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Research Article

Asymptotic Tracking Control for a Class of Nonlinear Systems with Unknown Failures of Hysteretic Actuators

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An adaptive failure compensation controller for a class of nonlinear systems preceded by hysteretic actuators is proposed in this paper. Three types of high-gain functions are constructed to counteract the effects of the hysteresis, bounded modeling errors, and bounded disturbances. It is shown that the proposed controller not only ensures bounded signals and asymptotic tracking but also avoids possible chattering, despite the presence of unknown hysteretic actuator failures. Simulation results verify the desired failure compensation performance.

1. Introduction

The hysteresis phenomenon occurs in all the smart material-based actuators and sensors. With such nonlinearity in control systems, it may lead to undesirable inaccuracies or oscillations. On the other hand, actuator failures seem inevitable in practical systems. Thus, failure compensation of hysteretic actuators is an important and challenging problem. Adaptive failure compensation has received great attention in recent years [1–12]. However, available results based on adaptive approaches to address hysteretic actuator failures are very limited [13–16].

To address such a challenging problem, it is important to find an appropriate model for the hysteresis. As mentioned in [17], several models were proposed such as Duhen model, Preisach model, Prandtl-Ishlinskii hysteresis operator, Bouc-Wen differential model [17, 18]. The Bouc-Wen differential model is one of the most widely accepted models of the hysteresis. Actually, it can be shown that the hysteresis model presented in [13–16] is a special case of the Bouc-Wen hysteresis model.

In [13], an adaptive failure compensation scheme for a class of nonlinear systems was studied, where control gains are constants. To avoid possible chattering, the sign $(\cdot)$ functions were involved in the backstepping controller. When the control gains are nonlinear functions of system states, the effects of the actuator hysteresis can no longer be assumed bounded as in [13]. How to handle such effects is a challenging issue especially when there are possible actuator failures. In [14, 15], the idea is to separate such effects into two parts by applying Young's inequality. It is noted that the control methods in [14, 15] are very complicated and the tracking error cannot asymptotically converge to zero but to the so-called predefined bound. Furthermore, the existence of the estimators increases the order of the closed-loop system.

In this paper, we develop a backstepping [19, 20] adaptive compensation controller for a class of nonlinear systems preceded by hysteretic actuators described by Bouc-Wen model. Three types of high-gain functions are incorporated into the controller to counteract the effects of the hysteresis, bounded modeling errors, and bounded disturbances, respectively. In our design, the sign $(\cdot)$ function and a priori knowledge on the bounds of control gains are not required. Besides showing the stability of the closed-loop system, the tracking error is also ensured to achieve zero asymptotically.

The rest of the paper is outlined as follows. In Section 2, the control problem is formulated. In Section 3, a robust adaptive compensation scheme with high-gain functions is proposed. In Section 4, the stability analysis is presented. Simulation results are presented to show the proposed scheme is effective in Section 5. Finally, this paper is concluded in Section 6.

2. Problem Formulation

Consider a class of nonlinear systems in the following form [15]:

$$\begin{aligned}
\dot{x}_i &= x_{i+1} + a^T \varphi_i(\overline{x}_i) \quad (i = 1, 2, \ldots, \rho - 1) \\
&\vdots \\
\dot{x}_\rho &= \varphi_0(x, \xi) + a^T \varphi_\rho(\overline{x}_\rho) + \sum_{j=1}^{m} b_j \beta_j(x) u_j + d(t) + \eta(x, t) \\
\dot{\xi} &= \psi(x, \xi) + a^T \phi(x, \xi), \\
y &= x_1,
\end{aligned} \tag{1}$$

where ρ is the relative degree of the system; $u_j \in R$, $j = 1, 2, \ldots, m$ are the inputs whose actuators may fail during system operation; $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T$ $(i = 1, \ldots, n-1)$, $x = [x_1, x_2, \ldots, x_n]^T$ are the state vectors; $a = [a_1, \ldots, a_q]^T$ are unknown constant parameters; $b_j \in R$ $(j = 1, 2, \ldots, m)$ are unknown constant parameters with known signs; $\psi_0 \in R$, $\varphi_j \in R^\rho$ $(j = 1, \ldots, \rho)$, $\beta_j(x) \neq 0 \in R$ $(j = 1, \ldots, m)$, $\psi \in R^{n-\rho}$, and $\phi \in R^{(n-\rho)\times\rho}$ are known smooth functions; $|d(t)| \leq \overline{D}$ denotes bounded disturbance; and $\eta(x,t)$ is an unknown nonlinear function representing system modeling errors. There exists a known function $\delta(x,t)$ such that $|\eta(x,t)| \leq \delta(x,t)$.

