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*Research Article*

# Stabilization of Networked Distributed Systems with Partial and Event-Based Couplings

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The stabilization problem of networked distributed systems with partial and event-based couplings is investigated. The channels, which are used to transmit different levels of information of agents, are considered. The channel matrix is introduced to indicate the work state of the channels. An event condition is designed for each channel to govern the sampling instants of the channel. Since the event conditions are separately given for different channels, the sampling instants of channels are mutually independent. To stabilize the system, the state feedback controllers are implemented in the system. The control signals also suffer from the two communication constraints. The sufficient conditions in terms of linear matrix equalities are proposed to ensure the stabilization of the controlled system. Finally, a numerical example is given to demonstrate the advantage of our results.

## 1. Introduction

Recent years have witnessed a thriving research activity on how to assemble and coordinate networked distributed systems (NDSs) into a coherent whole to perform a common task [1]. NDSs have obvious advantages in practice, such as energy saving, easy installation, and higher reliability [2–6]. Thus, studying stabilization of NDSs is of theoretical and practical importance. To realize the stabilization, a control strategy is needed. However, due to the absence of central data fusion, the classical centralized control scheme is not feasible for NDSs. Accordingly, the cooperative control strategy is a preferred choice. Since an NDS consists of a large number of agents, it is impossible and unnecessary to control every agent. An effective approach is to implement controllers for a fraction of the NDSs to stabilize the whole system, which is referred to as the pinning stabilization problem [7, 8].

To achieve stabilization, the communication in NDSs plays a crucial role. However, due to physical and environmental limitations, communication constraints, such as time delays [9–11] and noise [12, 13], are unavoidable. In fact, incomplete information is universal. As each agent of an NDS has multiple levels of information, the coupling has to be split into multiple channels to transmit the corresponding levels of information. Due to the physical limitatioins, only some parts of the channels can transmit information successfully, which brings the partial-coupling problem. Such a phenomenon can be observed in many real systems. For example, in brain networks, only 5% excitatory synapses sent from a cortical area can be received by another connected cortical area [14]; in sensor networks, the information packet of a target may be partly lost during communication between sensors [5]. Thus, it is highly desirable to analyze NDSs with partial couplings.

The sampling issue has received intense attention, ever since the rapid development of digital technology and intelligent equipment. A traditional sampling protocol is time-based sampling technique [15–17]. Recently, the event-based sampling has been investigated as an alternative to time-based sampling. The distinct feature of event-based sampling is the real-time scheduling algorithm. The information is sampled only when a certain event occurs, for example, when the system state exceeds a predefined threshold. The advantage of the event-based sampling is the capability of fast reacting to sudden events and, therefore, being more efficient [18, 19]. In an NDS, the event conditions are individually designed for each agent. Thus, the agents are sampled at mutually independent instants. This means that the event-based sampling scheme does not require a common sampling schedule, which makes it applicable for a system with large size. For the single system, an event-triggered method was

designed in [20], in which the lower bound of two successive sampling instants was given; in [21, 22], the event-triggered controller was designed for networked control systems with transmission delay. In [23], the event-based control was used in multiagent system drive the agents to average consensus. However, until now, few results have been given for the NDS with partial and event-based couplings. The difficulty of this problem is threefold. Firstly, two communication constraints are considered, both of which make less information available for communication. Secondly, each agent samples information separately; however, they should cooperatively converge. Thus, how to design the event conditions of the agents? Finally, the stabilization conditions should be given to guarantee the stabilization of the NDS.

In this paper, we focus on the stabilization problem of NDSs with partial and event-based couplings. By designing an event condition for each agent, an event-based sampling scheme is proposed for NDSs. Due to the constraint of partial information transmission, the channels are considered in the event condition. Thus, for different channels of one agent, the sampling instants are distinct. The sampled data are used for the communication among agents and building the feedback controllers. The sufficient conditions are given to ensure the stabilization of the controlled NDS with both communication constraints. Finally, a numerical example is given to demonstrate the advantage of our results.

*Notations*. The standard notations will be used in this paper. Throughout this paper, $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space. $\mathbb{R}^{n\times n}$ are the set of $n \times n$ real matrices. $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix. For real symmetric matrices $X$ and $Y$, the notation $X \leq Y$ (resp., $X < Y$) means that the matrix $X - Y$ is negative semidefinite (resp., negative definite). $\|\cdot\|$ denotes the Euclidean norm for vector or the spectral norm of matrix. $\mathrm{diag}\{\cdots\}$ denotes a diagonal matrix. The superscripts "$\top$" and "$-1$" represent the matrix transposition and matrix inverse. $*$ in a matrix represents the elements below the main diagonal of a symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

## 2. Problem Formulation and Preliminaries

An NDS with $N$ agents can be described by the following:

$$\dot{x}_i(t) = Ax_i(t) + u_{i1}(t), \quad i \in \{1, 2, \ldots, N\}, \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)] \in \mathbb{R}^n$ is the state of the $i$th system; $A = [a_{ij}]_{n\times n}$ is the system matrix. Assuming that each agent can only receive the information of its neighbors, the coupling control $u_{i1}(t)$ can be constructed as follows:

$$u_{i1}(t) = \alpha \sum_{j=1}^{N} g_{ij} R_{ij} \left(x_j(t) - x_i(t)\right), \tag{2}$$

where $\alpha$ is the coupling strength; $G = [g_{ij}]_{N\times N}$ is the Laplacian matrix representing the coupling structure of the NDS. The elements of $G$ are defined as follows: if there is a connection from the $j$th agent to $i$th agent, then $g_{ij} > 0$; otherwise, $g_{ij} = 0$, and the diffusive coupling condition $g_{ii} = \sum_{j=1, j\neq i}^{N} g_{ij}$ is satisfied. $R_{ij}$, named as *channel matrix*, is a diagonal matrix with the diagonal element $r_{ij}^k = 0$ or 1 ($k = 1, 2, \ldots, n$).

*Remark 1.* Since the state of each agent consists of $n$ levels of information, the couplings in the NDS have to be divided into $n$ channels to transmit the corresponding levels of information. Due to practical constraints, only part of the $n$ channels can work normally. The diagonal element $r_{ij}^k$ ($k = 1, 2, \ldots, n$) of the channel matrix $R_{ij}$ is employed to indicate the activity of the $k$th channel connecting agents $j$ and $i$. Specifically, $R_{ij}x_j = [r_{ij}^1 x_{j1}, \ldots, r_{ij}^n x_{jn}]^\top$. When $r_{ij}^k = 1$, $r_{ij}^k x_{jk} = x_{jk}$, the $k$th level of state of agent $j$ can be transmitted to agent $i$ (i.e., the $k$th channel of the connection is *active*); otherwise, when $r_{ij}^k = 0$, $r_{ij}^k x_{jk} = 0$, the $k$th level of state of $x_j$ is lost (i.e., the $k$th channel of the connection fails to transmit the information).

