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Research Article

Reliability-Based Robust Design Optimization of Structures Considering Uncertainty in Design Variables

Shujuan Wang,[1] Qiuyang Li,[1] and Gordon J. Savage[2]

[1] *Electrical Engineering, Harbin Institute of Technology, Harbin, Heilongjiang 150001, China*
[2] *Systems Design Engineering, University of Waterloo, Waterloo, ON, Canada N2L 3G1*

Correspondence should be addressed to Qiuyang Li; 12qiuy.li@gmail.com

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This paper investigates the structural design optimization to cover both the reliability and robustness under uncertainty in design variables. The main objective is to improve the efficiency of the optimization process. To address this problem, a hybrid reliability-based robust design optimization (RRDO) method is proposed. Prior to the design optimization, the Sobol sensitivity analysis is used for selecting key design variables and providing response variance as well, resulting in significantly reduced computational complexity. The single-loop algorithm is employed to guarantee the structural reliability, allowing fast optimization process. In the case of robust design, the weighting factor balances the response performance and variance with respect to the uncertainty in design variables. The main contribution of this paper is that the proposed method applies the RRDO strategy with the usage of global approximation and the Sobol sensitivity analysis, leading to the reduced computational cost. A structural example is given to illustrate the performance of the proposed method.

1. Introduction

The deterministic models do not account for parameter variation, which is poorly identified and provides an inaccurate picture of the problem in question. In practice, a theoretically excellent deterministic solution may be proved catastrophic due to uncertainty introduced by manufacturing process and environmental changes. To tackle the uncertainty in design variables and parameters, nondeterministic methods have been developed rapidly in the last twenty years [1, 2]. The aim of mechanical structure optimization is to find a high-performance system, which generally has two criteria, robustness and reliability. Nondeterministic methods thus can be classified into two approaches, namely, reliability-based design optimization (RBDO) and robust design optimization (RDO) [3]. RBDO concentrates on finding an optimal design with low probability of failure, while RDO aims to reduce the variability of the system performance. To obtain a reliable and robust product, a hybrid algorithm named reliability-based robust design optimization (RRDO) employs both RDO and RBDO techniques to search for robust optima while obeying reliability type of constraints.

In the present study, structural sizing RRDO problems aim to minimize both the weight and the response variance of the structure. A number of RRDO methods have been reported for structural optimization in the recent years [2, 4–9]. Stochastic design optimization employing Monte Carlo simulation (MCS) is known as the most adequate methodology, which can directly calculate the probability of failure and the response moments of system [2]. Lagaros et al. took into account the probabilistic constraints using MCS combined with Latin hypercube sampling in the framework of RBDO [5]. Then each optimal design was checked if it satisfies the RDO formulation (European design codes for structures) in order to select the final results. However, a large number of sample points need to be generated to ensure accuracy, which makes MCS time consuming for implementation when numerical simulations are involved.

Besides MCS, reliability level may be estimated through the first- or second-order reliability methods (FORM or SORM) in the process of RBDO. Reliability index is introduced as an optimization criterion instead of the probabilistic constraints [6]. In [7] and later in [8], a combined algorithm was discussed based on performance measure approach

(PMA), finding the reliable and robust Pareto optimal solutions. Although reliability methods reduce the sampling sets and simulation cost, they make the optimization structure more complex with double loops (the outer loop is the optimization loop and reliability analysis is performed in the inner loop), even triple loops (including robustness evaluation loop) as presented in [9]. Hence a common challenge for the designers is the computational expense of either RDO or RBDO, whose objective is distinctly different. Some attempts have been made to improve the optimization efficiency for a one desired characteristic (robustness or reliability) [10–13], and few of the existing works address the development of efficient techniques for both robustness and reliability.

The main purpose of this paper is to improve the optimization efficiency for both the reliability design and the robust design under uncertainty. To address this problem, a hybrid RRDO method is proposed combining the single-loop RBDO algorithm and monoobjective RDO formulation to improve efficiency. In the case of RBDO, the single-loop approach based on the conjugate gradient is employed to treat the constraints efficiently. The robustness evaluation is fulfilled considering both the mean and the variance of response represented by a weighting factor. The proposed RRDO method is applied to a "hook" problem with uncertainty in design variables. The results are compared with those from other methods in the literature, leading to a reduced computational cost.

