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Research Article

Adaptive Output Tracking Control for Nonlinear Systems with Failed Actuators and Aircraft Flight System Applications

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An adaptive failure compensation scheme using output feedback is proposed for a class of nonlinear systems with nonlinearities depending on the unmeasured states of systems. Adaptive high-gain K-filters are presented to suppress the nonlinearities while the proposed backstepping adaptive high-gain controller guarantees the stability of the closed-loop system and small tracking errors. Simulation results verify that the adaptive failure compensation scheme is effective.

1. Introduction

Actuator failure compensation has significant impact on control systems, such as aircraft flight systems and nuclear power systems. Due to the unknown failure patterns, times, and values, it has been an important and challenging research problem. Remarkable progress has been made in the area, such as adaptive control [1–10], sliding-mode based designs [11–14], switching based designs [15–17], fault-detection diagnosis based designs, fuzzy systems based designs, and neural-network based designs. Since adaptive designs use an adaptive controller to accommodate the uncertainties, it has been extensively employed. An adaptive state feedback controller is proposed to guarantee the performance requirements in presence of actuator failures [1–4]. Adaptive actuator failure compensation using output feedback is studied for a class of nonlinear systems [5–9, 14, 16, 18–21]. Unfortunately, if the systems have nonlinearities that cannot be bounded by any function of output, the existing methods fail to compensate the control systems. Therefore, new techniques need to be developed.

In this paper, an adaptive output feedback controller for a class of nonlinear systems with actuator failures is discussed. Motivated by the fact that the nonlinearities satisfying the growth condition [15, 22–25] can be suppressed by a dynamic high-gain output feedback controller, a modified compensation scheme is proposed where adaptive high-gain K-filters and an adaptive high-gain controller are introduced. The main contribution of our paper is as follows. (1) We relax the condition imposed by Tang et al. in [5–7]. (2) An adaptive output feedback controller is developed with switching laws [15, 16]. (3) The parameters of the K-filters [26] and the controller are adaptively tuned online depending on system nonlinearities and actuator failures. By applying the backstepping technique, the robust controller is recursively constructed step by step. Parameter update laws are addressed to ensure closed-loop signal boundedness and small tracking errors. The simulation results are presented to demonstrate the effectiveness of the scheme proposed in this paper.

The rest of the paper is outlined as follows. In Section 2, the control problem is formulated. In Section 3, a robust adaptive compensation scheme with switching laws via the backstepping design is proposed. In Section 4, the stability analysis is presented. Two detailed simulation examples show the proposed scheme is effective in Section 5. Finally, this paper is concluded by Section 6.

2. Problem Formulation

Consider a class of nonlinear dynamic systems in the form of

$$
\begin{aligned}
\dot{x}_i &= x_{i+1} + \varphi_{0,i}(y) + \phi_i(x) + a^T\varphi_i \\
&\qquad (i = 1, 2, \ldots, \rho - 1), \\
\dot{x}_\rho &= x_{\rho+1} + \varphi_{0,\rho}(y) + \phi_\rho(x) \\
&\quad + a^T\varphi_\rho + \sum_{j=1}^{m} b_{n^*,j}\beta_j(y) u_j, \\
&\vdots \\
\dot{x}_{n-1} &= x_n + \varphi_{0,n-1}(y) + \phi_{n-1}(x) \\
&\quad + a^T\varphi_{n-1} + \sum_{j=1}^{m} b_{1,j}\beta_j(y) u_j, \\
\dot{x}_n &= \varphi_{0,n}(y) + \phi_n(x) + a^T\varphi_n + \sum_{j=1}^{m} b_{0,j}\beta_j(y) u_j, \\
y &= x_1,
\end{aligned} \tag{1}
$$

where ρ is the relative degree of the system and $u_j \in R$, $j = 1, 2, \ldots, m$, are the control inputs whose actuators may fail during system operation; $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T$ $(i = 1, \ldots, n-1)$ are the state vector; $a = [a_1, \ldots, a_q]^T$, $b_{r,j}$ $(r = 0, 1, \ldots, n^* = n - \rho,\ j = 1, 2, \ldots, m)$ are unknown constant parameters. Only the output y is available for measurement. $\varphi_{0,i}(y)$ $(i = 1, 2, \ldots, n)$, $\beta_j(y) \neq 0$ (for $\forall y \in R, j = 1, 2, \ldots, m$), and $\varphi_{j,i}(y)$ $(j = 1, 2, \ldots, q,\ i = 1, 2, \ldots, n)$ are known smooth nonlinear functions; $\phi_i(x)$ $(i = 1, 2, \ldots, n)$ are continuous unknown functions satisfying the following assumption.

Assumption 1. For $i = 1, 2, \ldots, n$, there is an unknown constant $c \geq 0$ such that

$$
|\phi_i(x)| \leq c(|x_1| + \cdots + |x_i|). \tag{2}
$$

We denote u_i as the input of the ith $(i = 1, \ldots, m)$ actuator. Suppose the actuator failure can be modeled as

$$
\begin{aligned}
u_i &= \rho_i v_i + u_{ki}, \quad \forall t \geq t_{iF}, \\
\rho_i u_{ki} &= 0, \quad i = 1, \ldots, m,
\end{aligned} \tag{3}
$$

where $\rho_i \in [0, 1]$, u_{ki} and t_{iF} are all unknown constants, and $v_i(t)$ $(i = 1, 2, \ldots, m)$ are applied control inputs to be designed in Section 3. For different values of ρ_i, three types of failures are included as follows:

(1) $\rho_i = 1$; the actuator works normally, namely, $u_i = u_{ci}$, which is regarded as a failure-free actuator;

(2) $0 < \rho_i < 1$; it implies $u_i = \rho_i u_{ci}$. The ith actuator is called partial loss of effectiveness (PLOE);

(3) $\rho_i = 0$; it indicates $u_i = u_{ki}$. The ith actuator is called total loss of effectiveness (TLOE).

Remark 2. The values of ρ_i can change only from $\rho_i = 1$ to some values with $0 \leq \rho_i < 1$. This means that possible changes from normal to any one of the failure cases are unidirectional. The uniqueness of t_{iF} indicates that a failure occurs only once on the ith actuator.

Our objective is to design an output feedback controller for the nonlinear systems (1) with p unknown actuator failures when p changes at time instants t_k, $k = 1, 2, \ldots, q$, such that the plant output $y(t)$ tracks a given reference signal $y_r(t)$ with up to ρth-order derivatives bounded as close as possible and that all closed-loop signals are bounded despite the presence of unknown actuator failures and unknown plant parameters.

3. Adaptive Compensation Control Scheme

The zero dynamics of system (1) with actuator failures are only dependent on the failure pattern [1]. For a fixed failure pattern, there is a resulting pattern of zero dynamics. Since the failure pattern $j = j_1, \ldots, j_p$, $0 \leq p \leq m-1$ is unknown, a desirable adaptive design is expected to achieve the control objective for any possible failure pattern. For the closed-loop stability, all zero dynamics corresponding to the possible failure patterns need to be stable. To derive a suitable adaptive control scheme, the following assumptions are made.

Assumption 3. When TOLE type of actuator failures is up to $m - 1$, the remaining actuators can still achieve a desired control objective.

Assumption 4. The polynomials $\sum_{j \neq j_1, j_2, \ldots, j_p}^{m} \operatorname{sign}[b_{n^*,j}] B_j(s)$ are stable,

$$
\forall \{j_1, j_2, \ldots, j_p\} \subset \{1, 2, \ldots, m\}, \quad \forall p \subset \{0, 1, 2, \ldots, m-1\}, \tag{4}
$$

where

$$
\begin{aligned}
B_j(s) &= b_{n^*,j} s^{n^*} + b_{n^*-1,j} s^{n^*-1} + \cdots + b_{1,j} s + b_{0,j}, \\
&\qquad (j = 1, 2, \ldots, m).
\end{aligned} \tag{5}
$$

Assumption 5. The sign of $b_{n^*,j}$ is known for $j = 1, 2, \ldots, m$.

For adaptive actuator failure compensation, the proportional actuation scheme is

$$
v_j = \operatorname{sign}[b_{n^* j}] \frac{1}{\beta_j(y)} v_0, \quad j = 1, 2, \ldots, m, \tag{6}
$$

where v_0 is a control signal generated by a backstepping design procedure to be given in Section 3.3.

