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Research Article

Research on Evaluation Method Based on Modified Buckley Decision Making and Bayesian Network

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This work presents a novel evaluation method, which can be applied in the field of risk assessment, project management, cause analysis, and so forth. Two core technologies are used in the method, namely, modified Buckley Decision Making and Bayesian Network. Based on the modified Buckley Decision Making, the fuzzy probabilities of element factors are calibrated. By the forward and backward calculation of Bayesian Network, the structure importance, probability importance, and criticality importance of each factor are calculated and discussed. A numerical example of risk evaluation for dangerous goods transport process is given to verify the method. The results indicate that the method can efficiently identify the weakest element factor. In addition, the method can improve the reliability and objectivity for evaluation.

1. Introduction

The applications of evaluation models almost extend to every aspect of research, for example, forecasting traffic flows [1], analyzing the causes of traffic accidents [2], evaluating the risk of transport process [3], and discussing the origins of driver fatigue [4]. In these topics, two targets attract researchers' interest: investigating the top-event occurrence rate and exploring the influence degree of element factors. Analytic hierarchy process (AHP), fuzzy analytic hierarchy process (FAHP), fishbone diagram model (FDM), fault tree analysis (FTA), and Bayesian Network (BN) have proved to be efficient approaches for studying them [5–9]. In these models, the probabilities of element factors are the foundation of quantitative analysis. They are calibrated in the traditional approach by the following three steps. (1) Construct judgment matrix by introducing 1~9 contrast scaling (K). (2) Normalize processing of the matrix to calculate the eigenvector. (3) Assign each element factor a value of element in eigenvector as its probability value correspondingly. However, in the first step, it has natural deficiency by using 1~9 contrast scaling to construct the judgment matrix. To illustrate this point, an intensity of importance scale is given in Table 1.

Suppose $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are element factors. According to Table 1, if $\mathbf{b}$ is slightly more important than $\mathbf{a}$, then $\mathbf{b} : \mathbf{a} = 3 : 1$. Moreover, if $\mathbf{c} : \mathbf{b} = 9 : 3$ (in the view of scale table, the weight ratio of element factors "$\mathbf{c}$" and "$\mathbf{b}$" is $9 : 3$), then the importance of factor "$\mathbf{c}$" compared with "$\mathbf{b}$" belongs to the level of "strong importance." At the same time, $9 : 3$ is equal to $3 : 1$ mathematically. In other words, the weight ratio of factors "$\mathbf{c}$" and "$\mathbf{b}$" is $3 : 1$, so factor "$\mathbf{c}$" is a little more important than "$\mathbf{b}$." Therefore, it is conflicting between the above two standpoints. Furthermore, if the relationship between factors "$\mathbf{c}$" and "$\mathbf{b}$" applies to nexus between "$\mathbf{c}$" and "$\mathbf{a}$," then the weight ratio of factors "$\mathbf{c}$" and "$\mathbf{a}$" is $9 : 1$. Although factor "$\mathbf{c}$" is 9 times of "$\mathbf{a}$" in weight ratio, it is not definite that factor "$\mathbf{c}$" is extremely more important than "$\mathbf{a}$." Therefore, it inevitably causes the confusion when using the traditional approach. In addition, the judgment matrix constructed by the traditional approach is a deterministic matrix, which cannot well represent the fuzzy comparisons between element factors ideally.

Aiming at the defects of traditional method, the 9/9~9/1 contrast scaling will be introduced into the Buckley matrix; in addition, K-value will be replaced by trapezoidal fuzzy number [3, 10]. Then the modified Buckley Decision

Table 1: Scale and its implication.

1~9 contrast scaling (K)	Intensity of importance
1	Equal importance
3	Moderate importance
5	Obvious importance
7	Strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values between two adjacent judgments

Making method will be applied to calibrate the probabilities of element factors. Using this technique, the calculation becomes more rigorous and makes the importance degree much more scientific despite of its complexity.