2. Design Optimization under Uncertainty

2.1. Reliability-Based Design Method. The objective of RBDO is to find an optimal solution that verifies as a probability of failure lower or equal to the target probability, expressed as P_f^t. The classical RBDO is expressed in the form

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & f(\mathbf{v}) \\ \text{subject to:} \quad & \Pr\left[g_i(\mathbf{v}) < 0\right] \le P_f^t = \Phi\left(-\beta_i^t\right), \\ & \qquad i = 1, \ldots, m \\ & \mathbf{v}^L \le \mathbf{v} \le \mathbf{v}^U, \end{aligned} \tag{1}$$

where $\mathbf{v} \in \mathbf{R}^n$ is the design vector, $\mathbf{v}^L$ and $\mathbf{v}^U$ are the lower limit and upper limit of the design vector, respectively, $g_i(\cdot)$ is the ith limit state function dividing the safety region ($g > 0$) and failure region ($g < 0$), $\Phi(\cdot)$ is the standard cumulative function for the standard normal distribution, and β_i^t is target reliability index of the ith probabilistic constraint representing the reliability level.

In the field of RBDO considering uncertainty, reliability index approach (RIA) and performance measure approach (PMA) based on the reliability index have been mainly studied. The reliability index approach is a FORM-based method, which is a direct method in comparison to other optimization approaches. The basic formulation of RIA is written as

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & f(\mathbf{v}) \\ \text{subject to:} \quad & \beta_i(\mathbf{u}) \ge \beta_i^t, \quad i = 1, \ldots, m \\ & \mathbf{v}^L \le \mathbf{v} \le \mathbf{v}^U. \end{aligned} \tag{2a}$$

The suboptimization problem for the evaluation of reliability is defined as

$$\begin{aligned} \text{minimize:} \quad & \beta_i^t = \|\mathbf{u}\| \\ \text{subject to:} \quad & g_i(\mathbf{u}) = 0, \quad i = 1, \ldots, m, \end{aligned} \tag{2b}$$

where $\mathbf{u}$ is independent standard normal random variable vectors taking the following form:

$$\mathbf{u} = \frac{\mathbf{v} - \mu_{\mathbf{v}}}{\sigma_{\mathbf{v}}} \tag{3}$$

and $g(\mathbf{u})$ is the probabilistic constraint defined in $\mathbf{u}$-space.

Similarly, the PMA formulation is expressed as follows:

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & f(\mathbf{v}) \\ \text{subject to:} \quad & G_i(\mathbf{u}) \ge 0, \quad i = 1, \ldots, m \\ & \mathbf{v}^L \le \mathbf{v} \le \mathbf{v}^U, \end{aligned} \tag{4a}$$

where $G_i(\mathbf{u})$ is the minimum of the performance function among the points that have the target reliability index, which is obtained from a suboptimization problem. Consider

$$\begin{aligned} \text{minimize:} \quad & G_i = g_i(\mathbf{u}) \\ \text{subject to:} \quad & \|\mathbf{u}\|_i = \beta_i^t, \quad i = 1, \ldots, m. \end{aligned} \tag{4b}$$

It is necessary for both RIA and PMA to solve the suboptimization problem in order to estimate the reliability because of its double-loop structure.

2.2. Robust Design Method. A commonly used definition states that a robust design is a design that is insensitive (or less sensitive) to input variations. RDO aims to improve the quality of products by minimizing the performance variation without eliminating the existing uncertainty [14]. To achieve the robustness, there is no unified mathematical formulation. In most cases, designers emphasized obtaining the robustness of the objective performance rather than satisfying constraints with more robust limit state functions. Thus, the construction of a robust problem generally is based on the mean and standard deviation of objective functions. Consider

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & \mathbf{F}(\mathbf{v}) = \left[\mu_f(\mathbf{v}), \sigma_f(\mathbf{v})\right] \\ \text{subject to:} \quad & g_i(\mathbf{v}) \ge 0, \quad i = 1, \ldots, m \\ & \mathbf{v}^L \le \mathbf{v} \le \mathbf{v}^U, \end{aligned} \tag{5}$$

where $\mu_f(\mathbf{v})$ and $\sigma_f(\mathbf{v})$ are the mean and standard deviation of the objective function $f(\mathbf{v})$, respectively. Within this formulation, RDO is defined as a multiobjective problem (MOP) that gives a series of solutions, known as a set of optimal Pareto solutions. However, MOP suffers from a large degree of complexity, with regard to numerical implementation, and provides large numbers of candidate solutions for designers to make a decision. In this paper, the MOP is converted to a monoobjective problem by adding weighting factors before each entry. A normalized RDO therefore becomes

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & \mathbf{F}(\mathbf{v}) = w\frac{\mu_f(\mathbf{v})}{\mu_f^*} + (1-w)\frac{\sigma_f(\mathbf{v})}{\sigma_f^*}, \\ & 0 < w < 1 \\ \text{subject to:} \quad & g_i(\mathbf{v}) \geq 0, \quad i = 1, \ldots, m \\ & \mathbf{v}^L \leq \mathbf{v} \leq \mathbf{v}^U, \end{aligned} \tag{6}$$

where w is the positive weighting factor and μ_f^* and σ_f^* are the values that take the function f as their optimum considering only the response mean or standard deviation as objective functions (i.e., $w = 1$ and $w = 0$, resp.). The weighting factor is determined according to the importance between minimum performance and robustness.