3.1. Parameterized Model with Actuator Failures. To obtain a compact form of system (1) with actuator failures, we define

$$
\begin{aligned}
k_{1,r} &= \sum_{j \neq j_1, j_2, \ldots, j_p} \operatorname{sign}[b_{n^*,j}] b_{r,j} * \rho_j, \quad r = 0, 1, \ldots, n^*, \\
k_{2,rj} &= \begin{cases} b_{rj} u_{kj}, & r = 0, 1, \ldots, n^*,\ j = j_1, j_2, \ldots, j_p, \\ 0, & r = 0, 1, \ldots, n^*,\ j \neq j_1, j_2, \ldots, j_p. \end{cases}
\end{aligned} \tag{7}
$$

Applying (6)-(7) to (1), we have

$$\dot{x}_i = x_{i+1} + \varphi_{0i}(y) + \phi_i(x) + a^T\varphi_i \quad (i = 1, 2, \dots, \rho - 1),$$
$$\dot{x}_\rho = x_{\rho+1} + \varphi_{0,\rho}(y) + \phi_\rho(x) + a^T\varphi_\rho + \sum_{j=1}^{m} k_{2,n^* j}\beta_j(y) + k_{1,n^*}v_0$$
$$\vdots$$
$$\dot{x}_{n-1} = x_n + \varphi_{0,n-1}(y) + \phi_{n-1}(x) + a^T\varphi_{n-1} + \sum_{j=1}^{m} k_{2,1j}\beta_j(y) + k_{1,1}v_0,$$
$$\dot{x}_n = \varphi_{0,n}(y) + \phi_n(x) + a^T\varphi_n + \sum_{j=1}^{m} k_{2,0j}\beta_j(y) + k_{1,0}v_0,$$
$$y = x_1. \tag{8}$$

Let

$$k_1 = [k_{1,n^*}, \dots, k_{1,0}]^T \in R^{n^*+1},$$
$$k_{2,r} = [k_{2,r1}, k_{2,r2}, \dots, k_{2,rm}]^T \in R^m, \tag{9}$$
$$k_2 = [k_{2,n^*}, \dots, k_{2,0}]^T \in R^{(n^*+1)m},$$

where $r = 0, 1, \dots, n^*$, $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$. Substituting (9) into (8), we have

$$\dot{x} = Ax + \varphi_0(y) + \phi(x) + \varphi_a(y)a + \Phi_{k_2}(y)k_2 + \begin{bmatrix} 0_{(\rho-1)\times 1} \\ k_1 \end{bmatrix} v_0, \quad y = e_1^T x, \tag{10}$$

where

$$x = [x_1, x_2, \dots, x_n]^T,$$
$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix},$$
$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^T,$$
$$\varphi_0(y) = [\varphi_{0,1}(y), \varphi_{0,2}(y), \dots, \varphi_{0,n}(y)]^T,$$
$$\varphi_a(y) = [\varphi_1(y), \varphi_2(y), \dots, \varphi_n(y)]^T,$$
$$\Phi_{k_2}(y) = \begin{bmatrix} & 0_{(\rho-1)\times[(n^*+1)m]} & & \\ \beta^T(y) & 0 & \cdots & 0 \\ 0 & \beta^T(y) & \cdots & 0 \\ & \ddots & \ddots & \\ 0 & \cdots & 0 & \beta^T(y) \end{bmatrix}. \tag{11}$$

We rewrite the system (10) as

$$\dot{x} = Ax + \varphi_0(y) + \phi(x) + F(y, t, v_0)^T\theta, \quad y = e_1^T x, \tag{12}$$

where

$$\theta = \left[k_1^T, k_2^T, a^T\right]^T \in R^{(n^*+1)\times(m+1)+q},$$
$$\Phi(y, t) = \left[\Phi_{k_2}(y, t), \varphi_a(y)\right], \tag{13}$$
$$F(y, t, v_0)^T = \left[\begin{bmatrix} 0_{(\rho-1)\times 1} \\ k_1 \end{bmatrix} v_0, \Phi(y, t)\right].$$

Remark 6. It is significant to point that the parametric model (12) is very similar to (8.3) and (8.7) in [26] and (38) in [9]. However, the system has nonlinearity terms $\phi_i(x)$ which are functions of state vector. In other words, the design procedures in [9, 26] are not applicable for the system. In this paper, we will develop adaptive high-gain K-filters to deal with this problem.

3.2. State Observation and Switching Laws. Since the states of system (1) are not available, an observer is needed to provide auxiliary signals for handling the unmeasured states in control design. The conventional K-filters cannot deal with the nonlinearities depending on the system state. Motivated by [15, 16, 25], we develop adaptive high-gain K-filters which can be expressed as

$$\dot{\xi} = A_0(L)\xi + \varphi_0(y) + l(L)y,$$
$$\dot{\Xi} = A_0(L)\Xi + \Phi(y, t), \quad r = 0, 1, \dots, n^*, \; j = 1, 2, \dots, m,$$
$$\dot{\lambda} = A_0(L)\lambda + e_n v_0,$$
$$v_j = (A_0(L))^j \lambda, \quad j = 0, 1, \dots, n^*,$$
$$\Omega^T = [v_{n^*}, \dots, v_0, \Xi], \tag{14}$$

where $l = [l_1, l_2, \dots, l_n]^T$ is chosen such that $A_0 = A - le_1^T$ is Hurwitz, $l(L) = \left[Ll_1, L^2 l_2, \dots, L^n l_n\right]^T$, $A_0(L) = A - l(L)e_1^T$, and $L \geq 1$. As a result, $\hat{x} = \xi + \Omega^T\theta$ is an observer for x with the estimation error which satisfies

$$\dot{\varepsilon} = A_0(L)\varepsilon + \phi(x). \tag{15}$$

Remark 7. Since $L \geq 1$ and it varies with the system nonlinearities and actuator failures, we call it adaptive high-gain K-filters. But L is so complicated that we cannot solve an analytic value. Moreover, when actuator failures occur, the uncertainties will be also brought into the system. In the following, an adaptive controller is proposed which can tune the high-gain parameters online according to system nonlinearities and actuator failures.

Tuning mechanism and switching signal $\chi(t)$: motivated by [15, 16], we consider the integral of $\|e(t)\|$ as

$$\chi = \int_{t_0}^{t_0+T} \|e(t)\| \, dt, \qquad \chi(t_0) = \chi_0, \tag{16}$$

where

$$e(t) = y(t) - y_r(t). \tag{17}$$

The tuning steps are shown as follows.

Initialization. Set $\chi_0 = 0$, $L_0 = 1$.

Step 1. Obtain $S = a$, where a is positive constant.

Step 2. If $\chi \geq S$, $L_{i+1}(t) = KL_i(t)$, where $K > 1$, $\chi = 0 \rightarrow i = i + 1$. Go to Step 1.

Else $L_{i+1}(t) = L_i(t)$, $\chi = 0 \rightarrow i = i + 1$. Go to Step 2.

Remark 8. We briefly explain the idea of the tuning mechanism with the switching law. First, $L(t)$ is a monotonically nondecreasing function on the interval $[t_0, t_0 + T]$ by taking integral value of $\|e(t)\|$. Obviously, χ stops increasing only when $\|e(t)\| = 0$. Under this modified rule, once the system is stabilized, any integral of errors smaller than the prespecified value will not cause further switching. Also, from the switching logic, it is obvious that $L(t)$ is a piecewise constant function of time and increases stepwise with some time interval.

3.3. Adaptive Output Feedback Controller

To prepare for the backstepping procedure, we consider the equation for the output $y = x_1$:

$$\dot{y} = \omega_0 + \omega^T\theta + \varepsilon_2 + \phi_1 = k_{1,n^*} v_{n^*,2} + \omega_0 + \overline{\omega}^T\theta + \varepsilon_2 + \phi_1, \tag{18}$$

where the regressor ω and truncated regressor $\overline{\omega}$ are defined as

$$\begin{aligned} \omega &= \left[v_{n^*,2}, v_{n^*-1,2}, \ldots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}\right]^T, \\ \overline{\omega} &= \left[0, v_{n^*-1,2}, \ldots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}\right]^T, \\ \omega_0 &= \varphi_{0,1}(y) + \xi_2. \end{aligned} \tag{19}$$

Define a change of coordinates

$$\begin{aligned} z_1 &= y - y_r, \\ z_i &= v_{m,i} - \widehat{\rho}_0 y_r^{(i-1)} - \alpha_{i-1}, \quad i = 2, \ldots, \rho, \end{aligned} \tag{20}$$

where $\widehat{\rho}_0$ is an estimate of $\rho_0 = 1/k_{1,n^*}$.