The hysteresis nonlinearity can be described by Bouc-Wen model [17, 18]. Consider

$$\begin{aligned}
v_i &= \mu_i k_i u_{ci} + (1 - \mu_i) k_i \zeta_i = \mu_{i1} u_{ci} + \mu_{i2} \zeta_i \\
\dot{\zeta}_i &= \dot{u}_{ci} - \chi_{i1} |\dot{u}_{ci}| |\zeta_i|^{L_i - 1} - \chi_{i2} \dot{u}_{ci} |\zeta_i|^{L_i},
\end{aligned} \tag{2}$$

where $0 < \mu_i < 1$ $(i = 1, \ldots, m)$ are weighting parameters, k_i $(i = 1, \ldots, m)$ are stiffness coefficients, $\mu_{i1} = \mu_i k_i$, $\mu_{i2} = (1 - \mu_i) k_i$ $(i = 1, \ldots, m)$ are constants; u_{ci} is the input of the ith $(i = 1, \ldots, m)$ actuator, χ_{i1}, χ_{i2} $(i = 1, \ldots, m)$ describe the shape and amplitude of the ith hysteresis, respectively, L_i governs the smoothness of the transition from initial slope to the slope of the asymptote, and $\chi_{i1} \geq |\chi_{i2}|$, $L_i \geq 1$ $(i = 1, 2 \ldots, m)$. By Lemma 1 in [17], ζ_i $(i = 1, 2 \ldots, m)$ are bounded.

The actuator failure can be modeled as [8, 9, 13–15]

$$u_i = \rho_i \mu_{i1} v_i + d_i + u_{ki}, \quad \forall t \geq t_{iF}, \ (\rho_i u_{ki} = 0, i = 1, \ldots, m), \tag{3}$$

where $\rho_i \in [0, 1]$, u_{ki}, and t_{iF} are all unknown constants and $d_i = \rho_i \mu_{i2} \zeta_i$ $(i = 1, 2 \ldots, m)$ are bounded. For different values of ρ_i, three types of failures are included:

(1) $\rho_i = 1$, where the actuator works normally; namely, $u_i = u_{ci}$, which is regarded as a failure-free actuator;

(2) $0 < \rho_i < 1$; it implies $u_i = \rho_i u_{ci}$; the ith actuator is called partial loss of effectiveness (PLOE);

(3) $\rho_i = 0$; it indicates $u_i = u_{ki}$; the ith actuator is called total loss of effectiveness (TLOE).

Remark 1. The values of ρ_i can change only from $\rho_i = 1$ to some values with $0 \leq \rho_i < 1$. This means that possible changes from normal to any one of the failure cases are unidirectional. The uniqueness of t_{iF} indicates that a failure occurs only once on the ith actuator.

Substituting (2), (3) into (1), we have

$$\begin{aligned}
\dot{x}_i &= x_{i+1} + a^T \varphi_i(\overline{x}_i) \quad (i = 1, 2, \ldots, \rho - 1). \\
&\vdots \\
\dot{x}_\rho &= \varphi_0(x, \xi) + a^T \varphi_\rho(\overline{x}_\rho) \\
&\quad + \sum_{j=1}^{m} b_j \beta_j(x) \left(\rho_j \mu_{j1} u_{cj} + u_{kj} + d_j\right) \\
&\quad + d(t) + \eta(x, t) \\
\dot{\xi} &= \psi(x, \xi) + a^T \phi(x, \xi) \\
y &= x_1.
\end{aligned} \tag{4}$$

To derive a suitable adaptive control scheme, the following assumptions are made.

Assumption 2. When TOLE type of actuator failures up to $m-1$, the remaining actuators can still achieve a desired control objective.

Remark 3. Note that all actuators are allowed to have partial loss of effectiveness simultaneously.

Assumption 4. The zero dynamics of $\dot{\xi} = \psi(x, \xi) + a^T \phi(x, \xi)$ is input to state stable with respect to x as its input.