In the coupling control (2), each agent can receive the real-time information of its neighboring agents. However, the real-time information will increase the burden of the communication media. More importantly, it is unnecessary. For the sake of energy saving, the event-based control mechanism has been recently proposed as an effective alternative to the more conventional execution of control tasks. In this paper, the communication of agents will be carried out in an event-based manner; that is, the state of the agent will be sampled, if a given event is triggered.

To realize the event-based sampling, an event condition is designed for each agent. When the event condition is violated, the agent will sample its information and send it to its neighbors. Considering that the information is transmitted through channels, the event condition is given as follows:

$$\begin{gathered}\left|x_{ik}\left(t_m^{ik}\right) - x_{ik}\left(t_m^{ik} + lh\right)\right| \leq \gamma_{ik}\left|x_{ik}\left(t_m^{ik} + lh\right)\right|, \\ i \in \{1, 2, \ldots, N\}, \quad k \in \{1, 2, \ldots, n\},\end{gathered} \tag{3}$$

where $l = 1, 2, \ldots$; $\gamma_{ik}$ is a positive scalar; $h > 0$ is the sampling period; $t_m^{ik}$ is the latest sampling instant of the $k$th level of information of the $i$th agent. Thus, the next event-triggered instant is the time when event condition (3) is violated; that is,

$$\begin{aligned} t_{m+1}^{ik} = t_m^{ik} + h \inf \Big\{ l \in \mathbb{N} : &\left\|x_{ik}\left(t_m^{ik}\right) - x_{ik}\left(t_m^{ik} + lh\right)\right\| \\ &> \gamma_{ik}\left\|x_i\left(t_m^{ik} + lh\right)\right\| \Big\}. \end{aligned} \tag{4}$$

Without loss of generality, $t_0^{ik} = 0$ is the initial sampling time, for all $i$ and $k$.

Let $\hat{x}_{ik}(t) = x_{ik}(t_m^{ik})$, for $t \in [t_m^{ik}, t_{m+1}^{ik})$ and $\hat{x}_i(t) = [\hat{x}_{i1}(t), \hat{x}_{i2}(t), \ldots, \hat{x}_{in}(t)]$. Thus, NDS (1) with partial and event-based couplings can be described as

$$\dot{x}_i(t) = Ax_i(t) + \alpha \sum_{j=1, j\neq i}^{N} g_{ij} R_{ij} \left(\hat{x}_j(t) - \hat{x}_i(t)\right). \tag{5}$$

*Remark 2.* Due to the traffic jam and physical characteristics of transmission media, communication constraints, such as

transmission delay [24–26], data packet dropout [27], and noise [28], commonly happen in real systems. In this paper, two kinds of communication constraints including partial information transmission of system and event-based sampled data are simultaneously considered in the couplings of NDS (5). Although both constraints cause information loss, their mechanisms are different. The event-based sampled data makes the real-time information available at the instants when event condition (3) is violated. The constraint of partial information transmission implies that the information sent at every instant is a lack of integrity. When we consider these two communication constraints simultaneously in NDSs, only part of the sampled information can be used for coupling control. Thus, the stabilization of the NDS is much harder to be realized.

*Remark 3.* Event condition (3) governs the sampling operation of NDS (5). When a subsystem is diverging, that is, the left hand side of the event condition becomes large, the subsystem has to update its state by sampling. In other words, the sampling happens only when it is needed. Thus, the event-based sampling scheme is more flexible and effective. It is able to realize fast reaction to emergency and avoid redundant samplings. Besides energy saving, another advantage of event-based sampling scheme is feasibility for large-scale systems. The sampling instants are independently decided by the event conditions of different subsystems. Thus, the common sampling schedule is unnecessary, which is hardly carried out in a system with large number of components. These two advantages make the event-based sampling scheme more applicable in engineering.

In order to stabilize the NDS, the state feedback controllers will be implemented. Considering that it is difficult and unnecessary to install the controller for every agent, we only choose a small fraction of agents to be controlled. In addition, we also assume that the control signal suffers from the two communication constraints. Thus, the pinning controller of agent $i$ can be constructed as follows:

$$u_{i2}(t) = -d_i H_i \hat{x}_i(t), \tag{6}$$

where $d_i \geq 0$ is the control strength. In particular, when $d_i = 0$, it means that the $i$th agent will not be controlled. $H_i = \mathrm{diag}\{h_{i1}, h_{i2}, \ldots, h_{in}\}$ with $h_{ik} = 1$ or 0, which indicates that the $k$th level of the event-based sampled information of $\hat{x}(t)$ can be or cannot be sent from the controller.

Combining (5) and (6), the controlled NDS with partial and event-based couplings can be described by following equation:

$$\dot{x}_i(t) = Ax_i(t) + \alpha \sum_{j=1, j\neq i}^{N} g_{ij} R_{ij} \left(\hat{x}_j(t) - \hat{x}_i(t)\right) - d_i H_i \hat{x}_i(t). \tag{7}$$

Note that, for $t \in [t_m^{ik} + lh, t_m^{ik} + lh + h)$ ($l \geq 0$, $m \geq 0$, $i = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, n$),

$$\begin{aligned}
\hat{x}_{jk}(t) - \hat{x}_{ik}(t) &= x_{jk}\left(t_{m'}^{jk}\right) - x_{ik}\left(t_m^{ik}\right) \\
&= x_{jk}\left(t_{m'}^{jk}\right) - x_{jk}\left(t_m^{ik} + lh\right) - x_{ik}\left(t_m^{ik}\right) \\
&\quad + x_{ik}\left(t_m^{ik} + lh\right) + x_{jk}\left(t_m^{ik} + lh\right) \\
&\quad - x_{ik}\left(t_m^{ik} + lh\right) \\
&= e_{jk}\left(t_m^{ik} + lh\right) - e_{ik}\left(t_m^{ik} + lh\right) \\
&\quad + x_{jk}\left(t_m^{ik} + lh\right) - x_{ik}\left(t_m^{ik} + lh\right),
\end{aligned} \tag{8}$$

where $e_{ik}(t_m^{ik}+lh) = \hat{x}_{ik}(t_m^{ik}+lh) - x_{ik}(t_m^{ik}+lh)$, $t_{m'}^{jk} = \max\{t \mid t \in t_m^{jk}, m = 0, 1, \ldots, t \leq t_m^{ik} + lh\}$, and $x_{jk}(t_{m'}^{jk}) = x_{jk}(t_m^{jk} + lh)$. Let $e_i(t) = [e_{i1}(t), e_{i2}(t), \ldots, e_{in}(t)]^\top$. Thus, for $t \in [mh, (m+1)h)$, (7) can be written as

$$\begin{aligned}
\dot{x}_i(t) = Ax_i(t) &+ \alpha \sum_{j=1, j\neq i}^{N} g_{ij} R_{ij} \left(e_j(mh) + x_j(mh)\right) \\
&- d_i H_i \left(x_i(mh) + e_i(mh)\right).
\end{aligned} \tag{9}$$