2.3. Proposed RRDO Method. In order to improve the optimization efficiency, especially for the multicriteria problem, the single-loop approach (SLA) has been used to substitute the double-loop structure [15]. The SLA aims to replace the characteristic point with the minimum performance target point integrating system's reliability. The SLA formulation is as follows:

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & f(\mathbf{v}) \\ \text{subject to:} \quad & g_i(\mathbf{z}) \geq 0, \quad i = 1, \ldots, m \\ & \mathbf{v}^L \leq \mathbf{v} \leq \mathbf{v}^U \end{aligned} \tag{7a}$$

such that $\mathbf{z} = \mu_{\mathbf{v}} - \sigma_{\mathbf{v}}\beta^t\alpha$ with

$$\alpha = \frac{\sigma_{\mathbf{v}}\nabla_{\mathbf{v}}g_i(\mathbf{v})}{\left\|\sigma_{\mathbf{v}}\nabla_{\mathbf{v}}g_i(\mathbf{v})\right\|}, \tag{7b}$$

where $\mathbf{z}$ corresponds to the characteristic point connected to the target level of reliability β^t and $\mu_{\mathbf{v}}$ and $\sigma_{\mathbf{v}}$ are the mean and standard deviation of the design variables $\mathbf{v}$, respectively.

The RRDO problem therefore is formulated combining (5) and (7a) and (7b) in the following form:

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & \mathbf{F}(\mathbf{v}) = w\frac{\mu_f(\mathbf{v})}{\mu_f^*} + (1-w)\frac{\sigma_f(\mathbf{v})}{\sigma_f^*}, \\ & 0 < w < 1 \\ \text{subject to:} \quad & g_i(\mathbf{z}) \geq 0, \quad i = 1, \ldots, m \\ & \mathbf{v}^L \leq \mathbf{v} \leq \mathbf{v}^U, \\ \text{where} \quad & \mathbf{z} = \mu_{\mathbf{v}} - \sigma_{\mathbf{v}}\beta^t\frac{\sigma_{\mathbf{v}}\nabla_{\mathbf{v}}g_i(\mathbf{v})}{\left\|\sigma_{\mathbf{v}}\nabla_{\mathbf{v}}g_i(\mathbf{v})\right\|}. \end{aligned} \tag{8}$$

It can be seen from formulation (8) that two essential elements of the RRDO problem are (i) determining objective function and constraint function and (ii) estimating the moments (i.e., mean and standard deviation) of the performance function. To begin with, many approximated models have been developed for several decades. Detailed description of meta-modeling can be found in Section 3.1. On the other hand, the moments generally can be achieved in two ways: Taylor series expansion and Monte Carlo method. Taylor series expansion method omits higher-order indices of performance function and becomes difficult as the number and intersection of variables increase. Although Monte Carlo simulation gives more accurate results, the computation is expensive and time consuming. In this paper, the moments are calculated through a fully quantitative variance-based global analysis, which is also called Sobol method. The moments can be obtained analytically from Sobol functions for simple mathematical model or estimated from Monte Carlo calculation when the model is more complex and nonintegrable. By means of the Sobol method, the main sources of uncertainty also can be determined. This measure will be specifically discussed in Section 3.2.

In summary, the general framework of the proposed RRDO algorithm is depicted in Figure 1, and it can be described in the following steps.

(1) According to the information on the uncertain parameters, determine and generate the sets of random design variables $\mathbf{v}$.

(2) Get the studied responses from finite element analysis.

(3) Construct the approximate models of the objective function and constraint function in (8).

(4) Consider the less significant variables as deterministic without variation by the Sobol method and at the same time obtain mean and standard deviation of objective function.

(5) Define optimization problem (weighting factor, upper and lower limits, etc.)

(6) Check convergence. If there is a feasible result, go to the 7th step; otherwise, insert more sample sets and go to the 2nd step.

(7) Obtain the optimum design and the corresponding objective function value.

(8) Repeat from the 5th step to the 7th step, if another optimization with respect to a different weighting factor is needed.

3. Methodology Used in RRDO

3.1. Global Approximation. To optimize the performance of complex structures, the relationship between design variables and considered functions can be approximated by means of a pure mathematical model. The global approximation model technology presents an advantage that it can be used for optimization through a single meta-model rather than a sequence of fitted local meta-models. The parameters of the meta-model are defined on the basis of a limited number of simulations, such as finite element simulation.