Let $T_0 = 0$. Suppose that there are p_k $(0 \leq p_k \leq m)$ actuators failing at time instants t_k, $k = 1, 2, \ldots, q$, and $t_0 < t_1 < \cdots t_q < \infty$. In other words, all actuators work normally in time interval $[t_0, t_1)$ and no new failure will occur after time t_q. Let the set Q_{jT} denote the actuators of total failure in interval $[t_j, t_{j+1})$ and use the set $\overline{Q}_{jT}$ to represent other normal actuators. It can be concluded that $Q_{jT} \cup \overline{Q}_{jT} = \{1, 2, \ldots, m\}$.

Our objective is to design a control law $u_{ci}(t)$ for the nonlinear systems with p unknown actuator failures, when p changes at time instants t_k, $k = 1, 2, \ldots, q$, such that the output $y(t)$ asymptotically tracks a given reference signal $y_r(t)$ with up to ρth order derivatives bounded and that all closed-loop signals are bounded.

3. Adaptive Compensation Control Schemes

The backstepping technique [19, 20] is applied to derive an adaptive actuator failure compensation controller. The following change of coordinates is required:

$$z_1 = x_1 - y_r$$
$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1} \quad (i = 2, \ldots, \rho), \tag{5}$$

where z_1 is the tracking error and α_ρ is the ρth stabilizing function. To illustrate the backstepping procedures, only the last step of the design is elaborated in details.

Step i. Consider $i = 1, \ldots, \rho-1$; the ith stabilizing function α_i, the ith regressor ω_i, and the ith tuning function τ_i are chosen as

$$\begin{aligned}
\alpha_1 &= -c_1 z_1 - \varphi_1(x_1)^T \hat{a} \\
\alpha_i &= -z_{i-1} - c_i z_i - \omega_i^T \hat{a} \\
&\quad + \sum_{k=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)} \right) \\
&\quad + \frac{\partial \alpha_{i-1}}{\partial \hat{a}} \Gamma \tau_i + \sum_{k=2}^{i-1} \left(\frac{\partial \alpha_{k-1}}{\partial \hat{a}} \Gamma \omega_i z_k \right), \quad (i = 2, \ldots, \rho-1) \\
\omega_i &= \varphi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_k \quad (i = 1, \ldots, \rho-1) \\
\tau_i &= \tau_{i-1} + \omega_i z_i \quad (i = 1, \ldots, \rho-1),
\end{aligned} \tag{6}$$

where c_i $(i = 1, \ldots, \rho-1)$ are positive design parameters and $\hat{a} \in R^q$ is estimator of the unknown vector a.

Step ρ. From (6), the derivative of z_ρ is

$$\begin{aligned}
\dot{z}_\rho &= \varphi_0(x, \xi) + \varphi_\rho(\bar{x}_\rho)^T a \\
&\quad + \sum_{i=1}^{m} b_i \beta_i(x) \left(\rho_i \mu_{i1} u_{ci} + u_{ki} + d_i \right) + \eta(x, t) \\
&\quad + d(t) - \sum_{k=1}^{\rho-1} \left(\frac{\partial \alpha_{\rho-1}}{\partial x_k} x_{k+1} + \varphi_k^T a \right) \\
&\quad - \frac{\partial \alpha_{\rho-1}}{\partial \hat{a}} \dot{\hat{a}} - \sum_{k=1}^{\rho-1} \left(\frac{\partial \alpha_{\rho-1}}{\partial y_r^{(k-1)}} y_r^{(k)} \right) - y_r^{(\rho)}.
\end{aligned} \tag{7}$$

If the effects of hysteresis are treated as disturbances, it should be noted that the disturbances in (7) can be classified into three types: (1) $d(t)$ is bounded by an unknown constant D; (2) $\eta(x, t)$ is bounded by a known function; (3) $\sum_{j=1}^{m} b_j \beta_j(x) d_i$ cannot be bounded by any known function but $\beta_j(x)$ $(j = 1, \ldots, m)$ are known and $b_j d_j$ $(j = 1, 2 \ldots, m)$ are bounded. According to their different characteristics, three high-gain functions will be proposed to counteract the effects of the disturbances. At the final step, the stabilizing function α_ρ is given by

