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Research Article

Energy-Based Spectrum Sensing under Nonreconstruction Framework

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To reduce the computational complexity and rest on less prior knowledge, energy-based spectrum sensing under nonreconstruction framework is studied. Compressed measurements are adopted directly to eliminate the effect of reconstruction error and high computational complexity caused by reconstruction algorithm of compressive sensing. Firstly, we summarize the conventional energy-based spectrum sensing methods. Next, the major effort is placed on obtaining the statistical characteristics of compressed measurements and its corresponding squared form, such as mean, variance, and the probability density function. And then, energy-based spectrum sensing under nonreconstruction framework is addressed and its performance is evaluated theoretically and experimentally. Simulations for the different parameters are performed to verify the performance of the presented algorithm. The theoretical analysis and simulation results reveal that the performance drops slightly less than that of conventional energy-normalization method and reconstruction-based spectrum sensing algorithm, but its computational complexity decreases remarkably, which is the first thing one should think about for practical applications. Accordingly, the presented method is reasonable and effective for fast detection in most cognitive scenarios.

1. Introduction

Cognitive radio is an effective method to cope with spectral unbalanced utilization and low spectral efficiency [1, 2], and spectrum sensing is its basis and premise, which exploits the conventional signal detection schemes to determine the existence or absence of primary users in some radio spectrums. The most classical methods include energy detection, cyclic-stationary detection, matched-filter detection, and eigenvalue-based detection for single node and cooperative detection for multiple nodes over the fading channel [3–5]. In most cognitive scenarios, spectrum sensing requires that detection algorithms possess less detection time or lower computational complexity. In addition, most prior information is hard to know for cognitive users generally, especially for noncooperative communication and military communication. Energy-based detection method fits with these requirements of spectrum sensing, so it is widely studied in practical application, such as 802.22 protocol. Actually, the related work can be traced to [6] in 1965; more researches have been in detail carried out over the past decade because of emerging of cognitive radio. Many valuable results are acquired for the various communication environments; the corresponding performance of algorithms is justified, such as detection probability, false-alarm probability, missed probability, and ROC curve. Some factors affecting the detection performance are discussed including signal fading and SNR walls [7, 8].

However, wireless communications have experienced tremendous developments in the past decades. Many communication signals posses higher frequency and wider bandwidth, which result in high sampling rate and computational complexity for spectrum sensing. Compressive sensing (CS) is considered as a theoretical tool to deal with these challenges [9]. A considerable amount of research has been developed, especially for wide-band spectrum sensing [10–12]. Up to now, this topic is still extensively under investigation as well in methodological aspects as in some specific applications. According to CS theory, a sparse signal can be accomplished through the linear random projections; the sparsity

of signal denotes the number of nonzero values in some bases. Simply speaking, a high dimension sparse signal can be projected into a low dimension vector by exploiting incoherent measurement matrix. At the same time, these projections can retain most information to precisely reconstruct signal from low dimension vector with high probability. Therefore, sampling rate is determined by sparsity of signal but not bandwidth required by Shannon's sampling theorem. Compressive sensing ignores much unnecessary information. Thus, the number of compressed measurements is far less than that of conventional Shannon's sampling theorem, which breaks through the bottleneck and restriction of Shannon's sampling theorem and makes high resolution sampling possible. Compared with conventional sampling theorem, sampling and compression are performed simultaneously and information but not signal itself is sampled for compressive sensing; consequently it can decrease the sampling number of signal and extract much more information of interest. It is widely recognized that three aspects of topics for compressive sensing should be researched; they are the sparse representation, measurement matrix design, and signal reconstruction algorithm; therefore reconstruction method is an indispensable step for applications of compressive sensing. In general, reconstruction of signal is to recover the signal from less compressed measurements under the constraint of minimum norm. According to definition of sparsity, the condition is 0-norm restriction for reconstructing signal, but this condition is too strict not to obtain much effective and specific application algorithms. As a consequence, Candès et al. proved that restriction condition can be changed to 1-norm if the measurement matrix satisfies the restricted isometry property (RIP) [13].

Recently, compressive sensing is undergoing a great progress; some new methods and theories are introduced, such as adaptive compressed sensing [14], compressed sensing of 2D sparse signal [15], and method of basis selection [16]. More importantly, many advantages of compressive sensing are discovered, which draw extensive attentions from different academic communities. A considerable number of research results are inspired for applications of various research fields, such as video and image compression [17], remote sensing image [18], communication and radar [19], and signal processing [20]. Combined with increasing requirements of cognitive radio, compressive sensing is applied to spectrum sensing for resolving the high sampling rate and computational complexity. But these works almost exploit three aspects of compressive sensing; that is, firstly the signal is compressed and sampled by the measurement matrix, and then compressed measurements are recovered by virtue of reconstruction algorithm; finally the reconstructed signal is adopted to perform spectrum sensing. As we all know, reconstruction algorithm possesses high computational complexity. In addition, the error between the reconstructed signal and the noncompressed signal also will affect necessarily spectrum sensing performance; as a consequence it is resistant to fast detection of spectrum sensing. As far as spectrum sensing is concerned, it belongs to inference problem; we only wonder whether the signal exists and do not care about precise information; therefore the reconstruction of compressed measurements is not necessarily performed. If we completely copy all steps of compressive sensing to carry out spectrum sensing, the computational resources for some practical applications will be dramatically consumed. In other words, reconstruction-based spectrum sensing does not exploit the advantages of compressive sensing completely. Hence, it is considerable method to adopt directly compressed measurements to deal with spectrum sensing without resorting to a full-scale signal reconstruction. It needs to be explained that other aspects of the presented method are the same as conventional compressive sensing except for nonreconstruction of compressed measurements. As a consequence, several standard assumptions in compressive sensing are adopted in nonreconstruction framework.

In literature [21], the idea of nonreconstruction was presented firstly for spectrum sensing, and then related works have been reported in this topic [22–26]. Most of these methods learn from matched filter. In other words, the correlation operation is implemented for the compressed measurements and the known noncompressed signal; some performances, such as detection probability and false-alarm probability, are analyzed approximately in terms of RIP. As we all know, RIP is a condition for the measurement matrix to recover the signal under 1-norm constraint; however, it is not necessary condition for spectrum sensing. In addition, other restrictions are added to the measurement matrix. For example, Gram matrix of the measurement matrix should be unit matrix approximatively; these conditions hold impossibly in most cases. Therefore, all these problems need to be resolved for the applications of spectrum sensing. Except for these works, the effect of measurement matrix on spectrum sensing is analyzed in [27], and then Bayesian compressive sensing approach is introduced to spectrum sensing in [28].

Although much effort is being spent on improving the aforementioned weaknesses, the efficient and effective method has yet to be developed. Works in analyzing statistical characteristics, especially probability density function, are lacking. Little research has been conducted in applying energy-based detection to spectrum sensing under nonreconstruction framework.

According to principle of matched filter, the existing methods require much prior knowledge of signal. But, in fact, it is difficult to acquire some related prior information; therefore these methods are not implemented correctly in most cognitive radio environments. Consequently, we concentrate on gathering directly signal energy from the compressed measurements to infer whether the signal exists. To the best of our knowledge, a little work on energy detection under nonreconstruction framework has been reported. More importantly, we will derive some statistical characteristics of the compressed measurements, including mean, variance, and probability density function, which are relevant to general inference problem, such as signal detection, parameters identification, and feature extraction. Based on these results, the statistical characteristics of the sum of square of compressed measurements are derived by virtue of central limit theorem. And then, quite a few performances are compared with the conventional energy-based detection

method; we analyze the difference of the two algorithms and simulate the performance.

The structure of the paper is as follows. Conventional energy-based detection schemes are summarized in Section 2. A nonreconstruction energy-based detection algorithm is presented in Section 3; we provide the system model of algorithm and derive statistical characteristics and the probability density function and so on. Section 4 affords detailed simulation experiments and analysis to prove the presented algorithm and some researched results. We end the paper with a discussion and some conclusions.

2. Related Research Results of Conventional Energy-Based Detection Algorithm

For the convenience of describing the problem, a binary hypothesis test for the presence or absence of a signal in Gaussian noise channel can be obtained [6]:

$$\begin{aligned} y(t) &= n(t) \quad H_0 \\ y(t) &= s(t) + n(t) \quad H_1. \end{aligned} \tag{1}$$

Because we only process the signal in a confined time interval $0 \le t \le T$ in practical cases, therefore the signal energy in this time interval is

$$E_{Ty} = \int_0^T y^2(t)\,dt. \tag{2}$$

Correspondingly, the analog signal power is defined as

$$P_y = \frac{1}{T}\int_0^T y^2(t)\,dt. \tag{3}$$

2.1. The Relationship of Sampled Signal Energy and Continuous Signal Energy under the Sampling Theorem Framework. At present, what we process is the digital signal in most cases; Shannon's sampling theorem bridge the two types of signals. It is supposed that the digital signal is denoted as $y(mT_s)$; here T_s is the sampling period satisfying Shannon's sampling theorem. In the following, we will discuss the relationship of energy about the continuous signal and the digital signal. Assumed that we process the low-pass signal, its bandwidth is Δf Hz; the used filter is an ideal filter with the bandwidth of Δf Hz and amplitude T_s. It is expressed in the form

$$H(f) = \begin{cases} T_s, & |f| \le \Delta f, \\ 0, & |f| > \Delta f. \end{cases} \tag{4}$$

The corresponding system impulse response is

$$h(t) = 2T_s\Delta f \sin c(2\pi\Delta f t). \tag{5}$$

The sampling signal can be described by

$$y_s(t) = \sum_{m=-\infty}^{\infty} y(mT_s)\,\delta(t - mT_s). \tag{6}$$

Here, δ is an impulse function. Therefore the continuous signal is expressed by terms of impulse response and the sampling signal in the form

$$\begin{aligned} y(t) &= h(t) * y_s(t) \\ &= 2T_s\Delta f \sum_{m=-\infty}^{+\infty} y(mT_s) \sin c\left[2\Delta f(t - mT_s)\right]. \end{aligned} \tag{7}$$

The corresponding energy is defined as

$$\begin{aligned} E_y &= \int_{-\infty}^{\infty} y^2(t)\,dt = \int_{-\infty}^{\infty} \left\{2T_s\Delta f \sum_{m=-\infty}^{+\infty} y(mT_s) \right. \\ &\quad \left. \cdot \sin c\left[2\Delta f(t - mT_s)\right]\right\}^2 dt. \end{aligned} \tag{8}$$