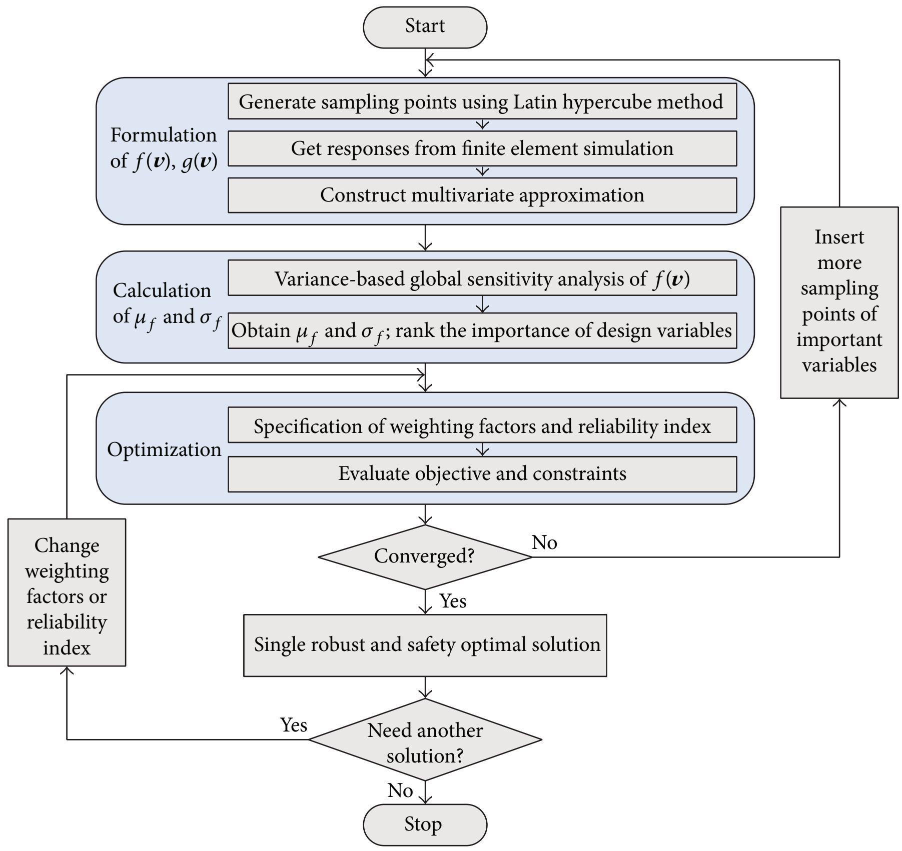

Figure 1: Flowchart of proposed method.

A linear or quadratic polynomial regression model has been used frequently in engineering due to its computational simplicity. The used multivariate polynomial expression considering the cross-product term is given as [16]

$$\tilde{Y}(x) = \sum C_{i_1,i_2,\ldots,i_n} \prod_{j=1}^{n} x_j^{i_j}, \qquad \sum_{j=1}^{n} i_j \le P, \tag{9}$$

where $\tilde{Y}(x)$ is the approximated function, C are the polynomial coefficients, n is the number of the polynomial variables $\mathbf{x}$, i_j is the degree of the polynomial variables, and P is the polynomial degree. The polynomial expression can be converted to the general form by grouping the fitting parameters $\mathbf{C}_a$ and the corresponding functions of variables $\mathbf{X}_a^T$ as shown:

$$\tilde{Y}(\mathbf{x}) = \mathbf{C}_a \mathbf{X}_a^T = \mathbf{C}_a \left[1, \mathbf{X}, f(\mathbf{X})\right]^T. \tag{10}$$

For the second-order approximation, the submatrix of functions of $\mathbf{X}$ and $f(\mathbf{X})$ represent the order and interaction terms of the vector, respectively, as shown:

$$\mathbf{X}_a^T = \left[1 \;\; x_1 \;\; \cdots \;\; x_n \;\; x_1^2 \;\; \cdots \;\; x_n^2 \;\; x_1x_2 \;\; \cdots \;\; x_ix_j\right]^T, \quad \forall i < j,$$

$$\mathbf{C}_a = \left[C_0 \;\; C_1 \;\; \cdots \;\; C_n \;\; C_{11} \;\; \cdots \;\; C_{nn} \;\; C_{12} \;\; \cdots \;\; C_{ij}\right], \quad \forall i < j. \tag{11}$$

To obtain polynomial coefficients, the least-square fitting method [17] is generally used by keeping a minimum distance (fitting error $\mathbf{e}$) between the original data and the approximated one. Consider

$$\min_C \mathbf{e} = \min_C \sum_{i=1}^{n_p} \left[Y_i(\mathbf{v}) - \tilde{Y}_i(\mathbf{v})\right]^2. \tag{12}$$

This method can provide a good fit for the low nonlinear functions with a small variable region. If the response data have multiple local extremes, then polynomial models cannot be used to construct an effective meta-model. In order to approximate globally nonlinear functions, other algorithms, for example, Kriging interpolation model [18, 19], radial basis function [20], and multilayer perceptron neural networks (MLPNN) [21], have been employed.