$$\begin{aligned}
\alpha_\rho &= -\varphi_0 - \omega_\rho^T \hat{a} - z_{\rho-1} - c_\rho z_\rho + y_r^{(\rho)} \\
&\quad + \sum_{k=1}^{\rho-1} \left(\frac{\partial \alpha_{\rho-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{\rho-1}}{\partial y_r^{(k-1)}} y_r^{(k)} \right) \\
&\quad + \frac{\partial \alpha_{\rho-1}}{\partial \hat{a}} \Gamma \tau_\rho - e^{f(t)} z_\rho - e^{f(t)} \delta(x)^2 z_\rho - e^{f(t)} \sum_{k=1}^{m} \beta_i^2(x) z_\rho,
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\omega_\rho &= \varphi_\rho - \sum_{k=1}^{\rho-1} \frac{\partial \alpha_{\rho-1}}{\partial x_k} \varphi_k \\
\tau_\rho &= \tau_{\rho-1} + \omega_\rho z_\rho.
\end{aligned} \tag{9}$$

$f(t)$ is bounded and $\lim_{t \to \infty} \int_0^t e^{-f(\tau)} d\tau$ exists, and c_ρ is a positive design parameter.

Let

$$\omega = \left[\alpha_\rho, \beta_1, \ldots, \beta_m \right]^T. \tag{10}$$

The control law and parameter update laws are obtained as follows

$$u_{ci} = \operatorname{sign}(b_i) \frac{1}{\beta_i(x)} \hat{k}^T \omega, \quad (i = 1, \ldots, m) \tag{11}$$

$$\dot{\hat{a}} = \Gamma \tau_\rho \tag{12}$$

$$\dot{\hat{k}} = -\Gamma_k \omega z_\rho, \tag{13}$$

where $\hat{k} = [\hat{k}_1, \hat{k}_2^T]^T$, $\hat{k}_2 = [\hat{k}_{21}, \hat{k}_{22}^T, \ldots, \hat{k}_{2m}^T]^T$, $\hat{k}$ and $\hat{a}$ are the estimates of k and a, respectively, and Γ, Γ_k are positive definite matrices chosen by users. If k is a desired constant vector which can be chosen to satisfy

$$\sum_{i=1, i \notin \bar{Q}_{jT}}^{m} |b_i| \rho_i k^T w = \alpha_\rho - \sum_{j \in Q_{jT}} b_j \beta_j u_{kj}, \tag{14}$$

this gives

$$k_1 = \frac{1}{\sum_{i=1, i \notin \bar{Q}_{jT}}^{m} |b_i| \rho_i}, \qquad k_{2,j} = -\frac{b_j u_{kj}}{\sum_{i=1, i \notin \bar{Q}_{jT}}^{m} |b_i| \rho_i}. \tag{15}$$

Substituting (11)–(13) and (8) into (7), we have

$$\begin{aligned}
\dot{z}_\rho &= -z_{\rho-1} - c_\rho z_\rho + \omega_\rho^T \tilde{a} \\
&\quad + \sum_{i \in \bar{Q}_{jT}} |b_i| \rho_i \mu_{i1} \tilde{k}^T \omega - \left(e^{f(t)} z_\rho - d(t) \right) \\
&\quad - \left(e^{f(t)} \delta(x)^2 z_\rho - \eta(x, t) \right) \\
&\quad - \left(e^{f(t)} \sum_{k=1}^{m} \beta_k^2(x) z_\rho - \sum_{k=1}^{m} \beta_k(x) b_k d_k \right).
\end{aligned} \tag{16}$$

4. Stability Analysis

To prepare for the stability analysis, we rewrite the error system as

$$\dot{z} = A_z(z,t)\,z + W_\theta(z,t)^T\tilde{a} + D(t)\,e_\rho, \tag{17}$$

where the system matrices $A_z(z,t)$, $W_\theta(z,t)$, $D(t)$, and e_ρ are given by

