Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 274713, 7 pages
http://dx.doi.org/10.1155/2015/274713

Research Article

Determination of Optimal Shipping Quantity for Perishable Goods under Probabilistic Supply

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Received 30 September 2015; Revised 28 November 2015; Accepted 6 December 2015

Academic Editor: Jei-Zheng Wu

The problems of shipping and transporting perishable goods are commonly considered in the literature as significant topics, but rarely did researchers adopt a probabilistic point of view in their models. It is common in SCM environments that the participating entities' behaviors are random and unpredictable and so can only be modeled in a probabilistic way. In this paper, we consider the shipping problem of determining the optimal quantity of perishable products with a limited time to be stored in the warehouse. The optimal quantity minimizes the overall operational costs including those of inventory and shipping. We develop a mathematical model, from which the probability distribution function, mean, and variance of the length of the build-up period are derived and we establish a cost function for determining the optimal shipping value.

1. Introduction

The shipment of goods, in general, aims for cost-effectiveness by loading as many items as possible in a restricted space of transportation. Another goal is the on-time delivery of specific amounts of goods. However, shipping and transporting of perishable goods such as food, biological materials (blood), and hazardous products (bacterial and fungal items) are quite different from shipping inert, nonperishable goods. Shipment of perishable items must involve consideration of the timeline of expiration. Thus, it also includes the goal of shipping and delivering the perishable items before expiration.

When several suppliers providing perishable products participate in supply chain management (SCM) circumstances, there can exist a trade-off between cost of shipping and risk of deterioration of the perishable goods. Specifically, suppose a typical SCM situation in which many suppliers use a warehouse to store their goods until they are delivered to another place, usually in batches. To reduce shipping costs, the amount of goods in shipment should be as large as feasible. It is therefore desirable to wait until enough products have accumulated in the warehouse to ship them together. However, this approach inevitably results in some products that arrive earlier at the warehouse deteriorating before being shipped. If the shipping quantity is large, shipping costs can be greatly reduced; however, the possibility of product deterioration will be much higher because of the time lag. Therefore, it is vital to determine a proper quantity of shipping that balances the two costs and minimizes the overall cost. In this study, we address this problem and study ways to find optimal shipping amounts of perishable goods under probabilistic supply in SCM.

There have been many studies on perishable goods in supply chain management (SCM), involving various perspectives, such as technical, heuristic, analytical, and strategic points of view. Grunow and Piramuthu [1] showed that, under some conditions, the incorporation of radio frequency identification (RFID) can benefit distributors, retailers, and consumers. Recently, Todorovic et al. [2] studied the use of RFID tags. These studies mainly focused on the usage of RFID technology for the logistics of perishable products, not on the mathematical analysis.

van Donselaar et al. [3] worked with two Dutch supermarket chains and improved the automated store ordering (ASO) systems by considering perishable items. They classified the items in the supermarkets and noted that different items require different inventory control rules, designing an inventory control strategy for perishable items. Ignaciuk [4] considered the problem of establishing an efficient control strategy for production-inventory systems in which the stock

replenishment process is unreliable. Considering supply chain strategies for perishable goods, Blackburn and Scudder [5] used melons and sweet corn as two examples and showed that an appropriate model to minimize loss in the supply chain is a hybrid of a responsive model from postharvest to cooling. In these studies, they introduced several inventory strategies for the perishable products but did not consider optimal shipping quantities for them.

A set of agricultural suppliers with low demand that could reduce long-haul transportation costs by consolidating their products was considered in Nguyen et al. [6]. They suggested a look-ahead heuristic algorithm taking advantage of economies of scale by aiming to ship larger quantities. Zanoni and Zavanella [7] examined the challenge of shipping a set of perishable products from a single vender to a common buyer. They developed a mixed integer linear programming model incorporating a well-known heuristic algorithm with some modification. Thangam [8] considered optimal price discounting and lot-sizing policies for perishable items in an SCM environment. In his research, an economic order quantity- (EOQ-) based model with perishable items was developed under an advance payment (AP) scheme and a two-echelon trade credit option. In these researches, they considered the control of perishable products by using the heuristic algorithms but failed to find any mathematical relationship between length of build-up period for shipping and expiration of products. Ali et al. [9] dealt with a supply chain of a food in a different perspective; that is, they showed that supply chain integration can mitigate the halal food integrity risk.