Step 1. Consider

$$\begin{aligned} z_1 &= y - y_r, \\ \overline{\alpha}_1 &= -c_1 z_1 - \left(d_1 + \frac{L\rho}{4}\right) z_1 - \omega_0 - \overline{\omega}^T \widehat{\theta} - \frac{W_0 z_1}{z_1^2 + \Delta_0^2}, \\ \alpha_1 &= \widehat{\rho}_0 \overline{\alpha}_1, \\ \tau_1 &= \Big(\omega - \widehat{\rho}_0 \left(\dot{y}_r + \overline{\alpha}_1\right) \\ &\quad \times \left[10.1\text{ cm} \;\; 00.1\text{ cm} \;\; \cdots \;\; 0.1\text{ cm} \;\; 0\right]^T\Big) z_1, \\ \dot{\widehat{\rho}}_0 &= -\gamma \operatorname{sgn}\left(k_{1,n^*}\right)\left(\dot{y}_r + \overline{\alpha}_1\right) z_1, \\ W_0 &= f(L)\left(\|\xi\| + \left\|\Omega_1^T\right\|\theta^* + \cdots + \left\|\Omega_n^T\right\|\theta^*\right), \end{aligned} \tag{21}$$

where α_1 is the first stabilizing function and τ_1 is the first tuning function, θ^*, Δ_0 are design parameters, and $f(L) \geq 1$ is an adaptive tuning function.

Step 2. Consider

$$\begin{aligned} z_2 &= v_{m,2} - \widehat{\rho}_0 \dot{y}_r - \alpha_1, \\ \alpha_2 &= -\widehat{k}_{1,n^*} z_1 - c_2 z_2 - \left(d_2 + \frac{L\rho}{4}\right)\left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 \\ &\quad + \left(\dot{y}_r + \frac{\partial \alpha_1}{\partial \widehat{\rho}_0}\right)\dot{\widehat{\rho}}_0 + \frac{\partial \alpha_1}{\partial \widehat{\theta}} T \tau_2 + \frac{\partial \alpha_1}{\partial y}\left(\omega_0 + \omega^T \widehat{\theta}\right) \\ &\quad + \frac{\partial \alpha_1}{\partial \xi}\left(A_0(L)\xi + l(L)y + \varphi(y)\right) \\ &\quad + \frac{\partial \alpha_1}{\partial \Xi}\left(A_0(L)\Xi + \Phi(y,t)\right) + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + l(L)_2 v_{n^*,1} \\ &\quad + \sum_{j=1}^{m+1} \frac{\partial \alpha_1}{\partial \lambda_j}\left(-l(L)_j \lambda_1 + \lambda_{j+1}\right) - \frac{W_0 z_2}{z_1^2 + \Delta_0^2}, \\ \tau_2 &= \tau_1 - \frac{\partial \alpha_1}{\partial y}\omega z_2. \end{aligned} \tag{22}$$

Step i, $i = 3, \ldots, \rho - 1$. Consider

$$\begin{aligned}
z_i &= v_{m,i} - \widehat{\rho}_0 y_r^{(i-1)} - \alpha_{i-1}, \\
\alpha_i &= -z_{i-1} - c_i z_i - \left(d_i + \frac{L\rho}{4}\right)\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i \\
&+ \left(y_r^{(i-1)} + \frac{\partial \alpha_{i-1}}{\partial \widehat{\rho}_0}\right)\dot{\widehat{\rho}}_0 + \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}} T \tau_i \\
&- \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \widehat{\theta}} T \frac{\partial \alpha_{j-1}}{\partial y} z_j + \frac{\partial \alpha_{i-1}}{\partial y}\left(\omega_0 + \omega^T \widehat{\theta}\right) \\
&+ \frac{\partial \alpha_{i-1}}{\partial \xi}\left(A_0(L)\xi + l(L)y + \varphi(y)\right) \\
&+ \frac{\partial \alpha_{i-1}}{\partial \Xi}\left(A_0(L)\Xi + \Phi(y,t)\right) \\
&+ \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} + l(L)_i v_{n^*,1} \\
&+ \sum_{j=1}^{m-1+i} \frac{\partial \alpha_{i-1}}{\partial \lambda_j}\left(-l(L)_j \lambda_1 + \lambda_{j+1}\right) - \frac{W_0 z_i}{z_1^2 + \Delta_0^2}, \\
\tau_i &= \tau_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y}\omega z_{i-1}.
\end{aligned} \tag{23}$$

Step ρ. Consider

$$\begin{aligned}
z_\rho &= v_{m,\rho} - \widehat{\rho}_0 y_r^{(\rho-1)} - \alpha_{\rho-1}, \\
\alpha_\rho &= -z_{\rho-1} - c_\rho z_\rho - \left(d_\rho + \frac{L\rho}{4}\right)\left(\frac{\partial \alpha_{\rho-1}}{\partial y}\right)^2 z_\rho \\
&+ \left(y_r^{(\rho-1)} + \frac{\partial \alpha_{\rho-1}}{\partial \widehat{\rho}_0}\right)\dot{\widehat{\rho}}_0 + \frac{\partial \alpha_{\rho-1}}{\partial \widehat{\theta}} T \tau_\rho \\
&+ \frac{\partial \alpha_{\rho-1}}{\partial y}\left(\omega_0 + \omega^T \widehat{\theta}\right) \\
&+ \frac{\partial \alpha_{\rho-1}}{\partial \xi}\left(A_0(L)\xi + l(L)y + \varphi(y)\right) \\
&+ \frac{\partial \alpha_{\rho-1}}{\partial \Xi}\left(A_0(L)\Xi + \Phi(y,t)\right) \\
&+ \sum_{j=1}^{\rho-1} \frac{\partial \alpha_{\rho-1}}{\partial y_r^{(j-1)}} y_r^{(j)} + l(L)_i v_{n^*,1} \\
&+ \sum_{j=1}^{m-1+\rho} \frac{\partial \alpha_{i-1}}{\partial \lambda_j}\left(-l(L)_j \lambda_1 + \lambda_{j+1}\right) \\
&- \sum_{j=2}^{\rho-1} \frac{\partial \alpha_{j-1}}{\partial \widehat{\theta}} T \frac{\partial \alpha_{j-1}}{\partial y} z_\rho - \frac{W_0 z_\rho}{z_1^2 + \Delta_0^2}, \\
\tau_\rho &= \tau_{\rho-1} - \frac{\partial \alpha_{\rho-1}}{\partial y}\omega z_{\rho-1},
\end{aligned} \tag{24}$$

where α_ρ is the ρth stabilizing function and τ_ρ is the ρth tuning function.

Finally, the actual control signal and parameter adaptive laws are, respectively, designed as

$$v_0 = \alpha_\rho - v_{n^*,\rho+1} + \widehat{\rho}_0 y_r^{(\rho)}, \tag{25}$$

$$\dot{\widehat{\theta}} = T\tau_\rho, \tag{26}$$

where $T = T^T > 0$ is the adaptive gain.

4. Stability Analysis

To prepare for the stability analysis, we rewrite the error system as

$$\begin{aligned}
\dot{z} &= \left(A_z(z,t) + A_c(z,t)\right) z + W_\varepsilon(z,t)\left(\varepsilon_2 + \phi_1\right) \\
&+ W_\theta(z,t)^T \widetilde{\theta} - k_{1,n^*}\left(\dot{y}_r + \overline{\alpha}_1\right) e_1 \widetilde{\rho}_0 - \Gamma_z W_0,
\end{aligned} \tag{27}$$

where the system matrices $A_z(z,t)$, $W_\varepsilon(z,t)$, $W_\theta(z,t)$, $W_\theta(z,t)$, $A_c(z,t)$, Γ_z are given by

$$A_z(z,t) = \begin{bmatrix} -c_1 - d_1 & \widehat{k}_{1,n^*} & 0 & \cdots & \cdots & 0 \\ \widehat{k}_{1,n^*} & -c_2 - d_2\left(\dfrac{\partial\alpha_1}{\partial y}\right)^2 & 1+\sigma_{23} & \sigma_{24} & \cdots & \sigma_{2\rho} \\ \vdots & -1-\sigma_{23} & \ddots & \ddots & \ddots & \vdots \\ \vdots & -\sigma_{24} & \ddots & \ddots & \ddots & \sigma_{\rho-2\rho} \\ \vdots & \vdots & \ddots & \ddots & \ddots & 1+\sigma_{\rho-1,\rho} \\ 0 & -\sigma_{2\rho} & \cdots & -\sigma_{\rho-2,\rho} & -1-\sigma_{\rho-1,\rho} & -c_\rho - d_\rho\left(\dfrac{\partial\alpha_{\rho-1}}{\partial y}\right)^2 \end{bmatrix},$$

$$W_\varepsilon(z,t) = \left[1 - \frac{\partial\alpha_1}{\partial y} - \frac{\partial\alpha_2}{\partial y} - \cdots - \frac{\partial\alpha_{\rho-1}}{\partial y}\right]^T \in R^\rho,$$

$$W_\theta(z,t) = W_\varepsilon(z,t)\,\omega^T - \widehat{\rho}_0\left(\dot{y}_r + \overline{\alpha}_1\right) e_1 e_1^T \in R^{\rho\times\rho}, \tag{28}$$

$$A_c(z,t) = \begin{bmatrix} -\dfrac{L\rho}{4} & 0 & 0 & \cdots & 0 \\ 0 & -\dfrac{L\rho}{4}\left(\dfrac{\partial\alpha_1}{\partial y}\right)^2 & 0 & \cdots & 0 \\ 0 & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\dfrac{L\rho}{4}\left(\dfrac{\partial\alpha_{\rho-1}}{\partial y}\right)^2 \end{bmatrix},$$

$$\Gamma_z = \left[\frac{z_1}{z_1^2+\Delta_0^2}, \frac{z_2}{z_1^2+\Delta_0^2}, \ldots, \frac{z_\rho}{z_1^2+\Delta_0^2}\right]^T.$$