$$\begin{gathered}
A_z(z,t) \\
= \begin{bmatrix} -c_1 & 1 & 0 & \cdots & \cdots & 0 \\ 1 & -c_2 & 1+\sigma_{23} & \sigma_{24} & \cdots & \sigma_{2\rho} \\ \vdots & -1-\sigma_{23} & \ddots & \ddots & \ddots & \vdots \\ \vdots & -\sigma_{24} & \ddots & \ddots & \ddots & \sigma_{\rho-2\rho} \\ \vdots & \vdots & \ddots & \ddots & \ddots & 1+\sigma_{\rho-1,\rho} \\ 0 & -\sigma_{2\rho} & \cdots & -\sigma_{\rho-2,\rho} & -1-\sigma_{\rho-1,\rho} & -c_\rho \end{bmatrix} \\
W_\varepsilon(z,t) = \left[\omega_1\omega_2,\ldots,\omega_\rho\right]^T \in R^\rho. \\
W_\theta(z,t) = W_\varepsilon(z,t)\,\omega^T - \hat{\rho}\left(\dot{y}_r + \overline{\alpha}_1\right)e_1e_1^T \in R^{\rho\times\rho} \\
D(t) = \sum_{i\in\overline{Q}_{jT}} |b_i|\,\rho_i\mu_{i1}\tilde{k}^T\omega - \left(e^{f(t)}z_\rho - d(t)\right) \\
- \left(e^{f(t)}\delta(x)^2 z_\rho - \eta(x,t)\right) \\
e_\rho = [0,\ldots,0,1]^T \in R^\rho \\
\sigma_{jk} = -\frac{\partial\alpha_{j-1}}{\partial\hat{a}}\Gamma\omega_k.
\end{gathered} \tag{18}$$

The closed-loop system has the following desired properties.

Theorem 5. *With the ISS of the zero dynamics of system* (1) *and m hysteretic actuators modeled in* (2) *with possible unknown failures by* (3), *the controller* (11) *with the adaptive laws* (12) *and* (13) *ensures the boundedness of the closed-loop signals and the asymptotic output tracking*: $\lim_{t\to\infty}(y-y_r) \to 0$.

Proof. For each time interval (t_k, t_{k+1}), $k = 0,1,2,\ldots,q$, we have a Lyapunov function V in the following form:

$$V = \frac{1}{2}\sum_{i=1}^{\rho} z_i^2 + \frac{1}{2}\tilde{a}^T\Gamma\tilde{a} + \sum_{i\in\overline{Q}_{jT}} \frac{\rho_i\mu_{i1}|b_i|}{2}\tilde{k}_j^T\Gamma_k\tilde{k}_j. \tag{19}$$

Taking the derivative of (19) yields

$$\dot{V} = \sum_{i=1}^{\rho} z_i\dot{z}_i - \tilde{a}^T\Gamma\dot{\hat{a}} - \sum_{i\in\overline{Q}_{jT}} \rho_i\mu_{i1}|b_i|\,\tilde{k}_j^T\Gamma_k\dot{\hat{k}}_j. \tag{20}$$

Substituting (11)–(13) and (8) into (7), we have

$$\begin{aligned}
\dot{V} = &-\sum_{i=1}^{\rho} c_i z_i^2 - \left(e^{f(t)}z_\rho^2 - d(t)\,z_\rho\right) \\
&- \left(e^{f(t)}\delta(x)^2 z_\rho^2 - \eta(x,t)\,z_\rho\right) \\
&- \left(e^{f(t)}\sum_{k=1}^{m}\beta_k^2(x)\,z_\rho^2 - \sum_{k=1}^{m}\beta_k(x)\,b_kd_kz_\rho\right).
\end{aligned} \tag{21}$$

From $|d(t)| \le \overline{D}$, $|\eta(x,t)| \le \delta(x,t)$, and bounded d_i ($i = 1,2,\ldots,m$), we get

$$\begin{aligned}
\dot{V} \le &-\sum_{i=1}^{\rho} c_i z_i^2 - \left(e^{f(t)}z_\rho^2 - \left|\overline{D}\right|\left|z_\rho\right|\right) \\
&- \left(e^{f(t)}\delta(x)^2 z_\rho^2 - |\delta(x)|\left|z_\rho\right|\right) \\
&- \left(e^{f(t)}\sum_{k=1}^{m}\beta_k^2(x)\,z_\rho^2 - \sum_{k=1}^{m}\left|\beta_k(x)\right|\left|b_kd_k\right|\left|z_\rho\right|\right) \\
\le &-\sum_{i=1}^{\rho} c_i z_i^2 + \frac{\overline{D}^2}{4}e^{-f(t)} + \frac{1}{4}e^{-f(t)} + \frac{1}{4}\sum_{k=1}^{m}\left|b_k^2d_k^2\right|e^{-f(t)} \\
= &-\sum_{i=1}^{\rho} c_i z_i^2 + \frac{\sum_{k=1}^{m}\left|b_k^2d_k^2\right| + \overline{D}^2 + 1}{4}e^{-f(t)} \\
= &-\sum_{i=1}^{\rho} c_i z_i^2 + D_0e^{-f(t)},
\end{aligned} \tag{22}$$

where

$$D_0 = \frac{\sum_{k=1}^{m}\left|b_k^2d_k^2\right| + \overline{D}^2 + 1}{4}. \tag{23}$$