Hsieh and Dye [10] suggested a deterministic, stock-dependent inventory model for deteriorating items and provided procedures for determining the maximum total profit per unit time. They assumed the storage space for deteriorating items is finite. Gumasta et al. [11] developed a transportation model mapped onto an inventory model with time-varying demand and two types of perishable goods. The model was developed to maximize the revenue and minimize transportation and inventory costs. They consider the relationships between the transportation problem and the inventory model assuming the fixed shortage cost, and selling price of products decreasing with time.

In these studies, the problems of shipping and transporting perishable goods are commonly considered as significant topics, but rarely did researchers adopt a probabilistic point of view in their models. It is common in SCM environments that the participating entities' behaviors are random and unpredictable and so can only be modeled in a probabilistic way. In this paper, we consider the shipping problem of determining the optimal quantity of perishable products with a limited time to be stored in the warehouse. The optimal quantity minimizes the overall operational costs including those of inventory and shipping. We develop a mathematical model by adopting an approach similar to the queuing system with what we call "impatient" products under the N-policy, which is described in the following sections.

The paper is organized as follows. Section 2 explains the mathematical model considered in this paper, and Section 3 derives the probability distributions of the build-up period conditioned on the quantity of shipping, with which mean and variance of the period are calculated. Section 4 establishes a cost function that includes inventory holding cost and one-time shipping cost as a function of shipping quantity. This cost function can be used to determine the optimal quantity of one-time shipping. A summary and conclusion are provided in Section 6.

2. Model

We consider the problem of determining the optimal shipping quantity of perishable goods that minimizes the overall inventory and shipping costs and model it similar to a queuing system. Perishable goods arriving at a warehouse are treated as impatient customers arriving at a service station, so that shipment of these goods is a service provided to the waiting customers in the system, as depicted in Figure 1.

In our model, perishable goods arrive at the warehouse according to the Poisson process with a known positive constant rate λ and are stored for later shipment with many other products. Shipping is performed right after the quantity of goods accumulated in the warehouse reaches a predetermined level, say N, and we call the storage duration the *build-up period*. The shipment time for the perishable goods arriving during the build-up period, however, is not certain because it depends on the arrival of goods, which is probabilistic. In this period, stored products might go bad and will be unusable if they have to wait in the warehouse for longer than a fixed time T. Those products must then be discarded, resulting in inventory loss.

Figure 2 illustrates a typical sample path of the number of products in the warehouse, assuming $N = 9$. At point ⓥ, the warehouse is empty, and a new build-up period starts. During the build-up period, new products are held in the warehouse until shipment. At point ①, the first product is removed from the warehouse because its waiting time exceeded the expiration date T. The second product is removed from the warehouse at point ② for the same reason. At point ③, the number of stored products reaches N before the third item's waiting time reaches T, so the items are shipped. In this example, a total of $N + 2 = 11$ products have arrived at the warehouse, and two products have deteriorated and been scrapped at points ① and ②. The length of the build-up period is the time it takes to accumulate N products in the warehouse and depends on the arrivals and T.

