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Research Article

Bilateral Coordination Strategy of Supply Chain with Bidirectional Option Contracts under Inflation

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As far as the price increase and the demand contraction caused by inflation are concerned, we establish a Stackelberg game model that incorporates bidirectional option contracts and the effect of inflation and derive the optimal ordering and production policies on a one-period two-stage supply chain composed of one supplier and one retailer. Through using the model of wholesale price contracts as the benchmark, we find that the introduction of bidirectional option contracts can benefit both the supplier and the retailer under inflation scenarios. Based on the conclusions drawn above, we design the bilateral coordination mechanism from the different perspective of two members involved and discuss how bidirectional option contracts should be set to achieve channel coordination under inflation scenarios. Through the sensitivity analysis, we illustrate the effect of inflation on the optimal decision variables and the optimal expected profits of the two parties with bidirectional option contracts.

1. Introduction

Due to the international financial crisis, most countries in the world have suffered from inflationary pressure during years. Inflation is not a good thing when inflation rate reaches above 2% [1, 2]. A lot of problems seem to appear as the inflationary pressure spreading to all aspects of the economy. Since the process of international integration results in one country's economy being susceptible to the state of other countries, inflation has an important effect on real economic growth not only in developing countries but also in developed countries [3, 4]. Let us take China as an example of a developing country. Since 2008, China has suffered from serious inflation. It is easy to observe that general and food prices have climbed to previously unknown heights. People need to spend more money to buy a smaller amount of a particular product or service than they did in the past. It means that the essentials for survival become more expensive and less attainable as the purchasing power of the money going down. Because China is still a relatively poor country in terms of its standard of living, middle-class groups fall into the low-income trap and the low-income groups get hit very hard under this situation. The depressed consumer spending causes the ongoing downturn of domestic consumer market and subsequently a large number of small and medium-sized companies resort to voluntary bankruptcy. Besides, as one of the largest exporters, the impact of rising inflation in China is propagated to the rest of the world. Let us take America as an example of a developed country. America makes increasing trade agreements with China in recent years [5]. US imports from China primarily focus on the labor-intensive goods such as apparel, fabric, and textiles which are closely related to the daily life of American people. Influenced by the inflation in China, we see the prices on low-end goods that are exported to America continue going up and the lives in the United States become more expensive. Besides, due to the soft economic conditions in the United States, growing sales are not as easy as it used to be. A fair number of American multinationals are now counting on Chinese consumers to fuel their growth. But higher inflation in China means there is less foreign demand for American goods. Furthermore, China is the largest owner of US government debt and the value of debt exceeds \$1.2 trillion [6, 7]. In the inflationary environment, China needs to stimulate their domestic economies. At this time, it is hardly feasible for Chinese government to use the dollar in reserves and buy more debt. Thus, inflation in China has a negative effect on the American economy. From the above description, we

know that inflation presents the downward trend in the economy and poses a threat to the enterprise operation. On the one hand, employers require higher pay to make up for the increase in their living expenses owing to sustained inflation. However, the raise in real average wages always falls behind with rising consumer prices. As the earned money can only buy less tangible goods, there is a corresponding reduction in purchasing power and then the slack consumer demand starts to come out. On the other hand, the ongoing rising prices of raw materials put pressure on the production and operation costs of the enterprise. It is hard for companies to transfer price increases to consumers. Thus, the business profit margins are squeezed further. Obviously, inflation has a negative effect on the daily operation of the companies. Since companies are inseparable from their partners, the effect of inflation can exert great influence on the daily operations of the supply chain. In the face of the price increase and the demand contraction caused by inflation, option contracts are regarded as one of the most effective tools to improve performance and adopted gradually by more and more enterprises in the supply chain. For example, HP obtains real business value through the portfolio purchase programs which contain derivatives of option contracts. The programs help HP reduce the procurement costs, control the operational risks and drive faster growth [8, 9]. Motivated by this, we plan to study how to use option contracts to mitigate the effect of inflation and make the enterprise involved obtain more incomes.

Option contracts are usually classified into three different types: call option contracts, put option contracts, and bidirectional option contracts [10, 11]. Call option contracts give the buyer the right to obtain some additional items. Put option contracts give the buyer the right to return some surplus items. Bidirectional option contracts which contain both call options and put options give the buyer the right to reorder or return some ordered items. It is easy to discover that bidirectional option contracts are the extension of both call option contracts and put option contracts [12]. When bidirectional option contracts are used, the buyer can ensure adequate supply from the upstream supplier and adjust the order quantity either upwards or downwards based on the demand information obtained. Obviously, the presence of bidirectional option contracts can make the procurement process more flexible. Moreover, when bidirectional option contracts are used, the seller can gain some profit through bidirectional options sales and adjust the production schedule based on the buyer's order requirement. Obviously, the presence of bidirectional option contracts can also make the manufacturing process more flexible. So far, a large number of papers focus on the issues of call option contracts under different settings. The studies relating to bidirectional option contracts are not comprehensive and some issues have not been addressed. Moreover, all these papers relating to option contracts do not consider the effect of inflation. Motivated by this, we plan to study how to use bidirectional option contracts to resolve channel conflicts and achieve channel coordination under inflation scenarios.

The problem of coordinating a channel has always been a hot topic in the management of supply chain activities which is associated with the decision making. Except for some scenarios such as random yield, most papers always assume that the seller adopts make-to-order production pattern and decides how many products to manufacture based on the buyer's order under various contract types such as buyback contracts, revenue sharing contracts and so on. Only when the buyer's decision making is the same as the system's decision making, the channel is said to attain coordination. At this time, how to make a nonintegrated supply chain coordinate is regarded as the unilateral coordination problem from the buyer's perspective. However, when bidirectional option contracts are used, there is no obligation for the buyer to execute all the options order. In this case, the seller does not commit to manufacturing the products to the buyer's order requirement. The seller has a strong incentive to decide the production quantity that maximizes its own profit. Thus, it poses a challenge on the implementation of the unilateral coordination mechanism in the presence of bidirectional option contracts. Motivated by this, we design the bilateral coordination mechanism by adopting the different perspectives from both the seller and the buyer in the presence of bidirectional option contracts.

In this paper, we consider the simplest structure of supply chain which contains one supplier and one retailer. Owing to the price increase and the demand contraction caused by inflation, bidirectional option contracts are introduced into the supply chain decision making process. The following problems will be solved in this paper.

(1) How to decide the optimal order quantity that maximizes the retailer's expected profit under inflation scenarios when bidirectional option contracts are considered?

(2) How to decide the optimal production quantity that maximizes the supplier's expected profit under inflation scenarios when bidirectional option contracts are considered?

(3) What effect does inflation have on the optimal ordering and production policies of two parties?

(4) What effect do bidirectional option contracts have on the supply chain under inflation scenarios?

(5) How to achieve supply chain coordination under inflation scenarios when bidirectional option contracts are considered?

The main contributions of our work are as follows.

(1) So far, the effect of inflation has not been addressed in supply chain applications. Moreover, there are no published papers that analyze how to use bidirectional option contracts to protect against the effect of inflation in supply chain applications. We develop supply chain models that integrate the effect of inflation and bidirectional option contracts in this paper. Our objective is to provide management insights into the effect of inflation and bidirectional option contracts on the supply chain.

(2) We refer to two different types of contracting arrangements: wholesale price contracts as well as portfolio contracts consisting of wholesale price contracts and bidirectional option contracts. We establish the Stackelberg game models and derive the optimal decision policies under these two contracts just mentioned under inflation scenarios. Then we explore the effect of inflation on supply chain and gain many management interesting results.

(3) Through taking wholesale price contracts model as the benchmark, we investigate the value of bidirectional option contracts on supply chain and discuss which kind of contract is more suitable for supply chain members under inflation scenarios. We find that portfolio contracts are more beneficial for the supply chain members and two parties involved are inclined to portfolio contracts under inflation scenarios.

(4) We design the bilateral coordination mechanism by adopting the different perspectives from both sides and derive the coordination conditions with bidirectional option contracts under inflation scenarios.

The rest part of this paper is organized as follows. The related literature is reviewed in Section 2. The formulation and assumptions throughout this paper are proposed in Section 3. In Section 4 we develop the benchmark model without considering bidirectional option contracts under inflation scenarios. In Section 5 we develop the model with considering bidirectional option contracts and derive the optimal decision policies for the supply chain members under inflation scenarios. In Section 6 we examine the role of bidirectional option contracts on the supply chain decisions and performance under inflation scenarios. Coordination conditions with bidirectional option contracts under inflation scenarios are discussed in Section 7. We provide a number example to illustrate the effect of inflation on the decision variables in Section 8. We conclude this paper and suggest the future work in Section 9.

2. Literature Review

We first review the literature relating to the effect of inflation on the daily operation management of companies. Dey et al. [13] apply the fast and elitist multiobjective genetic algorithm to deal with the deteriorating items inventory problem with two isolate warehouses under inflation. Jaggi and Khanna [14] derive the optimal ordering quantity of deteriorating items when demand rate is related to inflation rate and trade credit policy is considered. Yang et al. [15] investigate the inventory replenishment problem for deteriorating items under inflationary conditions. In this paper, shortages are allowed and consumption rate is dependent on stock. Hsieh and Dye [16] take a discounted cash flow method to study the lot-sizing problems with partial backlogging, where demand function is related to price and time. Tripathi [17] analyzes the effect of inflation dependent demand rate on the joint pricing and ordering policies with allowable delay in payments. Sarkar et al. [18] present the EMQ model to explore the effect of inflation in a finite time horizon. In this paper, demand rate is dependent on time and imperfect quality items are deemed to be unavoidable in the production process. Taheri-Tolgari et al. [19] extend the model with the imperfect inspection processes under inflationary conditions. Guria et al. [20] employ the generalized reduced gradient method to find the optimal solution to inventory problem when demand rate is dependent on inflation and selling price. Gilding [21] deduces the optimal replenishment schedule for the deteriorating items when demand rate is dependent on time. Although the influence of inflation and time value of money is considered, all these papers focus on the inventory control problem from a single enterprise perspective. The studies concerning the effect of inflation on the supply chain are still rare. Furthermore, the research integrating option contracts and the effect of inflation has not been covered.