From (22), we conclude that $V(t) \in L_\infty, t \in [t_0, t_1)$, so that z_i ($i = 1,\ldots,\rho$), $\dot{z}_i$ ($i = 1,\ldots,\rho$), and $\tilde{a}, \tilde{k}$ are bounded for $t \in [t_0, t_1)$. It follows from (8) and (10) that α_ρ and ω are bounded. Therefore, all closed-loop signals are bounded for (t_0, t_1). In order to prove the asymptotic tracking, considering the last time interval (t_q, ∞). From (22), we can obtain

$$c_i\int_0^t z_i^2(\tau)\,d\tau \le V(0) - V(t) + D_0\int_0^t e^{-f(\tau)}d\tau. \tag{24}$$

Because of the boundedness of $D_0\int_0^t e^{-f(\tau)}d\tau$, we have $z_i \in L_2$, ($i = 1,\ldots,\rho$). By Barbalat's lemma, $\lim_{t\to\infty}(z_i(t)) \to 0$ ($i = 1,\ldots,\rho$) can be obtained. This completes the proof. □

5. Simulations

We consider a second order nonlinear [15] system with two inputs described as

$$\begin{aligned}
\dot{x}_1 &= x_2 + \varphi_1^T(x_1)\,\theta \\
\dot{x}_2 &= \varphi_2^T(x)\,\theta + \sum_{i=1}^{2} b_i\beta_i(x)\,u_i + \eta(x,t),
\end{aligned} \tag{25}$$

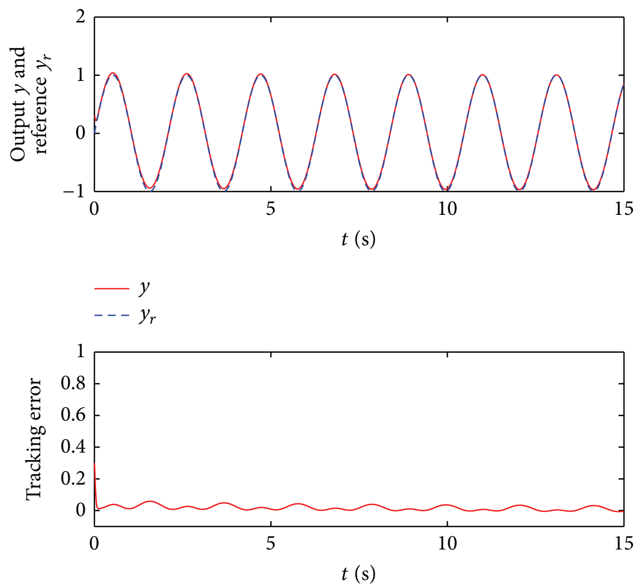

Figure 1: Output y, reference y_r, and error $e(t)$.

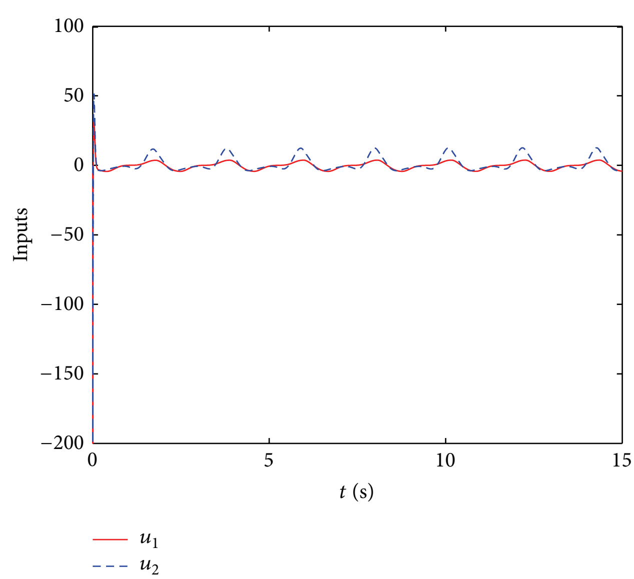

Figure 2: u_1 and u_2.