3. Duration of the Build-Up Period

In this section, we develop a mathematical model representing the number of products stored in the warehouse, from which the probability distribution and length of build-up period are derived. We introduce the following random variables for use in our model. Let A_0 be the arrival epoch of the first product after the system is empty, and let A_n ($n = 1, 2, \ldots$) be the time interval between the nth and $n + 1$th arrival epochs. A_n's are assumed to be independent of each other and exponentially distributed with a common mean of $1/\lambda$. Additionally, let X_τ be the random variable representing the total elapsed time, starting at τ, until the number of

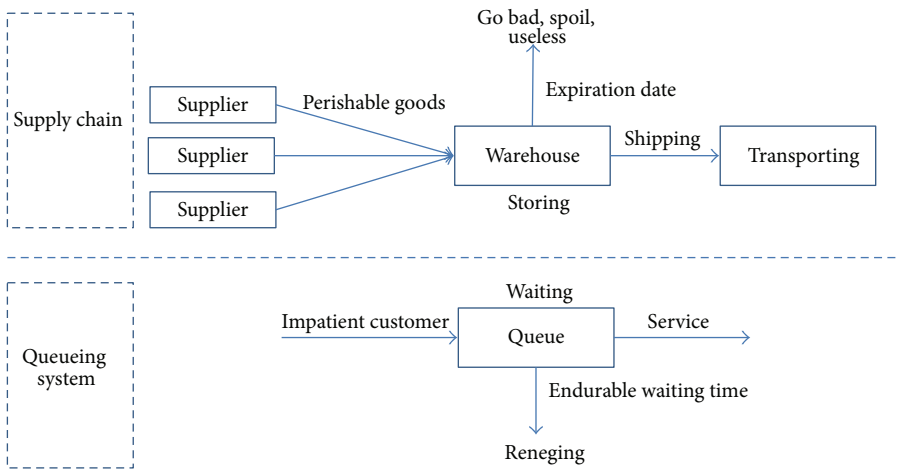

FIGURE 1: A simplified comparison of a supply chain with a queuing system.

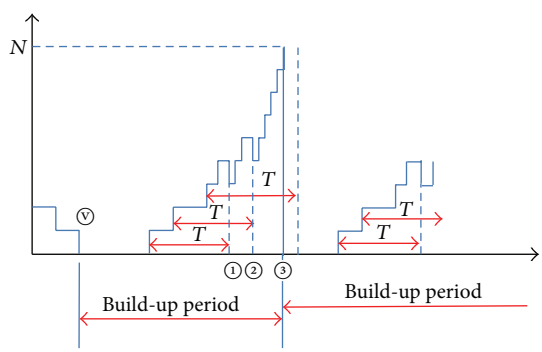

FIGURE 2: A typical sample path of the number of items in a warehouse (case $N = 9$).

products stored in the warehouse reaches N for the first time, and let $G_\tau^*(\theta)$ denote the Laplace transform (LT) of the probability density function of X_τ. Then, Y, the length of the build-up period, is the sum of A_0 and X_{A_0}, which are independent of each other.

We introduce the concept of a *leading product*, which is the one whose waiting time is the longest among all products in the warehouse. The first product arrives at the warehouse and becomes the leading product. Products continue to arrive at the warehouse, accumulating until a certain inventory threshold for shipping is reached. If time T has passed but the shipment has not yet occurred, the first product expires and is discarded; the second product then becomes the leading product. Such successive transfers of the leading product position continue until the build-up period ends.

We determine the probability distribution of X_{A_0} by conditioning it on two mutually exclusive events based on the status of the first leading products as follows:

> E_1: event in which the leading product has waited for time T but shipping has not yet occurred because there are still fewer than N products in the warehouse. When E_1 occurs, the leading product is discarded, and the next product becomes the leading product. We assume $N \geq 2$ because $E_1 = \emptyset$ if $N = 1$.
>
> E_0: complementary event of E_1. The event E_0 occurs when $N - 1$ or more products arrive at the system during time T after the leading product's arrival.

For notational convenience, we introduce the following notations:

$$\alpha_i \equiv \frac{e^{-\lambda T}(\lambda T)^i}{i!}, \quad i = 0, 1, \ldots,$$
$$\beta_j \equiv \sum_{i=0}^{j} \alpha_i, \tag{1}$$

where β_j is the probability that, at most, j products arrive during time T. Then, the probabilities of events E_0 and E_1 are given as follows:

$$\Pr(E_1) = \sum_{k=0}^{N-2} \left(\frac{e^{-\lambda T}(\lambda T)^k}{k!} \right) = \beta_{N-2},$$
$$\Pr(E_0) = 1 - \Pr(E_1) = \sum_{k=N-1}^{\infty} \left(\frac{e^{-\lambda T}(\lambda T)^k}{k!} \right) \tag{2}$$
$$= 1 - \beta_{N-2}.$$

Because the leading product is removed from the system without being shipped with probability β_{N-2}, the number of leading products that leave before shipping begins is geometrically distributed with an expected value of $\beta_{N-2}/(1-\beta_{N-2})$.