We now review the literature relating to the employment of option contracts in the supply chain applications. Wang and Tsao [10] consider the buyer can adjust the order upwards and downwards by using bidirectional options. The appliance of bidirectional option contracts can increase the buyer's incomes when stochastic demand follows a uniformly distribution. Wang and Liu [22] adopt option contracts to coordinate the supply chain in which the dominant market position is occupied by the retailer. Gomez-Padilla and Mishina [23] investigate that the employment of bidirectional option contracts can improve both the individual party profit and the system profit for the general case of one supplier and one retailer as well as the particular case of multiple suppliers and one retailer. Zhao et al. [24] propose both channel coordination and Pareto-improvement can be realized when a cooperative game method and option contracts are taken by both sides of the supply chain. Fu et al. [25] concentrate on the procurement problem with spot market when option contracts are used. Buzacott et al. [26] apply a variance trade-off analysis to study the optimal decisions under option supply contracts, where both the supply and the demand are uncertain as well as the advanced reservation is carried out. Chen and Shen [27] study how the service level constraint makes an effect on the ordering policy and the production policy when option contracts and wholesale price contracts are used to purchase items by the retailer. Zhao et al. [11] derive the optimal ordering quantity which contains the initial order and the options order and the conditions which make the channel coordinated, when bidirectional options contracts are used and the demand fits a general distribution. Chen et al. [28] analyze how option contracts improve the performance of both parties and achieve the coordination of the supply chain system when the retailer is supposed to be loss-averse. These papers mainly focus on the role of call option contracts on the optimal decision policies of supply chain members and the coordination strategy of entity channel. The studies involving bidirectional option contracts are relatively fewer. At the same time, there are no published papers that integrate the effect of inflation and bidirectional option contracts in the supply chain management appliances.

We eventually review the literature relating to supply chain coordination with contracts. Except for some specific reasons such as random yield, the supply-side is considered to deliver the full order of the demand-side under several contracts such as revenue sharing contracts, buyback contracts and so on. In this case, the supply-side is assumed to comply with the make-to-order production policy and only the order quantity of the demand-side needs coordination. Linh and Hong [29] study how revenue sharing contracts should be set to achieve the supply chain coordination in the two-period setting. Xiao et al. [30] focus on how to coordinate a supply chain under consumer return and partial refund policy and explore the effect of the consumer return on the coordination policy. Chiu et al. [31] describe how the strategy which contains wholesale price, channel rebate, and returns should

be designed to attain the channel coordination. However, when bidirectional option contracts are used, there is no obligation for the seller to execute all the options order and so the seller can plan the production schedule that maximizes its own profit. To the best of our knowledge, all the papers considering bidirectional option contracts assume that the supply-side adopts the make-to-order policy. In these papers, the optimal production decisions are neglected and the unilateral coordination mechanism is designed to attain the channel coordination. In addition, all these papers considering bidirectional option contracts do not discuss the effect of inflation on the supply chain coordination conditions.

3. Model Formulation and Assumptions

We consider a one-period two-stage supply chain which contains one supplier and one retailer. The supplier manufactures one type of seasonal products and sells them to the downstream retailer. The retailer purchases the products from the upstream and sells them to the end consumers. The retailer obtains the products through two different types of contracts, respectively: wholesale price contracts and portfolio contracts consisting of wholesale price contracts and bidirectional option contracts. Under wholesale price contracts, the retailer orders Q_{r0} products at unit wholesale price w. After receiving the firm order, the supplier decides the production quantity, denoted as Q_{s0}, at unit production cost c. At the start of the selling season, the retailer obtains the products through the firm order. Under portfolio contracts, the retailer orders Q_{r1} products, as well as purchases m_{r1} put options at unit put option price c_{op} and q_{r1} call options at unit call option price c_{oc}. After receiving the firm order and the bidirectional options order, the supplier decides the production quantity, denoted as Q_{s2}. At the start of the selling season, the retailer obtains the products through the firm order. Once the demand does not exceed the firm order, the retailer will exercise put options to return some surplus products at unit exercise price of put option c_{ep}. Once the demand exceeds the firm order, the retailer will excise call options to obtain some additional products at unit exercise price of call option c_{ec}. The supplier incurs a unit penalty cost g_s for each exercised call option that cannot be immediately filled. Thus, supplier's unit penalty cost represents the cost to obtain an additional unit of product by expediting production or buying from an alternative source. The retailer incurs a unit shortage cost g_r for each unsatisfied demand.

In the seasonal product industry, the length of selling period is short but the length of production lead-time is long [32]. Owing to the effect of inflation, there exists a remarkable increase in the retail price and a remarkable decrease in the market demand during the production lead-time. Similar to Shanthikumar and Sumita [33] and Karmarkar [34], we assume that the length of the production lead-time, denoted as t, is a random variable over $(0, T)$, whose probability density function (PDF) is $g(t)$ and cumulative distribution function (CDF) is $G(t)$. Similar to Jaggi and Khanna [14], we assume that the retail price increases exponentially over time during the production lead-time under inflation scenarios. That is, $p(t) = pe^{\gamma t}$, where p denotes the initial retail price and γ ($\gamma > 0$) denotes the price rising factor. Similar to Xiao et al. [30] and Tripathi [17], we assume that the market demand decreases exponentially over time during the production lead-time under inflation scenarios. That is, $D(t) = \lambda e^{-\alpha t} + \xi$, where λ denotes the initial demand scale, α ($\alpha > 0$) denotes the demand contraction factor, and ξ is a random variable over $(0, +\infty)$ with probability density function (PDF) $f(x)$ and strictly increasing cumulative distribution function (CDF) $F(x)$. In addition, $F(0) = 0$, $E(x) = \mu$, and $\overline{F}(x) = 1 - F(x)$ denotes the tail distribution.

To avoid trivialities, we assume that the supply chain members are rational and self-interested and all the information available is symmetric between the supplier and the retailer. Moreover, the retailer's initial inventory is supposed to be zero and any excess product either owned by the retailer or by the supplier is ignored. Furthermore, we assume that $pe^{\gamma t} > c_{\mathrm{oc}} + c_{\mathrm{ec}} > w > c > c_{\mathrm{ep}} - c_{\mathrm{op}}$, $c_{\mathrm{oc}} + c_{\mathrm{ec}} > w > c_{\mathrm{oc}}$ and $c_{\mathrm{ec}} > c_{\mathrm{ep}}$. These conditions ensure profit for each party.

4. Wholesale Price Contracts Model

4.1. Retailer's Optimal Ordering Policy. Wholesale price contracts are widely used in practice due to the simplicity and convenience. To begin, we establish the model of wholesale price contracts and use it as the benchmark to compare with the model of portfolio contracts.

Under wholesale price contracts, only products are ordered from the upstream and the retailer's expected profit, denoted as $\Pi_{r0}(Q_{r0})$, is given by

$$\begin{aligned}\Pi_{r0}\left(Q_{r0}\right) = \int_0^T \Big\{ & p\left(t\right) \min\left[D\left(t\right), Q_{r0}\right] \\ & - wQ_{r0} - g_r \left[D\left(t\right) - Q_{r0}\right]^+ \Big\}\, g\left(t\right) dt.\end{aligned} \tag{1}$$

The first term is the sales revenue. The second term is the costs of ordering products, and the last term is the shortage cost. With some algebra, the function stated above can be simplified as

$$\begin{aligned}\Pi_{r0}\left(Q_{r0}\right) = {} & Q_{r0} \int_0^T \left(pe^{\gamma t} + g_r - w\right) g\left(t\right) dt \\ & - g_r \int_0^T \lambda e^{-\alpha t} g\left(t\right) dt \\ & - g_r \mu - \int_0^T \int_0^{Q_{r0} - \lambda e^{-\alpha t}} \left(pe^{\gamma t} + g_r\right) \\ & \cdot F\left(x\right) g\left(t\right) dx\, dt.\end{aligned} \tag{2}$$

In this case, the retailer's problem is to determine how many products to order so as to maximize its expected profit function (2), which yields the following proposition.

Proposition 1. *Under wholesale price contracts, the retailer's optimal firm order quantity Q_{r0}^* satisfies*

$$\int_0^T \left(pe^{\gamma t} + g_r\right) \overline{F}\left(Q_{r0}^* - \lambda e^{-\alpha t}\right) g\left(t\right) dt = w. \tag{3}$$

Proof. From (2), we can derive that $d\Pi_{r0}(Q_{r0})/dQ_{r0} = \int_0^T (pe^{\gamma t}+g_r-w)g(t)dt - \int_0^T (pe^{\gamma t}+g_r)F(Q_{r0}-\lambda e^{-\alpha t})g(t)dt$ and $d^2\Pi_{r0}(Q_{r0})/dQ_{r0}^2 = -\int_0^T (pe^{\gamma t}+g_r) f(Q_{r0}-\lambda e^{-\alpha t})g(t)dt < 0$. It follows that $\Pi_{r0}(Q_{r0})$ is a strictly concave function of Q_{r0}. Thus, there exists a unique optimal firm order quantity Q_{r0}^* to maximize $\Pi_{r0}(Q_{r0})$. Let $d\Pi_{r0}(Q_{r0})/dQ_{r0} = 0$. We can characterize the retailer's optimal firm order quantity under wholesale price contracts as follows: $\int_0^T (pe^{\gamma t} + g_r)\overline{F}(Q_{r0}^* - \lambda e^{-\alpha t})g(t)dt = w$. This completes the proof. □

From Proposition 1, the following corollary can be obtained.

Corollary 2. *Under wholesale price contracts, Q_{r0}^* is decreasing in (α, w) and increasing in γ.*

Proof. Set $L_0(Q_{r0}^*) = \int_0^T (pe^{\gamma t} + g_r)\overline{F}(Q_{r0}^* - \lambda e^{-\alpha t})g(t)dt - w$, we can deduce that

$$\begin{aligned}
\frac{\partial Q_{r0}^*}{\partial \alpha} &= -\frac{\partial L_0(Q_{r0}^*)/\partial \alpha}{\partial L_0(Q_{r0}^*)/\partial Q_{r0}^*} \\
&= -\frac{\int_0^T \lambda t e^{-\alpha t}(pe^{\gamma t} + g_r) f(Q_{r0} - \lambda e^{-\alpha t}) g(t)\, dt}{\int_0^T (pe^{\gamma t} + g_r) f(Q_{r0} - \lambda e^{-\alpha t}) g(t)\, dt} < 0, \\
\frac{\partial Q_{r0}^*}{\partial w} &= -\frac{\partial L_0(Q_{r0}^*)/\partial w}{\partial L_0(Q_{r0}^*)/\partial Q_{r0}^*} \\
&= -\frac{1}{\int_0^T (pe^{\gamma t} + g_r) f(Q_{r0} - \lambda e^{-\alpha t}) g(t)\, dt} < 0, \\
\frac{\partial Q_{r0}^*}{\partial \gamma} &= -\frac{\partial L_0(Q_{r0}^*)/\partial \gamma}{\partial L_0(Q_{r0}^*)/\partial Q_{r0}^*} \\
&= \frac{\int_0^T pte^{\gamma t}\overline{F}(Q_{r0}^* - \lambda e^{-\alpha t}) g(t)\, dt}{\int_0^T (pe^{\gamma t} + g_r) f(Q_{r0} - \lambda e^{-\alpha t}) g(t)\, dt} > 0.
\end{aligned} \tag{4}$$

This completes the proof. □

From Corollary 2, we can see that when the demand contraction factor grows, the retailer will reduce the firm order quantity. When the price rising factor grows, the retailer will enlarge the firm order quantity. Since both price and demand change in two opposite directions due to the effect of inflation, it poses a challenge for the retailer to decide whether to increase or decrease the firm order. When the increase in the retail price is more obvious, the retailer will increase the firm order quantity. When the decrease in the market demand is more obvious, the retailer will decrease the firm order quantity. From the above corollary, we can also see that the supplier can improve and adjust the retailer's ordering strategy by changing the parameter of wholesale price contracts. When wholesale price w increase, the retailer will purchase fewer products. When unit wholesale price w decrease, the retailer will purchase more products. Obviously, wholesale price contracts cannot provide a flexible mechanism for the retailer to protect against the effect of inflation.