where $\varphi_1(x_1) = 1 + x_1^2$, $\varphi_2(x) = 1 + \sin(x_1)$, $\beta_1(x) = 1.9 + 0.1\sin x_1$, $\beta_2(x) = e^{x_1}$, $\theta = 2$, $b_1 = b_2 = 1$, b_1, b_2 are unknown constants, u_1, u_2 are the outputs of two hysteretic actuators, x_1, x_2 are the states, and $\eta(x) = 0.1x_1^2(1 + \sin t)(1 + \sin x_2)$ is bounded by $\delta(x) = 0.4x_1^2$. The reference signal is set as $y_r(t) = \sin(3t)$. The backlash-like hysteresis is described by (2) with parameters $\mu_{11} = \mu_{21} = \mu_{12} = \mu_{22} = 1$, $\chi_{11} = \chi_{21} = 1$, $\chi_{12} = \chi_{22} = 0$, and $L_1 = L_2 = 1$. The high-gain function is chosen as $f(t) = e^{2.5\arctan(t)}$. For simulation, we consider three actuator failure cases.

Case 1. There are no actuator failures.

By Theorem 5, we can obtain the actual control law and the update laws. The initial conditions are set as follows:

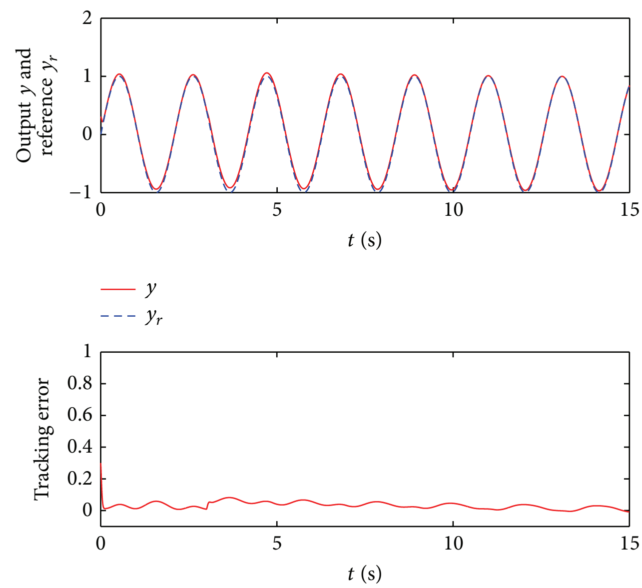

Figure 3: Output y, reference y_r, and error $e(t)$.

$$\begin{aligned} &\hat{a} = 1.5, &&\Gamma = 0.0005, \\ &\hat{k} = [0.4, 0, 0]^T, &&\Gamma_k = 0.0001 * I_3 \\ &x_0 = [0.3, 0]^T, &&c_1 = 60, \quad c_2 = 60, \end{aligned} \tag{26}$$

where I_3 is the 3rd order identity matrix.

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 1; the actuators outputs u_1, u_2 are shown in Figure 2. The system responses are as expected. At the beginning, there is a transient response in tracking errors. But, as time goes on, the tracking errors become smaller and ultimately vanish. The proposed controller guarantees that asymptotic tracking is achieved.

Case 2. Actuator u_1 is stuck at $u_1 = 45$ from $t = 3$ s, thus undergoing a TLOE type of failure. By Theorem 5, we can obtain the actual control law and the update laws. The initial conditions are set as follows.

The other parameters are the same as those in Case 1.

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 3; the actuators outputs u_1, u_2 are shown in Figure 4. The system responses are as expected. When one of the actuators fails, there is a transient response in tracking errors. But, as time goes on, the tracking errors become smaller and ultimately vanish. The proposed controller guarantees that asymptotic tracking is achieved.

Case 3. Actuator u_2 is stuck at $u_2 = 40$ from $t = 3$ s and actuator u_1 loses 60% from $t = 6$ s. Thus, u_1 undergoes a PLOE type of failure while u_2 is a TLOE type of failure.

The other parameters are the same as those in Case 1.

