

Research Article

Fuzzy Finite-Time Stability of Chaotic Systems with Time-Varying Delay and Parameter Uncertainties

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This paper discusses the finite-time stability of chaotic systems with time-varying delay and parameter uncertainties. A new model based on Takagi-Sugeno (T-S) model is proposed for representing chaotic systems. By the new model, finite-time stability of chaotic systems can be converted into stabilization of fuzzy T-S systems with parameter uncertainties. A sufficient condition is given in terms of matrix inequalities, which guarantees the finite-time stability for fuzzy systems can be achieved. Numerical simulations on the chaotic systems are presented to demonstrate the effectiveness of the theoretical results.

1. Introduction

In the last decades, the study of controlling chaotic systems has received considerable attention due to its broad applications in biological systems, information processing, secure communications, and so forth. In practical physical systems, the parameters of chaotic systems may not be known exactly. So, there are many papers concerning uncertain chaotic systems [1, 2]. For the character of the uncertain chaotic systems, many significant results have been obtained by using adaptive control technique, for example, output feedback control [3], fuzzy adaptive control [4], and optimal control [5].

In order to simplify the controller design of the chaotic systems, various schemes have been developed, among which fuzzy model is one of the successful approaches to obtain nonlinear chaotic systems. Specifically, a so-called Takagi-Sugeno (TS) fuzzy model was proposed in [6], where complex nonlinear systems are represented by some local linear subsystems. Based on the T-S fuzzy model, some chaotic control design methods have been developed [7–12]. In [7], a fuzzy model is presented to simulate two different chaotic systems with different numbers of nonlinear terms and a

new adaptive approach is proposed to synchronize these two different fuzzy chaotic systems. Predictive control for chaotic systems is based on a T-S fuzzy model in [8]. Wang and Wu [9] and Zhang et al. [10] investigate the problem of fuzzy impulsive control to stabilize chaotic systems.

Although there have been many works to discuss the stabilization of chaotic systems, most of the existing stabilization algorithms are asymptotically convergent algorithms [13–17], which means that the convergence rate is at best exponential with infinite settling time. Compared to the asymptotically convergent algorithms, the finite-time convergence algorithms demonstrate not only faster convergence rates, but also better disturbance rejection properties and robustness against uncertainties [18–20]. References [21–24] discuss finite-time control for chaotic systems. But there are few works considering finite-time control for chaotic systems based on the fuzzy models.

Motivated by the aforementioned analysis, the finite-time stabilization control for chaotic systems based on the T-S fuzzy models is a significative topic. Our main contribution in this paper is the introduction of the fuzzy model to discuss finite-time stabilization of chaotic systems in a general model. In this model, we not only consider time-varying delay, but

also study the chaotic system with parameter uncertainties. Obviously, on the one hand, our results have optimality in the convergence time of stabilization in [14, 15]; on the other hand, they are more practically applicable than those in [22–24]. Based on the Lyapunov stability theory, a feedback controller is designed to guarantee the stabilization in finite time. The example is given to illustrate our theoretical results.

The rest of the paper is organized as follows. In Section 2, some preliminaries are briefly given. Section 3 presents the main results. In Section 4, simulation results aiming at substantiating the theoretical analysis are presented. This paper is concluded in Section 5.

2. Preliminaries and Problem Formulation

In this paper, we want to develop a finite-time control law for a class of chaotic systems. In system analysis and design, it is important to select an appropriate model representing a real system. The T-S fuzzy model can express a highly nonlinear functional relation with a small number of rules. Many chaotic systems can be represented by fuzzy linear models, such as Lorenz, Rössler, Chua, Chen, and Lü systems. This fuzzy modeling method is simple and natural. So, consider a class of time-delayed chaotic systems represented by the following time-delayed T-S fuzzy model.

Plant Rule is as follows.

If $\theta_1(t)$ is M_{i1}, \dots , and $\theta_p(t)$ is M_{ip} , then

$$\dot{x}(t) = A_i x(t) + B_i x(t - \tau(t)), \quad i = 1, 2, \dots, N, \quad (1)$$

where $x(t) = (x_1(t), \dots, x_n(t))^T \in R^n$ is the state variable; the premise variables $\theta_1(t), \dots, \theta_p(t)$ are proper state variables; N is the number of the fuzzy rules; M_{ij} ($j = 1, 2, \dots, p$) are fuzzy sets; A_i and B_i are known constant matrices with appropriate dimensions; $\tau(t)$ is the transmission delay.