Let

$$E_L = \operatorname{diag}\left\{1, \frac{1}{L}\cdots\frac{1}{L^{n-1}}\right\}. \tag{29}$$

The candidate Lyapunov function for the closed-loop system is chosen as

$$V = \frac{1}{2}z^T z + \frac{1}{2}\widetilde{\theta}T^{-1}\widetilde{\theta}^T + \frac{\left|k_{1,n^*}\right|}{2\gamma}\widetilde{\rho}^2 + \varepsilon^T P_L \varepsilon, \tag{30}$$

where

$$A_0^T P + P A_0 = -I, \qquad P_L = E_L P E_L. \tag{31}$$

The proposed adaptive scheme has the following properties.

Theorem 9. *The adaptive output feedback control scheme consisting of the controller (25) and the filters (14) along with the parameter update laws (26) applied to the system (1) based on Assumptions 1–5 ensures global boundedness of all closed-loop signals and desired output tracking performance:*

$$\int_{t_1}^{t_2}\left(y(t) - y_r(t)\right)^2 dt \le \frac{\lambda}{c_0}\left(t_2 - t_1\right) + \gamma_0, \tag{32}$$

for any $t_2 > t_1 \ge 0$, where $\lambda > 0$, $\gamma_0 > 0$, $c_0 > 0$ are design parameters. The tracking error can be made as small as desired by choosing sufficiently large c_0 and L.

Proof. For each time interval (t_k, t_{k+1}), $k = 0, 1, 2, \ldots, q$, we have a Lyapunov function V defined in (30). Recalling (27), the derivative of V is

$$\begin{aligned} \dot{V} = {} & \frac{1}{2}z^T\left(A_z + A_z^T\right)z + z^T A_c(z,t)\,z + z^T W_\varepsilon \varepsilon_2 \\ & + z^T W\phi_1 + z^T W_\theta^T \widetilde{\theta} + 2\varepsilon^T E_L P E_L \phi \\ & - \widetilde{\theta}^T W_\theta z - z^T k_{1,n^*}\left(\dot{y}_r + \overline{\alpha}_1\right) e_1 \widetilde{\rho}_0 \\ & + \widetilde{\rho}_0 k_{1,n^*}\left(\dot{y}_r + \overline{\alpha}_1\right) e_1^T z + \varepsilon^T\left(A_0^T(L)\,P_L + P_L A_0(L)\right)\varepsilon \\ & - \frac{\|z\|^2}{z_1^2+\Delta_0^2} f(L)\,W_0. \end{aligned} \tag{33}$$

Then, observing that

$$2\varepsilon^T E_L P E_L \phi = 2L\varepsilon^T E_L P \begin{bmatrix} \dfrac{\phi_1}{L} \\ \dfrac{\phi_2}{L^2} \\ \vdots \\ \dfrac{\phi_n}{L^n} \end{bmatrix} \le 2L\left\|\varepsilon\right\|\left\|E_L P\right\|\left\|\varphi\right\|, \tag{34}$$

where $\varphi = [\varphi_1, \varphi_2, \ldots, \varphi_n]^T = [\phi_1/L, \phi_2/L^2, \ldots, \phi_n/L^n]^T$ and from Assumption 1, we estimate the elements of φ

$$\left|\frac{\phi_i}{L^i}\right| \leq \frac{c}{L^i}\left(|\hat{x}_1| + \cdots + |\hat{x}_i| + |\varepsilon_1| + \cdots + |\varepsilon_i|\right) \leq c_i\left(\|\hat{x}\| + \|\varepsilon\|\right), \tag{35}$$

where $c_i = c\sqrt{n}/L^i$.

Hence,

$$2L\|\varepsilon\|\|E_L P\|\|\varphi\| = 2L\|E_L P\|\|\varepsilon\|\|\varphi\| \leq 2C\|\varepsilon\|\left(\|\hat{x}\| + \|\varepsilon\|\right), \tag{36}$$

where C is a positive constant depending on $\hat{x}$, P, E_L, c, n, and L.

Note that

$$\begin{aligned} & z^T A_c(z,t) z + z^T W \phi_1 \\ &= \frac{\phi_1^2}{L} - \left(\frac{\sqrt{L\rho}}{2} z_1 - \frac{1}{\sqrt{L\rho}}\phi_1\right)^2 \\ &\quad - \sum_{i=2}^{\rho}\left(\frac{\sqrt{L\rho}}{2}\frac{\partial \alpha_{i-1}}{\partial y} z_i + \frac{1}{\sqrt{L\rho}}\phi_1\right)^2 \\ &\leq \frac{c^2}{L} y^2 \leq \frac{2c^2}{L}\left(y_r^2 + z_1^2\right). \end{aligned} \tag{37}$$

Substituting (34) and (36) into (33), we have

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^{\rho} c_i z_i^2 + \frac{2c^2}{L} z_1^2 - \sum_{i=1}^{\rho} d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right) z_i^2 \\ &\quad - \sum_{i=1}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y}\varepsilon_2 - L\|\varepsilon\|^2 + 2C\|\varepsilon\|\left(\|\hat{x}\| + \|\varepsilon\|\right) \\ &\quad + \frac{2c^2}{L} y_r^2 - \frac{\|z\|^2}{z_1^2 + \Delta_0^2} f(L) \\ &\leq -\sum_{i=1}^{\rho} c_i z_i^2 + \frac{2c^2}{L} z_1^2 - \frac{\|z\|^2}{z_1^2 + \Delta_0^2} f(L) W_0 \\ &\quad + \|\hat{x}\|^2 + \frac{2c^2}{L} y_r^2 - \left(L - 2C - 4C^2 - \sum_{i=1}^{\rho}\frac{1}{4d_i}\right)\|\varepsilon\|^2. \end{aligned} \tag{38}$$

With the choice of $L - 2C - 4C^2 - \sum_{i=1}^{\rho}(1/4d_i) > 0$, $c_1 - 2c^2 > 0$, and $f(L) \geq ((z_1^2 + \Delta_0^2)/\|z\|^2) \times (\|\theta\|_1/\theta^*)$,

$$\dot{V} \leq -\sum_{i=1}^{\rho} c_i z_i^2 + \frac{2c^2}{L} z_1^2 + \frac{2c^2}{L} y_r^2. \tag{39}$$

Let

$$\lambda = \frac{2c^2}{L} y_r^2. \tag{40}$$

It should be noted that λ can be made as small as desired by choosing sufficiently larger L. Recalling (39), we can obtain

$$\dot{V} \leq -c_0 z_1^2 - \sum_{i=2}^{\rho} c_i z_i^2 + \lambda, \tag{41}$$

where $c_1' = c_1 - 2c^2$.

Starting from the first interval, we conclude that $(t) \in L^\infty \forall t \in [t_0, t_1)$, so that $z, \hat{\theta}$, and ε are all bounded $\forall t \in [t_0, t_1)$. From (14) and (25), it follows that $\|\hat{x}\|$ is bounded. At time $t = t_1$, there occur p_1 actuator failures, which results in the abrupt change of the parameters. Since the change of values is finite, we have

$$V(t_1^+) = V(t_1^-) + \overline{V}_1, \tag{42}$$

where $\overline{V}_1$ is bounded.

Therefore, it can be concluded from (41) that $(t) \in L^\infty \forall t \in [t_1, t_2)$. By repeating the argument above, the boundedness of all the signals is proved for the time interval (t_1, t_2). Continuing in the same manner, finally we have that $\forall t \in (t_q, \infty)$, $V(t) \in L^\infty$ and so are the closed-loop signals.