4.2. Supplier's Optimal Production Policy. Under wholesale price contracts, the supplier decides how many products to manufacture according to the retailer's order. At this time, the supplier's optimal production quantity is $Q_{s0}^* = Q_{r0}^*$. Then, the supplier's expected profit, denoted as $\Pi_{s0}(Q_{s0}^*)$, is given by

$$\Pi_{s0}(Q_{s0}^*) = (w - c) Q_{s0}^* = (w - c) Q_{r0}^*. \tag{5}$$

5. Portfolio Contracts Model

5.1. Retailer's Optimal Ordering Policy. Under portfolio contracts, both products and bidirectional options are purchased from the upstream. In order to propose a more generic model, we assume that the call options order quantity is not equivalent to the put options order quantity. Then, the retailer's expected profit, denoted as $\Pi_{r1}(m_{r1}, Q_{r1}, q_{r1})$, is given by

$$\begin{aligned}
&\Pi_{r1}(m_{r1}, Q_{r1}, q_{r1}) \\
&\quad = \int_0^T \{p(t)\min[D(t), Q_{r1} + q_{r1}] - wQ_{r1} - c_{\mathrm{op}} m_{r1} \\
&\qquad + c_{\mathrm{ep}} \min[(Q_{r1} - D(t))^+, m_{r1}] - c_{\mathrm{oc}} q_{r1} \\
&\qquad - c_{\mathrm{ec}} \min[(D(t) - Q_{r1})^+, q_{r1}] \\
&\qquad - g_r [D(t) - (Q_{r1} + q_{r1})]^+\} g(t)\, dt.
\end{aligned} \tag{6}$$

The first term is the sales revenue. The second term is the costs of ordering products. The third term is the costs of purchasing put options. The fourth term is the incomes of exercising put options. The fifth term is the costs of purchasing call options. The sixth term is the costs of exercising call options, and the last term is the shortage cost. Let $Q_1 = Q_{r1} + q_{r1}$ and $R_1 = Q_{r1} - m_{r1}$. Note that determining (m_{r1}, Q_{r1}, q_{r1}) is equivalent to determining (R_1, Q_{r1}, Q_1). With some algebra, the function described above can be simplified as

$$\begin{aligned}
&\Pi_{r1}(R_1, Q_{r1}, Q_1) \\
&\quad = c_{\mathrm{op}} R_1 - c_{\mathrm{ep}} \int_0^T \int_0^{R_1 - \lambda e^{-\alpha t}} F(x) g(t)\, dx\, dt \\
&\qquad + (c_{\mathrm{oc}} + c_{\mathrm{ec}} - w - c_{\mathrm{op}}) Q_{r1} \\
&\qquad - (c_{\mathrm{ec}} - c_{\mathrm{ep}}) \int_0^T \int_0^{Q_{r1} - \lambda e^{-\alpha t}} F(x) g(t)\, dx\, dt \\
&\qquad + Q_1 \int_0^T (pe^{\gamma t} + g_r - c_{\mathrm{oc}} - c_{\mathrm{ec}}) g(t)\, dt \\
&\qquad - g_r \int_0^T \lambda e^{-\alpha t} g(t)\, dt \\
&\qquad - g_r \mu - \int_0^T \int_0^{Q_1 - \lambda e^{-\alpha t}} (pe^{\gamma t} + g_r - c_{\mathrm{ec}}) F(x) g(t)\, dx\, dt.
\end{aligned} \tag{7}$$

In this case, the retailer's problem is to determine how many products to order and how many bidirectional options to purchase so as to maximize its expected profit function (7), which yields the following proposition.

Proposition 3. *Under portfolio contracts, the retailer's optimal put options order quantity m_{r1}^*, the optimal firm order quantity Q_{r1}^*, and the optimal call options order quantity q_{r1}^* satisfy*

$$\begin{gathered}\int_0^T F\left(Q_{r1}^* - m_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt = \frac{c_{op}}{c_{ep}},\\ \int_0^T F\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt = \frac{c_{oc} + c_{ec} - w - c_{op}}{c_{ec} - c_{ep}},\\ \int_0^T \left(pe^{\gamma t} + g_r - c_{ec}\right) \overline{F}\left(Q_{r1}^* + q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt = c_{oc}.\end{gathered} \tag{8}$$

Proof. From (7), we can derive that

$$\begin{aligned}\frac{\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1} &= c_{op} - c_{ep} \int_0^T F\left(R_1 - \lambda e^{-\alpha t}\right) g(t)\, dt,\\ \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1^2} &= -c_{ep} \int_0^T f\left(R_1 - \lambda e^{-\alpha t}\right) g(t)\, dt < 0,\\ \frac{\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1}} &= \left(c_{oc} + c_{ec} - w - c_{op}\right) - \left(c_{ec} - c_{ep}\right) \int_0^T F\left(Q_{r1} - \lambda e^{-\alpha t}\right) g(t)\, dt,\\ \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1}^2} &= -\left(c_{ec} - c_{ep}\right) \int_0^T f\left(Q_{r1} - \lambda e^{-\alpha t}\right) g(t)\, dt < 0,\\ \frac{\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1} &= \int_0^T \left(pe^{\gamma t} + g_r - c_{oc} - c_{ec}\right) g(t)\, dt - \int_0^T \left(pe^{\gamma t} + g_r - c_{ec}\right) F\left(Q_1 - \lambda e^{-\alpha t}\right) g(t)\, dt,\\ \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1^2} &= -\int_0^T \left(pe^{\gamma t} + g_r - c_{ec}\right) f\left(Q_1 - \lambda e^{-\alpha t}\right) g(t)\, dt < 0.\end{aligned} \tag{9}$$

Besides, we can derive that

$$\begin{aligned}\frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1 \partial Q_{r1}} &= \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1} \partial R_1} = 0,\\ \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1 \partial Q_1} &= \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1 \partial R_1} = 0,\\ \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1} \partial Q_1} &= \frac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1 \partial Q_{r1}} = 0.\end{aligned} \tag{10}$$

Thus, the Hessian Matrix of the retailer's expected profit function under portfolio contracts is given by

$$\begin{aligned}&\begin{bmatrix}\dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1^2} & \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1 \partial Q_{r1}} & \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial R_1 \partial Q_1}\\ \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1} \partial R_1} & \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1}^2} & \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1} \partial Q_1}\\ \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1 \partial R_1} & \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1 \partial Q_{r1}} & \dfrac{\partial^2 \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1^2}\end{bmatrix}\\ &= \begin{bmatrix}-c_{ep}\int_0^T f\left(R_1 - \lambda e^{-\alpha t}\right) g(t)\, dt & 0 & 0\\ 0 & -\left(c_{ec} - c_{ep}\right)\int_0^T f\left(Q_{r1} - \lambda e^{-\alpha t}\right) g(t)\, dt & 0\\ 0 & 0 & -\int_0^T \left(pe^{\gamma t} + g_r - c_{ec}\right) f\left(Q_1 - \lambda e^{-\alpha t}\right) g(t)\, dt\end{bmatrix}.\end{aligned} \tag{11}$$

Obviously, this matrix is said to be negative definite. It follows that $\Pi_{r1}(R_1, Q_{r1}, Q_1)$ is a strictly concave function of R_1, Q_{r1}, and Q_1. Let $\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)/\partial R_1 = 0$, $\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)/\partial Q_{r1} = 0$, and $\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)/\partial Q_1 = 0$. We can deduce the equations as follows:

$$\int_0^T F\left(R_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt = \frac{c_{\text{op}}}{c_{\text{ep}}},$$

$$\int_0^T F\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt = \frac{c_{\text{oc}} + c_{\text{ec}} - w - c_{\text{op}}}{c_{\text{ec}} - c_{\text{ep}}}, \tag{12}$$

$$\int_0^T \left(pe^{\gamma t} + g_r - c_{\text{ec}}\right) \overline{F}\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt = c_{\text{oc}}.$$

Therefore, we have that $q_{r1}^* = Q_1^* - Q_{r1}^*$ and $m_{r1}^* = Q_{r1}^* - R_1^*$. This completes the proof. □

Proposition 3 shows that the options order quantity as either put or call must be greater than zero. Otherwise, the retailer has no right to excise bidirectional options. Note that $Q_1^* > Q_{r1}^* > R_1^*$ is equivalent to $c_{\text{ep}} - c_{\text{op}} + ((pe^{\gamma t} + g_r - c_{\text{ep}})/(pe^{\gamma t} + g_r - c_{\text{ec}}))c_{\text{oc}} < w < c_{\text{oc}} + c_{\text{ec}} - (c_{\text{ec}}/c_{\text{ep}})c_{\text{op}}$. This inequality implies that only when unit wholesale price is limited in a certain price range, the retailer feels willing to purchase bidirectional options. Besides, $Q_1^* > R_1^*$ is equivalent to $c_{\text{ep}} - c_{\text{op}} > (c_{\text{ep}}/(pe^{\gamma t} + g_r - c_{\text{ec}}))c_{\text{oc}}$. This inequality implies that unit exercise price of put option must be greater than unit put option price; otherwise the retailer will refuse to order any put option. The proposition mentioned above leads to the following corollary.