In practice, however, some parameters of chaotic systems cannot be exactly known a priori. If the information of uncertain matrices is considered, the results will be less conservative than those results that do not utilize the elemental uncertain information. Thus, it is very interesting to study the design of the fuzzy systems with parameter perturbations.

Plant Rule is as follows.

If $\theta_1(t)$ is M_{i1}, \dots , and $\theta_p(t)$ is M_{ip} , then

$$\dot{x}(t) = (A_i + \nabla A_i) x(t) + (B_i + \nabla B_i) x(t - \tau(t)), \quad (2)$$

$$i = 1, 2, \dots, N.$$

The parameter uncertainties are classically written as [10] $\nabla A_i = D_i^A F_i^A E_i^A$ and $\nabla B_i = D_i^B F_i^B E_i^B$ where $D_i^A, D_i^B, E_i^A, E_i^B$ are known real matrices of appropriate dimension and F_i^A, F_i^B are the unknown matrix functions with Lebesgue-measurable elements and satisfy the conditions $F_i^{AT} F_i^A \leq I, F_i^{BT} F_i^B \leq I$, where I is the identity matrix of appropriate dimension.

The overall fuzzy system with control is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^N h_i(\theta(t)) [(A_i + \nabla A_i) x(t) + (B_i + \nabla B_i) x(t - \tau(t)) + u(t)], \quad (3)$$

where $u(t)$ denotes the feedback control, $h_i(\theta(t)) = \omega_i(\theta(t)) / \sum_{i=1}^N \omega_i(\theta(t))$, $\omega_i(\theta(t)) = \prod_{j=1}^p M_{ij}(\theta_j(t))$, and $\omega_i(\theta(t)) \geq 0$, $\sum_{i=1}^N \omega_i(\theta(t)) \geq 0$; thus $h_i(\theta(t)) \geq 0$, $\sum_{i=1}^N h_i(\theta(t)) = 1$.

Assume $C([- \tau, 0], R^n)$ is a Banach space of continuous functions mapping the interval $[- \tau, 0]$ into R^n with the norm $\|\phi\| = \sup_{-\tau \leq \theta \leq 0} \|\phi(\theta)\|$. For the functional differential equation (3), its initial conditions are given by $x_i(t) = \phi_i(t) \in C([- \tau, 0], R^n)$. We always assume that (3) has a unique solution with respect to initial conditions.

For starting simplification, one has the following fundamental assumption.

Assumption 1. Consider $0 \leq \dot{\tau}(t) \leq h < 1$, where h is constant.

To end this section, we introduce the following lemmas which are useful in deriving sufficient conditions of finite-time stability.

Lemma 2 (see [25]). Consider system $\dot{x} = f(x)$, $f(0) = 0$, $x \in R^n$, where $f(\cdot) : R^n \rightarrow R^n$ is a continuous vector function. Suppose there exist a C^1 positive definite and proper function $V : R^n \rightarrow R$ and real numbers $\mu > 0$ and $\eta \in (0, 1)$ such that $\dot{V} + \mu V^\eta$ is negative semidefinite. Then the origin is a globally finite-time stable equilibrium of system $\dot{x} = f(x)$. Moreover, the settling time $T \leq V^{1-\eta}(0)/(1-\eta)\mu$.

Lemma 3 (see [26]). Given any real matrices A, B, Σ of appropriate dimensions and a scalar $s > 0$, such that $0 < \Sigma = \Sigma^T$, then the following inequality holds:

$$A^T B + B^T A \leq s A^T \Sigma A + s^{-1} B^T \Sigma^{-1} B. \quad (4)$$

Lemma 4 (see [27]). Let A, D, E , and F be real matrices of appropriate dimensions, with F satisfying $\|F\| \leq 1$. Then one has the following.

(a) For any scalar $\lambda > 0$,

$$DFE + E^T F^T D^T \leq \lambda^{-1} DD^T + \lambda EE^T. \quad (5)$$

(b) For any matrix $P > 0$ and scalar $\xi > 0$ such that $\xi I - E^T P E > 0$,

$$(A + DFE)^T P (A + DFE) \leq A^T P A + A^T P E (\xi I - E^T P E)^{-1} E^T P A + \xi^{-1} D^T D. \quad (6)$$

Lemma 5 ((Schur complement) [28]). For a given matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0, \quad (7)$$

where $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T$ are equivalent to any one of the following conditions:

- (a) $S_{22} < 0, S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0;$
 (b) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0.$

3. Finite-Time Stabilization

First, we present our main result on the finite-time stabilization of system (3).