Let $C_{ij} = g_{ij} R_{ij}$, for $i, j = 1, 2, \ldots, N$, $i \neq j$ and $C_{ii} = -\sum_{j=1, j\neq i}^{N} C_{ij}$. Thus, $C_{ij}$ are diagonal matrices with diagonal elements $c_{ij}^1, c_{ij}^2, \ldots, c_{ij}^n$ ($c_{ij}^k = g_{ij} \cdot r_{ij}^k$). For each $k = 1, 2, \ldots, n$, it can be followed from (9) that

$$\begin{aligned}
\dot{x}_{ik}(t) = \sum_{j=1}^{n} a_{kj} x_{ij}(t) &+ \alpha \sum_{j=1}^{N} c_{ij}^k \left(e_{jk}(mh) + x_{jk}(mh)\right) \\
&- d_i h_{ik} \left(e_{jk}(mh) + x_{jk}(mh)\right).
\end{aligned} \tag{10}$$

Let $\delta x_k(t) = [x_{1k}(t), x_{2k}(t), \ldots, x_{Nk}(t)]^\top$ and $\delta e_k(t) = [e_{1k}(t), e_{2k}(t), \ldots, e_{Nk}(t)]^\top$. From (10) we can get

$$\delta \dot{x}_k(t) = \sum_{j=1}^{n} a_{kj} \delta x_j(t) + \left(\alpha C_k - D_k\right)\left(\delta e_k(mh) + \delta x_k(mh)\right), \tag{11}$$

where $C_k = [c_{ij}^k]_{N\times N}$ and $D_k = \mathrm{diag}\{d_1 h_{1k}, d_2 h_{2k}, \ldots, d_N h_{Nk}\}$.

The following definition and lemma are needed for the derivation of our main results in this paper.

*Definition 4.* NDS (7) with partial and event-based couplings is said to achieve globally exponential stabilization, if there exist $M > 0$, $\varepsilon > 0$ such that $\|x_i(t)\|^2 < Me^{-\varepsilon t}$ is satisfied with any initial states $x_i(0)$ for $\forall i \in \{1, 2, \ldots, N\}$.

The purpose of this paper is to propose a set of sufficient conditions for controlled NDS (7) with partial and event-triggered communication to ensure the globally exponential stabilization.

**Lemma 5** (see [29]). *For any real vectors $a$, $b$ and scalar $\varepsilon > 0$, one has*

$$a^\top b + b^\top a \leq \varepsilon a^\top a + \varepsilon^{-1} b^\top b. \tag{12}$$

## 3. Main Results

In this section, the stabilization conditions will be derived for controlled NDS (7) with partial and event-based couplings.

**Theorem 6.** *For any $k = 1, 2, \ldots, n$, let $\Gamma_k = \operatorname{diag}\{\gamma_{1k}^2, \gamma_{2k}^2, \ldots, \gamma_{Nk}^2\}$. NDS (7) with partial and event-based couplings can be globally exponentially stabilized, if there exist positive scalars $\varepsilon_{1k}$, $\varepsilon_{2k}$ and matrices $P_k > 0$, $Q_k = \operatorname{diag}\{q_{1k}, q_{2k}, \ldots, q_{Nk}\} > 0$, $\mathscr{W}_k = \begin{bmatrix} W_{1k} & W_{2k} & W_{4k} \\ * & W_{3k} & 0 \\ * & * & W_{5k} \end{bmatrix} > 0$, $U_{1k}$, $U_{2k}$, $U_{3k}$, $U_{4k}$, $V_{1k}$, $V_{2k}$, $X_k$, $X_{1k}$, such that the following LMIs are satisfied:*

$$\begin{bmatrix} P_k + h\dfrac{X_k + X_k^\top}{2} + W_{1k} & -hX_k + hX_{1k} - W_{1k} & W_{2k} & W_{4k} \\ * & -hX_{1k} - hX_{1k}^\top + h\dfrac{X_k + X_k^\top}{2} + W_{1k} & -W_{2k} & -W_{4k} \\ * & * & W_{3k} & 0 \\ * & * & * & W_{5k} \end{bmatrix} > 0, \tag{13}$$

$$\begin{bmatrix} \Phi_{11}^k & \Phi_{12}^k & \Phi_{13}^k & \Phi_{14}^k & hU_{1k}^\top & nV_{1k}^\top & 0 \\ * & \Phi_{22}^k & \Phi_{23}^k & \Phi_{24}^k & hU_{2k}^\top & 0 & 0 \\ * & * & \Phi_{33}^k & \Phi_{34}^k & hU_{3k}^\top & 0 & nV_{2k}^\top \\ * & * & * & \Phi_{44}^k & hU_{4k}^\top & 0 & 0 \\ * & * & * & * & -hW_{1k} & 0 & 0 \\ * & * & * & * & * & -n\varepsilon_{1k}I_N & 0 \\ * & * & * & * & * & * & -n\varepsilon_{2k}I_N \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \Phi_{11}^k & \Phi_{12}^k & \Pi_{13}^k & \Phi_{14}^k & nV_{1k}^\top & 0 \\ * & \Pi_{22}^k & \Pi_{23}^k & \Phi_{24}^k & 0 & 0 \\ * & * & \Pi_{33}^k & \Pi_{34}^k & 0 & nV_{2k}^\top \\ * & * & * & \Pi_{44}^k & 0 & 0 \\ * & * & * & * & -n\varepsilon_{1k}I_N & 0 \\ & * & * & * & * & -n\varepsilon_{2k}I_N \end{bmatrix} < 0, \tag{15}$$