In the field of reasoning model, BN can be mapped easily from the AHP, FAHP, FDM, and FTA models. Owing to the conditional probability tables of BN, the diagram calculation process of AHP, FAHP, FDM, and FTA can be transformed into a logical table calculation process, which helps to calculate intelligently. Based on the chain rule in BN, the evaluation value of top-event as well as the structure (probability, criticality) importance of each factor will be obtained by the forward and backward calculation of BN. The influence degree of each element factor can be effectively indicated by the previous three importance indexes.

The remaining parts of this paper are organized as follows. Section 2 introduces the new evaluation method. In the method, modified Buckley Decision Making method is applied to calibrate the probability of each element factor; BN is served as the reasoning computation model; structure importance, probability importance and criticality importance are designed as the evaluation indicators; Bucket Elimination algorithm is modified to solve the BN. In Section 3, a numerical example is given to verify the method. Conclusions are finally drawn in Section 4, along with recommendations for future research.

2. The New Evaluation Method

2.1. Modified Buckley Decision Making. The probabilities of element factors are fundamental to the evaluation process, which rely on judgment matrix. Based on the Buckley Decision Making method, the trapezoidal fuzzy number is introduced to construct the judgment matrix. If element factor I_x is extremely more important than I_y, then the initialized fuzzy judgment matrix can be constructed as

$$\mathbf{A}_0 = \begin{array}{c} \\ I_x \\ \vdots \\ I_y \end{array} \begin{array}{c} \begin{array}{ccc} \quad I_x \quad & \cdots & \quad I_y \quad \end{array} \\ \begin{bmatrix} (1\ 1\ 1\ 1) & \cdots & (8\ 8\ 9\ 9) \\ \vdots & \ddots & \vdots \\ \left(\frac{1}{9}\ \frac{1}{9}\ \frac{1}{8}\ \frac{1}{8}\right) & \cdots & (1\ 1\ 1\ 1) \end{bmatrix} \end{array}. \tag{1}$$

Then, the formula $\mathbf{K}' = 9/(10 - \mathbf{K})$ is applied to modify each element in the upper right corner of the matrix [11]. The elements in the lower left corner of the matrix are derived by construction rules of Buckley matrix [10]. Therefore, the modified fuzzy judgment matrix is

$$\mathbf{A} = \begin{array}{c} \\ I_x \\ \vdots \\ I_y \end{array} \begin{array}{c} \begin{array}{ccc} \quad I_x \quad & \cdots & \quad I_y \quad \end{array} \\ \begin{bmatrix} (1\ 1\ 1\ 1) & \cdots & \left(\frac{9}{2}\ \frac{9}{2}\ \frac{9}{1}\ \frac{9}{1}\right) \\ \vdots & \ddots & \vdots \\ \left(\frac{1}{9}\ \frac{1}{9}\ \frac{2}{9}\ \frac{2}{9}\right) & \cdots & (1\ 1\ 1\ 1) \end{bmatrix} \end{array}. \tag{2}$$

Similarly, based on the modified Buckley Decision Making method, the matrix contained contrast between all the element factors which is defined as

$$\mathbf{B} = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nn} \end{bmatrix}, \tag{3}$$

where $x_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$, $x_{11} \sim x_{nn} = (1, 1, 1, 1)$, and $x_{ji} = 1/x_{ij} = (1/\delta_{ij}, 1/\gamma_{ij}, 1/\beta_{ij}, 1/\alpha_{ij})$.