Theorem 6. Suppose the positive constants $\lambda_i, i = 1, 2, 3, k_1, k_2,$ and positive definite matrix $P,$ such that

$$(1) \begin{pmatrix} A_i^T P + PA_i + (\lambda_2 + k_2 - k_1) P & E_i^A & PD_i^A \\ E_i^{A^T} & -\lambda_1 I & 0 \\ D_i^{A^T} P & 0 & -\lambda_1^{-1} I \end{pmatrix} \leq 0, \quad i = 1, \dots, N, \quad (8)$$

$$(2) \begin{pmatrix} B_i^T PB_i - k_2 \lambda_2 (1 - h) P & D_i^B & E_i^{B^T} PB_i \\ D_i^{B^T} & -\lambda_3^{-1} I & 0 \\ B_i^T PE_i^B & 0 & -(\lambda_3 I - E_i^{B^T} PE_i^B) \end{pmatrix} \leq 0, \quad i = 1, \dots, N. \quad (9)$$

Then, stabilization of system (3) under Assumption 1 can be achieved in finite time, if the control law u is designed as

$$u = -\frac{k_1}{2} x(t) - \frac{\mu}{2} \frac{\lambda_{\max}^{(\eta+1)/2}(P)}{\lambda_{\min}(P)} \text{sign}(x(t)) |x(t)|^\eta - \frac{\mu}{2\lambda_{\min}(P)} \left(k_2 \int_{t-\tau}^t x^T(s) Px(s) ds \right)^{(1+\eta)/2} \cdot \frac{x(t)}{\|x(t)\|^2}, \quad (10)$$

where μ is an arbitrary positive constant.

Proof. Construct a Lyapunov function:

$$V(t) = x^T(t) Px(t) + k_2 \int_{t-\tau}^t x^T(s) Px(s) ds, \quad (11)$$

so, we get

$$\begin{aligned} \dot{V}(t) \Big|_{(3)} &= \sum_{i=1}^N h_i(\theta(t)) \left\{ x^T(t) \right. \\ &\cdot \left(A_i^T P + PA_i + E_i^{A^T} F_i^{A^T} D_i^{A^T} P + PD_i^A F_i^A E_i^A \right) x(t) \\ &+ x^T(t-\tau) \left(B_i^T + E_i^{B^T} F_i^{B^T} D_i^{B^T} \right) Px(t) + x^T(t) \\ &\cdot P \left(B_i + D_i^A F_i^A E_i^A \right) x(t-\tau) + 2x^T(t) Pu(t) \Big\} \\ &+ k_2 x^T(t) Px(t) - k_2 x^T(t-\tau) Px(t-\tau) (1 - \dot{\tau}(t)). \end{aligned} \quad (12)$$

Using Lemma 4(a), we have

$$\begin{aligned} &E_i^{A^T} F_i^{A^T} D_i^{A^T} P + PD_i^A F_i^A E_i^A \\ &\leq \lambda_1^{-1} PD_i^A D_i^{A^T} P + \lambda_1 E_i^A E_i^{A^T}. \end{aligned} \quad (13)$$

Applying Lemmas 3 and 4(b), we get

$$\begin{aligned} &x^T(t-\tau) \left(B_i^T + E_i^{B^T} F_i^{B^T} D_i^{B^T} \right) Px(t) + x^T(t) P \left(B_i \right. \\ &+ D_i^B F_i^B E_i^B \Big) x(t-\tau) \leq \lambda_2^{-1} x^T(t-\tau) \left(B_i^T \right. \\ &+ E_i^{B^T} F_i^{B^T} D_i^{B^T} \Big) P \left(B_i + D_i^B F_i^B E_i^B \right) x(t-\tau) \\ &+ \lambda_2 x^T(t) Px(t) \leq \lambda_2^{-1} x^T(t-\tau) \left(B_i^T PB_i \right. \\ &+ B_i^T PE_i^B \left(\lambda_3 I - E_i^{B^T} PE_i^B \right)^{-1} E_i^{B^T} PB_i \\ &+ \lambda_3^{-1} D_i^{B^T} D_i^B \Big) x(t-\tau) + \lambda_2 x^T(t) Px(t), \end{aligned} \quad (14)$$

where $\lambda_i, i = 1, 2, 3,$ are positive constants.