A percentage error between Y_i and $\tilde{Y}_i$ is considered by using the n_p simulations that constitute the validation set. Percentage error is given as

$$E = \frac{\tilde{Y}_i - Y_i}{Y_i}. \tag{13}$$

3.2. Global Sensitivity Analysis. A fully quantitative variance-based global sensitivity method, namely, the Sobol method, can analyze the impact of the uncertainties of inputs on the output [22]. The Sobol decomposition was originally developed for the analysis with respect to uniformly distributed variables [23]. However, Sobol method can be used for functions of variables with any distribution. Here we use the extended Sobol formulation for normal distribution mentioned in [24].

Let the random design variables $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ with normal distribution functions $\Phi_1(X_1), \Phi_2(X_2), \ldots, \Phi_n(X_n)$. And then system response $F(\mathbf{X})$ in Sobol decomposition format is given by

$$F(\mathbf{X}) = F_0 + \sum_{1\le i\le n} F_i(X_i) + \sum_{1\le i<j\le n} F_{ij}(X_i, X_j) + \cdots + F_{1,2,\ldots,n}(X_i, \ldots, X_n). \quad (14)$$

In this expansion, the terms $F_i(X_i), F_{ij}(X_i, X_j), \ldots$, called the Sobol functions, can be calculated by integrating $F(\mathbf{X})$ according to

$$\begin{aligned}
&\int_{R^n} F(\mathbf{x})\,\varphi_n(\mathbf{x})\,\mathrm{d}\mathbf{x} = F_0,\\
&\int_{R^{n-1}} F(\mathbf{x}_{\neq i}, X_i)\,\varphi_{n-1}(\mathbf{x}_{\neq i})\,\mathrm{d}\mathbf{x}_{\neq i}\\
&\quad = (F_0 + F_i(X_i))\,\varphi_1(\mathbf{x}_i),\\
&\int_{R^{n-2}} F(\mathbf{x}_{\neq ij}, X_i, X_j)\,\varphi_{n-2}(\mathbf{x}_{\neq ij})\,\mathrm{d}\mathbf{x}_{\neq ij}\\
&\quad = (F_0 + f_i(X_i) + F_j(X_j) + F_{ij}(X_i, X_j))\,\varphi_2(\mathbf{x}_i, \mathbf{x}_j),\\
&\quad \vdots
\end{aligned} \quad (15)$$

where $\varphi_n(x)$ is the n-dimensional probability density function of normal distribution, $\mathbf{x}_{\neq i}$ implies a vector of variables corresponding to all but X_i and $\mathbf{x}_{\neq ij}$ notation implies the vector of variables corresponding to all but X_i and X_j, and F_0 is the mean value of the response function. The variance of $F(X)$, denoted by D, can be decomposed as well as the function $F(X)$ according to

$$\begin{aligned}
&\int_{R^n} F^2(\mathbf{x})\,\varphi_n(\mathbf{x})\,\mathrm{d}\mathbf{x} - F_0^2\\
&\quad = \int_{R^1} F_1^2(x_1)\,\varphi_1(x_1)\,\mathrm{d}x_1 + \int_{R^1} F_2^2(x_2)\,\varphi_1(x_2)\,\mathrm{d}x_2\\
&\quad + \cdots + \int_{R^2} f_{12}^2(x_1, x_2)\,\varphi_2(x_1, x_2)\,\mathrm{d}x_1 x_2 + \cdots;
\end{aligned} \quad (16)$$

thereby the partitioned variance has the form

$$D = \sum_{1\le i\le n} D_i + \sum_{1\le i<j\le n} D_{ij} + \cdots + D_{12\cdots n}. \quad (17)$$

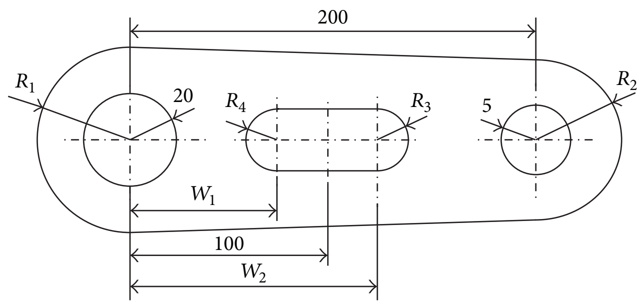

Figure 2: Dimensions of the section of the hook structure.