Corollary 4. *Under portfolio contracts, R_1^* is decreasing in (α, c_{ep}), increasing in c_{op}, and constant in γ; Q_{r1}^* is decreasing in (α, w, c_{op}), increasing in (c_{oc}, c_{ec}, c_{ep}), and constant in γ; Q_1^* is decreasing in (α, c_{oc}, c_{ec}), increasing in γ, and constant in (w, c_{op}, c_{ep}).*

Proof. Set $L_1(R_1^*) = c_{\text{op}} - c_{\text{ep}} \int_0^T F(R_1^* - \lambda e^{-\alpha t}) g(t) dt$, we can deduce that

$$\begin{aligned}
\frac{\partial R_1^*}{\partial \alpha} &= -\frac{\partial L_1(R_1^*)/\partial \alpha}{\partial L_1(R_1^*)/\partial R_1^*} \\
&= -\frac{\int_0^T \lambda t e^{-\alpha t} f\left(R_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt}{\int_0^T f\left(R_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt} < 0, \\
\frac{\partial R_1^*}{\partial c_{\text{op}}} &= -\frac{\partial L_1(R_1^*)/\partial c_{\text{op}}}{\partial L_1(R_1^*)/\partial R_1^*} \\
&= \frac{1}{c_{\text{ep}} \int_0^T f\left(R_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt} > 0, \\
\frac{\partial R_1^*}{\partial c_{\text{ep}}} &= -\frac{\partial L_1(R_1^*)/\partial c_{\text{ep}}}{\partial L_1(R_1^*)/\partial R_1^*} \\
&= -\frac{\int_0^T F\left(R_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt}{c_{\text{ep}} \int_0^T f\left(R_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt} < 0.
\end{aligned} \tag{13}$$

Set $L_2(Q_{r1}^*) = (c_{\text{oc}} + c_{\text{ec}} - w - c_{\text{op}}) - (c_{\text{ec}} - c_{\text{ep}}) \int_0^T F(Q_{r1}^* - \lambda e^{-\alpha t}) g(t) dt$, we can deduce that

$$\begin{aligned}
\frac{\partial Q_{r1}^*}{\partial \alpha} &= -\frac{\partial L_2(Q_{r1}^*)/\partial \alpha}{\partial L_2(Q_{r1}^*)/\partial Q_{r1}^*} \\
&= -\frac{\int_0^T \lambda t e^{-\alpha t} f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt}{\int_0^T f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt} < 0, \\
\frac{\partial Q_{r1}^*}{\partial c_{\text{oc}}} &= -\frac{\partial L_2(Q_{r1}^*)/\partial c_{\text{oc}}}{\partial L_2(Q_{r1}^*)/\partial Q_{r1}^*} \\
&= \frac{1}{\left(c_{\text{ec}} - c_{\text{ep}}\right) \int_0^T f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt} > 0, \\
\frac{\partial Q_{r1}^*}{\partial c_{\text{ec}}} &= -\frac{\partial L_2(Q_{r1}^*)/\partial c_{\text{ec}}}{\partial L_2(Q_{r1}^*)/\partial Q_{r1}^*} \\
&= \frac{\int_0^T \overline{F}\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt}{\left(c_{\text{ec}} - c_{\text{ep}}\right) \int_0^T f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt} > 0, \\
\frac{\partial Q_{r1}^*}{\partial w} &= -\frac{\partial L_2(Q_{r1}^*)/\partial w}{\partial L_2(Q_{r1}^*)/\partial Q_{r1}^*} \\
&= -\frac{1}{\left(c_{\text{ec}} - c_{\text{ep}}\right) \int_0^T f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt} < 0, \\
\frac{\partial Q_{r1}^*}{\partial c_{\text{op}}} &= -\frac{\partial L_2(Q_{r1}^*)/\partial c_{\text{op}}}{\partial L_2(Q_{r1}^*)/\partial Q_{r1}^*} \\
&= -\frac{1}{\left(c_{\text{ec}} - c_{\text{ep}}\right) \int_0^T f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt} < 0, \\
\frac{\partial Q_{r1}^*}{\partial c_{\text{ep}}} &= -\frac{\partial L_2(Q_{r1}^*)/\partial c_{\text{ep}}}{\partial L_2(Q_{r1}^*)/\partial Q_{r1}^*} \\
&= \frac{\int_0^T F\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt}{\left(c_{\text{ec}} - c_{\text{ep}}\right) \int_0^T f\left(Q_{r1}^* - \lambda e^{-\alpha t}\right) g(t)\, dt} > 0.
\end{aligned} \tag{14}$$

Set $L_3(Q_1^*) = \int_0^T (pe^{\gamma t} + g_r - c_{\text{ec}}) \overline{F}(Q_1^* - \lambda e^{-\alpha t}) g(t) dt - c_{\text{oc}}$, we can deduce that

$$\begin{aligned}
\frac{\partial Q_1^*}{\partial \alpha} &= -\frac{\partial L_3(Q_1^*)/\partial \alpha}{\partial L_3(Q_1^*)/\partial Q_1^*} \\
&= -\frac{\int_0^T \lambda t e^{-\alpha t} \left(pe^{\gamma t} + g_r - c_{\text{ec}}\right) f\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt}{\int_0^T \left(pe^{\gamma t} + g_r - c_{\text{ec}}\right) f\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\, dt} \\
&< 0, \\
\frac{\partial Q_1^*}{\partial \gamma} &= -\frac{\partial L_3(Q_1^*)/\partial \gamma}{\partial L_3(Q_1^*)/\partial Q_1^*}
\end{aligned}$$

$$= \frac{\int_0^T pte^{\gamma t}\overline{F}\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\,dt}{\int_0^T \left(pe^{\gamma t} + g_r - c_{\text{ec}}\right) f\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\,dt} > 0,$$

$$\frac{\partial Q_1^*}{\partial c_{\text{oc}}} = -\frac{\partial L_3\left(Q_1^*\right)/\partial c_{\text{oc}}}{\partial L_3\left(Q_1^*\right)/\partial Q_1^*}$$

$$= -\frac{1}{\int_0^T \left(pe^{\gamma t} + g_r - c_{\text{ec}}\right) f\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\,dt} < 0,$$

$$\frac{\partial Q_1^*}{\partial c_{\text{ec}}} = -\frac{\partial L_3\left(Q_1^*\right)/\partial c_{\text{ec}}}{\partial L_3\left(Q_1^*\right)/\partial Q_1^*}$$

$$= -\frac{\int_0^T \overline{F}\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\,dt}{\int_0^T \left(pe^{\gamma t} + g_r - c_{\text{ec}}\right) f\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\,dt} < 0. \tag{15}$$

This completes the proof. □

From Corollary 4, we find that influenced by an increase in retail price, the retailer will enlarge the call options order quantity to meet the needs of target market. It is worth noting that the retailer does not attempt to make any alteration in the put options order quantity and the firm order quantity under this situation. Influenced by a decrease in market demand, the retailer will lessen the optimal firm order quantity to reduce the purchasing costs. Besides, the retailer needs to narrow the gap between the put options order and the firm order as well as cut down the sum total of the firm order and the call options order. Because the price and the demand move in the two opposite direction to accommodate the inflationary pressure, it is important to strike a balance between surging price and slowing demand by adjusting the portfolio ordering strategy. At this time, the retailer needs to estimate the probability of the price uptrend and the demand downtrend. When the price has experienced a stronger upward momentum, the retailer just needs to increase the call options order. When the demand has experienced a stronger downward momentum, the retailer needs to decrease the firm order. At the same time, the retailer needs to reduce the distance between the put options order and the firm order as well as the entire quantity consisting of the firm order and the call options order. From the above corollary, we can also see that the supplier can improve and adjust the retailer's ordering strategy by changing the parameters of portfolio contracts. When unit call option price c_{oc} and unit exercise price of call option c_{ec} increase, the retailer will purchase fewer call options. When unit wholesale price w increase, the retailer will purchase more call options. Obviously, the presence of call options provides a flexible mechanism for the retailer to reduce risks. When unit put option price c_{op} increase, the retailer will purchase fewer put options. When unit exercise price of put option c_{ep} increase, the retailer will purchase more put options. Obviously, the presence of put options provides a flexible mechanism for the retailer to reduce risks. Comparing with the conclusion in Corollary 2, we find that portfolio contracts containing bidirectional options provide more flexibility for the retailer to protect against the effect of inflation than wholesale price contacts.

5.2. Supplier's Optimal Production Policy. Since the supplier is required to deliver the firm order to the downstream retailer at the start of the selling season, it is necessary for the supplier to make the production quantity consistent with the firm order quantity at least. Moreover, since the retailer is not asked to exercise all the call options, it is feasible for the supplier to make the production quantity less than the sum total of the firm order and the call options order. That is, $Q_{r1}^* \le Q_{s1} \le Q_1^*$. Then, the supplier's expected profit, denoted as $\Pi_{s1}(Q_{s1})$, is given by

$$\begin{aligned}
&\Pi_{s1}\left(Q_{s1}\right) \\
&\quad = \int_0^T \Big\{ wQ_{r1}^* + c_{\text{op}}\left(Q_{r1}^* - R_1^*\right) \\
&\qquad - c_{\text{ep}} \min\left[\left(Q_{r1}^* - D(t)\right)^+, Q_{r1}^* - R_1^*\right] \\
&\qquad + c_{\text{oc}}\left(Q_1^* - Q_{r1}^*\right) \\
&\qquad + c_{\text{ec}} \min\left[\left(D(t) - Q_{r1}^*\right)^+, Q_1^* - Q_{r1}^*\right] \\
&\qquad - g_s\left[\min\left(D(t), Q_1^*\right) - Q_{s1}\right]^+ - cQ_{s1} \Big\}\, g(t)\,dt.
\end{aligned} \tag{16}$$

The first term is the incomes realized from selling products. The second term is the incomes realized from selling put options. The third term is the costs of exercising put options. The fourth term is the incomes realized from selling call options. The fifth term is the incomes realized from exercising call options. The sixth term is the penalty cost, and the last term is the costs of manufacturing products. With some algebra, the function listed above can be simplified as

$$\begin{aligned}
\Pi_{s1}\left(Q_{s1}\right) &= \left(c_{\text{oc}} + c_{\text{ec}} - g_s\right) Q_1^* \\
&\quad + \left(g_s - c_{\text{ec}}\right) \int_0^T \int_0^{Q_1^* - \lambda e^{-\alpha t}} F(x)\, g(t)\,dx\,dt \\
&\quad + \left(w + c_{\text{op}} - c_{\text{oc}} - c_{\text{ec}}\right) Q_{r1}^* \\
&\quad + \left(c_{\text{ec}} - c_{\text{ep}}\right) \int_0^T \int_0^{Q_{r1}^* - \lambda e^{-\alpha t}} F(x)\, g(t)\,dx\,dt \\
&\quad - c_{\text{op}} R_1^* + c_{\text{ep}} \int_0^T \int_0^{R_1^* - \lambda e^{-\alpha t}} F(x)\, g(t)\,dx\,dt \\
&\quad + \left(g_s - c\right) Q_{s1} - g_s \int_0^T \int_0^{Q_{s1} - \lambda e^{-\alpha t}} F(x)\, g(t)\,dx\,dt.
\end{aligned} \tag{17}$$

Hence, the supplier's decision problem under portfolio contracts can be expressed as follows:

$$\begin{aligned}
&\max_{Q_{s1} > 0} \quad \Pi_{s1}\left(Q_{s1}\right) \\
&\text{s.t.} \quad Q_{r1}^* \le Q_{s1} \le Q_1^*.
\end{aligned} \tag{18}$$

Based on the above problem, we derive the following proposition.