According to property of $\sin c$ function, we have

$$\begin{aligned} &\int_{-\infty}^{+\infty} \sin c(2\Delta f t - 2i\Delta f T_s) \sin c(2\Delta f t - 2k\Delta f T_s)\,dt \\ &= \begin{cases} \dfrac{1}{2\Delta f}, & i = k, \\ 0, & i \ne k. \end{cases} \end{aligned} \tag{9}$$

Substituting (9) into (8), the energy is computed by

$$\begin{aligned} E_y &= \int_{-\infty}^{\infty} \left\{2T_s\Delta f \sum_{m=-\infty}^{+\infty} y(mT_s) \right. \\ &\quad \left. \cdot \sin c\left[2\Delta f(t - mT_s)\right]\right\}^2 dt = (2T_s\Delta f)^2 \\ &\quad \cdot \sum_{m=-\infty}^{+\infty} y^2(mT_s) \int_{-\infty}^{\infty} \left\{\sin c\left[2\Delta f(t - mT_s)\right]\right\}^2 dt \\ &= \frac{(2T_s\Delta f)^2}{2\Delta f} \sum_{m=-\infty}^{+\infty} y^2(mT_s) \\ &= 2T_s^2\Delta f \sum_{m=-\infty}^{+\infty} y^2(mT_s). \end{aligned} \tag{10}$$

As mentioned before, it is virtually impossible to exploit the entire signals in domain $(-\infty, \infty)$; we only process the signal in the time interval $0 \le t \le T$, and the corresponding number of sampling signal is $N = T/T_s$. The accumulative energy in the time interval $0 \le t \le T$ is

$$E_y = 2T_s^2\Delta f \sum_{m=0}^{N-1} y^2(mT_s). \tag{11}$$

The corresponding power is expressed as

$$P_y = \frac{E_y}{T} = \frac{2T_s\Delta f}{N} \sum_{m=0}^{N-1} y^2(mT_s). \tag{12}$$

For convenience, but without loss of generality, we take Nyquist frequency as sampling frequency; that is, $f_s = 2\Delta f$; (11) and (12) can be simplified to

$$E_y = T_s \sum_{m=0}^{N-1} y^2(mT_s) = \frac{1}{2\Delta f} \sum_{m=0}^{N-1} y^2(mT_s),$$
$$P_y = \frac{E_y}{T} = \frac{1}{N} \sum_{m=0}^{N-1} y^2(mT_s), \tag{13}$$

where $N = T/T_s = Tf_s = 2T\Delta f$. The binary hypothesis test for digital signal under the condition of Shannon's sampling theorem is

$$y(mT_s) = n(mT_s) \quad H_0$$
$$y(mT_s) = s(mT_s) + n(mT_s) \quad H_1. \tag{14}$$

For the present, energy and power or their varieties are adopted as test statistic for spectrum sensing; main methods consist of energy normalization-based and power-based schemes. In the case of energy normalization-based method, noise energy is normalized to make the noise fit with standard Gaussian distribution; therefore the chi-square distribution can be exploited to analyze the performances, such as detection probability and false-alarm probability. As far as power-based algorithm is concerned, signal power is exploited to detect whether the signal exists by terms of Gaussian distribution.

2.2. Energy Normalization-Based Spectrum Sensing Algorithm. For the binary test hypothesis (14), the energy is computed by

$$E_y = \frac{1}{2\Delta f} \sum_{m=0}^{N-1} y^2(mT_s) = \begin{cases} \dfrac{1}{2\Delta f} \displaystyle\sum_{m=0}^{N-1} [s(mT_s) + n(mT_s)]^2 & H_1 \\ \dfrac{1}{2\Delta f} \displaystyle\sum_{m=0}^{N-1} n^2(mT_s) & H_0. \end{cases} \tag{15}$$

To normalize the noise energy, $Z = E_y/(N_0/2)$ is used as test statistic; the corresponding binary test hypothesis is

$$Z = \frac{E_y}{N_0/2} = \frac{1}{N_0\Delta f} \sum_{m=0}^{N-1} y^2(mT_s) = \begin{cases} \dfrac{1}{N_0\Delta f} \displaystyle\sum_{m=0}^{N-1} [s(mT_s) + n(mT_s)]^2 & H_1 \\ \dfrac{1}{N_0\Delta f} \displaystyle\sum_{m=0}^{N-1} n^2(mT_s) & H_0. \end{cases} \tag{16}$$

For the noise with mean 0 and variance σ_n^2, the double-sideband power spectral density is $N_0/2$ W/Hz; therefore the noise power is expressed by

$$P = R_X(0) = E\left[n^2(t)\right] = \sigma_n^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_n(\omega)\, d\omega = \frac{N_0}{2} \cdot 2\Delta f = N_0 \cdot \Delta f. \tag{17}$$

The binary test hypothesis is transformed to

$$Z = \begin{cases} \displaystyle\sum_{m=0}^{N-1} \left[\frac{s(mT_s)}{\sigma_n} + \frac{n(mT_s)}{\sigma_n}\right]^2 & H_1 \\ \displaystyle\sum_{m=0}^{N-1} \left[\frac{n(mT_s)}{\sigma_n}\right]^2 & H_0. \end{cases} \tag{18}$$

Hence, $n(mT_s)/\sigma_n$ fits with the standard Gaussian distribution. Let $a(m) = s(mT_s)/\sigma_n$, $b(m) = n(mT_s)/\sigma_n$. Then (18) reduces to

$$Z = \begin{cases} \displaystyle\sum_{m=0}^{N-1} [a(m) + b(m)]^2 & H_1 \\ \displaystyle\sum_{m=0}^{N-1} [b(m)]^2 & H_0. \end{cases} \tag{19}$$

Because $b(m) = n(mT_s)/\sigma_n$ is a standard Gaussian distribution, $Z_n = \sum_{m=0}^{N-1}[b(m)]^2$ is a central chi-square distribution with degree of freedom N, and $Z_{sn} = \sum_{m=0}^{N-1}[a(m) + b(m)]^2$ is a noncentral chi-square distribution with degree of freedom N. They can be denoted as $Z_n \sim \chi_N^2$, $Z_{sn} \sim \chi_N^2(2\gamma)$; here $\gamma = \sum_{m=0}^{N-1} a^2(m) = \sum_{m=0}^{N-1}(s^2(mT_s)/\sigma_n^2) = \sum_{m=0}^{N-1} s^2(mT_s)/\sigma_n^2$; it is the ratio of signal energy to noise power, that is, SNR. If the noise power is fixed, γ will become bigger with the increasing of the number of signal N.

According to the probability density function of chi-square distribution, for a specific threshold λ, the detection probability and false-alarm probability for the Gaussian channel model are

$$P_d = P(Z > \lambda \mid H_1) = Q_{N/2}\left(\sqrt{2\gamma}, \sqrt{\lambda}\right),$$
$$P_f = P(Z > \lambda \mid H_0) = \frac{\Gamma(N/2, \lambda/2)}{\Gamma(N/2)}. \tag{20}$$

Here, $\Gamma(\cdot)$ is the gamma function, $Q_u(a, x)$ is the generalized Marcum's Q function, and $\Gamma(a, x)$ is the incomplete gamma function, which are, respectively, defined as

$$Q_u(a, x) = \frac{1}{a^{u-1}} \int_x^{\infty} t^u e^{-(a^2+t^2)/2} I_{u-1}(at)\, dt,$$
$$\Gamma(a, x) = \int_x^{+\infty} t^{a-1} e^{-t} dt. \tag{21}$$

2.3. Power-Based Spectrum Sensing Algorithm. According to (14), the power-based binary test hypothesis can be expressed as

$$Z = \frac{1}{N}\sum_{m=0}^{N-1} y^2(mT_s) = \begin{cases} \frac{1}{N}\sum_{m=0}^{N-1}\left[s(mT_s)+n(mT_s)\right]^2 & H_1 \\ \frac{1}{N}\sum_{m=0}^{N-1} n^2(mT_s) & H_0. \end{cases} \tag{22}$$

Generally, the number of signals is big enough; therefore test statistic can be modeled as a random Gaussian variable by virtue of central limit theorem. The corresponding detection probability and false-alarm probability, respectively, are

$$\begin{aligned} P_d &= P(Z > \lambda \mid H_1) = \frac{1}{2}\operatorname{erfc}\left[\frac{\lambda - (P_s+\sigma_n^2)}{2/\sqrt{N}(P_s+\sigma_n^2)}\right], \\ P_f &= P(Z > \lambda \mid H_0) = \frac{1}{2}\operatorname{erfc}\left(\frac{\lambda - \sigma_n^2}{2/\sqrt{N}\sigma_n^2}\right). \end{aligned} \tag{23}$$

Here, $P_s = \sum_{m=0}^{N-1} s^2(m)/N$; $\operatorname{erfc}(\cdot)$ is a complementary error function, which is defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}dt. \tag{24}$$

3. A Nonreconstruction Energy Detection Algorithm

3.1. The System Model of Energy-Based Spectrum Sensing under Nonreconstruction Framework. Suppose that the sparsity of signal is K, and the length of signal is N, the measurement matrix $\mathbf{\Phi} \in R^{M\times N}$ $(M \ll N)$. If the signal $\mathbf{s}$ is sparse, then $\mathbf{y} = \mathbf{\Phi s}$; each entry of $\mathbf{y}$ is the inner product of the row of matrix $\mathbf{\Phi}$ and $\mathbf{s}$; here $\mathbf{y} \in R^M$ are the compressed measurements. If the signal $\mathbf{s}$ itself is not sparse, $\mathbf{s}$ should be firstly represented in a basis sparsely; that is, $\mathbf{s} = \mathbf{\Psi}\boldsymbol{\alpha}$, where $\boldsymbol{\alpha}$ is the sparse representation of N-dimension vector and $\mathbf{\Psi}$ is a $N \times N$ matrix consisting of the sparse basis; then the compressed measurements are obtained by

$$\mathbf{y} = \mathbf{\Phi s} = \mathbf{\Phi\Psi}\boldsymbol{\alpha} = \mathbf{\Theta}\boldsymbol{\alpha}. \tag{25}$$