*where*

$$\begin{aligned}
\Phi_{11}^k &= -\frac{X_k + X_k^\top}{2} - U_{1k} - U_{1k}^\top + \sum_{j=1}^{n} \left(\varepsilon_{1j} + \varepsilon_{2j}\right) a_{jk}^2 I_n, \\
\Phi_{12}^k &= X_k - X_{1k} - W_{2k} + U_{1k}^\top - U_{2k} + V_{1k}^\top \left(\alpha C_k - D_k\right), \\
\Phi_{13}^k &= P_k - U_{3k} - V_{1k}^\top, \\
\Phi_{14}^k &= -W_{4k} - U_{4k} + V_{1k}^\top \left(\alpha C_k - D_k\right), \\
\Phi_{22}^k &= X_{1k} + X_{1k}^\top - \frac{X_k + X_k^\top}{2} + W_{2k} + W_{2k}^\top \\
&\quad + U_{2k} + U_{2k}^\top + Q_k\Gamma_k - hW_{3k}, \\
\Phi_{23}^k &= U_{3k} + \left(\alpha C_k^\top - D_k\right) V_{2k}, \\
\Phi_{24}^k &= W_{4k} + U_{4k}, \\
\Phi_{33}^k &= -V_{2k} - V_{2k}^\top, \\
\Phi_{34}^k &= V_{2k}^\top \left(\alpha C_k - D_k\right), \\
\Phi_{44}^k &= -Q_k - hW_{5k}, \\
\Pi_{13}^k &= h\frac{X_k + X_k^\top}{2} + P_k - U_{3k} - V_{1k}^\top, \\
\Pi_{22}^k &= X_{1k} + X_{1k}^\top - \frac{X_k + X_k^\top}{2} + W_{2k} + W_{2k}^\top \\
&\quad + U_{2k} + U_{2k}^\top + Q_k\Gamma_k + hW_{3k}, \\
\Pi_{23}^k &= hW_{2k}^\top + hX_{1k}^\top - hX_k^\top + U_{3k} + \left(\alpha C_k^\top - D_k\right) V_{2k}, \\
\Pi_{33}^k &= hW_{1k} - V_{2k} - V_{2k}^\top, \\
\Pi_{34}^k &= hW_{4k} + V_{2k}^\top \left(\alpha C_k - D_k\right), \\
\Pi_{44}^k &= -Q_k + hW_{5k}.
\end{aligned} \tag{16}$$

*Proof.* For $t \in [mh, (m+1)h)$, define the Lyapunov functional $V(t) = \sum_{k=1}^{n} V_k(t)$, where

$$\begin{aligned} V_k(t) &= \delta x_k^\top(t) P_k \delta x_k(t) \\ &\quad + \varsigma(t) \int_{mh}^{t} \begin{bmatrix} \delta \dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix}^\top \mathscr{W}_k \begin{bmatrix} \delta \dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix} ds \\ &\quad + \varsigma(t) \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \mathscr{X}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}, \end{aligned} \tag{17}$$

and $\varsigma(t) = (m+1)h - t$, $\mathscr{X}_k = \begin{bmatrix} (X_k + X_k^\top)/2 & -X_k + X_{1k} \\ * & -X_{1k} - X_{1k}^\top + (X_k + X_k^\top)/2 \end{bmatrix}$.

Before proceeding our proof, it should be pointed out that $V(t)$ is well defined. Based on $\mathscr{W}_k > 0$ and Schur complement [30], we have that

$$\widehat{W}_k \triangleq W_{1k} - W_{2k} W_{3k}^{-1} W_{2k}^\top - W_{4k} W_{5k}^{-1} W_{4k}^\top. \tag{18}$$

Thus, it follows that

$$\begin{aligned} &\mathscr{W}_k - \begin{bmatrix} \widehat{W}_k & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \\ &= \begin{bmatrix} W_{2k} W_{3k}^{-1} W_{2k}^\top + W_{4k} W_{5k}^{-1} W_{4k}^\top & W_{2k} & W_{4k} \\ * & W_{3k} & 0 \\ * & * & W_{5k} \end{bmatrix} \geq 0, \end{aligned} \tag{19}$$

which implies

$$\mathscr{W}_k \geq \begin{bmatrix} \widehat{W}_k & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}. \tag{20}$$

Based on the above inequality, it can be got that

$$\begin{bmatrix} \delta \dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix}^\top \mathscr{W}_k \begin{bmatrix} \delta \dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix} \geq \delta \dot{x}_k^\top(s) \widehat{W}_k \delta \dot{x}_k(s). \tag{21}$$

Thus, it follows from (17) and (21) that

$$\begin{aligned} V_k(t) &\geq \delta x_k^\top(t) P_k \delta x_k(t) \\ &\quad + \varsigma(t) \int_{mh}^{t} \delta \dot{x}_k^\top(s) \widehat{W}_k \delta \dot{x}_k(s) ds \\ &\quad + \varsigma(t) \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \mathscr{X}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}. \end{aligned} \tag{22}$$

By applying the Jensen inequality [31], we have

$$\begin{aligned} &\int_{mh}^{t} \delta \dot{x}_k^\top(s) \widehat{W}_k \delta \dot{x}_k(s) ds \\ &\geq \frac{1}{\tau(t)} \int_{mh}^{t} \delta \dot{x}_k^\top(s) ds \widehat{W}_k \int_{mh}^{t} \delta \dot{x}_k(s) ds \\ &\geq \frac{1}{h} \left[\delta x_k(t) - \delta x_k(mh)\right]^\top \widehat{W}_k \left[\delta x_k(t) - \delta x_k(mh)\right], \end{aligned} \tag{23}$$

where $\tau(t) = t - mh$. From (22) and (23), it can be obtained that

$$V_k(t) \geq \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \mathscr{H}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}, \tag{24}$$

where $\mathscr{H}_k = (\varsigma(t)/h)[\mathscr{P}_k + h\mathscr{X}_k + \widetilde{\mathscr{W}}_k] + (\tau(t)/h)\mathscr{P}_k$, $\mathscr{P}_k = \begin{bmatrix} P_k & 0 \\ * & 0 \end{bmatrix}$ and $\widetilde{\mathscr{W}}_k = \begin{bmatrix} \widehat{W}_k & -\widehat{W}_k \\ * & \widehat{W}_k \end{bmatrix}$. Based on the Schur complement [30], it follows from (13) that $\mathscr{P}_k + h\mathscr{X}_k + \widetilde{\mathscr{W}}_k > 0$. Thus, we can always find a positive scalar $\beta < 1$, such that $\mathscr{P}_k + h\mathscr{X}_k + \widetilde{\mathscr{W}}_k > \beta \mathscr{P}_k$, which further means that

$$\begin{aligned} V(t) &= \sum_{k=1}^{n} V_k(t) > \sum_{k=1}^{n} \beta \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \mathscr{P}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix} \\ &= \beta \sum_{i=1}^{N} \left\| x_i(t) \right\|^2. \end{aligned} \tag{25}$$

Therefore, $V(t)$ defined in (17) is a valid Lyapunov functional.