To prove tracking performance, consider the last time interval (t_q, ∞). From (41), we see that $\dot{V} \leq -c_0 z_1^2 - \sum_{i=2}^{\rho} c_i z_i^2 + \lambda$. In particular, $\dot{V} \leq -c_0 z_1^2 + \lambda$. Integrating both sides from $t = t_1$ to $t = t_2$, where $t_2 > t_1 \geq 0$, we have $\int_{t_1}^{t_2}(y(t) - y_r(t))^2 dt \leq (\lambda/c_0)(t_2 - t_1) + V(t_2) - V(t_1)$. Since $V(t) \in L_\infty$, there exists $\gamma_0 > 0$ such that the desired output tracking given by (32) is achieved. This completes the proof. □

5. Examples and Simulations

In this section, we present two examples to illustrate the applications of Theorem 9.

Example 1. The nonlinear longitudinal dynamics of the twin otter aircraft [1, 7, 27] is used for our actuator failure compensation study. Choosing the velocity, angle of attack, pitch angle, and pitch rate as the states x_1, x_2, x_3, and x_4 and the elevator angles of an augmented two-piece elevator as the inputs u_1, u_2, the system can be modeled as

$$\begin{aligned} \dot{x}_1 &= \left(c_1^T \varphi_0(x_2) x_1^2 + \varphi_1(x)\right)\cos(x_2) \\ &\quad + \left(c_2^T \varphi_0(x_2) x_2^2 + \varphi_2(x)\right)\sin(x_2) \\ &\quad + d_1 g_1(x) u_1 + d_2 g_1(x) u_2, \\ \dot{x}_2 &= x_4 - \left(c_1^T \varphi_0(x_2) x_1^2 + \varphi_1(x)\frac{1}{x_1}\right)\sin(x_2) \\ &\quad + \left(c_2^T \varphi_0(x_2) x_2^2 + \varphi_2(x)\frac{1}{x_1}\right)\cos(x_2) \\ &\quad + d_1 g_2(x) u_1 + d_2 g_2(x) u_2, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \theta^T \phi(x) + b_1 x_1^2 u_1 + b_2 x_1^2 u_2, \end{aligned} \tag{43}$$

where

$$\varphi_0(x_2) = [x_2, x_2^2, 1]^T,$$
$$\varphi_1(x) = p_{11} + p_{12}x_4x_1^2 - p_0\sin(x_3),$$
$$\varphi_2(x) = p_{21} + p_{22}x_4x_1^2 + p_0\cos(x_3),$$
$$g_1(x) = a_1x_1^2\cos(x_2) + a_2x_1^2\sin(x_2),$$
$$g_2(x) = -a_1x_1\sin(x_2) + a_2x_1\cos(x_2),$$
$$\phi(x) = [x_1^2x_2, x_1^2x_2^2, x_1^2, x_1^2x_4]. \quad (44)$$

For this application, x_3 is considered as the output; that is, $y = x_3$, x_1, and the velocity is measured while x_2, x_4 are unmeasured. Apparently, $[x_3, x_4]^T$ subsystem can be described as

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = \theta^T\phi(x) + b^Tx_1^2u, \quad (45)$$

with some unknown parameters $b \in R^2$ and $u = [u_1, u_2]^T$. The $[x_1, x_2]^T$ subsystem constructs the zero dynamics, which is input-to-state stability [1, 7]. The unmeasured state x_4 exhibits the appearance of a linear form $\phi(x)$, which satisfies Assumption 1. Taking advantage of this property, we design adaptive high-gain K-filters to estimate x_4.

The control objective is to design an adaptive scheme to control the elevator angles such that the pitch angle $y = x_3$ tracks a reference signal y_r as close as desired even if one piece of the elevator is stuck at an unknown angle at an unknown time instant. For simulation, we consider two actuator failure cases.

Case 1. We consider the case where u_1 fails at the 50th second. Thus, u_1 undergoes a TLOE type of failure:

$$u_1(t) = v_1(t),$$
$$u_2(t) = \rho_2v_2 + u_{k_2}, \quad (46)$$

where $\rho_2 = 1$, $u_{k_2} = 0$, and $t \in [0, 100)$; $\rho_2 = 0$, $u_{k_2} = -0.08$, and $t \in [100, \infty)$.

By Theorem 9, we can obtain the actual control law, the high-gain K-filters, and the update laws. The initial conditions are set as

$$c_1 = 0.01, \quad c_2 = 0.01, \quad d_1 = 1,$$
$$d_2 = 1, \quad l = [6, 8]^T, \quad \hat{\theta}_0 = [18, 0, 0]^T,$$
$$x_0 = [72, 0.00, 0.001, 0.05]^T, \quad T = \mathrm{diag}([5, 1, 1]) * 50. \quad (47)$$

The switching law parameters are chosen as

$$T = 1, \quad a = 0.000003, \quad K = 1.2. \quad (48)$$

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 1,

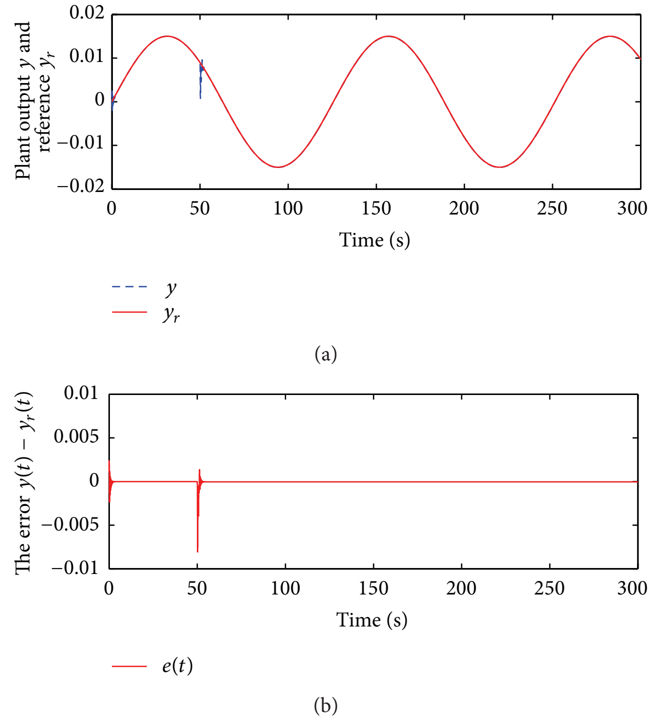


FIGURE 1: Plant output y, reference y_r, and error $e(t)$.

the control inputs u_1, u_2 are shown in Figure 2, the adaptive parameter L and unmeasured state x_4 are shown in Figure 3, and the zero dynamics x_1 and x_2 are shown in Figure 4. The system responses are as expected. When one of the actuators fails, there is a transient response in tracking errors. But as time goes on, the tracking errors become smaller and ultimately vanish. The adaptive controller can tune its parameters according to the actuator failures, while the adaptive high-gain K-filters also change their value if necessary. Furthermore, it is easily seen that the unmeasured state x_4 and the zero dynamics x_1, x_2 are stable even though there is an actuator failing during operation.

Case 2. We consider the case where u_2 (TLOE) fails at the 100th second and u_1 (PLOE) loses 50% of its effectiveness from $t = 160$ s. That is,

$$u_1(t) = \rho_1v_1,$$
$$u_2(t) = \rho_2v_2 + u_{k_2}, \quad (49)$$

where

$$\rho_1 = 1, \quad t \in [0, 160); \quad \rho_1 = 0.5, \quad t \in [160, \infty),$$
$$\rho_2 = 1, \quad t \in [0, 100);$$
$$\rho_2 = 0, \quad u_{k_2} = -0.08, \quad t \in [100, \infty). \quad (50)$$

The other parameters are the same as those in Case 1.

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 5, the control inputs u_1, u_2 are shown in Figure 6, the adaptive

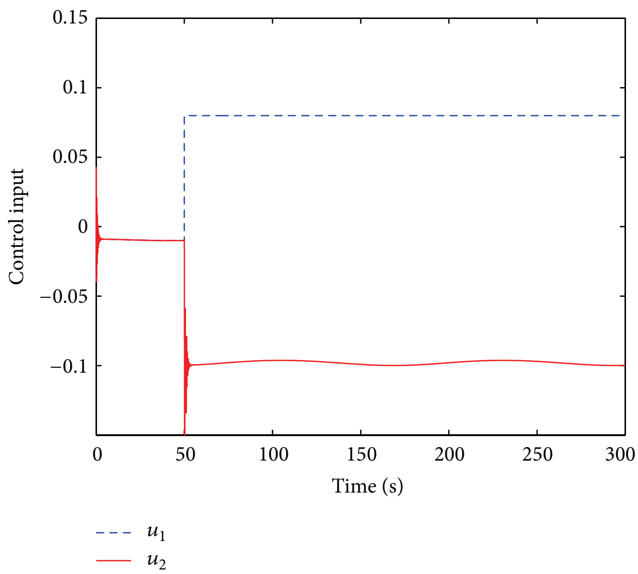


Figure 2: Plant inputs u_1, u_2.