Proposition 5. *Under portfolio contracts, the supplier's optimal production quantity* Q_{s1}^* *satisfies*

$$Q_{s1}^* = \begin{cases} Q_{r1}^* & Q_{s1}^{\varphi} \le Q_{r1}^* \\ Q_{s1}^{\varphi} & Q_{r1}^* < Q_{s1}^{\varphi} < Q_1^* \\ Q_1^* & Q_{s1}^{\varphi} \ge Q_1^*, \end{cases} \tag{19}$$

where $\int_0^T F(Q_{s1}^{\varphi} - \lambda e^{-\alpha t}) g(t)dt = (g_s - c)/g_s$.

Proof. From (17), we can derive that $d\Pi_{s1}(Q_{s1})/dQ_{s1} = (g_s - c) - g_s \int_0^T F(Q_{s1} - \lambda e^{-\alpha t}) g(t)dt$ and $d^2\Pi_{s1}(Q_{s1})/dQ_{s1}^2 = -g_s \int_0^T f(Q_{s1} - \lambda e^{-\alpha t}) g(t)dt < 0$. It follows that $\Pi_{s1}(Q_{s1})$ is a strictly concave function of Q_{s1}. Let $d\Pi_{s1}(Q_{s1})/dQ_{s1} = 0$. We can derive the optimal solution of the unconstrained problem as $\int_0^T F(Q_{s1}^{\varphi} - \lambda e^{-\alpha t}) g(t)dt = (g_s - c)/g_s$. Considering the constraint in (18), the supplier's optimal production quantity under portfolio contracts satisfies

$$Q_{s1}^* = \begin{cases} Q_{r1}^* & Q_{s1}^{\varphi} \le Q_{r1}^* \\ Q_{s1}^{\varphi} & Q_{r1}^* < Q_{s1}^{\varphi} < Q_1^* \\ Q_1^* & Q_{s1}^{\varphi} \ge Q_1^*. \end{cases} \tag{20}$$

This completes the proof. □

Proposition 5 shows that since there is no guarantee that all the call options will be exercised by the retailer during the selling season, the supplier does not adopt the make-to-order production policy but determine the production quantity with the aim of maximizing profit. Considering the existing production constraint, the optimal production quantity is an interval value under portfolio contracts. If $Q_{s1}^{\varphi} \le Q_{r1}^*$, the best output choice for the supplier is the quantity that is equivalent to the firm order. If $Q_{r1}^* < Q_{s1}^{\varphi} < Q_1^*$, the constraint condition has no effect on the solution. If $Q_{s1}^{\varphi} \ge Q_1^*$, the best output choice for the supplier is the quantity that is equivalent to the total amount consisting of the firm order and the call options order.

6. The Role of Bidirectional Option Contracts

Based on the above demonstration, we acquire the optimal order amount for the retailer and the optimal production amount for the supplier under two different contracts. In this section, we will discuss the role of bidirectional option contracts on supply chain decisions and performance through a comparative analysis.

6.1. The Role of Bidirectional Option Contracts on Supply Chain Decisions. By comparing with the retailer's optimal order quantity before and after the introduction of bidirectional option contracts, we explore the role of this contractual arrangement on the retailer's ordering decision.

Proposition 6. *The retailer's optimal ordering policies under different contracts satisfy the following.* (1) *When* $c_{ep} - c_{op} + ((pe^{\gamma t} + g_r - c_{ep})/(pe^{\gamma t} + g_r - c_{ec}))c_{oc} < w < ((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r + c_{ep} - c_{ec}))(c_{oc} + c_{ep} - c_{op})$, *it follows that* $Q_1^* > Q_{r1}^* > Q_{r0}^*$; (2) *when* $((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r + c_{ep} - c_{ec}))(c_{oc} + c_{ep} - c_{op}) < w < c_{oc} + c_{ec} - (c_{ec}/c_{ep})c_{op}$, *it follows that* $Q_1^* > Q_{r0}^* > Q_{r1}^*$.

Proof. First, we compare Q_1^* and Q_{r0}^*. Through the above observation, we find that it is fairly obvious that $w > ((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r - c_{ec}))c_{oc}$ given $w > c_{ep} - c_{op} + ((pe^{\gamma t} + g_r - c_{ep})/(pe^{\gamma t} + g_r - c_{ec}))c_{oc}$ and $c_{ep} - c_{op} > (c_{ep}/(pe^{\gamma t} + g_r - c_{ec}))c_{oc}$. Then

$$\begin{aligned} &\left.\frac{\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_1}\right|_{Q_1 = Q_{r0}^*} \\ &= \int_0^T \left(pe^{\gamma t} + g_r - c_{oc} - c_{ec}\right) g(t)\,dt \\ &\quad - \int_0^T \left(pe^{\gamma t} + g_r - c_{ec}\right) F\left(Q_{r0}^* - \lambda e^{-\alpha t}\right) g(t)\,dt \\ &> \int_0^T \frac{pe^{\gamma t} + g_r - c_{ec}}{pe^{\gamma t} + g_r} \\ &\quad \cdot \left[\left(pe^{\gamma t} + g_r - w\right) \right. \\ &\quad \left. - \left(pe^{\gamma t} + g_r\right) F\left(Q_{r0}^* - \lambda e^{-\alpha t}\right)\right] g(t)\,dt = 0. \end{aligned} \tag{21}$$

Recall $\Pi_{r1}(R_1, Q_{r1}, Q_1)$ is a strictly concave function of Q_1. It follows that $Q_1^* > Q_{r0}^*$.

Next, we compare the sizes of Q_{r0}^* and Q_{r1}^*. If

$$\begin{aligned} &c_{ep} - c_{op} + \frac{pe^{\gamma t} + g_r - c_{ep}}{pe^{\gamma t} + g_r - c_{ec}} c_{oc} \\ &\quad < w < \frac{pe^{\gamma t} + g_r}{pe^{\gamma t} + g_r + c_{ep} - c_{ec}} \left(c_{oc} + c_{ep} - c_{op}\right), \end{aligned} \tag{22}$$

then

$$\begin{aligned} &\left.\frac{\partial \Pi_{r1}(R_1, Q_{r1}, Q_1)}{\partial Q_{r1}}\right|_{Q_{r1} = Q_{r0}^*} \\ &= \left(c_{oc} + c_{ec} - w - c_{op}\right) \\ &\quad - \left(c_{ec} - c_{ep}\right) \int_0^T F\left(Q_{r0}^* - \lambda e^{-\alpha t}\right) g(t)\,dt \\ &> \int_0^T \frac{c_{ec} - c_{ep}}{pe^{\gamma t} + g_r} \\ &\quad \cdot \left[\left(pe^{\gamma t} + g_r - w\right) \right. \\ &\quad \left. - \left(pe^{\gamma t} + g_r\right) F\left(Q_{r0}^* - \lambda e^{-\alpha t}\right)\right] g(t)\,dt = 0. \end{aligned} \tag{23}$$

Recall $\Pi_{r1}(R_1, Q_{r1}, Q_1)$ is a strictly concave function of Q_{r1}. It follows that $Q^*_{r1} > Q^*_{r0}$. If

$$\frac{pe^{\gamma t} + g_r}{pe^{\gamma t} + g_r + c_{\mathrm{ep}} - c_{\mathrm{ec}}}\left(c_{\mathrm{oc}} + c_{\mathrm{ep}} - c_{\mathrm{op}}\right) < w < c_{\mathrm{oc}} + c_{\mathrm{ec}} - \frac{c_{\mathrm{ec}}}{c_{\mathrm{ep}}} c_{\mathrm{op}}, \tag{24}$$

then

$$\begin{aligned}
&\left.\frac{d\Pi_{r0}\left(Q_{r0}\right)}{dQ_{r0}}\right|_{Q_{r0}=Q^*_{r1}} \\
&\quad = \int_0^T \left(pe^{\gamma t} + g_r - w\right) g(t)\, dt \\
&\quad\quad - \int_0^T \left(pe^{\gamma t} + g_r\right) F\left(Q^*_{r1} - \lambda e^{-\alpha t}\right) g(t)\, dt \\
&\quad > \int_0^T \frac{pe^{\gamma t} + g_r}{c_{\mathrm{ec}} - c_{\mathrm{ep}}} \\
&\quad\quad \cdot \left[\left(c_{\mathrm{oc}} + c_{\mathrm{ec}} - w - c_{\mathrm{op}}\right)\right. \\
&\quad\quad \left. - \left(c_{\mathrm{ec}} - c_{\mathrm{ep}}\right) F\left(Q^*_{r1} - \lambda e^{-\alpha t}\right)\right] g(t)\, dt = 0.
\end{aligned} \tag{25}$$

Recall $\Pi_{r0}(Q_{r0})$ is a strictly concave function of Q_{r0}. It follows that $Q^*_{r0} > Q^*_{r1}$. This completes the proof. □

This proposition implies that when bidirectional option contracts are employed, the retailer can obtain additional products by excising call options when the demand exceeds the firm order and return surplus products by excising put options when the demand does not exceed the firm order. It means the retailer can adjust the order quantity in either direction under portfolio contracts. Thus, the aggregate quantity which contains the firm order and the call options order under portfolio contracts are always higher than the firm order under wholesale price contracts. Besides, it is noteworthy that call options give the retailer the right to reorder and its existence will suppress the purchasing behavior, meanwhile put options give the retailer the right to return and its existence will stimulate the buying behavior. However, bidirectional options give the retailer the right to reorder and return at the same time. Thus, we can see that only when certain conditions are satisfied, the firm order quantity under portfolio contracts is higher than that under wholesale price contracts.

In comparison with the supplier's optimal output quantity before and after the introduction of bidirectional option contracts, we explore the role of this contractual arrangement on the supplier's production decision.