Let

$$\begin{aligned} \alpha_{(i=k)} &= \left(\prod_{j=1}^{n} \alpha_{kj}\right)^{1/n}, & \alpha &= \sum_{i=1}^{n} \left[\alpha_{(i=k)}\right]; \\ \beta_{(i=k)} &= \left(\prod_{j=1}^{n} \beta_{kj}\right)^{1/n}, & \beta &= \sum_{i=1}^{n} \left[\beta_{(i=k)}\right]; \\ \gamma_{(i=k)} &= \left(\prod_{j=1}^{n} \gamma_{kj}\right)^{1/n}, & \gamma &= \sum_{i=1}^{n} \left[\gamma_{(i=k)}\right]; \\ \delta_{(i=k)} &= \left(\prod_{j=1}^{n} \delta_{kj}\right)^{1/n}, & \delta &= \sum_{i=1}^{n} \left[\delta_{(i=k)}\right]. \end{aligned} \tag{4}$$

Then the fuzzy weight x_k of element factor I_k is calculated:

$$x_k = \left(\frac{\alpha_{(i=k)}}{\delta}, \frac{\beta_{(i=k)}}{\gamma}, \frac{\gamma_{(i=k)}}{\beta}, \frac{\delta_{(i=k)}}{\alpha}\right). \tag{5}$$

Performing the same procedure for i from 1 to n, we have

$$\mathbf{X} = \{x_1, x_2, \ldots, x_k, \ldots, x_n\}. \tag{6}$$

Then, the value of element in $\mathbf{X}$ is assigned to each element factor as its fuzzy probability value correspondingly.

2.2. Bayesian Network. Bayesian Network is a directed acyclic graph with a series of conditional probability tables (CPTs) [3, 12, 13]. It has become widely used in the field of evaluation because of the conditional independence of BN nodes and

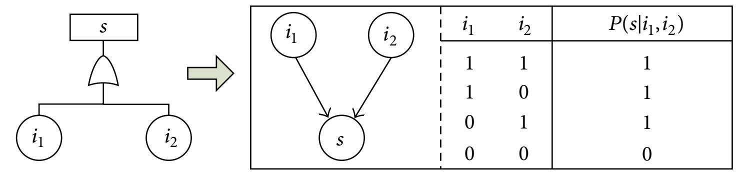


Figure 1: OR gate.

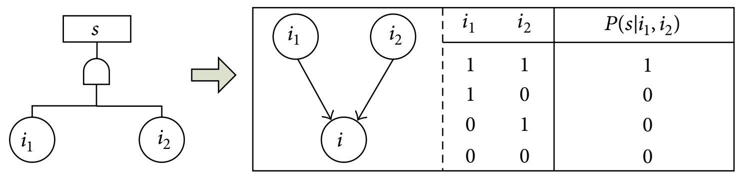


Figure 2: AND gate.

bidirectional reasoning mechanism. In addition, AHP, FAHP, FDM, and FTA models can be mapped to a BN easily, which helps to establish BN models. Roughly speaking, in most of AHP, FAHP, FDM, and FTA models, the relationship between subnodes and parent nodes only involves “OR gate” and “AND gate” (the logical relationships between events and causes in AHP, FAHP, and FDM models are represented by means of logical “AND” and “OR” gates). Therefore, it is critical to discuss how an “OR gate” and an “AND gate” convert into an equivalent BN. The conversions are shown in Figures 1 and 2.

According to Figure 1, we have

$$P\left(s=1 \mid i_1=0,\ i_2=0\right)=0, \qquad P\left(s=1 \mid \text{else}\right)=1. \tag{7}$$

According to Figure 2, we have

$$P\left(s=1 \mid i_1=1,\ i_2=1\right)=1, \qquad P\left(s=1 \mid \text{else}\right)=0. \tag{8}$$

Based on the conversions, each gate (OR or AND) in the AHP, FAHP, FDM, and FTA assigns the equivalent CPT to the corresponding node in the BN and then the whole BN is established [14]. Taking the conditional independence of BN nodes into consideration, we get the joint probability distribution function using chain rule, which is the foundation of forward and backward calculation.