The derivative of $V_k(t)$ can be got that

$$\begin{aligned} \dot{V}_k(t) &= 2\delta \dot{x}_k(t) P_k \delta x_k(t) \\ &\quad - \int_{mh}^{t} \begin{bmatrix} \delta \dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix}^\top \widetilde{\mathscr{W}}_k \begin{bmatrix} \delta \dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix} ds \\ &\quad + \varsigma(t) \begin{bmatrix} \delta \dot{x}_k(t) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix}^\top \mathscr{W}_k \begin{bmatrix} \delta \dot{x}_k(t) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix} \\ &\quad - \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \mathscr{X}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix} \\ &\quad + 2\varsigma(t) \begin{bmatrix} \delta \dot{x}_k(t) \\ 0 \end{bmatrix}^\top \mathscr{X}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix} \\ &\quad - \chi_1 \int_{mh}^{t} \left\| \delta \dot{x}_k(s) \right\|^2 ds, \end{aligned} \tag{26}$$

where $\widetilde{\mathscr{W}}_k = \mathscr{W}_k - \mathrm{diag}\{\chi_1 I_N, 0, 0\}$ and $\chi_1$ is a positive scalar such that $\widetilde{\mathscr{W}}_k > 0$. By employing the Jensen inequality [31], it follows that

$$\begin{aligned}
&\int_{mh}^{t} \begin{bmatrix} \delta\dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix}^{\top} \widetilde{\mathscr{W}}_k \begin{bmatrix} \delta\dot{x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix} ds \\
&\quad \geq \tau(t)\, \phi_k^{\top}(t) \left(W_{1k} - \chi_1 I_N\right) \phi_k(t) \\
&\qquad + \tau(t)\, \delta x_k^{\top}(mh)\, W_{3k} \delta x_k(mh) \\
&\qquad + \tau(t)\, \delta e_k^{\top}(mh)\, W_{5k} \delta e_k(mh) \\
&\qquad + 2\left[\delta x_k(t) - \delta x_k(mh)\right]^{\top} W_{2k} \delta x_k(mh) \\
&\qquad + 2\left[\delta x_k(t) + \delta x_k(mh)\right]^{\top} W_{4k} \delta e_k(mh),
\end{aligned} \tag{27}$$

where $\phi(t) = (1/\tau(t)) \int_{mh}^{t} \delta\dot{x}_k(s) ds$.

According to the Newton-Leibnitz formula, it can be obtained that

$$\begin{aligned}
0 = 2 &\left[\delta x_k^{\top}(t)\, U_{1k}^{\top} + \delta x_k^{\top}(mh)\, U_{2k}^{\top} + \delta\dot{x}_k(t)\, U_{3k}^{\top} \right. \\
&\left. + \delta e_k^{\top}(mh)\, U_{4k}^{\top}\right] \\
&\cdot \left[-\delta x_k(t) + \delta x_k(mh) + \tau(t)\, \phi_k(t)\right].
\end{aligned} \tag{28}$$

From (11), one can have that

$$\begin{aligned}
0 = 2 &\left[\delta x_k^{\top}(t)\, V_{1k}^{\top} + \delta\dot{x}_k(t)\, V_{2k}^{\top}\right] \\
&\cdot \left[-\delta\dot{x}_k(t) + \sum_{j=1}^{n} a_{kj} \delta x_j(t) \right. \\
&\left. + \left(\alpha C_k - D_k\right)\left(\delta e_k(mh) + \delta x_k(mh)\right)\right].
\end{aligned} \tag{29}$$

By employing Lemma 5, it follows that

$$\begin{aligned}
&2\delta x_k^{\top}(t)\, V_{1k}^{\top} \sum_{j=1}^{n} a_{kj} \delta x_j(t) \\
&\quad \leq n\varepsilon_{1k}^{-1} \delta x_k^{\top}(t)\, V_{1k}^{\top} V_{1k} \delta x_k(t) + \varepsilon_{1k} \sum_{j=1}^{n} a_{kj}^2 \delta x_j^{\top}(t)\, \delta x_j(t), \\
&2\delta\dot{x}_k^{\top}(t)\, V_{2k}^{\top} \sum_{j=1}^{n} a_{kj} \delta x_j(t) \\
&\quad \leq n\varepsilon_{2k}^{-1} \delta\dot{x}_k^{\top}(t)\, V_{2k}^{\top} V_{2k} \delta\dot{x}_k(t) + \varepsilon_{2k} \sum_{j=1}^{n} a_{kj}^2 \delta x_j^{\top}(t)\, \delta x_j(t).
\end{aligned} \tag{30}$$

From event condition (3), it can be got that $|e_{ik}(mh)| \leq \gamma_{ik}|x_{ik}(mh)|$, which is equivalent to $\sqrt{q_{ik}}|e_{ik}(mh)| \leq \sqrt{q_{ik}}\gamma_{ik}|x_{ik}(mh)|$. Thus, we have that

$$\delta e_k^{\top}(mh)\, Q_k \delta e_k(mh) \leq \delta x_k^{\top}(mh)\, Q_k \Gamma_k \delta x_k(mh). \tag{31}$$

Combining (26)–(31) yields that

$$\begin{aligned}
\dot{V}(t) = \sum_{k=1}^{n} &\left\{\eta_k^{\top}(t) \left[\frac{\varsigma(t)}{h} \overline{\Psi}_k + \frac{\tau(t)}{h} \Psi_k\right] \eta_k(t) \right. \\
&\left. - \chi_1 \int_{mh}^{t} \left\|\delta\dot{x}_k(s)\right\|^2 ds\right\},
\end{aligned} \tag{32}$$

where $\eta_k(t) = \left[\delta x_k(t) \;\; \delta x_k(mh) \;\; \delta\dot{x}_k(t) \;\; \delta e_k(mh) \;\; \phi_k(t)\right]^{\top}$ and $\overline{\Psi}_k = \left[\begin{smallmatrix} \Psi_k' & 0 \\ * & 0 \end{smallmatrix}\right]$,

$$\Psi_k = \begin{bmatrix} \Phi_{11}^k + n\varepsilon_{1k}^{-1} V_{1k}^{\top} V_{1k} & \Phi_{12}^k & \Phi_{13}^k & \Phi_{14}^k & hU_{1k}^{\top} \\ * & \Phi_{22}^k & \Phi_{23}^k & \Phi_{24}^k & hU_{2k}^{\top} \\ * & * & \Phi_{33}^k + n\varepsilon_{2k}^{-1} V_{2k}^{\top} V_{2k} & \Phi_{34}^k & hU_{3k}^{\top} \\ * & * & * & \Phi_{44}^k & hU_{4k}^{\top} \\ * & * & * & * & -hW_{1k} + h\chi_1 I_N \end{bmatrix},$$

$$\Psi_k' = \begin{bmatrix} \Phi_{11}^k + n\varepsilon_{1k}^{-1} V_{1k}^{\top} V_{1k} & \Phi_{12}^k & \Pi_{13}^k & \Phi_{14}^k \\ * & \Pi_{22}^k & \Phi_{23}^k & \Phi_{24}^k \\ * & * & \Pi_{33}^k + n\varepsilon_{2k}^{-1} V_{2k}^{\top} V_{2k} & \Pi_{34}^k \\ * & * & * & \Pi_{44}^k \end{bmatrix}. \tag{33}$$