If the signal is sampled in terms of compressive sensing, the corresponding binary test hypothesis of spectrum sensing is written as

$$\begin{aligned} \mathbf{y} &= \mathbf{\Phi n} \quad H_0 \\ \mathbf{y} &= \mathbf{\Phi}[\mathbf{s}+\mathbf{n}] \quad H_1, \end{aligned} \tag{26}$$

where $\mathbf{y}$ is a vector of compressed measurements with the length of M and $\mathbf{s}$ and $\mathbf{n}$ are noncompressed signal vector and noise vector with the length of N, respectively. Besides, entries of noise vector $\mathbf{n}$ are i.i.d random Gaussian variables. Entries in $\mathbf{y}$ can be expressed by

$$\begin{aligned} Y_k &= \sum_{i=1}^{N}\phi_{ki}n(i), \quad 1 \le k \le M \ (H_0) \\ Y_k &= \sum_{i=1}^{N}\phi_{ki}[s(i)+n(i)], \quad 1 \le k \le M \ (H_1), \end{aligned} \tag{27}$$

where ϕ_{ki} is the entry of measurement matrix $\mathbf{\Phi}$ and $n(i)$ denotes the entry of noise vector. The noise is a random variable for each time; therefore the compressed measurements are also random variables whether the measurement matrix is the random matrix or the deterministic matrix. From (27), we can observe that each entry of compressed measurements can be obtained by the inner product of the row vector of measurement matrix and the noncompressed signal vector when the noncompressed signal is digital, which is the case we study in this paper. If the noncompressed signal is analog, obtaining the compressive measurement data is by virtue of analog to information converter (AIC), which is another important and open problem for compressive sensing. However, we mainly focus on the spectrum sensing by exploiting directly compressed measurements when the noncompressed signal is digital. Therefore, inner product may satisfy our requirements; no other particular method of obtaining compressed measurements is introduced.

For measurement matrix $\mathbf{\Phi}$, it consists of deterministic matrix and random matrix. Up to now, random matrix is widely used because of incoherence of columns of measurement matrix; only limited works about deterministic matrix were reported. On the other hand, the entry of measurement matrix is a constant if the deterministic matrix is adopted. It is obvious that Y_k is a random Gaussian variable because it is a linear combination of the random Gaussian variables. This case can be analyzed easily. To generalize our conclusion, the random matrix is used to analyze statistical characteristics of compressed measurements in our paper. Besides, we assume that entries of random matrix are i.i.d random variables, which are independent with entries of noise vector.

For the convenience of following derivation, X_{ki} is defined as

$$\begin{aligned} X_{ki} &= \phi_{ki}n(i), \quad 1 \le k \le M,\ 1 \le i \le N \ (H_0) \\ X_{ki} &= \phi_{ki}[s(i)+n(i)], \\ &\qquad 1 \le k \le M,\ 1 \le i \le N \ (H_1). \end{aligned} \tag{28}$$

According to principles of statistical signal processing, X_{ki} is i.i.d random variable because entry of measurement matrix $\mathbf{\Phi}$ and entry of noise vector $\mathbf{n}$ are i.i.d random variables, and they are independent of each other. Therefore each entry in $\mathbf{y}$ can be denoted by

$$Y_k = \sum_{i=1}^{N} X_{ki}, \quad 1 \le k \le M. \tag{29}$$

The column of the measurement matrix $\mathbf{\Phi}$ is normalized; $D(\phi_{ki}) = 1/M$. the main purpose of normalization is to

remove the effect of measurement matrix on energy of the compressed measurements, which offer agreed standard for the comparison of different algorithms. In addition, the noise is also normalized to compare with traditional energy normalization scheme. Hence $Z = \sum_{k=0}^{M-1} Y_k^2/\sigma_n^2$ is adopted as test statistic for compressive energy-based detection algorithm under nonreconstruction framework.

3.2. Acquiring Probability Density Function by Virtue of Direct Method. According to the principles of random signal processing, the probability density function of Y_k should be obtained firstly to derive the probability density function of Y_k^2. It can be seen from (29) that Y_k is the sum of N random variables, each random variable can be obtained by the product of matrix entry ϕ_{ki} and $n(i)$. As declared above, $n(i)$ and the measurement matrix entry ϕ_{ki} are independently and identically distributed random Gaussian variables. Up to now, the distribution of two random variables possesses a concrete closed-form expression when only they are all random Gaussian variables. It is very difficult to derive the distribution function for other types of random variables. Now we analyze the distribution of the product when the entries of the measurement matrix are the random Gaussian variables. The two cases H_0 and H_1 on the basis of the binary test hypothesis will be discussed. For two independent random Gaussian variables $X \sim N(0, \sigma_X^2)$ and $Y \sim N(0, \sigma_Y^2)$, the probability density function of their product $Z = XY$ is

$$f_Z(z) = \frac{1}{\pi\sigma_X\sigma_Y} K_0\left(\frac{|z|}{\sigma_X\sigma_Y}\right), \tag{30}$$

where K_0 is the modified Bessel function of the second kind. Applying $D(\phi_{ki}) = 1/M$ and $D[n(i)] = \sigma_n^2$ to (30) yields the probability density function of X_{ki}:

$$f_{X_{ki}}(x) = \frac{M}{\pi\sigma_n} K_0\left(\frac{M|x|}{\sigma_n}\right). \tag{31}$$

To derive the probability density function of $Y_k = \sum_{i=1}^{N} X_{ki}$, $1 \le k \le M$ from X_{ki}, we assume that $Y_{k1} = X_{k1}$, $Y_{k2} = X_{k2}, \ldots, Y_{kN} = Y_k = \sum_{i=1}^{N} X_{ki}$; they form the multidimensional vectors $\mathbf{Y} = [Y_{k1}Y_{k2}\cdots Y_{kN}]^T$ and $\mathbf{X}_k = [X_{k1}X_{k2}\cdots X_{kN}]^T$. Therefore, the entries of $\mathbf{X}_k$ can be expressed in terms of $\mathbf{Y}$ in the form

$$\begin{aligned} X_{k1} &= h_1(Y_{k1}, Y_{k2}, \ldots, Y_{kN}) = Y_{k1} \\ X_{k2} &= h_2(Y_{k1}, Y_{k2}, \ldots, Y_{kN}) = Y_{k2} \\ &\vdots \\ X_{kN} &= h_N(Y_{k1}, Y_{k2}, \ldots, Y_{kN}) = Y_{kN} - \sum_{i=1}^{N-1} Y_{ki}. \end{aligned} \tag{32}$$

Their corresponding Jacobian is

$$|J| = \begin{vmatrix} \frac{\partial h_1}{\partial y_{k1}} & \frac{\partial h_1}{\partial y_{k2}} & \cdots & \frac{\partial h_1}{\partial y_{kN}} \\ \frac{\partial h_2}{\partial y_{k1}} & \frac{\partial h_2}{\partial y_{k2}} & \cdots & \frac{\partial h_2}{\partial y_{kN}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_N}{\partial y_{k1}} & \frac{\partial h_N}{\partial y_{k2}} & \cdots & \frac{\partial h_N}{\partial y_{kN}} \end{vmatrix} = \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & 1 \end{vmatrix} = 1. \tag{33}$$

Accordingly, the relation of joint probability density function of $\mathbf{Y}$ and joint probability density function of $\mathbf{X}_k$ is

$$\begin{aligned} f_Y(y_{k1}, y_{k2}, \ldots, y_{kN}) &= |J| \cdot f_{X_k}(x_{k1}, x_{k2}, \ldots, x_{kN}) \\ &= |J| \cdot f_{X_k}(h_1(y_{k1}, y_{k2}, \ldots, y_{kN}), \\ &\quad h_2(y_{k1}, y_{k2}, \ldots, y_{kN}), \ldots, h_N(y_{k1}, y_{k2}, \ldots, y_{kN})). \end{aligned} \tag{34}$$

Because the entries of $\mathbf{X}_k$ are independently and identically distributed random variables, the joint PDF of (34) can be rewritten by

$$\begin{aligned} f_Y(y_{k1}, y_{k2}, \ldots, y_{kN}) &= |J| \\ &\cdot f_{X_{k1}}(h_1(y_{k1}, y_{k2}, \ldots, y_{kN})) \\ &\cdot f_{X_{k2}}(h_2(y_{k1}, y_{k2}, \ldots, y_{kN})) \\ &\cdots f_{X_{kN}}(h_N(y_{k1}, y_{k2}, \ldots, y_{kN})). \end{aligned} \tag{35}$$

Consequently, the probability density function of Y_k is derived by virtue of that of $\mathbf{Y}$:

$$\begin{aligned} f_{Y_{kN}}(y_{kN}) &= \iiint_{N-1} \big(f_{X_{k1}}(h_1(y_{k1}, y_{k2}, \ldots, y_{kN})) \\ &\quad \cdot f_{X_{k2}}(h_2(y_{k1}, y_{k2}, \ldots, y_{kN})) \\ &\quad \cdots f_{X_{kN}}(h_N(y_{k1}, y_{k2}, \ldots, y_{kN}))\big)\, dy_{k1} dy_{k2} \cdots dy_{k(N-1)} \\ &= \iiint_{N-1} \left(f_{X_{k1}}(x_{k1}) f_{X_{k2}}(x_{k2}) \right. \\ &\quad \left. \cdots f_{X_{kN}}\left(\sum_{i=1}^{N} x_{ki}\right)\right) dx_{k1} dx_{k2} \cdots dx_{k(N-1)} \\ &= f_{X_{k1}}(h_1(y_{k1}, y_{k2}, \ldots, y_{kN})) * f_{X_{k2}}(h_2(y_{k1}, y_{k2}, \ldots, \\ &\quad y_{kN})) * \cdots * f_{X_{kN}}(h_N(y_{k1}, y_{k2}, \ldots, y_{kN})). \end{aligned} \tag{36}$$

As aforementioned, $E(\phi_{ki}) = E[n(i)] = 0$, $D(\phi_{ki}) = 1/M$, and $D[n(i)] = \sigma_n^2$. Applying these results and (31) to

(36), the specific expression can be achieved. When N is even number, the probability density function of Y_{kN} is

$$f_{Y_{kN}}(y_{kN}) = \frac{\sqrt{M}}{\sqrt{\sigma_n^2}(N/2-1)!}\exp\left(-\frac{\sqrt{M}(y_{kN})}{\sqrt{\sigma_n^2}}\right) \cdot \sum_{i=0}^{N/2-1}\frac{(N/2+i-1)!}{2^{N/2+i}i!(m-i-1)!} \cdot \left[\frac{\sqrt{M}(y_{kN})}{\sqrt{\sigma_n^2}}\right]^{N/2-1-i}. \tag{37}$$