Substituting (13)-(14) into (12) and using Assumption 1, we have

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^N h_i(\theta(t)) \left\{ x^T(t) \left(A_i^T P + PA_i \right. \right. \\ &+ \lambda_1^{-1} PD_i^A D_i^{A^T} P + \lambda_1 E_i^A E_i^{A^T} + \lambda_2 P + k_2 P \Big) x(t) \\ &+ 2x^T(t) Pu(t) + x^T(t-\tau) \left[\lambda_2^{-1} \left(B_i^T PB_i \right. \right. \\ &+ B_i^T PE_i^B \left(\lambda_3 I - E_i^{B^T} PE_i^B \right)^{-1} E_i^{B^T} PB_i \\ &+ \lambda_3^{-1} D_i^{B^T} D_i^B \Big) - k_2 P (1 - h) \Big] x(t-\tau) \Big\}. \end{aligned} \quad (15)$$

Applying Schur criterion on conditions (1) and (2) yields

$$\begin{aligned} &A_i^T P + PA_i + \lambda_1^{-1} PD_i^A D_i^{A^T} P + \lambda_1 E_i^A E_i^{A^T} + \lambda_2 P \\ &+ k_2 P \leq k_1 P, \\ &B_i^T PB_i + B_i^T PE_i^B \left(\lambda_3 I - E_i^{B^T} PE_i^B \right)^{-1} E_i^{B^T} PB_i \\ &+ \lambda_3^{-1} D_i^{B^T} D_i^B \leq k_2 \lambda_2 (1 - h) P. \end{aligned} \quad (16)$$

Substituting (16) into (15) and using control inputs in (10), one obtains

$$\begin{aligned} \dot{V}(t) &\leq -\mu \frac{\lambda_{\max}^{(1+\eta)/2}(P)}{\lambda_{\min}(P)} x^T(t) P \operatorname{sign}(x(t)) |x(t)|^\eta \\ &\quad - \frac{\mu}{\lambda_{\min}(P)} \left(k_2 \int_{t-\tau}^t x^T(s) P x(s) ds \right)^{(1+\eta)/2} \\ &\quad \cdot \frac{x(t)^T P x(t)}{\|x(t)\|^2} \\ &\leq -\mu \left[\lambda_{\max}^{(1+\eta)/2}(P) (x^T(t) x(t))^{(1+\eta)/2} \right. \\ &\quad \left. + \left(k_2 \int_{t-\tau}^t x^T(s) P x(s) ds \right)^{(1+\eta)/2} \right] \\ &\leq -\mu (V(t))^{(1+\eta)/2}. \end{aligned} \quad (17)$$

Therefore, by Lemma 2 the synchronization error system (3) is globally finite-time stable and the finite time is estimated by

$$T = t_0 + \frac{V^{1-(1+\eta)/2}(t_0)}{\mu(1-(1+\eta)/2)}. \quad (18)$$

This completes the proof of the theorem. \square

Remark 7. In [10, 14, 15], the authors investigated the stabilization of chaotic systems via fuzzy models. But these results were all based on the stability time in large enough. In Theorem 6, we introduce the feedback control method to guarantee finite-time stability.

Remark 8. Theorem 6 provides a sufficient condition of finite-time stabilization of chaotic systems. In [22–24], the authors also study the finite-time control for chaotic systems, but the delay is a constant or there is no delay; moreover the parameters of chaotic systems are known exactly. Obviously, the results of this section extend and improve existing results.

Remark 9. The magnitude of $x(t)/\|x(t)\|^2$ in the controller $u(t)$ will turn to infinity as $x(t) \rightarrow 0$; in order to avoid the occurrence of this phenomenon, we can add a sufficient small positive constant ε to its denominator in practice [2, 29].

Similar to Huang et al. [30] and Khoo et al. [31], the candidate Lyapunov function $V(t)$ can be chosen as $V(t) = x^T(t)x(t)$. A summary is shown in the next corollary.