We observe some properties of X_{A_0} under the conditions of either E_1 or E_0. When E_0 occurs, the following holds.

Property 1. For a given E_0, X_{A_0} is the sum of $N - 1$ independent interarrivals; that is, $X_{A_0} = A_1 + A_2 + \cdots + A_{n-1}$.

For E_1, the leading product is changed, and the following holds.

Property 2. $X_{A_0} \mid E_1$ and $(A_1 + X_{A_0+A_1}) \mid E_1$ are identically distributed.

Property 3. A_1 and $X_{A_0+A_1}$ are independent of each other.

Property 4. $A_1 \mid E_1$ and A_1 are identically distributed.

Property 5. Event E_1 does not affect $X_{A_0+A_1}$.

Property 6. $G^*_{A_0}(\theta) = G^*_{A_0+A_1}(\theta)$ or X_{A_0} and $X_{A_0+A_1}$ are identically distributed.

Property 2 holds because the second product is now the new leading product whose arrival epoch becomes the new starting point. Properties 3, 5, and 6 are obvious when the arrivals follow a Poisson process and the interarrival times are exponential. Therefore, the probabilistic behavior repeats itself immediately after each arrival. Finally, Property 4 holds because, as long as $N \geq 2$, the first arrival epoch after the leading product is independent of event E_1.

Then, by the above properties, we have the following:

$$\begin{aligned}
G^*_{A_0}(\theta) &= \mathrm{E}\left(e^{-\theta X_{A_0}}\right) \\
&= \mathrm{E}\left(e^{-\theta X_{A_0}} \mid E_1\right) \cdot \Pr\left(E_1\right) + \mathrm{E}\left(e^{-\theta X_{A_0}} \mid E_0\right) \\
&\quad \cdot \Pr\left(E_0\right) \\
&= \mathrm{E}\left(e^{-\theta(A_1+X_{A_0+A_1})} \mid E_1\right) \cdot \Pr\left(E_1\right) \\
&\quad + \mathrm{E}\left(e^{-\theta(A_1+\cdots+A_{N-1})} \mid E_0\right) \\
&\quad \cdot \Pr\left(E_0\right) \ \text{(by Properties 1 and 2)} \\
&= \mathrm{E}\left(e^{-\theta A_1}\right) \cdot E\left(e^{-\theta X_{A_0+A_1}}\right) \cdot \Pr\left(E_1\right) \\
&\quad + \mathrm{E}\left(e^{-\theta(A_1+\cdots+A_{N-1})} \mid E_0\right) \\
&\quad \cdot \Pr\left(E_0\right) \ \text{(by Property 3)} \\
&= \mathrm{E}\left(e^{-\theta A_1}\right) \cdot G^*_{A_0+A_1}(\theta) \cdot \Pr\left(E_1\right) \\
&\quad + \mathrm{E}\left(e^{-\theta(A_1+\cdots+A_{N-1})} \mid E_0\right) \\
&\quad \cdot \Pr\left(E_0\right) \ \text{(by Properties 4 and 5)} \\
&= \mathrm{E}\left(e^{-\theta A_1}\right) \cdot G^*_{A_0}(\theta) \cdot \Pr\left(E_1\right) \\
&\quad + \mathrm{E}\left(e^{-\theta(A_1+\cdots+A_{N-1})} \mid E_0\right) \\
&\quad \cdot \Pr\left(E_0\right) \ \text{(by Property 6)}.
\end{aligned} \tag{3}$$