When N is odd number, the probability density function of Y_{kN} is

$$f_{Y_{kN}}(y_{kN}) = \frac{\sqrt{M}\left[|y_{kN}|\sqrt{M}/2\sqrt{\sigma_n^2}\right]^{(N-1)/2}}{\sqrt{\pi\sigma_n^2}\Gamma[(N-1)/2+1/2]} \cdot K_{(N-1)/2}\left[\frac{|y_{kN}|\sqrt{M}}{\sqrt{\sigma_n^2}}\right]. \tag{38}$$

Now we analyze probability density function of the H_1 case. According to the binary test hypothesis, it follows that $Y_k = \sum_{i=1}^{N}\phi_{ki}[s(i)+n(i)]$, $1 \le k \le M$; the statistical parameters are with $E[\phi_{ki}] = 0$, $E[s(i)+n(i)] = s$, $D(\phi_{ki}) = 1/M$, and $D[n(i)] = \sigma_n^2$. If $D[\phi_{ki}] = D[s(i)+n(i)]$, that is, $\sigma_n^2 = 1/M$, the probability density function of Y_{kN} has a closed-form expression. Let $\sigma_n^2 = 1/M = \sigma^2$. When N is even number, the probability density function of Y_{kN} is

$$f_{Y_{kN}}(y_{kN}) = \frac{1}{2\sigma^2}\left[\frac{|y_{kN}|}{2\sigma^2}\right]^{N/2-l}\exp\left[-\frac{|y_{kN}|+s^2/2}{\sigma^2}\right] \cdot \sum_{i=0}^{\infty}\sum_{l=0}^{N/2+i-1}\frac{(m+i+l-1)!}{2^l i!(m+i-1)!} \cdot \frac{1}{l!(m+i-l-1)!}\left[\frac{s^2}{4\sigma^2}\right]^i\left[\frac{|y_{kN}|}{\sigma^2}\right]^{i-l}. \tag{39}$$

When N is odd number, the probability density function of Y_{kN} is

$$f_{Y_{kN}}(y_{kN}) = \frac{1}{\sqrt{\pi}\sigma^2}\left[\frac{y_{kN}}{2\sigma^2}\right]^{(N-1)/2}\exp\left[-\frac{s^2}{2\sigma^2}\right] \cdot \sum_{i=0}^{\infty}\frac{1}{i!\Gamma(m+i+1/2)} \cdot \left[\frac{s^2|y_{kN}|}{4\sigma^2}\right]^{(N-1)/2+i} \cdot K_{(N-1)/2+i}\left[\frac{|y_{kN}|}{\sigma^2}\right]. \tag{40}$$

In fact, it is difficult to satisfy the condition $\sigma_n^2 = 1/M = \sigma^2$. Therefore we cannot derive the specific closed form of probability density function under a condition of $\sigma_n^2 \ne 1/M$.

Through the previous analysis, we can observe that it is difficult or even impossible to derive the probability density function of the square of Y_{kN}, because the probability density function of Y_{kN} consists of multiple accumulations and factorial, and some operations are implemented in the range $(0,\infty)$. In addition, the most troublesome thing to us is that the aforementioned derivation can be carried out because the entries of measurement matrix are supposed as random Gaussian variables. But, in practical application, measurement matrix may fit with other distributions. Consequently, it is formidable to generalize the previous results to other distributions. In order to obtain general conclusions, probability distribution of compressed measurements is modeled as Gaussian distribution in terms of central limit theorem. In this case, we derive some related closed-form expressions when the entries of the measurement matrix are only independently and identically distributed.

3.3. Derivation of Probability Density Function of Test Statistic Using Central Limit Theorem. The probability density function is only determined by mean and variance for the random Gaussian variable. Therefore, we firstly derive their expression for two cases H_0 and H_1 of the binary test hypothesis under nonreconstruction framework. It is assumed that each entry of the measurement matrix and noise is the statistically independent random variable.

Assume that U and V are statistically and independently random variables; we give the result about mean and variance of product T of two random variables U and V directly. So the mean of product T of two random variables U and V is expressed by

$$E[T] = E[UV] = E[U]E[V]. \tag{41}$$

The variance of the product T is

$$\begin{aligned} D[T] &= D[UV] = \sigma_U^2\sigma_V^2 + \left(\sigma_U^2 m_V^2 + m_U^2\sigma_V^2\right) \\ &= D(U)D(V) + \left(\sigma_U^2 m_V^2 + m_U^2\sigma_V^2\right). \end{aligned} \tag{42}$$

We observe from (42) that $D[UV] \ge D(U)D(V)$, when the random variable has zero mean. Applying these results to (28), we can obtain mean and variance, respectively:

$$\begin{aligned} E[X_{ki}] &= E[\phi_{ki}n(i)] = E[\phi_{ki}]E[n(i)], \quad 1 \le k \le M \ (H_0) \\ E[X_{ki}] &= E[\phi_{ki}[s(i)+n(i)]] \\ &= E[\phi_{ki}]E[s(i)+n(i)], \quad 1 \le k \le M \ (H_1) \end{aligned} \tag{43}$$

$$\begin{aligned} D[X_{ki}] &= D[\phi_{ki}n(i)], \quad 1 \le k \le M \ (H_0) \\ D[X_{ki}] &= D[\phi_{ki}[s(i)+n(i)]], \quad 1 \le k \le M \ (H_1). \end{aligned} \tag{44}$$

Because $E[\phi_{ki}] = E[n(i)] = 0$, the mean of (43) can be calculated by

$$\begin{aligned} E\left[X_{ki}\right] &= 0, \quad 1 \le k \le M \ \left(H_0\right) \\ E\left[X_{ki}\right] &= 0, \quad 1 \le k \le M \ \left(H_1\right). \end{aligned} \tag{45}$$

The variance of H_0 case is computed by

$$D\left[X_{ki}\right] = D\left[\phi_{ki} n(i)\right] = D\left[\phi_{ki}\right] D\left[n(i)\right]. \tag{46}$$

Next we analyze the variance for H_1 case; it follows that $E[s(i) + n(i)] = E[s(i)] + E[n(i)] = E[s(i)] \neq 0$. According to (42), we have

$$\begin{aligned} D\left[\phi_{ki}\left[s(i) + n(i)\right]\right] &= D\left[\phi_{ki}\right] D\left[s(i) + n(i)\right] \\ &\quad + D\left[\phi_{ki}\right] E^2\left[s(i) + n(i)\right] \\ &= D\left[\phi_{ki}\right] D\left[n(i)\right] \\ &\quad + D\left[\phi_{ki}\right] E^2\left[s(i)\right]. \end{aligned} \tag{47}$$

So the variance of (44) is

$$\begin{aligned} D\left[X_{ki}\right] &= D\left[\phi_{ki} n(i)\right] = D\left[\phi_{ki}\right] D\left[n(i)\right], \\ &\qquad 1 \le k \le M \ \left(H_0\right) \\ D\left[X_{ki}\right] &= D\left[\phi_{ki}\left[s(i) + n(i)\right]\right] \\ &= D\left[\phi_{ki}\right] D\left[n(i)\right] + D\left[\phi_{ki}\right] E^2\left[s(i)\right], \\ &\qquad 1 \le k \le M \ \left(H_1\right). \end{aligned} \tag{48}$$

Because, for all k and i, ϕ_{ki} and $n(i)$ are statistically independent, ϕ_{ki} is identically distributed, and equivalently $n(i)$ is also identically distributed. Hence, for all k and i, each X_{ki} is statistically independent; the mean can be computed by

$$\begin{aligned} E\left[Y_k\right] &= E\left(\sum_{i=1}^{N} X_{ki}\right) = \sum_{i=1}^{N} E\left(X_{ki}\right) = 0, \quad H_0 \\ E\left[Y_k\right] &= E\left(\sum_{i=1}^{N} X_{ki}\right) = \sum_{i=1}^{N} E\left(X_{ki}\right) \\ &= \sum_{i=1}^{N} E\left[\phi_{ki}\left[s(i) + n(i)\right]\right] = 0, \quad H_1. \end{aligned} \tag{49}$$

And the variance can be calculated by

$$\begin{aligned} D\left[Y_k\right] &= D\left(\sum_{i=1}^{N} X_{ki}\right) = \sum_{i=1}^{N} D\left(X_{ki}\right) \\ &= \sum_{i=1}^{N} D\left[\phi_{ki}\right] D\left[n(i)\right], \quad H_0 \\ D\left[Y_k\right] &= D\left(\sum_{i=1}^{N} X_{ki}\right) = \sum_{i=1}^{N} D\left(X_{ki}\right) \\ &= \sum_{i=1}^{N} \left\{D\left[\phi_{ki}\right] D\left[n(i)\right] + D\left[\phi_{ki}\right] E^2\left[s(i)\right]\right\}, \\ &\qquad H_1. \end{aligned} \tag{50}$$

Supposed that $E[s(i)] = s$, $1 \le i \le N$. In addition, in order to eliminate the effects of measurement matrix to the energy of compressed measurements, column entries of measurement matrix are normalized; that is, $D[\phi_{ki}] = 1/M$; the variance in (50) can be simplified to

$$\begin{aligned} D\left[Y_k\right] &= D\left(\sum_{i=1}^{N} X_{ki}\right) = \sum_{i=1}^{N} D\left(X_{ki}\right) = \frac{N}{M}\sigma_n^2, \quad H_0 \\ D\left[Y_k\right] &= D\left(\sum_{i=1}^{N} X_{ki}\right) = \sum_{i=1}^{N} D\left(X_{ki}\right) = \frac{N}{M}\left\{\sigma_n^2 + s^2\right\}, \\ &\qquad H_1. \end{aligned} \tag{51}$$

Due to $M \ll N$, $D(Y_k) \ge D[n(i)]$ holds for the two cases of binary test hypothesis; namely, the variance of compressed measurements is bigger than that of initial noise and signal. After deriving its mean and variance, we can acquire the closed-form expression of probability density function for compressed measurements.