Corollary 10. Suppose the positive constants λ_i , $i = 1, 2, 3$, k_1, k_2 , and positive constant p , such that

$$\begin{aligned} (1) \quad & A_i^T p + p A_i + \lambda_1^{-1} p^2 D_i^A D_i^{A^T} + \lambda_1 E_i^A E_i^{A^T} \\ & + (\lambda_2 + k_2 - k_1) p I \leq 0, \quad i = 1, \dots, N, \end{aligned} \quad (19)$$

(2)

$$\begin{aligned} & \lambda_2^{-1} \left(p B_i^T B_i + p^2 B_i^T E_i^B (\lambda_3 I - p E_i^{B^T} E_i^B)^{-1} E_i^{B^T} B_i \right. \\ & \left. + \lambda_3^{-1} D_i^{B^T} D_i^B \right) - k_2 (1-h) p I \leq 0, \quad i = 1, \dots, N. \end{aligned} \quad (20)$$

Then, stabilization of system (3) under Assumption 1 can be achieved in finite time, if the control law u is designed as

$$\begin{aligned} u &= -\frac{k_1}{2} x(t) - \frac{\mu}{2} p^{(\eta-1)/2} \operatorname{sign}(x(t)) |x(t)|^\eta - \frac{\mu}{2} \\ & \cdot p^{(\eta-1)/2} \left(k_2 \int_{t-\tau}^t x^T(s) x(s) ds \right)^{(1+\eta)/2} \\ & \cdot \frac{x(t)}{\|x(t)\|^2}, \end{aligned} \quad (21)$$

where μ is an arbitrary positive constant.

Proof. Letting $P = pI$, we can obtain Corollary 10 directly from Theorem 6. \square

4. Numerical Example

In this section, simulation example is presented to illustrate the utility of theoretical analysis in this paper.

Example 1. We consider the following time-delayed Lorenz system [32]:

$$\begin{aligned} \dot{x}_1(t) &= -10x_1(t) + 10x_2(t-\tau), \\ \dot{x}_2(t) &= 28x_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) &= x_1(t)x_2(t) - \frac{8}{3x_3(t-\tau)}, \end{aligned} \quad (22)$$

where $\tau = 1/6$. Figure 1 is the state trajectory of system (22).

Then we have the following fuzzy control model with parameter uncertainties.

Plant Rule is as follows. If $x_1(t)$ is M_i , then

$$\begin{aligned} \dot{x}(t) &= (A_i + \nabla A_i) x(t) + (B_i + \nabla B_i) x(t-\tau(t)) \\ & + u(t), \quad i = 1, 2, \end{aligned} \quad (23)$$

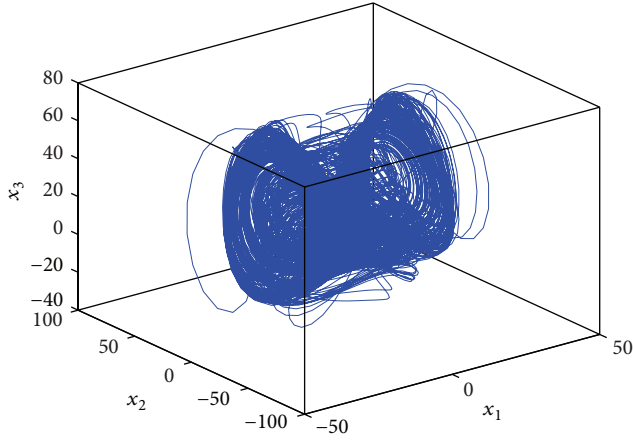


FIGURE 1: The state trajectory of system (22).

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$, $M_1 = (1/2)(1 + x_1(t)/30)$, $M_2 = (1/2)(1 - x_1(t)/30)$,

$$\begin{aligned} A_1 &= \begin{pmatrix} -10 & 0 & 0 \\ 28 & -1 & -30 \\ 0 & 30 & 0 \end{pmatrix}, \\ A_2 &= \begin{pmatrix} -10 & 0 & 0 \\ 28 & -1 & 30 \\ 0 & -30 & 0 \end{pmatrix}, \\ B_1 = B_2 &= \begin{pmatrix} 0 & 10 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{8}{3} \end{pmatrix}. \end{aligned} \quad (24)$$