Because $A_1 + A_2 + \cdots + A_{N-1}$ is a truncated Erlang random variable when event E_0 occurs, we obtain the following:

$$\begin{aligned}
&\mathrm{E}\left(e^{-\theta(A_1+\cdots+A_{N-1})} \mid E_0\right) \\
&\quad = \frac{\int_0^T e^{-\theta x}\left(\lambda^{N-1} x^{N-2} e^{-\lambda x}/(N-2)!\right) dx}{1-\sum_{n=0}^{N-2}\left(e^{-\lambda T}(\lambda T)^n/n!\right)} \\
&\quad = \left(\frac{\lambda}{\theta+\lambda}\right)^{N-1} \\
&\qquad \cdot \frac{1-\sum_{n=0}^{N-2} e^{-(\theta+\lambda)T}((\theta+\lambda)\cdot T)^n/n!}{1-\beta_{N-2}}.
\end{aligned} \tag{4}$$

Then, LT $G^*_{A_0}(\theta)$ of X_{A_0} is given by

$$\begin{aligned}
G^*_{A_0}(\theta) &= \mathrm{E}\left(e^{-\theta X_{A_0}}\right) = \frac{\mathrm{E}\left(e^{-\theta(A_1+\cdots+A_{N-1})} \mid E_0\right) \cdot \Pr\left(E_0\right)}{1-\mathrm{E}\left(e^{-\theta A_1}\right) \cdot \Pr\left(E_1\right)} \\
&= \frac{(\lambda/(\theta+\lambda))^{N-1} \cdot \left(1-\sum_{k=0}^{N-2} e^{-(\theta+\lambda)T}((\theta+\lambda)\cdot T)^k/k!\right)}{1-(\lambda/(\lambda+\theta))\cdot\beta_{N-2}}.
\end{aligned} \tag{5}$$

Mean value and variance of X_{A_0} are as follows:

$$\begin{aligned}
\mathrm{E}\left(X_{A_0}\right) &= -\frac{d}{d\theta} G^*_{A_0}(\theta)\Big|_{\theta=0} \\
&= \frac{\beta_{N-2}}{1-\beta_{N-2}} \cdot \frac{1}{\lambda} + (N-1)\cdot\frac{1}{\lambda} - \frac{1}{1-\beta_{N-2}} \\
&\quad \cdot T \cdot \alpha_{N-2}, \\
\mathrm{Var}\left(X_{A_0}\right) &= \frac{N-1}{\lambda^2} - \frac{T^2}{1-\beta_{N-2}}\left(\alpha_{N-2}-\alpha_{N-3}\right) \\
&\quad - \frac{T^2}{\left(1-\beta_{N-2}\right)^2}\alpha^2_{N-2} + \frac{\beta^2_{N-2}}{\left(1-\beta_{N-2}\right)^2}\cdot\frac{1}{\lambda^2} \\
&\quad + \frac{\beta_{N-2}}{1-\beta_{N-2}}\cdot\frac{2}{\lambda^2}.
\end{aligned} \tag{6}$$

In order to obtain the probability distribution of Y, we divide the build-up period into two periods: *unsuccessful build-up period* and *successful build-up period*. The former is the sum of interarrival times of leading products, and the latter is the duration from the arrival of the last leading product to the start of the busy period, as shown in Figure 3.

Therefore, X_{A_0} in the above equations can be expressed as a sum of two random variables, $X_{A_0,\text{fail}}$, representing the unsuccessful build-up period, and $X_{A_0,\text{success}}$, representing the successful build-up period. As indicated, $\mathrm{E}(X_{A_0,\text{fail}})$ is the expected number of discarded products multiplied by the expected interarrival time. Then, we have the following:

$$\begin{aligned}
\mathrm{E}\left(X_{A_0,\text{fail}}\right) &= \frac{\beta_{N-2}}{1-\beta_{N-2}}\cdot\frac{1}{\lambda}, \\
\mathrm{E}\left(X_{A_0,\text{success}}\right) &= \mathrm{E}\left(X_{A_0}\right) - \mathrm{E}\left(X_{A_0,\text{fail}}\right) \\
&= (N-1)\cdot\frac{1}{\lambda} - \frac{1}{1-\beta_{N-2}}\cdot T\cdot\alpha_{N-2}.
\end{aligned} \tag{7}$$