3.4. Performance Analysis of Energy-Based Spectrum Sensing Algorithm under Nonreconstruction Framework. For the convenience of analysis, the sum of square of the compressed measurements is expressed in terms of X_{ki} in the form

$$\sum_{k=0}^{M-1} Y_k^2 = \begin{cases} \displaystyle\sum_{k=0}^{M-1}\left[\sum_{i=1}^{N}\phi_{ki} n(i)\right]^2 = \sum_{k=0}^{M-1}\left[\sum_{i=1}^{N} X_{ki}\right]^2, & H_0 \\ \displaystyle\sum_{k=0}^{M-1}\left[\sum_{i=1}^{N}\phi_{ki}\left[s(i) + n(i)\right]\right]^2 = \sum_{k=0}^{M-1}\left[\sum_{i=1}^{N} X_{ki}\right]^2, & H_1. \end{cases} \tag{52}$$

It can be seen from (51) that the compressed measurements no longer fit with the standard Gaussian distribution because the variance is not equal to 1. So the sum of square of the compressed measurements does not satisfy the condition of chi-square distribution. For convenience of computation and comparisons, the noise is normalized; therefore the test statistic of energy-based spectrum sensing algorithm under nonreconstruction framework is taken as $Z = \sum_{k=0}^{M-1} Y_k^2/\sigma_n^2$.

Applying this expression to the binary test hypothesis (26), the test statistic is denoted as

$$Z = \frac{\sum_{k=0}^{M-1} Y_k^2}{\sigma_n^2} = \sum_{k=0}^{M-1} \left(\frac{Y_k}{\sigma_n}\right)^2 = \begin{cases} \sum_{k=0}^{M-1} \left[\sum_{i=1}^{N} \phi_{ki} \frac{n(i)}{\sigma_n}\right]^2, & H_0 \\ \sum_{k=0}^{M-1} \left\{\sum_{i=1}^{N} \left[\phi_{ki} \frac{s(i)}{\sigma_n} + \phi_{ki} \frac{n(i)}{\sigma_n}\right]\right\}^2, & H_1. \end{cases} \tag{53}$$

We can see from (53) that compressed measurements can be exploited to perform spectrum sensing, where spectrum sensing and compressive sensing are integrated. The mean and variance of test statistic are calculated, respectively, by

$$\begin{aligned} E\left[\frac{Y_k}{\sigma_n}\right] &= E\left(\sum_{i=1}^{N} \frac{X_{ki}}{\sigma_n}\right) = \frac{1}{\sigma_n} \sum_{i=1}^{N} E(X_{ki}) \\ &= \frac{1}{\sigma_n} \sum_{i=1}^{N} E[\phi_{ki} n(i)] = 0, \quad H_0 \\ E\left[\frac{Y_k}{\sigma_n}\right] &= E\left(\sum_{i=1}^{N} \frac{X_{ki}}{\sigma_n}\right) = \frac{1}{\sigma_n} \sum_{i=1}^{N} E(X_{ki}) \\ &= \frac{1}{\sigma_n} \sum_{i=1}^{N} E[\phi_{ki}[s(i) + n(i)]] = 0, \quad H_1 \\ D\left[\frac{Y_k}{\sigma_n}\right] &= D\left(\sum_{i=1}^{N} \frac{X_{ki}}{\sigma_n}\right) = \frac{1}{\sigma_n^2} \sum_{i=1}^{N} D(X_{ki}) = N\sigma_m^2, \\ & \qquad H_0 \\ D\left[\frac{Y_k}{\sigma_n}\right] &= D\left(\sum_{i=1}^{N} \frac{X_{ki}}{\sigma_n}\right) = \frac{1}{\sigma_n^2} \sum_{i=1}^{N} D(X_{ki}) \\ &= N\sigma_m^2 \left\{1 + \frac{s^2}{\sigma_n^2}\right\}, \quad H_1. \end{aligned} \tag{54}$$

Consequently, $\sum_{i=1}^{N} (n(i)/\sigma_n)$ fits with the standard Gaussian distribution. But, for the compressed measurements, their variance no longer equals 1 in most cases; the test statistic fits with gamma distribution; therefore the probability density function is

$$f_Z(z) = \begin{cases} \dfrac{z^{M/2-1} \exp\left(-z/2N\sigma_m^2\right)}{\left(2N\sigma_m^2\right)^{M/2} \Gamma((1/2)M)}, & H_0 \\ \dfrac{z^{M/2-1} \exp\left(-z\sigma_n^2/2N\sigma_m^2\left(\sigma_n^2 + s^2\right)\right)}{\left[2N\sigma_m^2\left(1 + s^2/\sigma_n^2\right)\right]^{M/2} \Gamma((1/2)M)}, & H_1. \end{cases} \tag{55}$$

Here, $\Gamma(\cdot)$ is gamma function. The corresponding cumulative distribution function (CDF) for the binary test hypothesis is

$$F_Z(z) = \begin{cases} \int_0^z \dfrac{u^{M/2-1} \exp\left(-u/2N\sigma_m^2\right)}{\left(2N\sigma_m^2\right)^{M/2} \Gamma((1/2)M)} du, & H_0 \\ \int_0^z \dfrac{u^{M/2-1} \exp\left(-\sigma_n^2 u/2N\sigma_m^2\left(\sigma_n^2 + s^2\right)\right)}{\left[2N\sigma_m^2\left(1 + s^2/\sigma_n^2\right)\right] 2N\left(\sigma_n^2 + s^2\right)^{M/2} \Gamma((1/2)M)} du, & H_1. \end{cases} \tag{56}$$

For any $M/2$, there is no closed-form expression for this integral; however, if $M/2$ is an integer, that is, M is even, (56) can be simplified to

$$F_Z(z) = \begin{cases} 1 - \exp\left(-\dfrac{z}{2N\sigma_m^2}\right) \sum_{k=0}^{M/2-1} \dfrac{1}{k} \left(\dfrac{z}{2N\sigma_m^2}\right)^k, & H_0 \\ 1 - \exp\left(-\dfrac{\sigma_n^2 z}{2N\sigma_m^2\left(\sigma_n^2 + s^2\right)}\right) \sum_{k=0}^{M/2-1} \dfrac{1}{k} \left(\dfrac{\sigma_n^2 z}{2N\sigma_m^2\left(\sigma_n^2 + s^2\right)}\right)^k, & H_1. \end{cases} \tag{57}$$

For a given threshold λ, the detection probability of Gaussian channel is expressed by

$$\begin{aligned} P_d = P(Z > \lambda \mid H_1) &= 1 - \left[1 - \exp\left(-\frac{\sigma_n^2 \lambda}{2N\sigma_m^2\left(\sigma_n^2 + s^2\right)}\right) \cdot \sum_{k=0}^{M/2-1} \frac{1}{k} \left(\frac{\sigma_n^2 \lambda}{2N\sigma_m^2\left(\sigma_n^2 + s^2\right)}\right)^k\right] \\ &= \exp\left(-\frac{\sigma_n^2 \lambda}{2N\sigma_m^2\left(\sigma_n^2 + s^2\right)}\right) \cdot \sum_{k=0}^{M/2-1} \frac{1}{k} \left(\frac{\sigma_n^2 \lambda}{2N\sigma_m^2\left(\sigma_n^2 + s^2\right)}\right)^k. \end{aligned} \tag{58}$$

The false-alarm probability is

$$P_f = P(Z > \lambda \mid H_0) = 1 - \left[1 - \exp\left(-\frac{\lambda}{2N\sigma_m^2}\right) \sum_{k=0}^{M/2-1} \frac{1}{k} \left(\frac{\lambda}{2N\sigma_m^2}\right)^k\right]$$

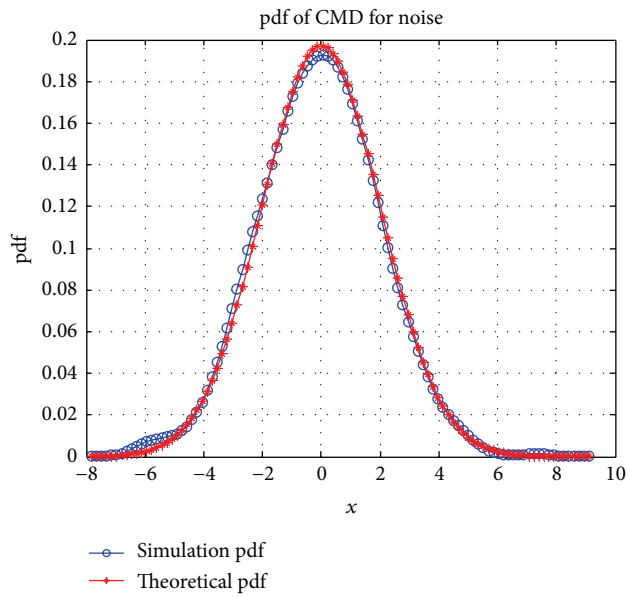


Figure 1: pdf of CMD for noise; $N = 2048$, $M = 500$.

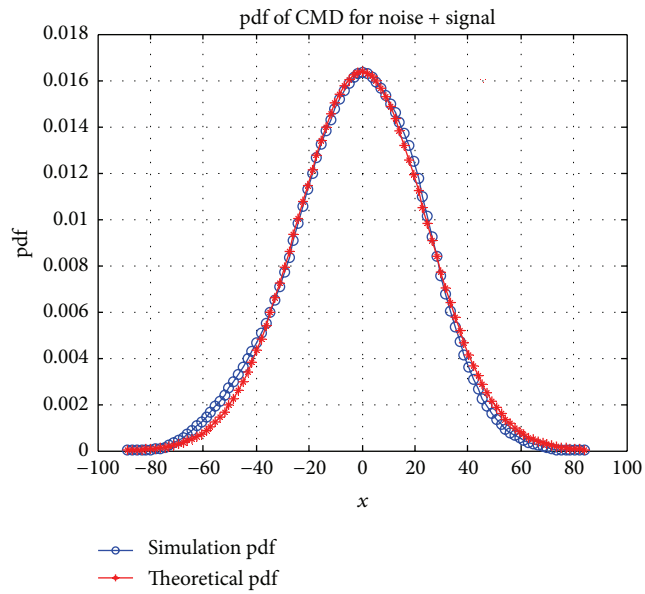


Figure 2: pdf of CMD for noise plus signal; $N = 2048$, $M = 500$, and $E(\mathbf{s}) = 12$.

$$= \exp\left(-\frac{\lambda}{2N\sigma_m^2}\right)\sum_{k=0}^{M/2-1}\frac{1}{k}\left(\frac{\lambda}{2N\sigma_m^2}\right)^k. \tag{59}$$

The missed probability is computed by $P_m = P(Z < \lambda \mid H_1) = 1 - P_d$.