The elements of ∇A_i and ∇B_i are randomly chosen within 20% of their nominal values corresponding to A_i and B_i , respectively. Based on assumption of uncertainty, we define F_i^A and F_i^B to be random matrices and satisfy the conditions $F_i^{A^T} F_i^A \leq I$, $F_i^{B^T} F_i^B \leq I$,

$$\begin{aligned} D_i^A &= D_i^B = \text{diag}(0.2, 0.2, 0.2), \\ E_i^A &= \begin{pmatrix} -10 & 0 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (25)$$

$$E_i^B = B_i,$$

$$i = 1, 2.$$

Select $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 15$, $k_1 = 100$, $k_2 = 20$, and $\eta = 0.25$. By the simulation of the Matlab LMI Toolbox, $P = [0.1345, -0.2493, 0.0000; -0.2493, 10.2021, -0.0000; 0.0000, -0.0000, 1.3286]$. The trajectory of system states $x_i(t)$ under finite-time control is shown in Figure 2. Then the simulation results show the correctness of Theorem 6.

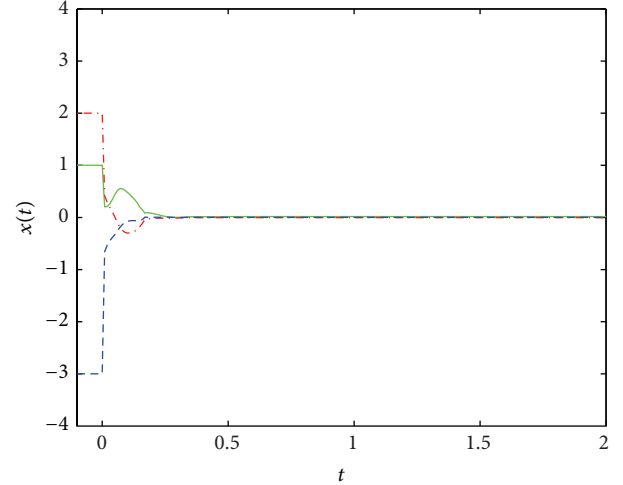


FIGURE 2: The trajectory of system states $x_i(t)$ with finite-time control.

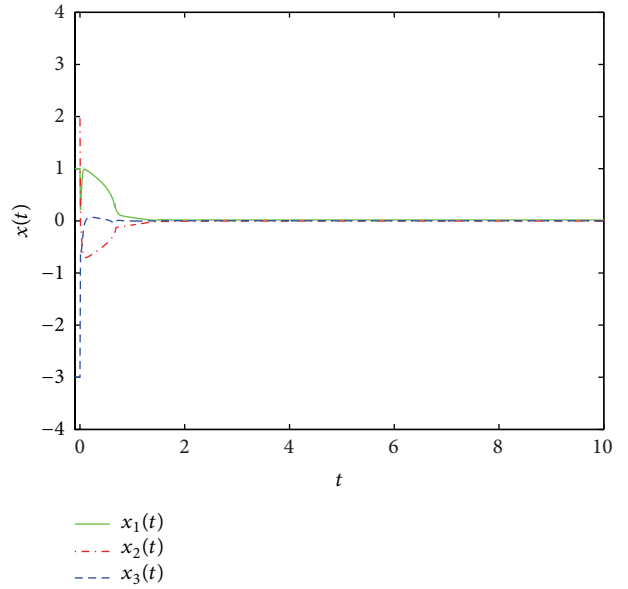


FIGURE 3: The trajectory of system (22) with $\tau(t) = e^t/(1 + e^t)$ via finite-time control.

More generally, we can choose a time-varying delay to verify the correctness of the theory. We choose the delay as $\tau(t) = e^t/(1 + e^t)$ in system (22). Obviously, $0 < \dot{\tau}(t) < 1/2 < 1$. By the simulation of the Matlab LMI Toolbox, $P = [0.1439, -0.3021, 0.0004; -0.3021, 7.8229, 0.0856; 0.0004, 0.0856, 1.7595]$. The trajectory of system states $x_i(t)$ under finite-time control is shown in Figure 3.

Remark 11. When τ is a time-varying delay, system (22) may not be a chaotic system. In [33], when the systems have no

chaotic phenomenon, the method is not feasible. However, our results do not need this restriction.

5. Conclusions

This paper presents finite-time stability of chaotic systems with time-varying delay by fuzzy-model-based controllers. In order to obtain better robustness results, we discuss the fuzzy system with parameter uncertainties. Various sufficient conditions for finite-time control are derived by Lyapunov methods and linear matrix inequalities (LMI) techniques. These results are novel; some numerical examples are given to verify our theoretical results. Next, we will continue the finite-time control problem for coupled chaotic systems based on the T-S fuzzy models.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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