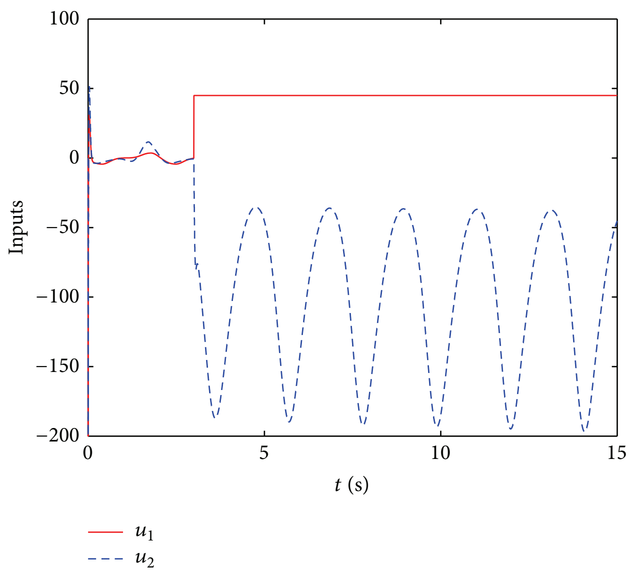

Figure 4: u_1 and u_2.

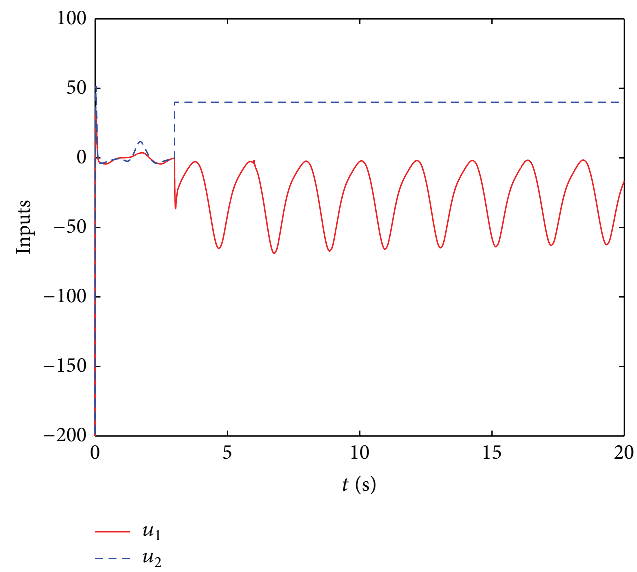

Figure 6: u_1 and u_2.

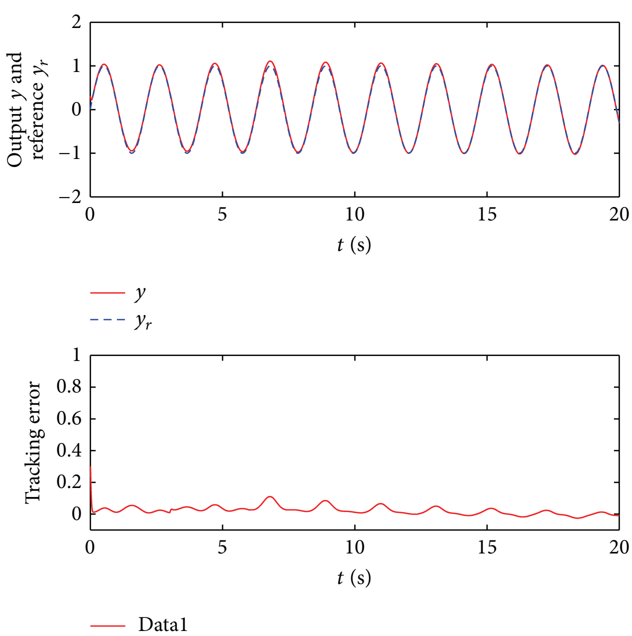

Figure 5: Output y, reference y_r, and error $e(t)$.

The simulation results including output $y(t)$, reference output $y_r(t)$, and tracking error $e(t)$ are shown in Figure 5; the actuators outputs u_1, u_2 are shown in Figure 6. The system responses are as expected. When the actuators fail, there is a transient response in tracking errors. But, as time goes on, the tracking errors become smaller and ultimately vanish. The proposed controller guarantees that asymptotic tracking is achieved.

6. Conclusions

This paper presents an adaptive failure compensation controller for a class of uncertain nonlinear systems dominated by the hysteresis actuator nonlinearity. We propose three types of high-gain functions to deal with the unknown bounded disturbances, unknown modeling errors, and unknown actuator failures. It has been shown that the tracking errors can converge to zero asymptotically while all the closed-loop signals remain bounded. Furthermore, the proposed scheme can avoid possible chattering. Simulation results illustrate the effectiveness of our proposed scheme.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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