4. Simulation Results and Discussion

To prove the theoretical analysis, we will evaluate the presented algorithm and related conclusions by terms of Monte Carlo simulation in this section, including the statistical characteristics of compressed measurements and the performance of spectrum sensing algorithm under nonreconstruction framework. For simplicity, the abbreviation of the compressed measurements is CMD, the abbreviation of the noncompressed signal is IND, and pdf, Pd, and Pf are the abbreviated notations of probability density function, detection probability, and false-alarm probability, respectively, in the simulation figure.

4.1. Statistic Characteristics of Compressed Measurements. To verify our assumption of Gaussian distribution for the compressed measurements, Figures 1 and 2 demonstrate simulation result and theoretical result in the case of noise and noise plus signal. We observe that simulation result can match theoretical analysis, which proves that Gaussian distribution is reasonable and suitable model for the compressed measurements.

Next, we begin with considering the statistical characteristics of compressed measurements and test statistic, such as pdf, mean, and variance. The pdf of compressed measurements and noncompressed signal are illustrated in Figures 3 and 4. A random Gaussian matrix is adopted as the measurement matrix; the number of noncompressed signals and compressed measurements are N and M, respectively. The compressed measurements and the noncompressed signal have same mean, but it is obvious that they possess different variances, and the variance of the compressed measurements is bigger than that of the noncompressed signal.

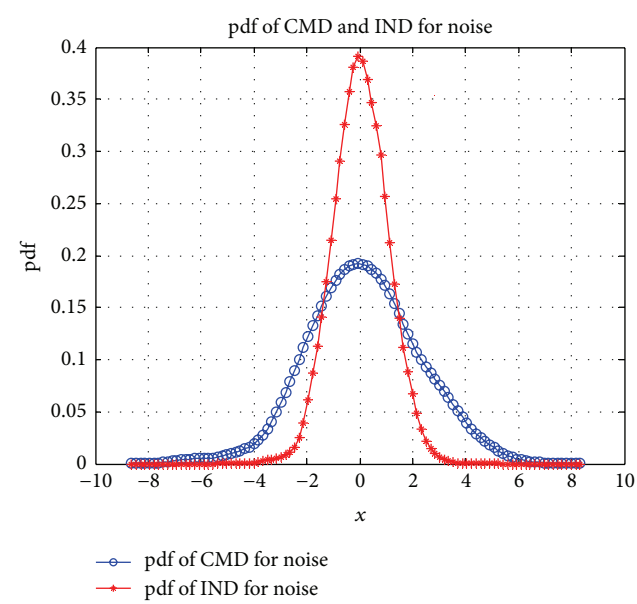


Figure 3: pdf of CMD and IND for noise; $N = 2048$, $M = 500$, $E(\mathbf{n}) = 0$, and $\sigma^2 = 1$.

Now, let us analyze the theoretical results about mean and variance through (49) and (51). It can be seen from (49) that the mean of compressed measurements is zero no matter what the mean of noncompressed signal is. For variance, we can observe from (51) that σ_n^2 is the variance of initial noise, and $\sigma_n^2 + s^2$ is the variance of initial noise plus signal. Due to $N \gg M$, it follows that $(N/M)\sigma_n^2$ is larger

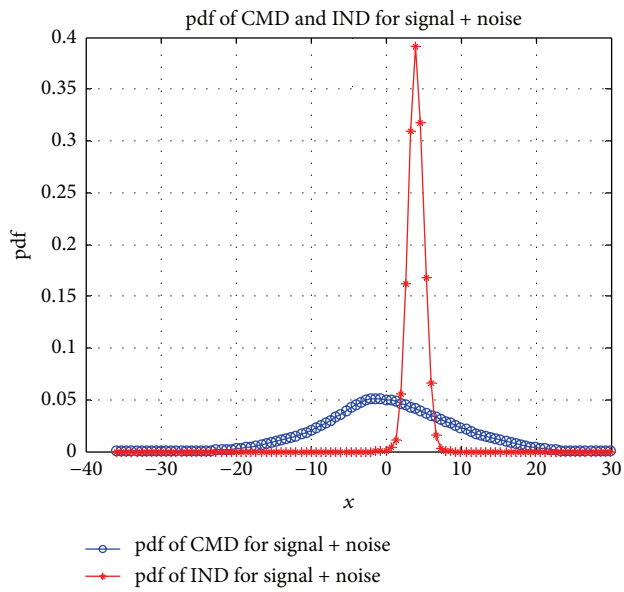
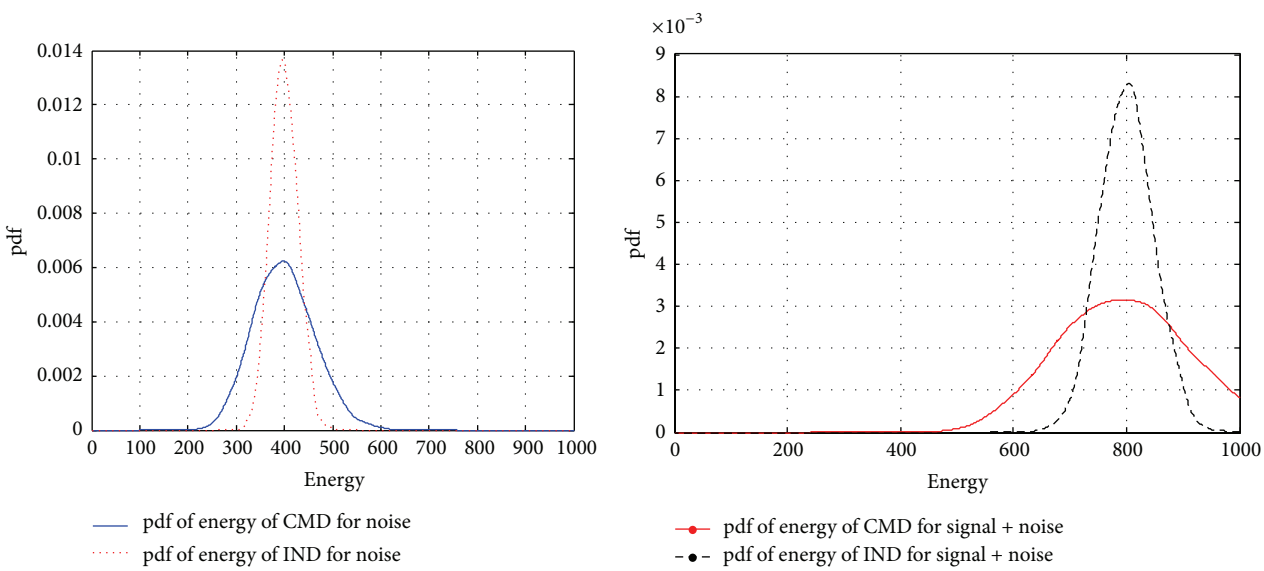


Figure 4: pdf of CMD and IND for noise plus signal; $N = 2048$, $M = 500$, $E(\mathbf{n}) = 0$, $\sigma^2 = 1$, and $E(\mathbf{n} + \mathbf{s}) = 4$.



(a) Energy of CMD and IND for noise

(b) Energy of CMD and IND for noise + signal

Figure 5: pdf of energy of CMD and IND; $N = 400$, $M = 100$, variance of measurement is $1/M$, $E(\mathbf{n}) = 0$, $\sigma^2 = 1$, and energy of signal = 2.

than σ_n^2 and $(N/M)\{\sigma_n^2 + s^2\}$ is larger than $\sigma_n^2 + s^2$; namely, the variance of compressed measurements becomes bigger than that of noncompressed signal. As mentioned before, simulation results of mean and variance are coincident with their theoretical analysis.

Below, we discuss the mean and variance of sum of square (energy); the simulation parameters are the same as that of Figure 3. As illustrated in Figure 5, there are the same mean and various variances for the compressed measurements and the noncompressed signal, but the variance becomes bigger after the signal is compressed by compressive sensing methods, which enable the energy of compressed measurements to be more dispersed.

To prove the simulation results in Figure 5 and generalize the theoretical results, the mean and variance of $\sum_{k=0}^{M-1} Y_k^2$ are calculated instead of test statistic $Z = \sum_{k=0}^{M-1} Y_k^2/\sigma_n^2$. We deal with firstly the mean of sum of square; it follows that

$$\begin{aligned} E\left[\sum_{k=0}^{M-1} Y_k^2\right] &= \sum_{k=0}^{M-1} E\left[Y_k^2\right] = M\frac{N}{M}\sigma_n^2 = N\sigma_n^2, \quad H_0 \\ E\left[\sum_{k=0}^{M-1} Y_k^2\right] &= \sum_{k=0}^{M-1} E\left[Y_k^2\right] = M\frac{N}{M}\left(\sigma_n^2 + s^2\right) \\ &= N\left\{\sigma_n^2 + s^2\right\}, \quad H_1. \end{aligned} \tag{60}$$