The Sobol index for a group of indices $\{i_1, i_2, \ldots, i_t\}$ with $1 \le i_1 < \cdots < i_t \le n$ is defined as

$$S_{i_1\cdots i_t} = \frac{D_{i_1\cdots i_t}}{D}. \quad (18)$$

All the $S_{i_1,\ldots,i_t}$ are nonnegative and their sum equals 1. In practice, the sum of first-order Sobol index, denoted by $S^{(1)}$, usually makes up a large part of the whole variance. Thus if all or part of the first-order Sobol index satisfies the condition $1 - S^{(1)} < \varepsilon$, where ε is some small number, the rest of Sobol index is negligible. When a relative simple model (e.g., linear or quadratic regression) is insufficient for the response function, the analytic estimating method may become difficult to be implemented. Hence, for engineering problems, it is convenient to estimate all the Sobol indices from Monte Carlo simulations. The specified Monte Carlo method for calculating Sobol indices is detailed in [23].

4. Case Study

In this section, a "hook" structure from [25] is selected to examine the performance of the proposed method, as shown in Figure 2. While uncertainty is considered in geometric dimensions of the hook, no uncertainty is considered in material properties or load condition. It is rational since manufacturing tolerance is the most likely source of uncertainty at the conceptual design stage. In later design phase, variation introduced by other sources (changes in temperature and fluctuation in load) can be taken into account to search a more thorough optimal design.

The hook's load environment is composed of a welded constraint around the left hole and even pressure $P = 50\,\mathrm{N/mm^2}$ applied to the lower half of the right side. The material has the following mechanical properties: elastic modulus $E = 71.018\,\mathrm{MPa}$, Poisson ratio $\nu = 0.33$, and the density $\rho = 8.25\,\mathrm{kg/m^3}$.

The goal of this problem is to find an optimal shape of hook so that the area of hook should be as small and robust as possible while keeping the maximum von Mises stress less than the admissible constraint $\sigma_{\mathrm{ad}} = 150\,\mathrm{MPa}$ with a probability of 99.73% (i.e., the target reliability index $\beta_c = 3$). The shape of hook can be defined using six design variables that are modeled by a vector $\mathbf{v} = \{R_1, R_2, R_3, R_4, W_1, W_2\}$. All random design variables are statistically independent and distributed normally with a standard deviation of 0.1. The

Table 1: Properties of design variables.

Variables	R_1	R_2	R_3	R_4	W_1	W_2
Initial value (mm)	45	20	10	20	75	125
Lower limit (mm)	30	6	5	10	65	110
Upper limit (mm)	60	30	30	30	85	140

Table 2: Approximation errors (mean $\|E\| \times 100$).

	QPA	RBF	MLPNN
Area	4.4×10^{-11}	1.64×10^{-8}	8.47×10^{-8}
Stress	7.2	3.62	0.89

initial values and upper and lower limits of design variables (nominal value) are presented in Table 1. The optimization problem is therefore expressed in the following form:

$$\begin{aligned} \underset{\mathbf{v}}{\text{minimize:}} \quad & w\frac{\mu_A(\mathbf{v})}{\mu_A^*} + (1-w)\frac{\sigma_A(\mathbf{v})}{\sigma_A^*}, \quad 0 < w < 1 \\ \text{subject to:} \quad & \sigma_{\text{eq}}(\mathbf{z}) \le \sigma_{\text{ad}} \\ & \mathbf{v}^L \le \mathbf{v} \le \mathbf{v}^U, \\ \text{where} \quad & \mathbf{z} = \mu_{\mathbf{v}} - \sigma_{\mathbf{v}}\beta^t \frac{\sigma_{\mathbf{v}}\nabla_{\mathbf{v}}\sigma_{\text{eq}}(\mathbf{v})}{\left\|\sigma_{\mathbf{v}}\nabla_{\mathbf{v}}\sigma_{\text{eq}}(\mathbf{v})\right\|}. \end{aligned} \tag{19}$$

In numerical implementation, a set of area and stress values are obtained through finite element simulations at 200 sampling points generated using Latin hypercube method based on orthogonal array. The minimum number of simulation points is chosen equal to 10 times the number of variables as recommended in [26]; thus 200 samples are enough for the case of six design variables. Then these sampling points and their corresponding function values are used to construct the approximation models of the area and the maximum stress. Since the complexity of the response behavior in the design space is unknown beforehand, a family of approximating algorithms, namely, quadratic polynomial approximation (QPA), radial basis functions (RBF), and multilayer perceptron neural networks (MLPNN), are used and compared. The approximation errors (percentage errors) are summarized in Table 2. It is evident that the area indices are approximated with a sufficient accuracy in the shape of polynomial expression. Sobol sensitivity analysis thus can be performed on the polynomial model, resulting in the response mean μ_A and standard deviation σ_A as a function of design variables. For the maximum stress data, the MLPNN model results in the most accurate fit with the maximum error equal to 0.89%.