Now, LT, $Y^*(\theta)$; the expected value, $\mathrm{E}(Y)$; and the variance, $\mathrm{Var}(Y)$, of the build-up period $Y = A_0 + X_{A_0,\text{fail}} + X_{A_0,\text{success}}$ are as follows:

$$\begin{aligned}
Y^*(\theta) &= \frac{\lambda}{\theta+\lambda}\cdot G^*_{A_0}(\theta) \\
&= \frac{(\lambda/(\theta+\lambda))^N \cdot \left(1-\sum_{k=0}^{N-2} e^{-(\theta+\lambda)T}((\theta+\lambda)\cdot T)^k/k!\right)}{1-(\lambda/(\lambda+\theta))\cdot\beta_{N-2}}, \\
\mathrm{E}(Y) &= \frac{1}{\lambda} + \mathrm{E}\left(X_{A_0}\right)
\end{aligned}$$

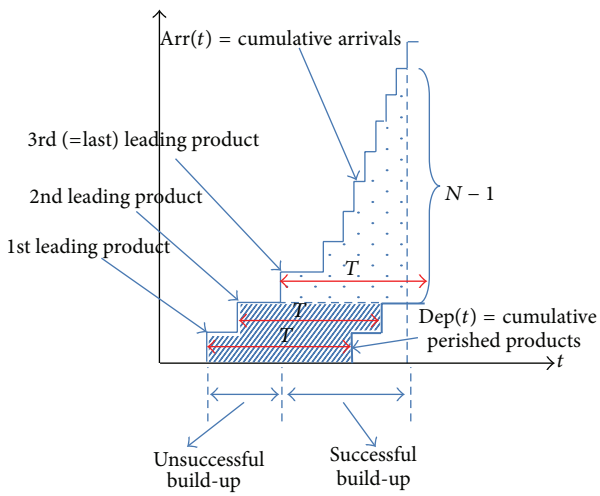

FIGURE 3: Unsuccessful build-up period and successful build-up period.

$$= \frac{N}{\lambda} - \frac{1}{1-\beta_{N-2}} \cdot T \cdot \alpha_{N-2} + \frac{\beta_{N-2}}{1-\beta_{N-2}} \cdot \frac{1}{\lambda},$$

$$\begin{aligned}\operatorname{Var}(Y) &= \frac{N}{\lambda^2} - \frac{T^2}{1-\beta_{N-2}}(\alpha_{N-2}-\alpha_{N-3}) - \frac{T^2}{(1-\beta_{N-2})^2}\alpha_{N-2}^2 \\ &\quad + \frac{\beta_{N-2}^2}{(1-\beta_{N-2})^2} \cdot \frac{1}{\lambda^2} + \frac{\beta_{N-2}}{1-\beta_{N-2}} \cdot \frac{2}{\lambda^2}.\end{aligned} \tag{8}$$

4. Cost Function for the Optimal Value of N

In this section, we introduce a cost function to determine the optimal value of N by minimizing the total cost per unit time during a cycle. Let C_s represent the shipping cost incurred whenever the products are shipped after the number of waiting products reaches N. Also, let C_h represent the inventory holding cost per product per unit time in the warehouse. A larger N value incurs a smaller shipping cost per product with a larger holding cost per unit time, and vice versa. Therefore, the optimal value of N should establish a balance between these two costs in order to minimize the long-run expected total cost per unit time.

For the total holding cost during the build-up period, we consider the two periods $X_{A_0,\text{fail}}$ and $X_{A_0,\text{success}}$, introduced in the previous section. Figure 3 illustrates the cumulative numbers of arrivals ($\text{Arr}(t)$) and departures ($\text{Dep}(t)$), respectively, up to time t during the build-up period. The area between $\text{Arr}(A_0 + X_{A_0})$ and $\text{Dep}(A_0 + X_{A_0})$ represents the total cumulative waiting time of the products during the build-up period.