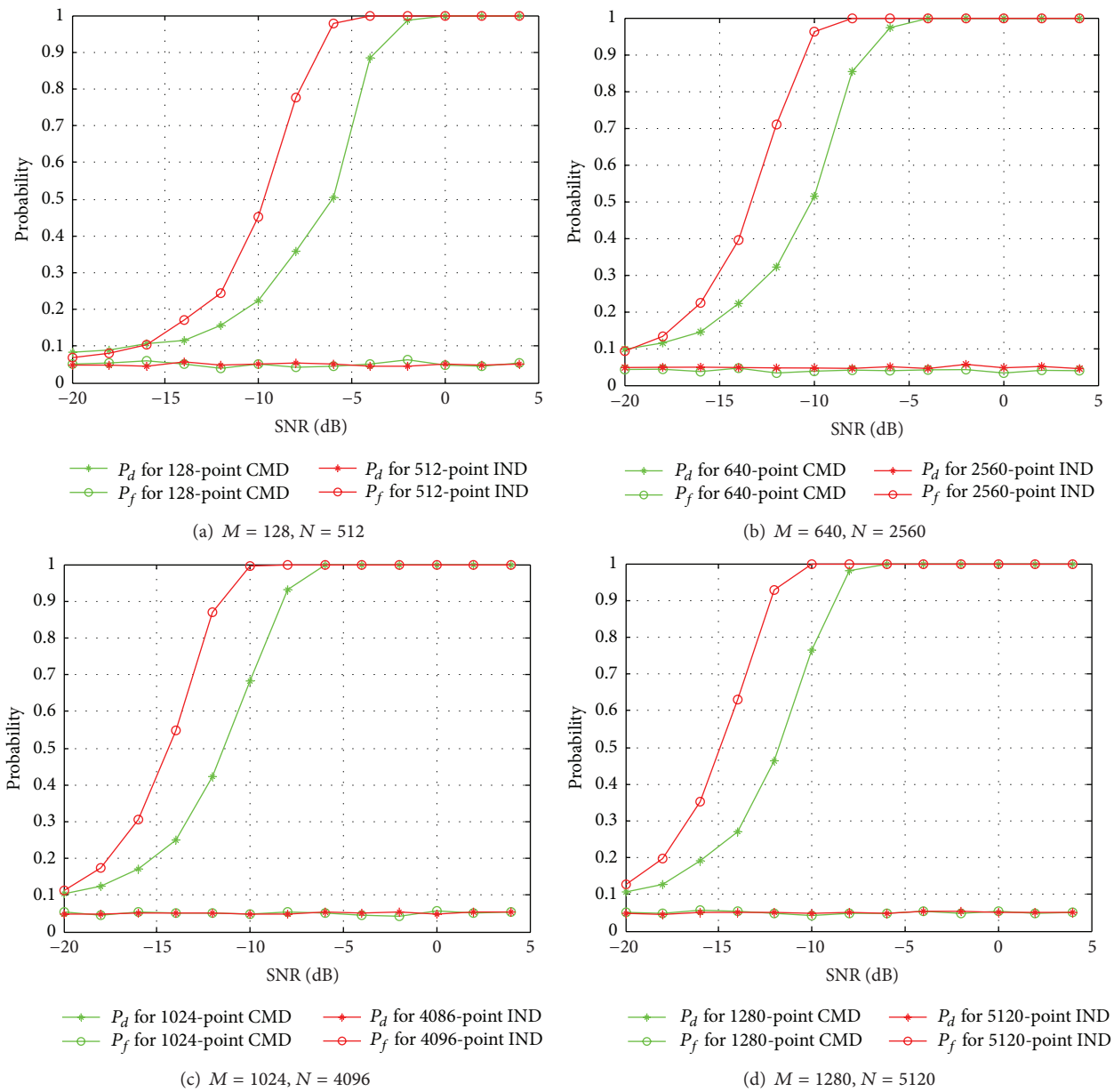


Figure 6: pd and pf for CMD and IND for fixed compressive rate.

We observe that the mean remains unchanged. The variance of $\sum_{k=0}^{M-1} Y_k^2$ is achieved by

$$\begin{aligned} D\left[\sum_{k=0}^{M-1} Y_k^2\right] &= \sum_{k=0}^{M-1} D\left[Y_k^2\right] = 2M\left(\frac{N}{M}\sigma_n^2\right)^2 = \frac{2N^2}{M}\sigma_n^4 \\ &= \frac{N}{M}\left(2N\sigma_n^4\right), \quad H_0 \\ D\left[\sum_{k=0}^{M-1} Y_k^2\right] &= \sum_{k=0}^{M-1} D\left[Y_k^2\right] = 2M\left[\frac{N}{M}\left(\sigma_n^2 + s^2\right)\right]^2 \\ &= \frac{N}{M}\left[2N\left\{\sigma_n^2 + s^2\right\}^2\right], \quad H_1, \end{aligned} \tag{61}$$

where $2N\sigma_n^4$ and $2N\{\sigma_n^2 + s^2\}^2$ are variance of noncompressed signal for H_0 and H_1 cases, respectively. Similarly, due to $N \ll M$, $(N/M)(2N\sigma_n^4)$ is bigger than $2N\sigma_n^4$, and $(N/M)[2N\{\sigma_n^2 + s^2\}^2]$ is bigger than $2N\{\sigma_n^2 + s^2\}^2$. Form (60) and (61), the simulation results in Figure 5 fit with completely the theoretical analysis.

For the test statistic $Z = \sum_{k=0}^{M-1} Y_k^2/\sigma_n^2$, it is special case of $\sum_{k=0}^{M-1} Y_k^2$; namely, $\sigma_n^2 = 1$. As a result, the mean and variance possess the same results as $\sum_{k=0}^{M-1} Y_k^2$.

4.2. Energy-Based Spectrum Sensing Performance for the Compressed Measurements and the Noncompressed Signal. According to Newman-Pearson criterion, the first step to evaluate the performance of spectrum sensing is determining

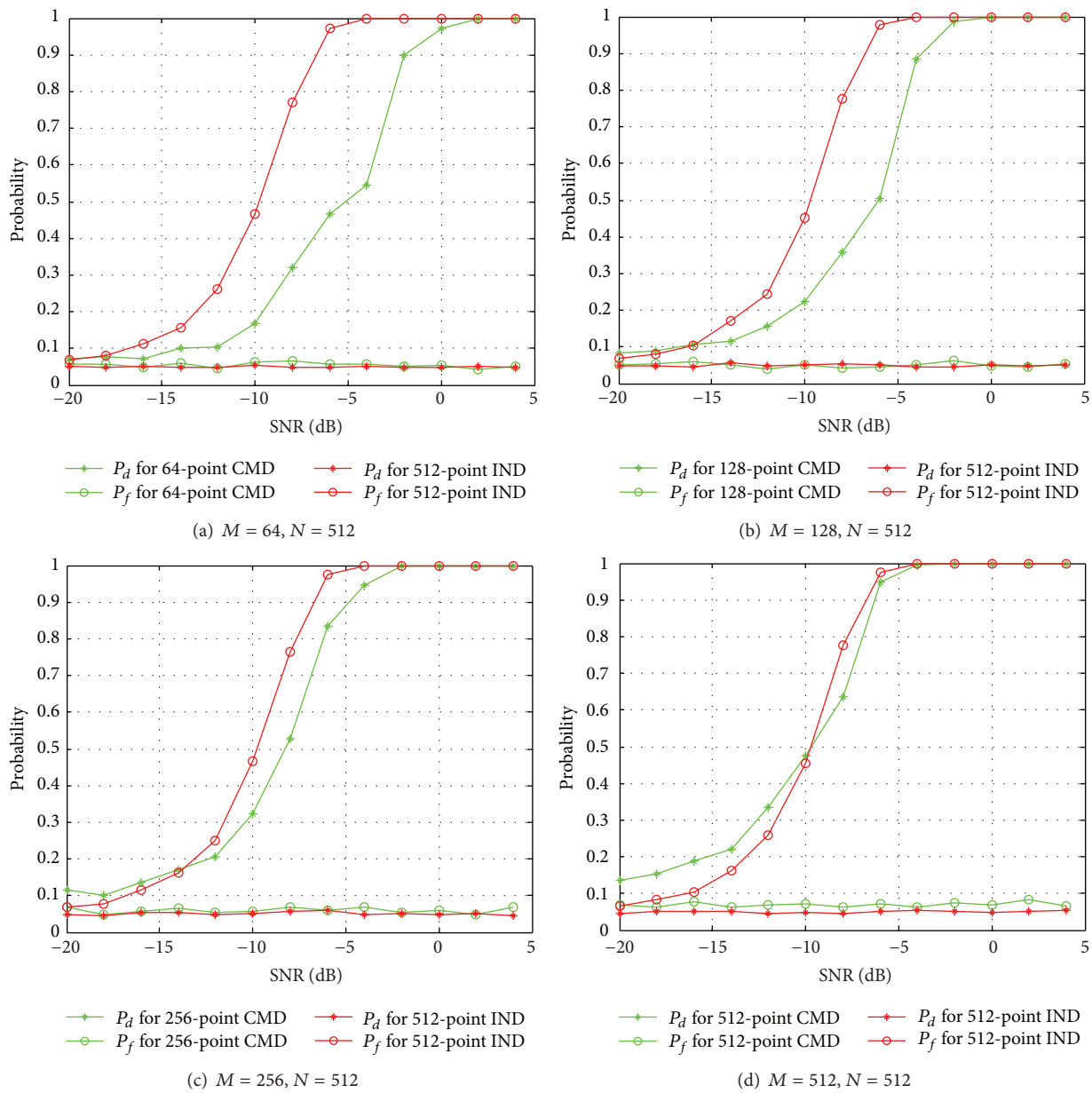


Figure 7: pd and pf for CMD and IND for various compressive rate.

the threshold in terms of (59) when the false-alarm probability is fixed. The key performance parameters for spectrum sensing are detection probability, false-alarm probability, and ROC curve. We will discuss the detection probability for the different cases. Firstly, we fix the compressive rate $N/M = 4$; other simulation parameters are as follows: false-alarm probability is 0.05, the SNR is varied from −20 dB to 5 dB, the variance of measurement matrix is $1/M$, and the noise is standard normally distributed with the mean 0 and the variance 1. The number of noncompressed signals and compressed measurements are, respectively, $N = 512$ and $M = 128$ in Figure 6(a), $N = 2560$ and $M = 640$ in Figure 6(b), $N = 4096$ and $M = 1024$ in Figure 6(b), and $N = 5120$ and $M = 1280$ in Figure 6(d). In these figures, the simulated false-alarm probability varies around the assumed false-alarm probability slightly.

We notice from Figure 6 that detection probability increases as N and M rise. Equivalently, increasing the number means improving the SNR, which is also verified by virtue of (58). In other words, the variance varies with N inverse proportionally, which will result in higher detection probability for the same threshold.