The global sensitivity analysis is performed by means of the simulations based on the approximated model. The first-order Sobol indices are computed by considering the design variables at their initial values reported in Table 1. Each index is obtained by varying one single variable, the other ones being constant and equal to those of the reference value. The results of the variable first-order sensitivity indices for the function of area are shown in Figure 3.

Table 3: Results of the different solutions.

	R_1 (mm)	R_2 (mm)	R_3 (mm)	R_4 (mm)	W_1 (mm)	W_2 (mm)
Deterministic	40.63	12.51	10.00	18.90	69.70	115.0
RBDO	41.49	12.64	10.00	19.33	69.83	115.0
RRDO	41.51	12.55	10.00	19.20	70.07	115.0

Table 4: Comparison of different designs.

	Deterministic	RBDO	RRDO
Objective (area A)			
μ_A (mm^2)	10104.6	10375.5	10382.6
σ_A	1817.2	1839.1	1835.8
Constraint (stress σ_{eq})			
μ_σ (MPa)	150.0	145.7	145.8
$\Pr\{F\}$	52.1%	0.17%	0.17%
Number of function calls			
Objective	157	237	822
Constraint	1,467	8,462	1,778

By inspection of Figure 3 it is noticed that the area function is not influenced significantly by W_1 and W_2. In the current case, an obvious truncation would include only the uncertainty in input variables corresponding to the four radius values, R_1, R_2, R_3, and R_4, which account, in only their first-order forms, for 98.03% of the total response variance. Thus W_1 and W_2 can be regarded as two deterministic parameters, which can efficiently reduce the sample size and variable dimension in the later optimization process. Since genetic algorithm (GA) works well in obtaining global optimum, it is chosen as the solver in the proposed RRDO optimization. The optimal results from the hybrid RRDO ($w = 0.5$) using (19), deterministic design, and RBDO approach are presented in Table 3.

Table 4 compares the results of the different optimal designs. As shown in Table 4, the deterministic design gives the smallest values in both the area response and its standard deviation, which is right because the purpose of deterministic optimization is to obtain the minimal value without regard for uncertainties. Consequentially, it has a relatively large probability of failure value of 52.1% when the maximum stress μ_σ satisfies the constraint condition. At the same time, both RRDO and RBDO let the mean values of area increase to meet the requirements of structural safety. To clearly show the difference between RBDO and RRDO results, the range of area response can be briefly estimated based on the moments of area value (mean and standard deviation). By using 3-σ rule applied for normal distribution, the ranges of area from RBDO and RRDO are [4858, 15893] and [4875, 15890], respectively. Thus RRDO gives a feasible result with better robustness as the range is smaller.

As to the constraint of stress, three probability density functions (PDFs) of maximum stress are compared with each other in Figure 4. Note that these three PDFs are expanded across just for better showing each failure region, and it cannot tell the exact number of probability density. From