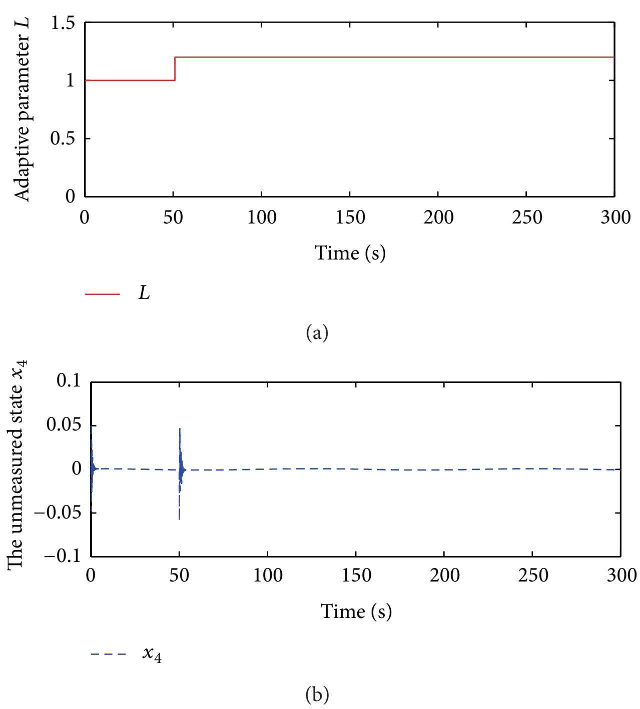


Figure 3: The adaptive parameter L and the unmeasured state x_4.

parameter L and unmeasured state x_4 are shown in Figure 7, and the zero dynamics x_1 and x_2 are shown in Figure 8. The system responses are as expected. When one of the actuators fails, there is a transient response in tracking errors. But as time goes on, the tracking errors become smaller and ultimately vanish. The adaptive controller can tune its parameters according to the actuator failures, while the adaptive high-gain K-filters also change their value if necessary. Furthermore, it is easily seen that the unmeasured

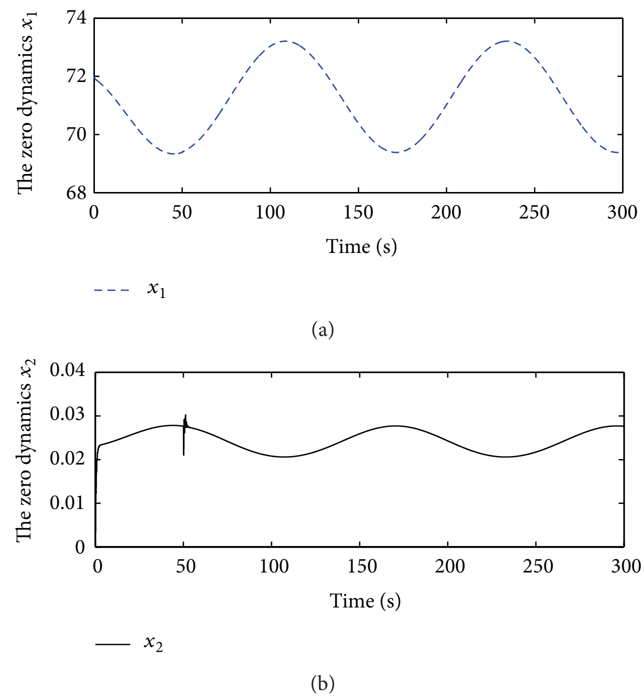


Figure 4: The zero dynamics x_1 and x_2.

state x_4 and the zero dynamics x_1, x_2 are stable even though there is an actuator failing during operation.

Example 2. A rigid-body longitudinal model of a hypersonic aircraft cruising at a velocity of 15 Mach and at an altitude of 110,000 ft [1, 11] is considered:

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_3, y), \\ \dot{x}_2 &= b_1 u_1 + b_2 u_2 + f_2(x_3, y), \\ \dot{x}_3 &= f_3(x_3, y), \end{aligned} \tag{51}$$

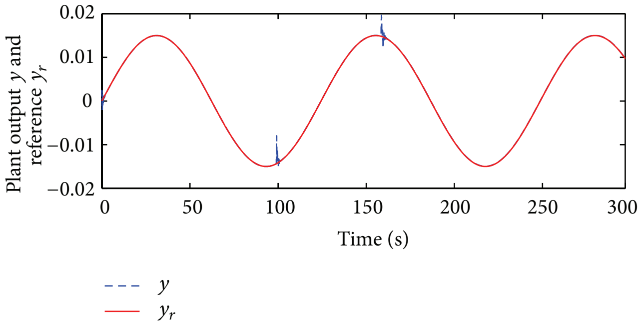
where

$$\begin{aligned} f_1(x_3, y) &= \theta_1 y + \theta_2 \sin(y) + \theta_3 y^2 \sin(y) + \theta_4 \cos(x_3), \\ f_2(x_2, y) &= \theta_5 y^2 + \theta_6 y + \theta_7 y^2 x_2 + \theta_8 y x_2 + \theta_9 x_2 + \theta_{11}, \\ f_3(x_3, y) &= \theta_{10} \cos(x_3) - \left(\theta_1 y + \theta_2 \sin(y) + \theta_3 y^2 \sin(y)\right). \end{aligned} \tag{52}$$

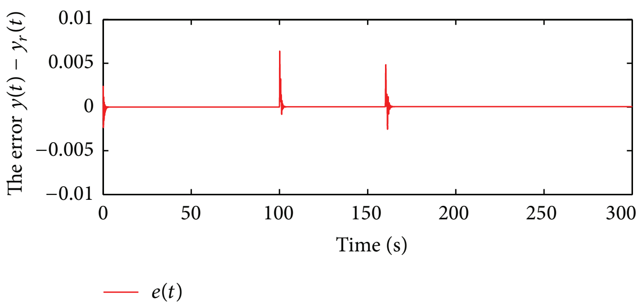
We rewrite the plant (51) as

$$\begin{aligned} \dot{x}_1 &= x_2 + \phi_1(x_3) + \varphi_1(y), \\ \dot{x}_2 &= \phi_2(y) x_2 + \varphi_2(y) + b_1 u_1 + b_2 u_2, \\ \dot{x}_3 &= \theta_{10} \cos(x_3) - \varphi_1(y), \\ y &= x_1, \end{aligned} \tag{53}$$

where x_1 is the angle of attack, x_2 is the pitch rate and unmeasured, x_3 is the flight-path angle and unmeasured, and $u_1(t)$ and $u_2(t)$ are the elevator segment deflection angles. b_1



(a)



(b)

FIGURE 5: Plant output y, reference y_r, and error $e(t)$.

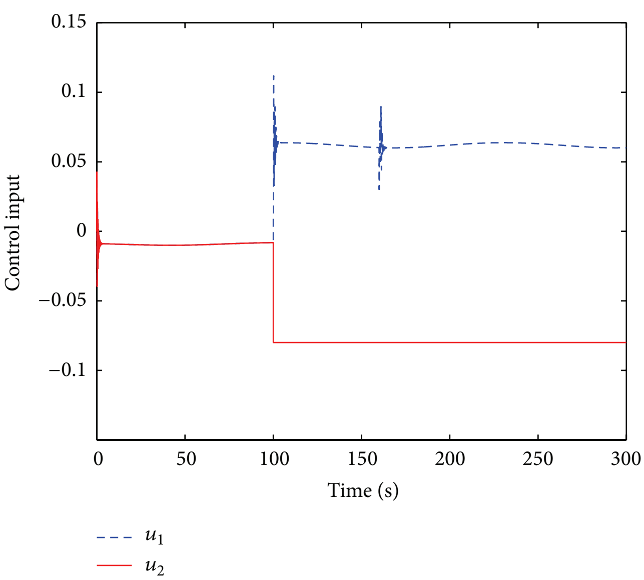
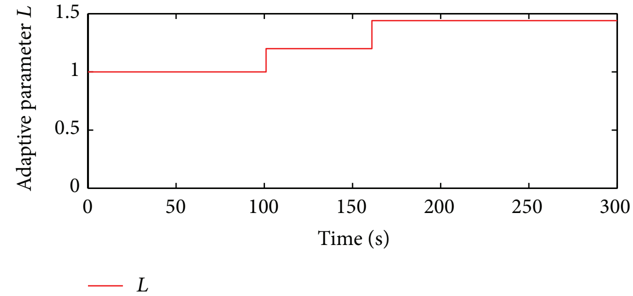