Proposition 7. *The supplier's optimal production policies under different contracts satisfy the following.* (1) *When* $((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r + c_{ep} - c_{ec}))(c_{oc} + c_{ep} - c_{op}) < w < c_{oc} + c_{ec} - (c_{ec}/c_{ep})c_{op}$*, the following conclusions can be derived: if* $Q^{\varphi}_{s1} \in (0, Q^*_{r0})$*, then* $Q^*_{s1} < Q^*_{s0}$*; If* $Q^{\varphi}_{s1} \in (Q^*_{r0}, +\infty)$*, then* $Q^*_{s1} > Q^*_{s0}$*.* (2) *When* $c_{ep} - c_{op} + ((pe^{\gamma t} + g_r - c_{ep})/(pe^{\gamma t} + g_r - c_{ec}))c_{oc} < w < ((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r + c_{ep} - c_{ec}))(c_{oc} + c_{ep} - c_{op})$*, it follows that* $Q^*_{s1} > Q^*_{s0}$*.*

Proof. When $((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r + c_{\mathrm{ep}} - c_{\mathrm{ec}}))(c_{\mathrm{oc}} + c_{\mathrm{ep}} - c_{\mathrm{op}}) < w < c_{\mathrm{oc}} + c_{\mathrm{ec}} - (c_{\mathrm{ec}}/c_{\mathrm{ep}})c_{\mathrm{op}}$ is satisfied, we see that $Q^*_1 > Q^*_{r0} > Q^*_{r1}$. At this time, if $Q^{\varphi}_{s1} \in (0, Q^*_{r1})$, then $Q^*_{s1} = Q^*_{r1}$ and $Q^*_{s0} = Q^*_{r0}$. We can deduce that $Q^*_{s1} < Q^*_{s0}$. If $Q^{\varphi}_{s1} \in (Q^*_{r1}, Q^*_{r0})$, then $Q^*_{s1} = Q^{\varphi}_{s1}$ and $Q^*_{s0} = Q^*_{r0}$. We can deduce that $Q^*_{s1} < Q^*_{s0}$. If $Q^{\varphi}_{s1} \in (Q^*_{r0}, Q^*_1)$, then $Q^*_{s1} = Q^{\varphi}_{s1}$ and $Q^*_{s0} = Q^*_{r0}$. We can deduce that $Q^*_{s1} > Q^*_{s0}$. If $Q^{\varphi}_{s1} \in (Q^*_1, +\infty)$, then $Q^*_{s1} = Q^*_1$ and $Q^*_{s0} = Q^*_{r0}$. We can deduce that $Q^*_{s1} > Q^*_{s0}$. When $c_{\mathrm{ep}} - c_{\mathrm{op}} + ((pe^{\gamma t} + g_r - c_{\mathrm{ep}})/(pe^{\gamma t} + g_r - c_{\mathrm{ec}}))c_{\mathrm{oc}} < w < ((pe^{\gamma t} + g_r)/(pe^{\gamma t} + g_r + c_{\mathrm{ep}} - c_{\mathrm{ec}}))(c_{\mathrm{oc}} + c_{\mathrm{ep}} - c_{\mathrm{op}})$ is satisfied, we realize that $Q^*_1 > Q^*_{r1} > Q^*_{r0}$. It follows that $Q^*_{s1} > Q^*_{s0}$. This completes the proof. □

Owing to the supplier adopts the make-to-order production strategy after receiving the relevant order under wholesale price contracts, the optimal production quantity is represented as the value of a fixed point at this moment. Owing to the fact that the supplier can make an adjustment in the production schedule on the basis of the relevant order under portfolio contracts, the optimal production quantity is expressed as the values of an interval at this point. This proposition implies that the presence of bidirectional option contracts will make the supplier's production decision more complex and more flexibility. Besides, we find that the production quantity directly links to the order quantity under arbitrary contracts and the order size depends on the contract parameters. Thus, we see that only when certain conditions are satisfied, the production quantity under portfolio contracts is higher than that under wholesale price contracts.

6.2. The Role of Bidirectional Option Contracts on Supply Chain Performance. By means of comparing the retailer's optimal expected profit before and after the introduction of bidirectional option contracts, we consider the role of this contractual arrangement on the retailer's performance.

Proposition 8. *The retailer's optimal expected profit under portfolio contracts is greater than that under wholesale price contracts; that is,* $\Pi_{r1}(R^*_1, Q^*_{r1}, Q^*_1) > \Pi_{r0}(Q^*_{r0})$*.*

Proof. Let $\Delta_1(Q^*_{r0}) = \Pi_{r1}(R^*_1, Q^*_{r0}, Q^*_1) - \Pi_{r0}(Q^*_{r0})$, we get

$$\begin{aligned}
\Delta_{\mathbf{1}}\left(Q^*_{r0}\right) = {} & \left(Q^*_1 - Q^*_{r0}\right)\int_0^T \left(pe^{\gamma t} + g_r - c_{\mathrm{oc}} - c_{\mathrm{ec}}\right) g(t)\, dt \\
& - \int_0^T \int_{Q^*_{r0}-\lambda e^{-\alpha t}}^{Q^*_1-\lambda e^{-\alpha t}} \left(pe^{\gamma t} + g_r - c_{\mathrm{ec}}\right) F(x)\, g(t)\, dx\, dt \\
& - c_{\mathrm{op}}\left(Q^*_{r0} - R^*_1\right) \\
& + c_{\mathrm{ep}} \int_0^T \int_{R^*_1-\lambda e^{-\alpha t}}^{Q^*_{r0}-\lambda e^{-\alpha t}} F(x)\, g(t)\, dx\, dt.
\end{aligned} \tag{26}$$

Then, from

$$(Q_1^* - Q_{r0}^*) \int_0^T \left(pe^{\gamma t} + g_r - c_{\mathrm{oc}} - c_{\mathrm{ec}}\right) g(t)\,dt - \int_0^T \int_{Q_{r0}^*-\lambda e^{-\alpha t}}^{Q_1^*-\lambda e^{-\alpha t}} \left(pe^{\gamma t} + g_r - c_{\mathrm{ec}}\right) F(x)\,g(t)\,dx\,dt > 0, \tag{27}$$

we can get that

$$\int_0^T \int_{Q_{r0}^*-\lambda e^{-\alpha t}}^{Q_1^*-\lambda e^{-\alpha t}} \left(pe^{\gamma t} + g_r - c_{\mathrm{ec}}\right) F(x)\,g(t)\,dx\,dt < (Q_1^* - Q_{r0}^*) \int_0^T \left(pe^{\gamma t} + g_r - c_{\mathrm{ec}}\right) \cdot F\left(Q_1^* - \lambda e^{-\alpha t}\right) g(t)\,dt. \tag{28}$$

Besides, $-c_{\mathrm{op}}(Q_{r0}^* - R_1^*) + c_{\mathrm{ep}} \int_0^T \int_{R_1^*-\lambda e^{-\alpha t}}^{Q_{r0}^*-\lambda e^{-\alpha t}} F(x)g(t)dx\,dt > 0$ can be calculated owing to $\int_0^T \int_{R_1^*-\lambda e^{-\alpha t}}^{Q_{r0}^*-\lambda e^{-\alpha t}} F(x)g(t)dx\,dt > (Q_{r0}^* - R_1^*) \int_0^T F(R_1^* - \lambda e^{-\alpha t})g(t)dt$. Hence $\Pi_{r1}(R_1^*, Q_{r0}^*, Q_1^*) > \Pi_{r0}(Q_{r0}^*)$. Since $\Pi_{r1}(R_1^*, Q_{r1}^*, Q_1^*) > \Pi_{r1}(R_1^*, Q_{r0}^*, Q_1^*)$, we get that $\Pi_{r1}(R_1^*, Q_{r1}^*, Q_1^*) > \Pi_{r0}(Q_{r0}^*)$. This completes the proof. □

This proposition implies that the retailer can ensure an adequate supply through purchasing bidirectional options and adjust its procurement plan according to the actual market demand under portfolio contracts. Thus, the presence of bidirectional option contracts is favorable for the retailer. In contrast with wholesale price contracts, the supplier is inclined to portfolio contracts with bidirectional options in order to gain more profits under inflation scenarios.

Through a comparative analysis on the supplier's optimal expected profit before and after the introduction of bidirectional option contracts, we consider the role of this contractual arrangement on the supplier's performance.

Proposition 9. *The supplier's optimal expected profit under portfolio contracts is higher than that under wholesale price contracts; that is,* $\Pi_{s1}(Q_{s1}^*) > \Pi_{s0}(Q_{s0}^*)$.

Proof. Set $\Delta_2(w) = \Pi_{s1}(Q_{s1}^*) - \Pi_{s0}(Q_1^*)$, we can derive that

$$\begin{aligned}\Delta_2(w) = {} & \left(c_{\mathrm{oc}} + c_{\mathrm{ec}} - g_s - w + c\right) Q_1^* + \left(g_s - c_{\mathrm{ec}}\right) \int_0^T \int_0^{Q_1^*-\lambda e^{-\alpha t}} F(x)\,g(t)\,dx\,dt \\ & + \left(w + c_{\mathrm{op}} - c_{\mathrm{oc}} - c_{\mathrm{ec}}\right) Q_{r1}^* + \left(c_{\mathrm{ec}} - c_{\mathrm{ep}}\right) \int_0^T \int_0^{Q_{r1}^*-\lambda e^{-\alpha t}} F(x)\,g(t)\,dx\,dt \\ & - c_{\mathrm{op}} R_1^* + c_{\mathrm{ep}} \int_0^T \int_0^{R_1^*-\lambda e^{-\alpha t}} F(x)\,g(t)\,dx\,dt + \left(g_s - c\right) Q_{s1}^* - g_s \int_0^T \int_0^{Q_{s1}^*-\lambda e^{-\alpha t}} F(x)\,g(t)\,dx\,dt.\end{aligned} \tag{29}$$

Let $w^0 = c_{\mathrm{ep}} - c_{\mathrm{op}} + ((pe^{\gamma t} + g_r - c_{\mathrm{ep}})/(pe^{\gamma t} + g_r - c_{\mathrm{ec}}))c_{\mathrm{oc}}$. If $w = w^0$, $Q_1^* = Q_{r1}^* = R_1^*$, and $Q_{s1}^* = Q_{r1}^*$, we can see $\Delta_2(w) = 0$. Notice $(d\Delta_4(w)/dw)|_{w=w^0} = g_s[\int_0^T F(Q_{s1}^{\varphi} - \lambda e^{-\alpha t})g(t)dt - \int_0^T F(Q_{s1}^* - \lambda e^{-\alpha t})g(t)dt](dQ_{s1}^*/dw)$. If $Q_{s1}^{\varphi} > Q_{r1}^*$, then $dQ_{s2}^*/dw = 0$. We can obtain that $(d\Delta_4(w)/dw)|_{w=w^0} = 0$. If $Q_{s1}^{\varphi} \le Q_{r1}^*$, then $Q_{s1}^* = Q_{r1}^*$, $\int_0^T F(Q_{s1}^{\varphi} - \lambda e^{-\alpha t})g(t)dt \le \int_0^T F(Q_{s1}^* - \lambda e^{-\alpha t})g(t)dt$, and $dQ_{s1}^*/dw < 0$. We can derive that $(d\Delta_4(w)/dw)|_{w=w^0} \ge 0$. Hence $\Pi_{s1}(Q_{s1}^*) > \Pi_{s0}(Q_1^*)$. Since $\Pi_{s0}(Q_1^*) > \Pi_{s0}(Q_{s0}^*)$, it follows that $\Pi_{s1}(Q_{s1}^*) > \Pi_{s0}(Q_{s0}^*)$. This completes the proof. □

This proposition implies that the supplier can get some benefits through selling bidirectional options and adjust its production schedule according to the retailer's order under portfolio contracts. Thus, the presence of bidirectional option contracts is advantageous for the supplier. As contrasted to wholesale price contracts, the supplier prefer portfolio contracts with bidirectional options in order to pursue more profits under inflation scenarios.