Next, we evaluate the performance for the various compressive rate and SNR, but other parameters remain unchanged. The false-alarm probability is 0.05, the number of noncompressed signals N is 512, the SNR is varied from −20 dB to 5 dB, the variance of measurement matrix is $1/M$, and the noise is normally distributed with the mean 0 and the variance 1. The compressive rate is, respectively, $N/M = 8$ in

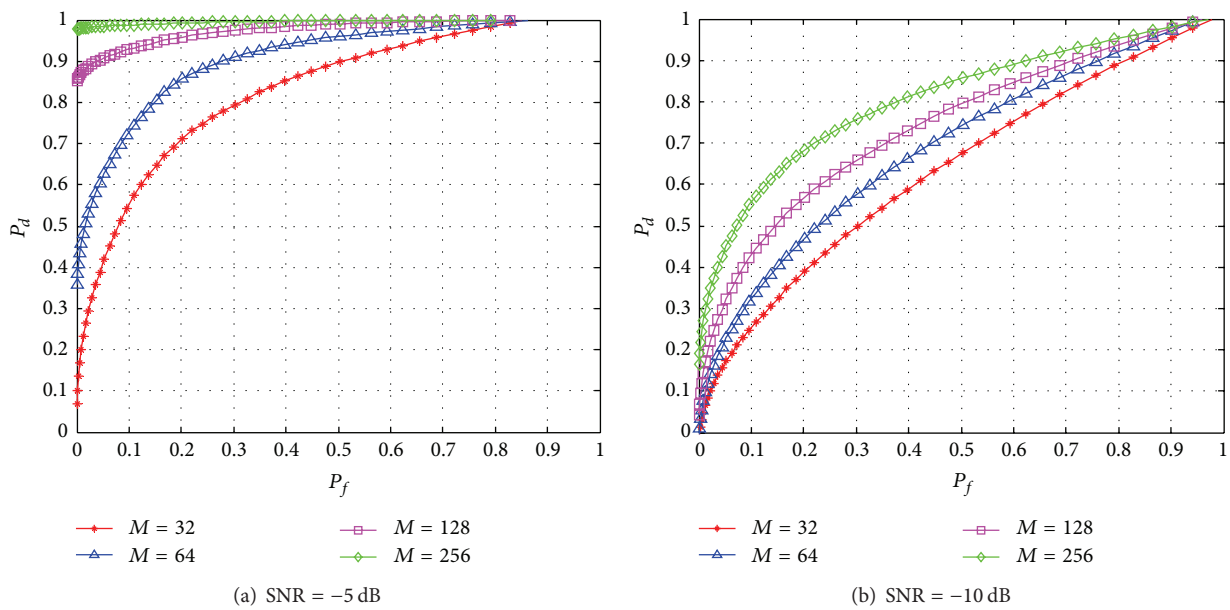


Figure 8: ROC curve for energy-based spectrum sensing algorithm under nonreconstruction framework.

Figure 7(a), $N/M = 4$ in Figure 7(b), $N/M = 2$ in Figure 7(c), and $N/M = 1$ in Figure 7(d).

The simulation results show that sensing performance of compressive sensing approaches that of conventional energy-based method gradually with the increasing of M.

And then the detection performance is quantified by depicting the receiver operating characteristic (ROC) for various SNR, as shown in Figure 8 for SNR = −5 dB and SNR = −10 dB. The simulation parameters are as follows: $N = 512$ and M is 32, 64, 128, and 256. The variance of measurement matrix is $1/M$; the noise is standard normally distributed with the mean 0 and the variance 1. The figure asserts the fact that the performance of ROC improves as M increases. On the other hand, ROC varies with SNR proportionally.

Finally, we evaluate the computational complexity of energy-based spectrum sensing and nonreconstruction-based spectrum sensing. If the real-valued signal is sampled, there are totally N multiplications and $N - 1$ additions, but, for the compressed measurements, there are only M multiplications and $M - 1$ additions. If the complex-valued signal is sampled, there exist totally $2N$ multiplications and $2N - 1$ additions, but, for the compressed measurements, there exist only $2M$ multiplications and $2M - 1$ additions. As mentioned before, $M \ll N$; the computational complexity of energy-based spectrum sensing under nonreconstruction framework is less than that of traditional energy-based spectrum sensing, which will result in faster detection and less detection time.

More importantly, we compare the computational complexity of conventional reconstruction-based spectrum sensing with that of nonreconstruction-based spectrum sensing. We take the MP reconstruction algorithm as example because it posseses lower computational complexity than that of other reconstruction algorithms. Assume that the times of iteration are the sparsity K, which is the fewest times required by the reconstruction algorithm. The computational complexity of each iteration is computing the N times of inner production; each inner production needs M multiplications and $M - 1$ addition. As a consequence, the total computational complexity for times of N inner production is $N \times M$ multiplications and $N \times (M-1)$ additions; moreover, there are $(N-1)$ times comparisons. The entire computational complexity for MP algorithm is $K \times (N \times M)$ multiplications, $K \times (N \times (M-1))$ additions, and $K(N-1)$ times comparisons. Even the ideal MP reconstruction algorithm posseses higher extremely computational complexity than that of nonreconstruction-based spectrum sensing.

5. Conclusion

Signal processing under nonreconstruction framework, especially inference problem, is very valuable and interesting topic due to its low computational complexity. The most important thing to us is that the nonreconstruction framework can eliminate the effect of reconstruction algorithm. Firstly, we discuss the two conventional types of energy-based detection methods, energy normalization and power-based spectrum sensing. Certainly, we focus on deriving statistical characteristics of compressed measurements and its squared form (energy). Their mean and variance and the corresponding approximated PDF are obtained in terms of central limit theorem, which are basis of future work. From the previous analysis, we notice that the mean of compressed measurements is still zero if the noise has zero mean, but its variance will become bigger clearly, which leads to decreasing of detection performance in essence. To evaluate the performance of energy-based spectrum sensing algorithm under nonreconstruction framework, some simulations are performed for the various simulation parameters; all simulation results coincide with completely theoretical analysis.

Through the simulation results and analysis, it can be seen that the performance drops slightly, but its computational complexity decreases evidently comparing with conventional energy-based spectrum sensing and reconstruction-based spectrum sensing, which is crucial for the case of requiring less detection time in cognitive radio. As a consequence, the presented algorithm provides a way for spectrum sensing to reduce the computational time. However, we need trade-off computational complexity and performance in practical application.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] J. Mitola III and G. Q. Maguire Jr., "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, 1999.

[2] S. Haykin, "Cognitive radio brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005.

[3] D. A. Guimarães and G. P. Aquino, "Resource-efficient fusion over fading and non-fading reporting channels for cooperative spectrum sensing," *Sensors*, vol. 15, no. 1, pp. 1861–1884, 2015.

[4] J. Chen, A. Gibson, and J. Zafar, "Cyclostationary spectrum detection in cognitive radios," in *Proceedings of the IET Seminar on Cognitive Radio and Software Defined Radios: Technologies and Techniques*, pp. 1–5, 2008.

[5] N. T. Do and B. An, "A soft-hard combination-based cooperative spectrum sensing scheme for cognitive radio networks," *Sensors*, vol. 15, no. 2, pp. 4388–4407, 2015.

[6] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523–531, 1967.

[7] R. Tandra and A. Sahai, "Fundamental limits on detection in low SNR under noise uncertainty," in *Proceedings of the International Conference on Wireless Networks, Communications and Mobile Computing (WirelessCom '05)*, vol. 1, pp. 464–469, June 2005.

[8] F. F. Digham, M.-S. Alouini, and M. K. Simon, "On the energy detection of unknown signals over fading channels," *IEEE Transactions on Communications*, vol. 55, no. 1, pp. 21–24, 2007.

[9] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[10] Y. L. Polo, Y. Wang, A. Pandharipande, and G. Leus, "Compressive wide-band spectrum sensing," in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '09)*, pp. 2337–2340, April 2009.

[11] F. Zeng, C. Li, and Z. Tian, "Distributed compressive spectrum sensing in cooperative multihop cognitive networks," *IEEE Journal on Selected Topics in Signal Processing*, vol. 5, no. 1, pp. 37–48, 2011.

[12] H.-F. Hu, Z. Yang, and J.-M. Bao, "Wavelet transform-based distributed compressed sensing in wireless sensor networks," *China Communications*, vol. 9, no. 2, pp. 1–12, 2012.

[13] E. J. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 489–509, 2006.

[14] M. L. Malloy and R. D. Nowak, "Near-optimal adaptive compressed sensing," *IEEE Transactions on Information Theory*, vol. 60, no. 7, pp. 4001–4012, 2014.

[15] H. Fang, S. A. Vorobyov, H. Jiang, and O. Taheri, "Permutation meets parallel compressed sensing: how to relax restricted isometry property for 2D sparse signals," *IEEE Transactions on Signal Processing*, vol. 62, no. 1, pp. 196–210, 2014.

[16] D. Bi, Y. Xie, X. Li, and Y. R. Zheng, "A sparsity basis selection method for compressed sensing," *IEEE Signal Processing Letters*, vol. 22, no. 10, pp. 1738–1742, 2015.

[17] E. J. Candes and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, 2008.

[18] L. Wang, K. Lu, and P. Liu, "Compressed sensing of a remote sensing image based on the priors of the reference image," *IEEE Geoscience and Remote Sensing Letters*, vol. 12, no. 4, pp. 736–740, 2014.

[19] P. B. Tuuk and S. L. Marple, "Compressed sensing radar amid noise and clutter using interference covariance information," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 2, pp. 887–897, 2014.

[20] T. Wei and H. Wang, "Research on application of compressed sensing based on signal decomposition," in *Proceedings of the IEEE International Conference on Communication Problem-Solving (ICCP '14)*, pp. 326–331, 2014.

[21] M. A. Davenport, M. B. Wakin, and R. G. Baraniuk, "Detection and estimation with compressive measurements," Tech. Rep. TREE, Department of Electrical and Computer Engineering, Rice University, 2006, http://dsp.rice.edu/cs.

[22] M. A. Davenport, P. T. Boufounos, M. B. Wakin, and R. G. Baraniuk, "Signal processing with compressive measurements," *IEEE Journal on Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 445–460, 2010.

[23] Z. Wang, G. R. Arce, and B. M. Sadler, "Subspace compressive detection for sparse signals," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 3873–3876, April 2008.

[24] J. Haupt and R. Nowak, "Compressive sampling for signal detection," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP '07)*, vol. 3, pp. III-1509–III-1512, April 2007.

[25] W. Wang, Z. Yang, B. Gu, and H. Hu, "A non-reconstruction method of compressed spectrum sensing," in *Proceedings of the 7th International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM '11)*, pp. 1–4, September 2011.

[26] R. Anupama, S. M. Jattimath, B. M. Shruthi, and P. Sure, "On the performance comparison of compressed sensing based detectors for sparse signals: Compressive detectors for sparse signals," in *Proceedings of the International Conference on Advances in Electronics, Computers and Communications (ICAECC '14)*, pp. 1–5, IEEE, Bangalore, India, October 2014.

[27] R. Zahedi, A. Pezeshki, and E. K. P. Chong, "Measurement design for detecting sparse signals," *Physical Communication*, vol. 5, no. 2, pp. 64–75, 2012.

[28] S. Hong, "Direct spectrum sensing from compressed measurements," in *Proceedings of the IEEE Military Communications Conference (MILCOM '10)*, pp. 1187–1192, IEEE, San Jose, Calif, USA, November 2010.