This area can be divided into two parts: the area of the perished products (corresponding to the shaded area in Figure 3) and the area of the products existing in the warehouse when shipping begins (corresponding to the dotted area in Figure 3). The first part is simply T multiplied by the expected number of perished products $\beta_{N-2}/(1-\beta_{N-2})$ because all perished products have waited in the warehouse for exactly time T. The second area is determined as follows: given $X_{A_0} = x$, the $N-2$ arrival epochs are uniformly distributed on $[0, x]$, and if we denote τ_n $(n = 0, 1, \ldots, N-1)$ as the nth arrival epoch after the previous leading product, then

$$\begin{aligned}\mathrm{E}\left(\sum_{k=0}^{N-1}(\tau_{N-1}-\tau_k)\right) &= \sum_{k=0}^{N-1}\mathrm{E}(\tau_{N-1}-\tau_k) \\ &= \int_{x=0}^{\infty}\left(\sum_{k=0}^{N-1}\mathrm{E}\left(\tau_{N-1}-\tau_k \mid X_{A_0,\text{success}}\right)\right) \\ &\quad \cdot \Pr\left(X_{A_0,\text{success}} = x\right) \\ &= \int_0^{\infty}\left(\sum_{k=0}^{N-2}\frac{N-1-k}{N-1}\cdot x\right)\cdot \Pr\left(X_{A_0,\text{success}}\right) \\ &= \int_0^{\infty}\frac{N}{2}\cdot x \cdot \Pr\left(X_{A_0,\text{success}} = x\right) \\ &= \frac{N}{2}\mathrm{E}\left(X_{A_0,\text{success}}\right).\end{aligned} \tag{9}$$

Therefore, the total holding cost ($C_{h\text{-build-up}}$) during the build-up period is given as

$$C_{h\text{-build-up}} = C_h \cdot \left(\frac{\beta_{N-2}}{1-\beta_{N-2}}\cdot T + \frac{N}{2}\cdot \mathrm{E}\left(X_{A_0,\text{success}}\right)\right). \tag{10}$$

Note that when $T \to \infty$, products do not expire. In this case, $E_1 = \emptyset$; therefore, $\alpha_i = \beta_j = 0$, for all i and j. Then, $Y^*(\theta) = (\lambda/(\lambda+\theta))^N$ and $C_{h\text{-build-up}} = C_h \cdot N(N-1)/(2\lambda)$, which coincide with the result obtained for the M/G/1 queueing system under N-policy without perishable products [12].

Now, the long-run total expected cost per unit time, $\text{TC}(N)$, $N \geq 2$, is given as

$$\begin{aligned}\text{TC}(N) &= \frac{\text{holding cost} + \text{shipping cost}}{\mathrm{E}(\text{one cycle})} \\ &= \frac{C_h \cdot \left[\left(\beta_{N-2}/(1-\beta_{N-2})\right)\cdot T + (N/2)\cdot\left\{(N-1)\cdot(1/\lambda) - \left(1/(1-\beta_{N-2})\right)\cdot T\cdot\alpha_{N-2}\right\}\right] + C_s}{\left(\beta_{N-2}/(1-\beta_{N-2})\right)\cdot(1/\lambda) + (N/\lambda) - \left(1/(1-\beta_{N-2})\right)\cdot T\cdot\alpha_{N-2}}.\end{aligned} \tag{11}$$