By using the Schur complement [30], it follows from (14) and (15) that $\Psi_k|_{\chi_1=0} < 0$ and $\Psi_k' < 0$. Furthermore, we can always choose sufficiently small scalars $\chi_1$, $\chi_2$, $\chi_3$, and $\chi_4$ to ensure $\Psi_k + \mathrm{diag}\{\chi_2 I_N, \chi_3 I_N, 0, \chi_4 I_N, 0\} < 0$ and

$\Psi_k' + \mathrm{diag}\{\chi_2 I_N, \chi_3 I_N, 0, \chi_4 I_N\} < 0$. It follows from (32) that

$$\dot{V}(t) < \sum_{k=1}^{n} \left\{ -\chi_2 \|\delta x_k(t)\|^2 - \chi_3 \|\delta x_k(mh)\|^2 - \chi_4 \|\delta e_k(mh)\|^2 - \chi_1 \int_{mh}^{t} \|\dot{\delta x}_k(s)\|^2 ds \right\}. \tag{34}$$

From (17), we have that

$$\begin{aligned} &\delta x_k^\top(t) P_k \delta x_k(t) + \varsigma(t) \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \mathscr{X}_k \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix} \\ &= \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \left\{ \frac{\varsigma(t)}{h} [\mathscr{P}_k + h\mathscr{X}_k] + \frac{\tau(t)}{h} \mathscr{P}_k \right\} \cdot \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix} \\ &\le \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}^\top \cdot \left\{ \frac{\varsigma(t)}{h} [\mathscr{P}_k + h\mathscr{X}_k + \mathscr{U}_k] + \frac{\tau(t)}{h} \mathscr{P}_k \right\} \begin{bmatrix} \delta x_k(t) \\ \delta x_k(mh) \end{bmatrix}, \end{aligned} \tag{35}$$

where $\mathscr{U}_k = \begin{bmatrix} W_{1k} & -W_{1k} \\ & W_{1k} \end{bmatrix} \ge 0$. From (13), we have that there exists a positive scalar $\rho$ such that

$$\mathscr{P}_k + h\mathscr{X}_k + \mathscr{U}_k < \rho_1 I_{2N}, \qquad P_k < \rho_1 I_N. \tag{36}$$

Furthermore, a positive number $\rho_2$ can be chosen such that

$$\begin{aligned} &\varsigma(t) \int_{mh}^{t} \begin{bmatrix} \dot{\delta x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix}^\top \mathscr{W}_k \begin{bmatrix} \dot{\delta x}_k(s) \\ \delta x_k(mh) \\ \delta e_k(mh) \end{bmatrix} ds \\ &\le \rho_2 \int_{mh}^{t} \|\dot{\delta x}_k(s)\|^2 ds + \rho_2 \|\delta x_k(mh)\|^2 + \rho_2 \|\delta e_k(mh)\|^2. \end{aligned} \tag{37}$$

Thus, from (35) to (37), it follows that

$$V(t) \le \sum_{k=1}^{n} \left\{ \rho_1 \|\delta x_k(t)\|^2 + (\rho_1 + \rho_2) \|\delta x_k(mh)\|^2 + \rho_2 \|\delta e_k(mh)\|^2 + \rho_2 \int_{mh}^{t} \|\dot{\delta x}_k(s)\|^2 ds \right\}. \tag{38}$$

Let $\varepsilon > 0$ such that

$$\begin{aligned} &\varepsilon\rho_1 - \chi_2 \le 0, \qquad \varepsilon(\rho_1 + \rho_2) - \chi_3 \le 0, \\ &\varepsilon\rho_2 - \chi_4 \le 0, \qquad \varepsilon\rho_2 - \chi_1 \le 0. \end{aligned} \tag{39}$$

According to (34), (38), and (39), we can get $\dot{V}(t) + \varepsilon V(t) \le 0 \forall t \in [mh, (m+1)h)$. Thus, $V(t) \le e^{-\varepsilon(t-mh)} V(mh)$. By mathematical induction, it can be concluded that $V(t) \le e^{-\varepsilon t} V(0), \forall t > 0$. From (25), we yield that

$$\|x_i(t)\|^2 \le \sum_{i=1}^{N} \|x_i(t)\|^2 \le \beta^{-1} V(t) \le \beta^{-1} V(0) e^{-\varepsilon t}, \quad i \in \{1, 2, \ldots, N\}. \tag{40}$$

Hence, from Definition 4, controlled NDS (1) with partial and event-based couplings can achieve globally exponentially stabilization under event condition (3) and pinning controllers (6). This completes the proof. □

When all the channel matrices $R_{ij}$ are identity matrices, that is, $R_{ij} = I_n$ $(i, j = 1, 2, \ldots, N)$, the communication constraint of partial information transmission is removed. Furthermore, we assume that if an agent is controlled, every level of information of the controller can be well transmitted by the controller; that is, $H_i$ $(i = 1, 2, \ldots, N)$ in (6) are also identity matrices. Overall, the controlled NDS with event-based couplings can be constructed by following differential equation:

$$\dot{x}_i(t) = Ax_i(t) + \alpha \sum_{j=1, j\ne i}^{N} g_{ij} \left( \hat{x}_j(t) - \hat{x}_i(t) \right) - d_i \hat{x}_i(t). \tag{41}$$

The main difference between NDS (7) and (41) is that the channels are not considered in (41). Thus, the event condition of each system can be described as

$$\left\| x_i\left(t_m^i\right) - x_i\left(t_m^i + lh\right) \right\| \le \gamma_i \left\| x_i\left(t_m^i + lh\right) \right\|, \quad i \in \{1, 2, \ldots, N\}, \tag{42}$$

where $l = 1, 2, \ldots$, $\gamma_i > 0$. Similarly, the next event-triggered instant is $t_{m+1}^i = t_m^i + h \inf\{l \in \mathbb{N} : \|x_i(t_m^i) - x_i(t_m^i + lh)\| > \gamma_i \|x_i(t_m^i + lh)\|\}$. We also assume that $t_0^i = 0$ for each agent $i$.

Based on Theorem 6, the stabilization conditions of NDS (41) with event condition (42) can be easily obtained as follows.