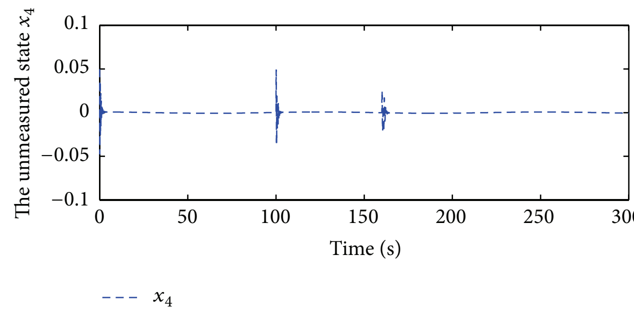
FIGURE 6: Plant inputs u_1, u_2.

and b_2 are unknown constants with known signs, and ϕ_1, ϕ_2, φ_1, and φ_2 are known functions given by

$$\begin{aligned} \phi_1(x_3) &= \theta_4 \cos(x_3), \\ \phi_2(y) &= \theta_7 y^2 + \theta_8 y + \theta_9, \\ \varphi_1(y) &= \theta_1 y + \theta_2 \sin(y) + \theta_3 y^2 \sin(y), \\ \varphi_2(y) &= \theta_5 y^2 + \theta_6 y + \theta_{11}. \end{aligned} \quad (54)$$



(a)



(b)

FIGURE 7: The adaptive parameter L and the unmeasured state x_4.

The control objective for actuator failure compensation of aircraft model is the angle of attack x_1 that tracks a reference signal y_r with a sufficiently small error, while the closed-loop system is stabilized in the presence of an actuator failure in either one of the elevator segments. Since $y = x_1$ is the output, equation $\dot{x}_3 = \theta_{10} \cos(x_3) - \varphi_1(y)$ constructs the zero dynamics. For simulation, we also consider two actuator failure cases.

Case 1. We consider the case where u_2 (TLOE) fails at the 50th second. That is,

$$\begin{aligned} u_1(t) &= v_1(t), \\ u_2(t) &= \rho_2 v_2 + u_{k_2}, \end{aligned} \quad (55)$$

where $\rho_2 = 1$, $u_{k_2} = 0$, and $t \in [0, 50)$; $\rho_2 = 0$, $u_{k_2} = 0.1$, and $t \in [50, \infty)$. From Theorem 9, we can obtain the actual control law, the high-gain K-filters, and the update laws. The simulation parameters are as follows:

$$\begin{gathered} c_1 = 0.001, \qquad c_2 = 0.001, \qquad d_1 = 1, \\ d_2 = 15, \qquad l = [6, 8]^T, \\ \hat{\theta}_0 = [0, 0, 0]^T, \qquad x_0 = [0, 0, 1.57]^T, \\ T = \mathrm{diag}([100, 10, 10]), \qquad T = 10, \\ a = 0.004, \qquad K = 1.2. \end{gathered} \quad (56)$$

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 9, the control inputs u_1, u_2 are shown in Figure 10, the adaptive

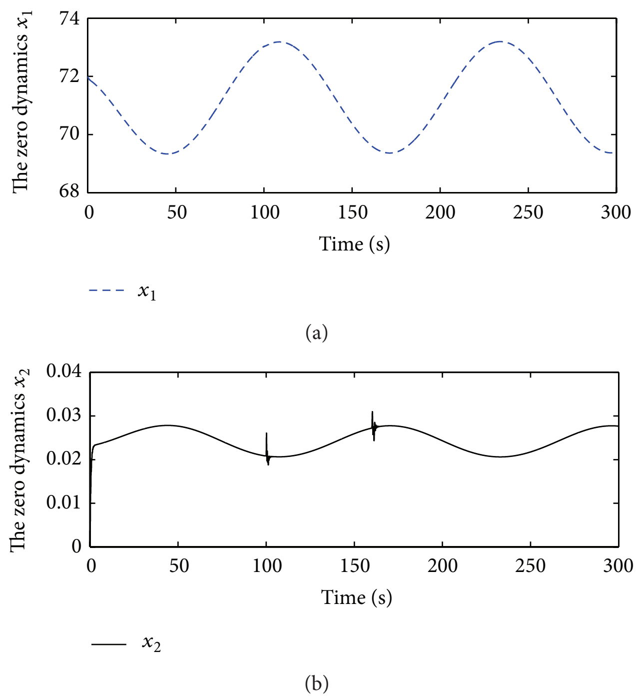


FIGURE 8: The zero dynamics x_1 and x_2.

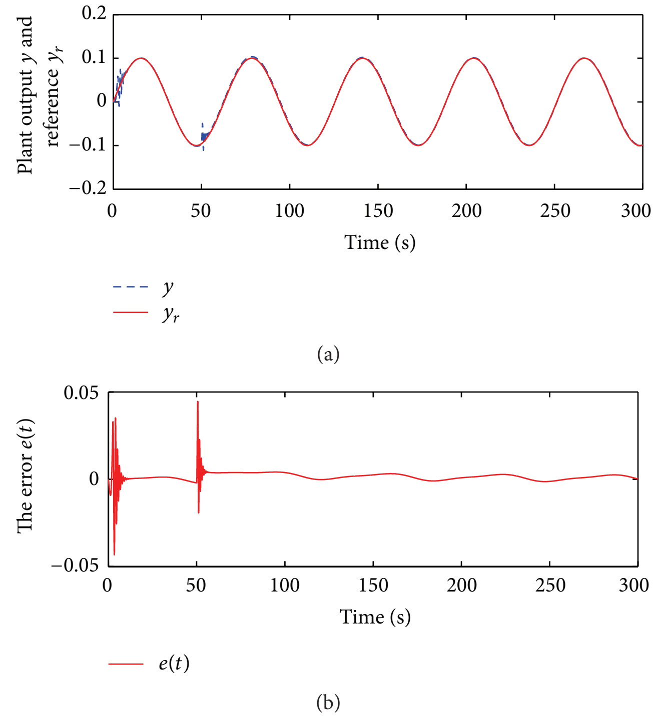


FIGURE 9: Plant output y, reference y_r, and error $e(t)$.

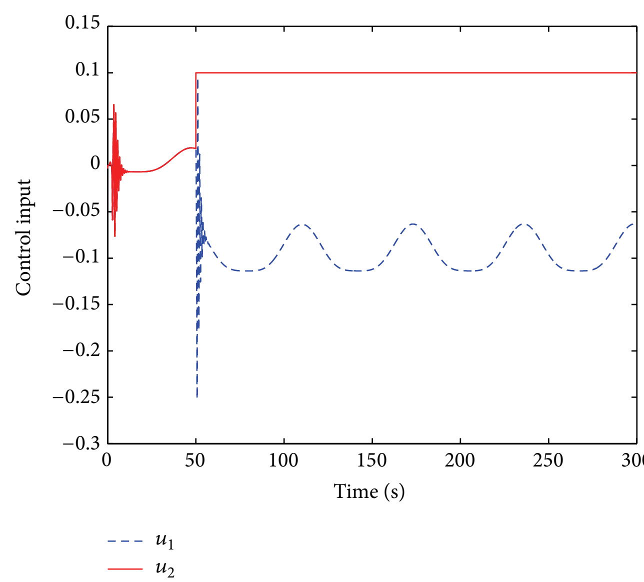


FIGURE 10: Plant inputs u_1, u_2.

parameter L is shown in Figure 11, and the adaptive parameter L is shown in Figure 12. The system responses are as expected. When one of the actuators fails, there is a transient response in tracking errors. But as time goes on, the tracking errors become smaller and ultimately vanish. The adaptive controller can tune its parameters according to the actuator failures, while the adaptive high-gain K-filters do not change their value. Furthermore, it is easily seen that the unmeasured state x_2 is stable even though there is an actuator failing during operation.

Case 2. We consider the case where u_1 (TLOE) fails at the 100th second and u_2 (PLOE) loses 40% of its effectiveness from $t = 150$ s:

$$u_1(t) = \rho_1 v_1(t) + u_{k_1}, \quad u_2(t) = \rho_2 v_2(t), \tag{57}$$

where

$$\begin{aligned} &\rho_1 = 1, \quad u_{k_1} = 0, \quad t \in [0, 100); \\ &\rho_1 = 0, \quad u_{k_1} = -0.1, \quad t \in [100, \infty), \\ &\rho_2 = 1, \quad t \in [0, 150); \quad \rho_2 = 0.6, \quad t \in [150, \infty). \end{aligned} \tag{58}$$

The other parameters are the same as those in Case 1.

