b) $|e_2(t)|$; (c) $|e_3(t)|$.

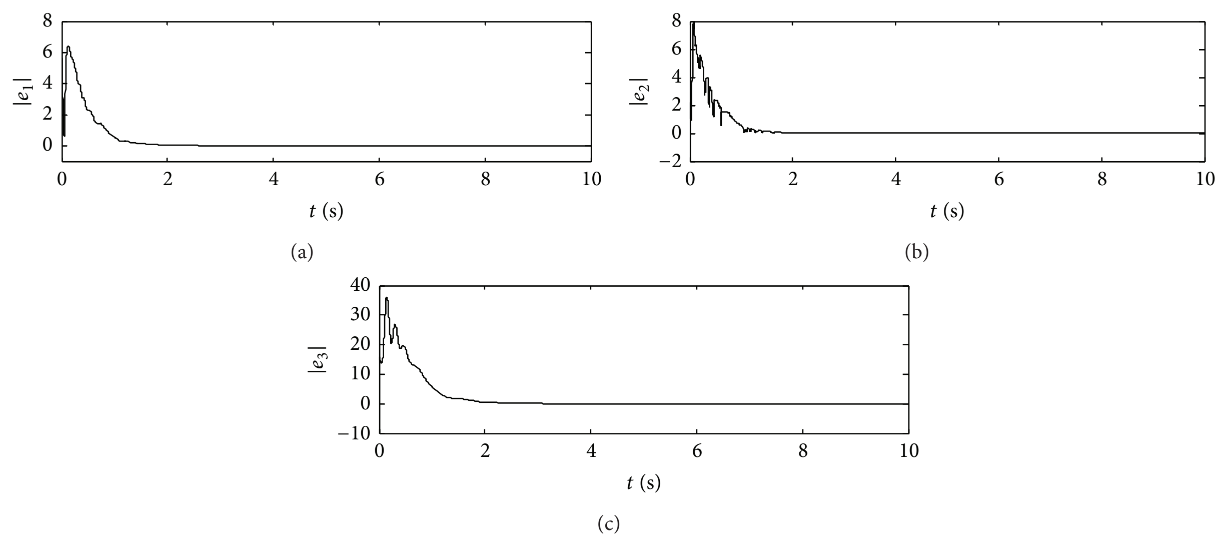

Figure 4: The synchronization error curves of the complex value chaotic system with parameter disturbance. (a) $|e_1(t)|$; (b) $|e_1(t)|$; (c) $|e_3(t)|$.

4 in [34]; the proposed controller in this paper has faster synchronization and better performance.

Then the external disturbance is considered. Let the parameter of the system (17) be $a = 35 + 0.5\sin(t)$ and let the other initial conditions be the same. The simulation results are shown in Figure 4.

From Figure 4, for the three-dimensional complex value chaotic system with parameter disturbance, the designed controller has the better performance. It shows that the proposed controller enhances the robustness and has better effectiveness.

4. Conclusion

This paper studies the dislocation synchronization of three-dimensional different complex value chaotic systems. A new method of synchronization controller based on active control and adaptive sliding mode variable structure control is proposed. Synchronization controller contains two parts. One is the active controller and the other part only contains a driving variable adaptive sliding mode variable structure controller. The single sliding mode controller expands the application of the chaotic system synchronization, which is suitable not only for the same complex value chaotic systems, but also for different complex value chaotic systems. Moreover this method simplifies the controller design and enhances the robustness of the system. It has important theory significance and practical value. For the complex state of the driving system and response system, the synchronization for the complex state of the two different chaotic systems is achieved according to the dislocation relationship, not in accordance with the corresponding relationship. Complexity of the complex chaotic system and diversity of the dislocation synchronization scheme make the chaotic secure communication more secure. Finally take the complex Chen chaotic system and complex Lü chaotic system as an example; the simulation results show that the designed controller in this paper is feasible and effective. Since the complex value chaotic system is the generalization of the chaotic system, the controller proposed in this paper is applied not only in complex value chaotic system, but also in real chaotic systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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