**Corollary 7.** *Let $\Gamma = \mathrm{diag}\{\gamma_1^2, \gamma_2^2, \ldots, \gamma_N^2\}$ and $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$. NDS (41) with event-based couplings can be exponentially stabilized, if there exist positive scalars $\varepsilon_1$, $\varepsilon_2$ and matrices $P > 0$, $Q = \mathrm{diag}\{q_1, q_2, \ldots, q_N\} > 0$, $\mathscr{W} = \begin{bmatrix} W_1 & W_2 & W_4 \\ * & W_3 & 0 \\ * & * & W_5 \end{bmatrix} > 0$, $U_1$, $U_2$, $U_3$, $U_4$, $V_1$, $V_2$, $X$, $X_1$, such that the following LMIs are satisfied:*

$$\begin{bmatrix} P_k + h\dfrac{X + X^\top}{2} + W_1 & -hX + hX_1 - W_1 & W_2 & W_4 \\ * & -hX_1 - hX_1^\top + h\dfrac{X + X^\top}{2} + W_1 & -W_2 & -W_4 \\ * & * & W_3 & 0 \\ * & * & * & W_5 \end{bmatrix} > 0,$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & hU_1^{\top} & nV_1^{\top} & 0 \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & hU_2^{\top} & 0 & 0 \\ * & * & \Phi_{33} & \Phi_{34} & hU_3^{\top} & 0 & nV_2^{\top} \\ * & * & * & \Phi_{44} & hU_4^{\top} & 0 & 0 \\ * & * & * & * & -hW_1 & 0 & 0 \\ * & * & * & * & * & -n\varepsilon_1 I_N & 0 \\ * & * & * & * & * & * & -n\varepsilon_2 I_N \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Pi_{13} & \Phi_{14} & nV_1^{\top} & 0 \\ * & \Pi_{22} & \Pi_{23} & \Phi_{24} & 0 & 0 \\ * & * & \Pi_{33} & \Pi_{34} & 0 & nV_2^{\top} \\ * & * & * & \Pi_{44} & 0 & 0 \\ * & * & * & * & -n\varepsilon_1 I_N & 0 \\ & * & * & * & * & -n\varepsilon_2 I_N \end{bmatrix} < 0, \tag{43}$$

*where*

$$\begin{aligned}
\Phi_{11} &= -\frac{X+X^{\top}}{2} - U_1 - U_1^{\top} + \sum_{j=1}^{n} \left(\varepsilon_{1j} + \varepsilon_{2j}\right) a_j^2, \\
\Phi_{12} &= X - X_1 - W_2 + U_1^{\top} - U_2 + V_1^{\top} (\alpha G - D), \\
\Phi_{13} &= P - U_3 - V_1^{\top}, \\
\Phi_{14} &= -W_4 - U_4 + V_1^{\top} (\alpha G - D), \\
\Phi_{22} &= X_1 + X_1^{\top} - \frac{X+X^{\top}}{2} + W_2 + W_2^{\top} \\
&\quad + U_2 + U_2^{\top} + Q\Gamma - hW_3, \\
\Phi_{23} &= U_3 + \left(\alpha G^{\top} - D\right) V_2, \\
\Phi_{24} &= W_4 + U_4, \\
\Phi_{33} &= -V_2 - V_2^{\top}, \\
\Phi_{34} &= V_2^{\top} (\alpha G - D), \\
\Phi_{44} &= -Q - hW_5, \\
\Pi_{13} &= h\frac{X+X^{\top}}{2} + P - U_3 - V_1^{\top}, \\
\Pi_{22} &= X_1 + X_1^{\top} - \frac{X+X^{\top}}{2} + W_2 + W_2^{\top} \\
&\quad + U_2 + U_2^{\top} + Q\Gamma + hW_3, \\
\Pi_{23} &= hW_2^{\top} + hX_1^{\top} - hX_k^{\top} + U_3 + \left(\alpha G^{\top} - D\right) V_2, \\
\Pi_{33} &= hW_1 - V_2 - V_2^{\top}, \\
\Pi_{34} &= hW_4 + V_2^{\top} (\alpha G - D), \\
\Pi_{44} &= -Q + hW_5.
\end{aligned} \tag{44}$$

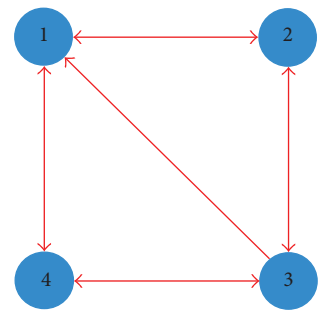


Figure 1: The topological structure of the NDS with 4 subsystems.

## 4. Numerical Example

In this section, a numerical example will be given to demonstrate the effectiveness of our main results.

An NDS with 4 agents is constructed ($N = 4$), the topological structure of which is depicted in Figure 1. The Laplacian matrix of the NDS is given as $G = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & -2 \end{bmatrix}$. The other parameters of the NDS are $\alpha = 0.35$ and $A = \begin{bmatrix} 0.12 & -0.08 \\ -0.06 & 0.12 \end{bmatrix}$. Thus, each agent of the NDS has 2 levels of information ($n = 2$). Due to the constraint of the partial information transmission, the couplings have to be divided into 2 levels of channels to transmit the information. For each coupling of the NDS, a channel matrix is given to reflect the work state of the channels of the coupling. The channel matrices of the couplings are listed as follows:

$$\begin{aligned}
R_{12} &= \operatorname{diag}\{0,1\}, & R_{13} &= \operatorname{diag}\{0,1\}, \\
R_{14} &= \operatorname{diag}\{1,0\}, & R_{21} &= \operatorname{diag}\{1,0\}, \\
R_{23} &= \operatorname{diag}\{0,1\}, & R_{32} &= \operatorname{diag}\{1,0\}, \\
R_{34} &= \operatorname{diag}\{0,1\}, & R_{41} &= \operatorname{diag}\{0,1\}, \\
& & R_{43} &= \operatorname{diag}\{1,0\}.
\end{aligned} \tag{45}$$

Thus, the Laplacian matrices of the channels of the 2 levels can be, respectively, built:

$$C_1 = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}. \tag{46}$$

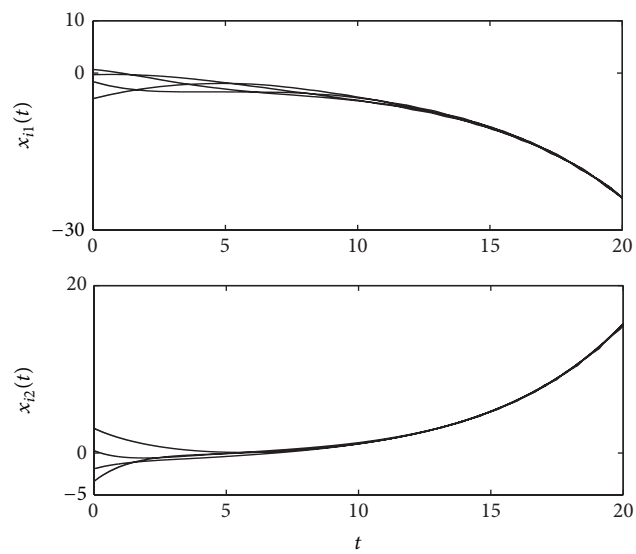


Figure 2: Without controllers, the trajectories of the NDS are divergent.