7. Supply Chain Coordination

In order to make channel coordination possible, we must ensure that the decentralization decision is identical to the centralization decision. We take both sides of the supply chain as an entity and consider a vertical integrated firm where one central controller determine the production quantity in order to maximize the profit. We assume that the production quantity under the centralized setting is Q_I. Then, the expected total profit of the centralized system, denoted as $\Pi_I(Q_I)$, is given by

$$\Pi_I(Q_I) = \int_0^T \left\{p(t) \min\left[D(t), Q_I\right] - g_r\left[D(t) - Q_I\right]^+ - cQ_I\right\} g(t)\,dt. \tag{30}$$

The first term is the sales revenue. The second term is the shortage cost, and the last term is the costs of manufacturing products. With some algebra, the function described above can be simplified as

$$\Pi_I(Q_I) = Q_I \int_0^T \left(pe^{\gamma t} + g_r - c\right) g(t)\,dt - g_r \int_0^T \lambda e^{-\alpha t} g(t)\,dt - g_r\mu - \int_0^T \int_0^{Q_I-\lambda e^{-\alpha t}} \left(pe^{\gamma t} + g_r\right) F(x)\,g(t)\,dx\,dt. \tag{31}$$

In this case, the system's problem is to decide how many items to manufacture so as to maximize its expected profit function (31), which yields the following proposition.

Proposition 10. *The central controller's optimal production quantity Q_I^* satisfies*

$$\int_0^T \left(pe^{\gamma t} + g_r\right) \overline{F}\left(Q_I^* - \lambda e^{-\alpha t}\right) g(t)\, dt = c. \tag{32}$$

Proof. From (31), we can derive that $d\Pi_I(Q_I)/dQ_I = \int_0^T (pe^{\gamma t} + g_r - c) g(t) dt - \int_0^T (pe^{\gamma t} + g_r) F(Q_I - \lambda e^{-\alpha t}) g(t) dt$ and $d^2\Pi_I(Q_I)/dQ_I^2 = -\int_0^T (pe^{\gamma t} + g_r) f(Q_I - \lambda e^{-\alpha t}) g(t) dt < 0$. It follows that $\Pi_I(Q_I)$ is a strictly concave function of Q_I. Let $d\Pi_I(Q_I)/dQ_I = 0$. We can present the system's optimal production quantity under centralized decision-making as follows: $\int_0^T (pe^{\gamma t} + g_r)\overline{F}(Q_I^* - \lambda e^{-\alpha t}) g(t) dt = c$. This completes the proof. □

Comparing (3) with (32), it can be shown that $Q_I^* > Q_{r0}^*$ due to $w > c$. Wholesale price contracts are an inactive mechanism to help attain supply chain coordination. According to the above analysis, we find the presence of bidirectional option contracts is a competitive advantage for both the supplier and the retailer. In the remainder of this paper, the conditions to realize the supply chain coordination will be discussed.

To the best of our knowledge, except for some scenarios such as random yield, a supply shortage is not allowed to occur and the supplier is assumed to commit to manufacturing the products up to the retailer's requirement under various contract types such as buyback contracts and revenue sharing contracts in the tradition approach [35]. At this time, the problem of coordinating a channel is classed as the unilateral coordination from the retailer's perspective. However, there is no assurance that the retailer will exercise all the options as either call or put. If the make-to-order production policy is used, the supplier has to face the risk of overproduction in the presence of bidirectional option contracts. For the sake of maximum profit, the supplier does not arrange the production planning and scheduling according to the mutual agreement of two parties, but decide the production quantity from its own point of view. On the premise that the supplier decides the production quantity that maximizes its own profit, the risk-sharing gains are unrealized in the traditional unilateral coordinated method. Similar to Chen et al. [27, 28], we take the different perspectives from both sides as the key entry point and design the bilateral coordination mechanism.

Proposition 11. *When $g_s > pe^{\gamma t} + g_r - c_{ec}$ and $\int_0^T F(Q_I^* - \lambda e^{-\alpha t}) g(t) dt = (c_{oc} + c_{ec} - c)/c_{ec}$ are satisfied, the supply chain can be coordinated in the presence of bidirectional option contracts under inflation scenarios.*

Proof. In order to attain channel coordination and achieve the maximum channel profit when both bidirectional option contracts and the effect of inflation are considered, both sides of the supply chain need to be coordinated. From Propositions 5 and 10, we find a solution to motivate the supplier to produce the same amount of goods as in the coordinated supply chain. Because $Q_{s1}^{\varphi} \geq Q_1^*$, it follows that $g_s > pe^{\gamma t} + g_r - c_{ec}$. From Propositions 3 and 10, we see that to ensure the retailer order is also coordinated, the retailer's order quantity must satisfy $\int_0^T (pe^{\gamma t} + g_r - c_{ec})\overline{F}(Q_I^* - \lambda e^{-\alpha t}) g(t) dt = c_{oc}$. From (32), we can derive that $\int_0^T F(Q_I^* - \lambda e^{-\alpha t}) g(t) dt = (c_{oc} + c_{ec} - c)/c_{ec}$. Portfolio contracts in which the two aforementioned conditions are satisfied can realize the supply chain coordination under inflation scenarios.

Proposition 11 identifies key sufficient conditions for the supply chain coordination considering bidirectional option contracts and the effect of inflation. Set $\Delta\pi = \Pi_I(Q_I^*) - \Pi_{r1}(R_1^*, Q_{r1}^*, Q_1^*)$, then we can see

$$\begin{aligned}\Delta\pi = {} & \left(c_{oc} + c_{ec} - c\right) Q_I^* - c_{ec} \int_0^T \int_0^{Q_I^* - \lambda e^{-\alpha t}} F(x)\, g(t)\, dt \\ & - \left(c_{oc} + c_{ec} - w - c_{op}\right) Q_{r1}^* \\ & + \left(c_{ec} - c_{ep}\right) \int_0^T \int_0^{Q_{r1}^* - \lambda e^{-\alpha t}} F(x)\, g(t)\, dx\, dt - c_{op} R_1^* \\ & + c_{ep} \int_0^T \int_0^{R_1^* - \lambda e^{-\alpha t}} F(x)\, g(t)\, dx\, dt.\end{aligned} \tag{33}$$

Now let $\eta = \Delta\pi/\Pi_I(Q_I^*)$ $(0 < \eta < 1)$. We can deduce that the expected profits of both the supplier and the retailer after coordinating are $\Pi_{s1} = \eta\Pi_I$ and $\Pi_{r1} = (1-\eta)\Pi_I$, respectively. This shows that portfolio contracts with bidirectional options can coordinate the channel and arbitrarily allocate profits between two parties under inflation scenarios. This completes the proof. □

This proposition shows that the sufficient conditions for channel coordination are determined by unit call option price c_{oc}, unit exercise price of call option c_{ec} and unit production cost c, and is not related to unit wholesale price w, unit put option price c_{op} and unit exercise price of put option c_{ep}. This indicates that unit wholesale price, unit put option price and unit exercise price of put option cannot used to control the division of profit between the two members involved under portfolio contracts and cannot influence the expected profit of the centralized system. Moreover, we see that unit call option price is negatively related to unit exercise price of call option when the channel coordination is achieved. The main reason is that if there is an increase in both unit call option price and unit exercise price of call option, it presents an advantage to the supplier and a disadvantage to the retailer. Only when there is an inverse relationship between these two parameters, it is feasible to reconcile the conflicting interests between the retailer and the supplier under portfolio contracts. Furthermore, we observe that when there has been a coordinating contract, the expected profits of both sides do not decrease and at least one of them is strictly better off. Therefore, the existence of a Pareto contract is demonstrated when the coordinating contract is used to compare with the noncoordinating contract.

Table 1: The effect of α on the optimal decision variables.

α	Q^*_{r0}	Q^*_{r1}	q^*_{r1}	m^*_{r1}	Q^*_{s0}	Q^*_{s1}
0.0005	176.1	128.52	56.80	10	176.10	185.31
0.001	174.59	127.06	56.73	10	174.59	183.79
0.0015	173.11	125.63	56.67	10	173.11	182.30
0.002	171.66	124.23	56.61	10	171.66	180.84
0.0025	170.24	122.86	56.55	10	170.24	179.41
0.003	168.84	121.52	56.49	10	168.84	178.01
0.0035	167.48	120.20	56.44	10	167.48	176.64
0.004	166.14	118.91	56.39	10	166.14	175.29
0.0045	164.83	117.64	56.33	10	164.83	173.97
0.005	163.54	116.39	56.28	10	163.54	172.68
0.0055	162.28	115.18	56.24	10	162.28	171.40
0.006	161.05	113.98	56.19	10	161.05	170.17
0.0065	159.84	112.81	56.14	10	159.84	168.95
0.007	159.84	111.66	56.10	10	159.84	167.75
0.0075	157.48	110.53	56.06	10	157.48	166.58

Table 2: The effect of γ on the optimal decision variables.

γ	Q^*_{r0}	Q^*_{r1}	q^*_{r1}	m^*_{r1}	Q^*_{s0}	Q^*_{s1}
0.0005	154.54	109.42	55.07	10	154.54	164.49
0.001	154.65	109.42	55.13	10	154.65	164.55
0.0015	154.77	109.42	55.2	10	154.77	164.62
0.002	154.89	109.42	55.26	10	154.89	164.68
0.0025	155.01	109.42	55.32	10	155.01	164.74
0.003	155.13	109.42	55.39	10	155.13	164.81
0.0035	155.25	109.42	55.45	10	155.25	164.87
0.004	155.37	109.42	55.52	10	155.37	164.94
0.0045	155.49	109.42	55.58	10	155.48	165.00
0.005	155.61	109.42	55.64	10	155.61	165.06
0.0055	155.73	109.42	55.71	10	155.73	165.13
0.006	155.85	109.42	55.77	10	155.85	165.19
0.0065	155.97	109.42	55.83	10	155.97	165.25
0.007	156.10	109.42	55.89	10	156.10	165.31
0.0075	156.22	109.42	55.95	10	156.22	165.37

8. Numerical Example

In this section, a numerical number is provided to demonstrate the feasibility of the mathematical model and analyze the effect of inflation on the optimal decision variables and the optimal expected profits of the two parties.