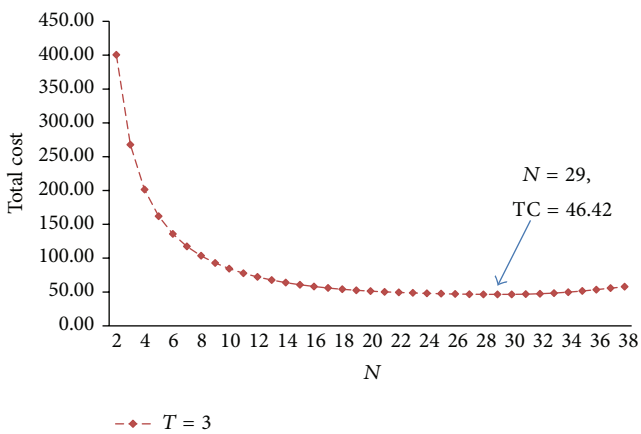

Figure 4: Total cost per unit time when $\lambda = 8$, $C_s/C_h = 100$, and $T = 3$.

The behavior of the function TC(N) with regard to N is not easily determined because of its complicated structure. However, the global point minimizing TC(N) can be obtained numerically. In practical situations, if the empirical range $N_1 \leq N \leq N_2$ is given, then the following can be used to identify the set M (if $M = \emptyset$, then let $M = \{N_1, N_2\}$), and TC(m) is compared to each $m \in M$ in order to obtain the local optimal value of threshold N within the range:

$$M = \{N_1 + 1 \leq m \leq N_2 - 1 : \mathrm{TC}(m-1) - \mathrm{TC}(m) \geq 0,\ \mathrm{TC}(m) - \mathrm{TC}(m+1) \leq 0\}. \tag{12}$$

5. Numerical Illustration

We present some numerical examples to illustrate the analysis in this section. Figure 4 shows the change of long-run expected total cost TC(N) per unit time as a function of N when $C_s/C_h = 100$, $\lambda = 8$, and $T = 3$. By plugging the numbers into (1) and (11), we obtain the curve of total cost per unit time, from which the optimal $N^* = 29$ and the optimal total cost TC(N^*) = 46.42 can be calculated.

More results of the numerical examples are provided in Table 1 under various parameter values. Specifically, two different values of cost ratio $C_s/C_h = 10, 100$, two different values of perishable products' endurable waiting time $T = 1, 3$, and two different Poisson arrival rates $\lambda = 5, 8$ are considered.

From Table 1, we can see that when the ratio C_s/C_h is bigger, the optimal N^* also becomes larger. This is because less shipping is preferable in order to reduce the shipping cost. If we compare the cases $T = 1$ and $T = 3$, it can be seen that the perishable products' endurable waiting time does not give much effect on N^* when C_s/C_h is small. That is, when $C_s/C_h = 10$, then $N^* = 8$ (case $\lambda = 5$) and 10 (case $\lambda = 8$) under $T = 1$ and these optimal values become $N^* = 10$ and 13, respectively, as T increases to 3. But when $C_s/C_h = 100$, remarkable increases in N^* are shown if T changes from 1 to 3, that is, 14 to 23, and 19 to 29. Therefore, we conclude that the optimal N^* becomes more sensitive to the perishable products' endurable waiting time T as the ratio C_s/C_h gets bigger. Lastly, we can see by comparing the cases of $\lambda = 5$ (medium traffic) and $\lambda = 8$ (heavy traffic) that optimal N^* should be obviously bigger as the products arrive more often.

Table 1: Optimal N^* for various parameter values.

Parameter			Optimal N^*
C_s/C_h	T	λ	
10	1	5	8
		8	10
	3	5	10
		8	13
100	1	5	14
		8	19
	3	5	23
		8	29

6. Conclusion

In this paper, we considered a mathematical model to derive the optimal shipping quantity of perishable goods in SCM circumstances. We modeled the system using an approach similar to the queuing system under N-policy with impatient customers. All products that wait for longer than a fixed time expire and are removed from the warehouse as loss.

In this model, we derive the probability distribution function, mean, and variance of the length of the build-up period and establish a cost function for determining the optimal shipping value N. Because an analytical solution for the optimal value of N is not provided, extensive numerical experiments are necessary to reveal the relations among parameters; such experiments are currently being performed by the authors.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by the research fund of Hanyang University (HY-2013-P).

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