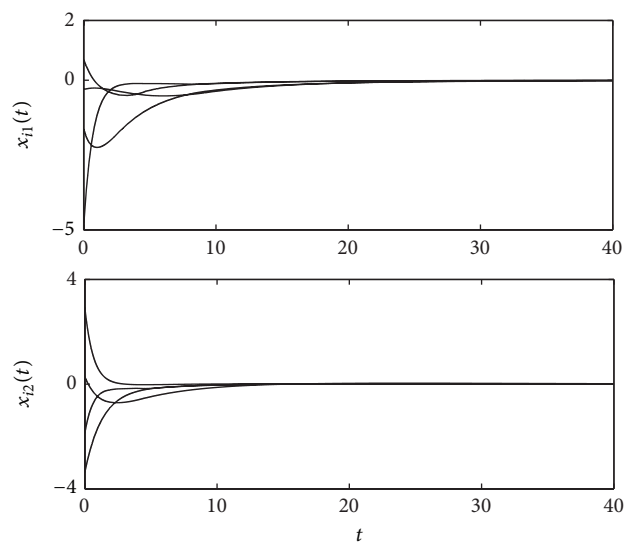


Figure 3: When the NDS is equipped with controllers, the trajectories of NDS tend to zero.

As discussed above, the event-based sampling scheme is used in the communication among agents of the NDS and the control signals. We assume that, in event condition (3), the sampling period $h = 0.1$ and $\gamma_{ik} = 0.15$, for $i = 1, 2, 3, 4$ and $k = 1, 2$. Let the coupling strength $\alpha = 0.35$. Without controllers, the constructed NDS is not stable. The trajectories of the uncontrolled NDS are shown in Figure 2, where $i = 1, 2, 3, 4$.

To achieve stabilization, controllers are distributed, implemented in the NDS with the control strengths: $d_1 = 1$, $d_2 = 1$, $d_3 = 1$, and $d_4 = 0$; that is, except the 4th agent, the other agents are controlled. Furthermore, for the controlled agents, only part of the control signals can be well received. Let $H_1 = \mathrm{diag}\{1, 0\}$, $H_2 = \mathrm{diag}\{0, 1\}$, and $H_3 = \mathrm{diag}\{1, 1\}$. Thus, $D_1 = \mathrm{diag}\{1, 0, 1, 0\}$ and $D_2 = \mathrm{diag}\{0, 1, 1, 0\}$. Based on the above parameters, a feasible solution can be found for conditions (13)–(15). The solution is listed as follows:

$$P_1 = \begin{bmatrix} 6.7804 & 0.9268 & -0.0993 & -1.2156 \\ 0.9268 & 6.5528 & -1.2156 & -1.0043 \\ -0.0993 & -1.2156 & 6.7804 & 0.9268 \\ -1.2156 & -1.0043 & 0.9268 & 6.5528 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 9.9491 & 0.8285 & 1.4249 & 0.2072 \\ 0.8285 & 10.3109 & 0.9803 & -0.2967 \\ 1.4249 & 0.9803 & 8.2046 & -1.1263 \\ 0.2072 & -0.2967 & -1.1263 & 7.4406 \end{bmatrix}, \tag{47}$$

$$Q_1 = \mathrm{diag}\{12.7657, 8.8809, 12.7657, 8.8809\},$$

$$Q_2 = \mathrm{diag}\{13.1103, 22.4816, 14.3887, 12.1669\}.$$

According to Theorem 6, the controlled NDS can be stabilized by the partial and event-based couplings. To show this fact, the trajectories of the controlled NDS are depicted in Figure 3. It can be found that all the trajectories tend to zero.

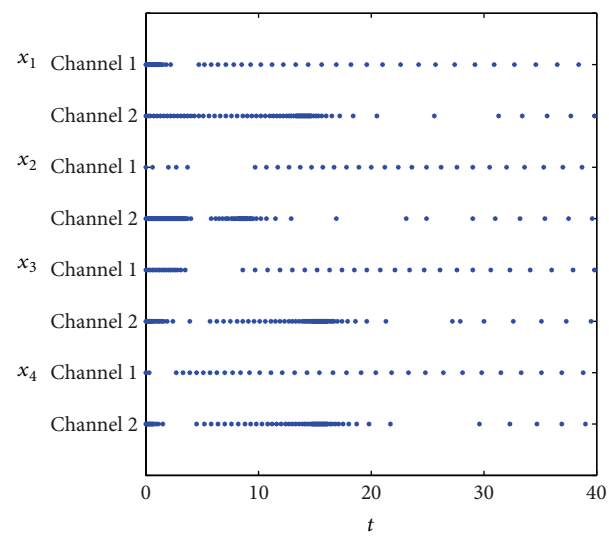


Figure 4: The sampling instants of all the channels of the NDS.

Figure 4 shows the sampling instants of the 2 channels of all the 4 agents of the NDS. The total number of the sampling instants of the NDS is 415. If the NDS employed the time-based sampling scheme with sampling period $h = 0.1$, the sampling number would be 3200. Thus, the sampling number of the event-based sampling scheme is only 12.97% of that of the time-based sampling scheme. In other words, the event-based sampling scheme can save 87% communication resource.

## 5. Conclusion

The stabilization problem of NDSs with partial and event-based couplings has been investigated. The communication

among the agents has been assumed to suffer from two communication constraints: partial information transmission and event-based sampling. The former constraint leads to the information packet of each agent lack of integrity. However, the constraint of sampling makes the real-time information available at some discrete sampling instants. Furthermore, the sampling instants cannot be figured out in advance. Thus, the two constraints make the stabilization problem much harder. Furthermore, the control signals also suffered from these two constraints. By building the channel Laplacian matrices for different levels of information, the stabilization condition has been derived for the NDS with partial and event-based couplings. The numerical simulation has been given to show the effectiveness of our theoretical results. In this paper, the partial information transmission is considered as the communication constraint. However, some other constraints are also significant, such as the quantization and saturation. When we consider the event-based sampling scheme simultaneously, how would they impact the dynamical behaviour of NDSs? It is an interesting yet challenging problem. We will investigate it in our future work.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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