We assume that the defaults values of parameters are used as $p = 5$, $w = 3$, $c = 1.2$, $c_{\text{oc}} = 1.5$, $c_{\text{ec}} = 2$, $c_{\text{op}} = 0.2$, $c_{\text{ep}} = 1$, $g_s = 20$, $g_r = 7$, $\lambda = 100$, $T = 60$, $t \sim U(0, 60)$, and $\xi \sim U(0, 100)$. The above values of parameters satisfy the basic assumptions of this paper.

8.1. The Effect of Inflation on the Optimal Decision Variables. The effect of demand contraction factor (α) on the optimal decision variables is shown in Table 1.

From Table 1, given the price rising factor γ, we derive the following observation. (1) When the demand contraction factor α increases, the retailer will face a smaller target market. Thus, the retailer will reduce the optimal firm order quantity under wholesale price contracts. At the same time, the supplier will decrease the optimal production quantity due to the declining order requirement of the retailer under wholesale price contracts. (2) When the demand contraction factor α increases, the retailer will not change the optimal put options order quantity, but the retailer will reduce the optimal firm order quantity and the optimal call options quantity under portfolio contracts. At this moment, the supplier will decrease the production quantity to reduce the production cost.

The effect of price rising factor (γ) on the optimal decision variables is shown in Table 2.

From Table 2, given the demand contraction factor α, we derive the following observation. (1) When the price rising factor γ increase, the optimal firm order quantity under wholesale price contracts increase, and the supplier will increase the optimal production quantity to satisfy the retailer's order requirement. (2) When the price rising factor γ increase, the optimal firm order quantity and the optimal

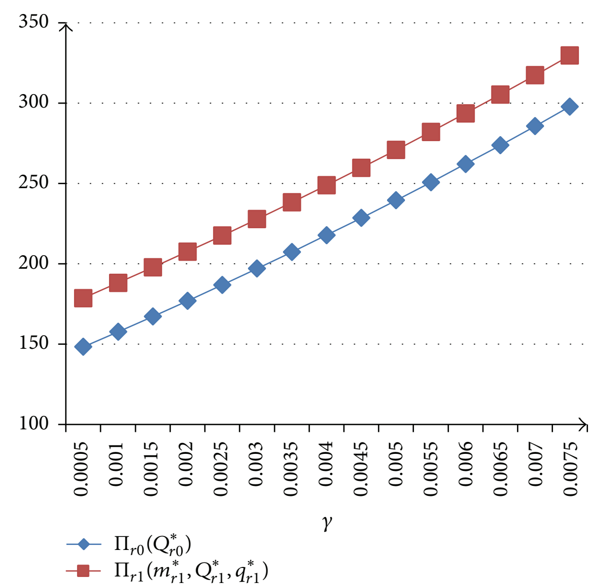

Figure 1: The effect of γ on the retailer's optimal expected profit.

put options order quantity under portfolio contracts remain constant. The retailer will raise the optimal call options quantity to reduce the probability that the market demand cannot be satisfied. At this moment, the supplier will increase the production quantity to reduce the probability that exercised call options cannot be filled immediately from inventory.

8.2. The Effect of Inflation on the Optimal Expected Profits of the Two Parties. The effect of price rising factor (γ) on the retailer's optimal expected profit is shown in Figure 1.

From Figure 1, we know that whether or not bidirectional option contracts are used, the expected profit of the retailer is increasing in γ. The main reason is that when the price rising factor increases, the retailer will face a bigger target market. Moreover, comparing with the case of wholesale price

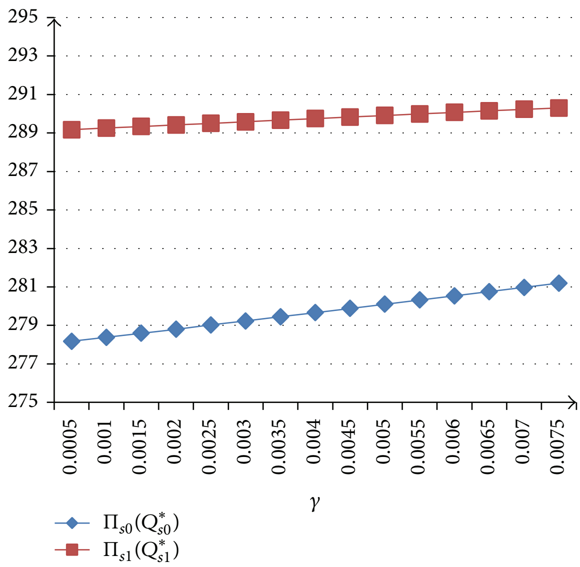

FIGURE 2: The effect of γ on the supplier's optimal expected profit.

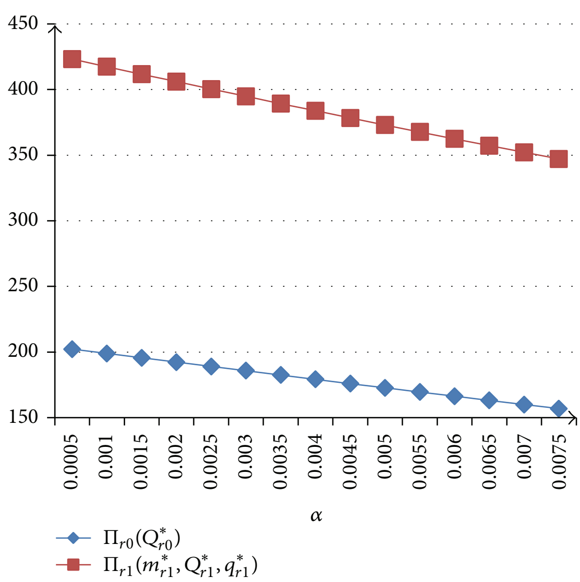

FIGURE 3: The effect of α on the retailer's optimal expected profit.

contracts, the retailer will obtain more profit under portfolio contracts when there is a remarkable increase in retail price.

The effect of price rising factor (γ) on the supplier's optimal expected profit is shown in Figure 2.

From Figure 2, we know that whether or not bidirectional option contracts are used, the expected profit of the supplier is increasing in γ. The main reason is that when the price rising factor increases, the supplier will receive a larger order requirement of the retailer. Moreover, comparing with the case of wholesale price contracts, the supplier will obtain more profit under portfolio contracts when there is a remarkable increase in retail price.

The effect of demand contraction factor (α) on the retailer's optimal expected profit is shown in Figure 3.

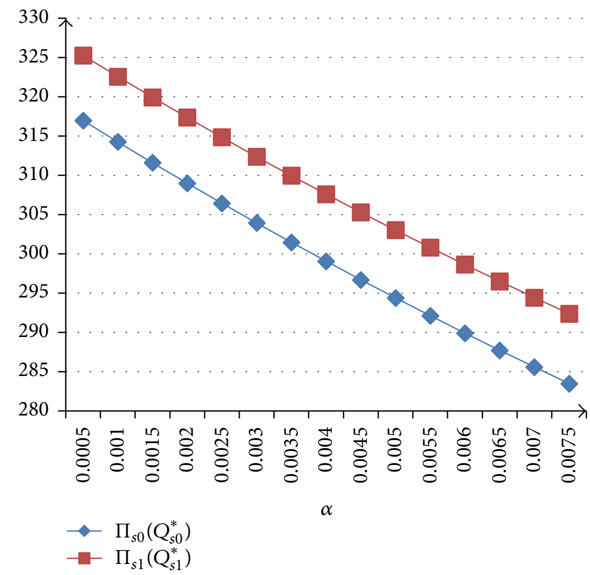

FIGURE 4: The effect of α on the supplier's optimal expected profit.

From Figure 3, we know that whether or not bidirectional option contracts are used, the expected profit of the retailer is decreasing in α. The main reason is that when the demand contraction increases, the retailer will face a smaller target market. Moreover, comparing with the case of wholesale price contracts, the retailer will obtain more profit under portfolio contracts when there is a remarkable decrease in market demand.

The effect of demand contraction factor (α) on the supplier's optimal expected profit is shown in Figure 4.

From Figure 4, we know that whether or not bidirectional option contracts are used, the expected profit of the supplier is decreasing in α. The main reason is that when the demand contraction increases, the supplier will receive a smaller order requirement of the retailer. Moreover, comparing with the case of wholesale price contracts, the supplier will obtain more profit under portfolio contracts when there is a remarkable decrease in market demand.

9. Conclusion

Inflation is defined as a sustained increase in price, which will lead to a continuous decline in demand. In this case, enterprises always modify the operation strategies to protect against the effect of inflation. Although there is a rapid increase in the theoretical support for the optimal solution to the inventory control problem under inflation scenarios, there has been little research directly assessing the effect of inflation on the optimal decision strategies in the supply chain applications. Given that, we explore the effect of inflation on supply chain. Moreover, bidirectional option contracts are deemed to be an effective tool to protect against the effect of inflation. However, the problems relating to bidirectional option contracts are not completely solved so far. At the same time, the studies which incorporate bidirectional option contracts and the effect of inflation are

rare. Given that, we introduce bidirectional option contracts into the effective decision making process in the enterprise of supply chain under inflation scenarios, and then study how bidirectional option contracts and the effect of inflation make an impact on the optimal decision policies and the optimal expected profits of the supply chain members. In comparison with the situation under wholesale price contracts, we find that the presence of bidirectional option contracts can smooth over the contradictions between the supplier and the retailer. It helps the retailer ensure an adequate supply and modify the order either upwards or downwards after the demand information is available. Besides, it helps the supplier get some benefits through bidirectional options sales and adjust its production plan after the ordering information is provided. Thus, the appliance of bidirectional options contracts can improve the expected profits of both the supplier and the retailer. These conclusions are advantageous for most companies to better understand the effect of inflation on the ordering and manufacturing decisions. Besides, these conclusions can help the enterprises utilize bidirectional options contracts to hedge against the effect of inflation. Furthermore, we design the bilateral coordination mechanism from the different perspective of two parties. These conclusions can help the corporations better optimize their operational decisions, increase their revenues and manage the risks when they face inflationary pressure and option games.

This paper only considers a one-period setting, and then we plan to extend our model to a multi-period setting. Besides, the retailer and the supplier are supposed to be risk-neutral in the current model. Considering the enterprise's risk preferences is viewed as the next thing to be done, such as loss-averse retailer [36]. Furthermore, we assume that the market demand and the retail price are mutually independent under inflation. In reality, the demand is determined by many different factors and one of the biggest factors is the price. We plan to resolve all the related problems in the future. At last, we assume that the power structure is Supplier-Stackelberg. More market power structures, such as Retailer-Stackelberg and Nash power structures [37, 38] can be take into consideration in the future.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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