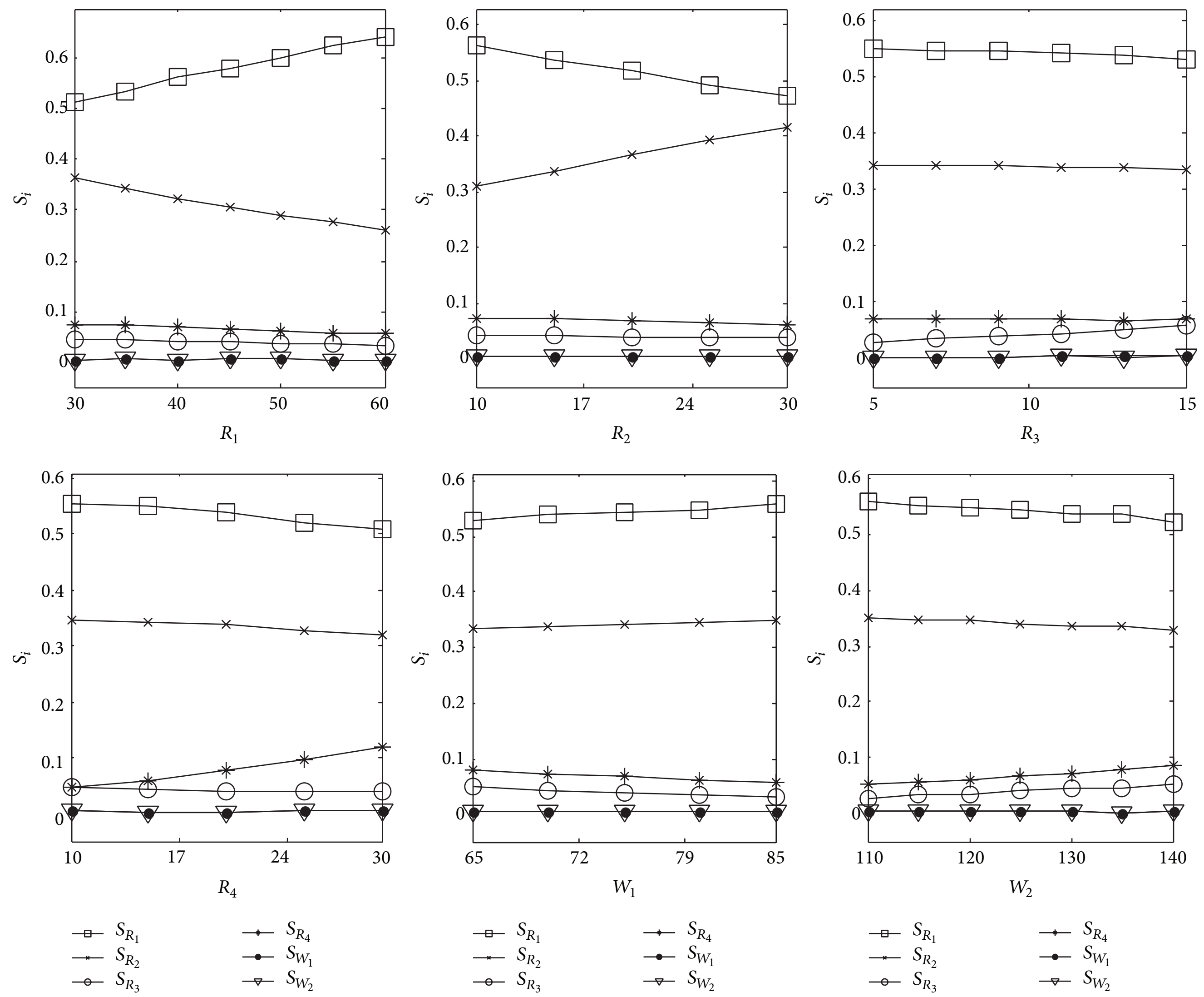

Figure 3: Nondimensional first-order sensitivity indices.

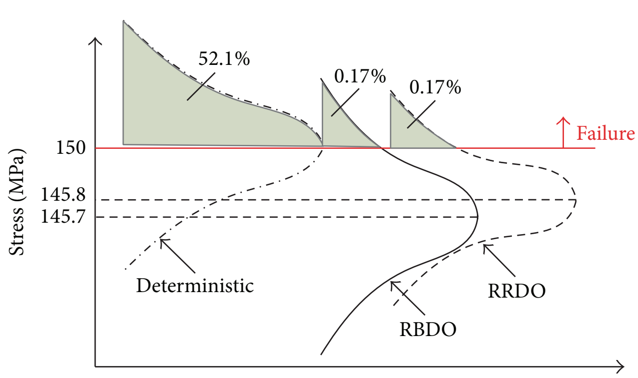

Figure 4: Probability density functions of maximum stress σ_{eq}.

Figure 4, failure occurs over the red line whose value is 150 MPa, and the PDF area belonging to the failure region corresponds to the probability of failure. It is seen that both RBDO and RRDO ensure reliability with the target probability of failure (i.e., 0.17%). Considering the number of function evaluations for all the approaches, the proposed method shows an economical solution with much lower computational cost over RBDO. From the results, it is evident that the proposed method reduces the computational cost, while maintaining the accuracy of reliability and robustness analysis.

5. Conclusions

In this study, a hybrid RRDO approach is presented to find a reliable and robust solution under uncertainty in design variables. This work combined the single-loop approach and monoobjective robust design (weighted sum) method, applying a RRDO strategy with the usage of global approximation and global sensitivity analysis. The central advantage of the proposed method is reducing the computational cost, which is implemented through three ways. First, using a family of approximated models to replace the computational expensive FE simulations, the response model was accurately constructed. Second, global sensitivity analysis ensured a relatively small sample size, because some insensitive design variables were considered deterministic without uncertainty. Third, the optimization structure was converted into single-loop process by the adoption of single-loop approach. A structural example was tested and the results have been compared with those from RBDO and deterministic case. The accuracy and efficiency of the proposed method have been demonstrated.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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