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 13, the control inputs u_1, u_2 are shown in Figure 14, the adaptive parameter L is shown in Figure 15, and the adaptive parameter L is shown in Figure 16. The system responses are as expected. When one of the actuators fails, there is a transient response in tracking errors. But as time goes on, the tracking errors become smaller and ultimately vanish. The adaptive controller can tune its parameters according to the actuator failures, while the adaptive high-gain K-filters do not change their value. Furthermore, it is easily seen that the unmeasured state x_2 is stable even though there is an actuator failing during operation.

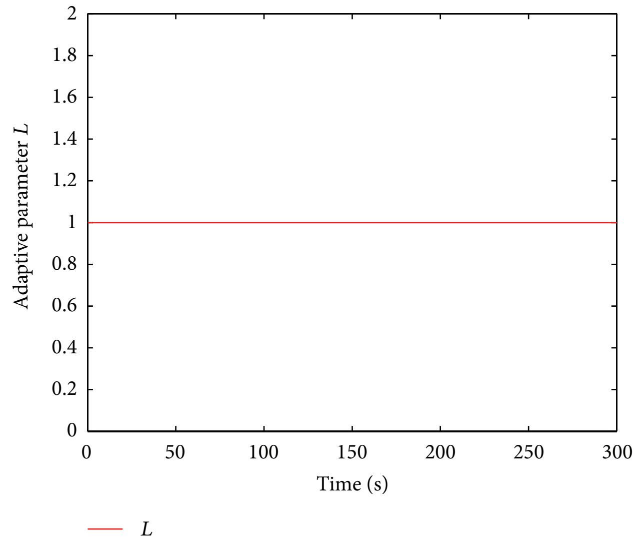


FIGURE 11: The adaptive parameter L and the unmeasured state x_4.

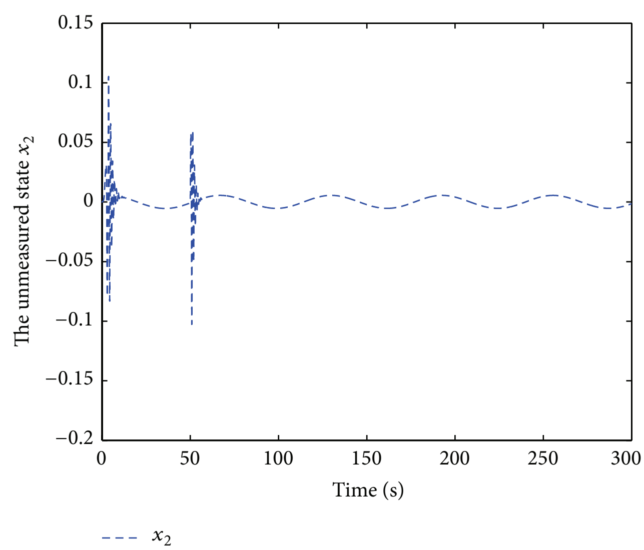


FIGURE 12: The zero dynamics x_1, x_2.

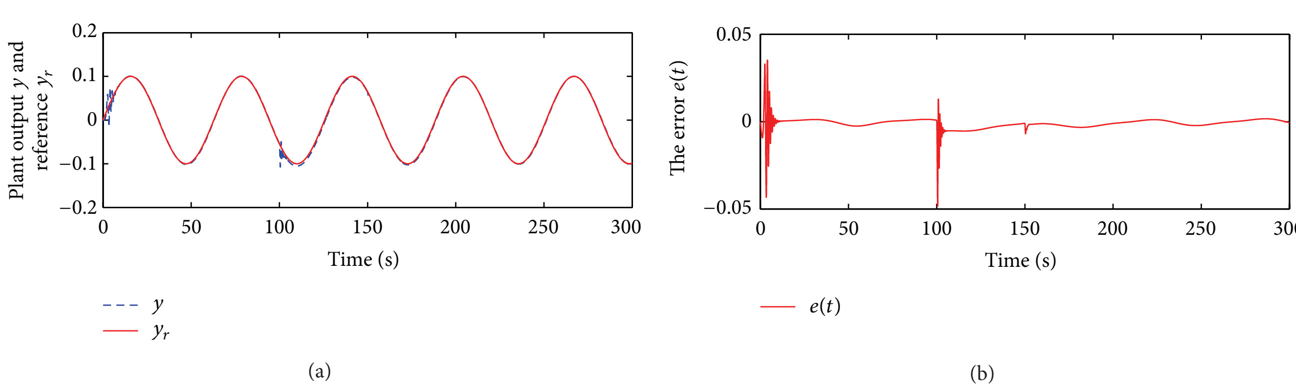


FIGURE 13: Plant output y, reference y_r, and error $e(t)$.

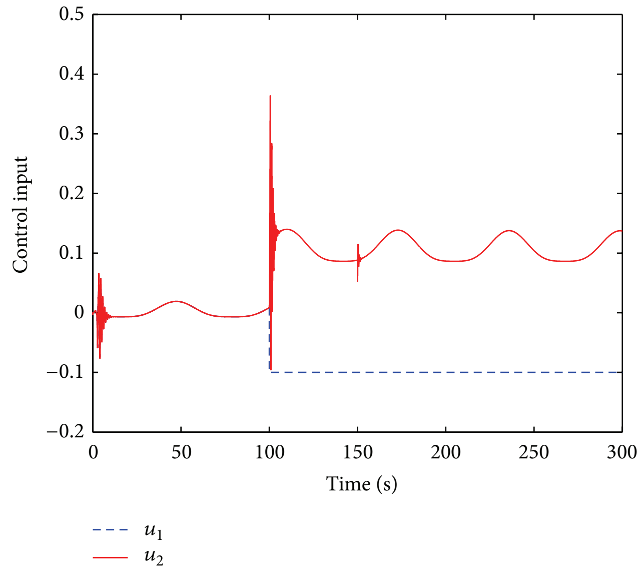


Figure 14: Plant inputs u_1, u_2.

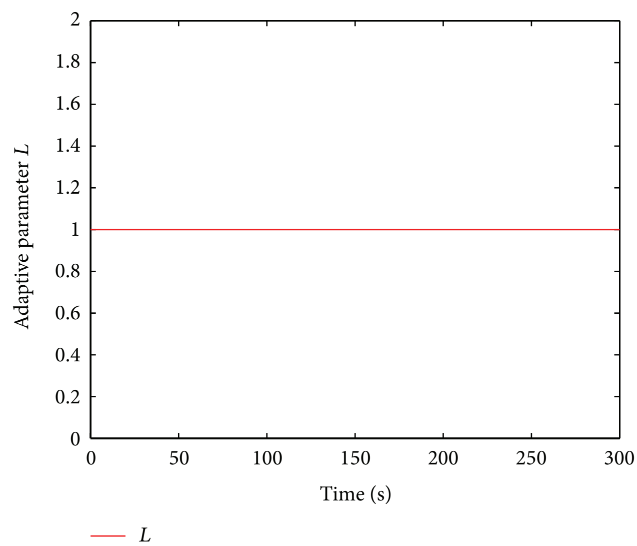


Figure 15: The adaptive parameter L and the unmeasured state x_4.

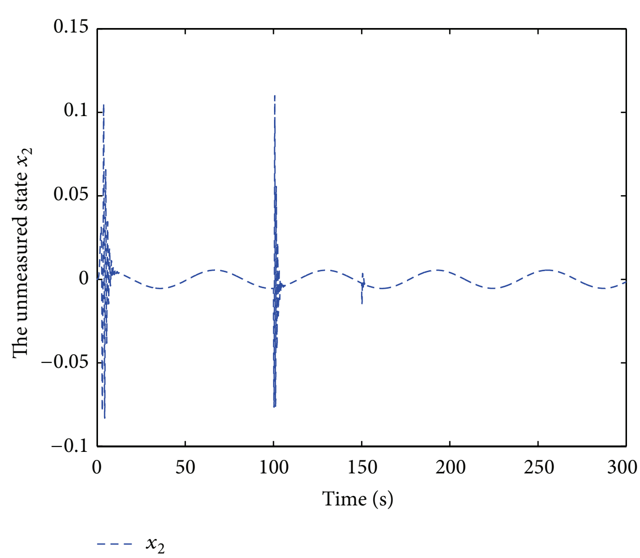


Figure 16: The zero dynamics x_1, x_2.

6. Conclusions

In this paper, adaptive high-gain K-filters and adaptive high-gain controller are applied to handle a class of nonlinear systems uncertainties in the presence of uncertain actuator failures. The adaptive high-gain K-filters can suppress the nonlinearities while the adaptive controller guarantees the closed-loop signals stability and small tracking errors. Simulation results verify the effectiveness of the adaptive actuator failure compensation for desired performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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