2.3. Evaluation Indicators. In the evaluation method, three indicators, that is, structure importance (Q_i^{St}), probability importance (Q_i^{Pt}), and criticality importance (Q_i^{Ct}), are designed to evaluate the importance of each factor. Q_i^{St} analyzes the influence of each factor with respect to model structure. Q_i^{Pt} analyzes the influence of each factor with respect to probability of each factor and model structure. Q_i^{Ct} analyzes the influence of each factor with respect to sensitivity and probability, which can reflect the fact that reducing the occurrence probability of a large probability event is easier than a rare event [3]. The calculation formulas of the above three indicators are defined by (9), where the parameters are shown in Table 2:

Table 2: Parameters of calculation formulas.

Parameters	Standing for
T	Top-event
I_i	Element factor i
I_j	Element factor j
n	Total numbers of all element factors
$I_i = 1$	Element factor i occurs
$I_j = 1$	Element factor j occurs
$p(T = 1)$	Occurrence probability of top-event
$p(I_i = 1)$	Occurrence probability of element factor i
$p(I_j = 1)$	Occurrence probability of element factor j
$p(T = 1 \mid \bullet)$	Conditional probability of top-event when it happens

$$\begin{aligned} Q_i^{\mathrm{St}} &= p\left(T=1 \mid I_i=1, p\left(I_j=1\right)=0.5, 1 \le j \ne i \le n\right) \\ &\quad - p\left(T=1 \mid I_i=0, p\left(I_j=1\right)=0.5, 1 \le j \ne i \le n\right), \\ Q_i^{\mathrm{Pr}} &= \left|p\left(T=1 \mid I_i=1\right) - p\left(T=1 \mid I_i=0\right)\right|, \\ Q_i^{\mathrm{Cr}} &= \frac{p\left(I_i=1\right) * \left|p\left(T=1 \mid I_i=1\right) - p\left(T=1 \mid I_i=0\right)\right|}{p\left(T=1\right)}. \end{aligned} \tag{9}$$

2.4. Evaluation Methodology. Among various techniques for solving BN, Bucket Elimination has proved to be one of the most efficient approaches [15]. Considering the dimorphism of BN mapped from AHP, FAHP, FDM, and FTA model, the Bucket Elimination algorithm can be modified to reduce the computational difficulty and to improve the efficiency of calculation. Therefore, we define calculation rules of the modified Bucket Elimination algorithm firstly; then, based on the rules and modified Buckley Decision Making, put forward the evaluation process.

Table 3: Parameters of calculation rules.

Parameters	Standing for	Note
C_i	The subnode event	If the node represents element factor, $C_i = I_i$
C_j	The parent node event	/
P_{ij}	The connection event	/
p_{ij}	The conditional probability table of P_{ij}	/
$p(C_i = 1)$	The occurrence probability of C_i	$p(C_i = 1) = x_i, p(C_i = 0) = 1 - x_i$
$p(C_j = 1)$	The occurrence probability of C_j	$p(C_j = 1) = x_i, p(C_j = 0) = 1 - x_i$

(1) Calculation Rules. In the modified Bucket Elimination algorithm, three calculation rules are defined as follows (see (10) ~ (12)), where the parameters are shown in Table 3.

Rule 1. To deal with "singer-factor," if we have $C_i \xrightarrow{P_{ij}} C_j$, then

$$P\left(C_j = 1\right) = p\left(C_i\right) \times p_{ij} = x_i p_{ij}, \quad \text{where } p_{ij} = 0 \text{ or } 1. \tag{10}$$

Rule 2. To deal with "AND gate," if we have $\bigcap_{i=k-m}^{i=k+n} C_i \xrightarrow{P_{ij}} C_j$, then

$$p\left(C_j = 1\right) = \sum_{i=k-m}^{i=k+n} p\left(C_i\right) \times p_{ij} = x_{k-m} \times x_{k-m+1} \times \cdots \times x_k \times \cdots \times x_{k+n} \times 1. \tag{11}$$

Rule 3. To deal with "OR gate," if we have $\bigcup_{i=k-m}^{i=k-m} C_i \xrightarrow{P_{ij}} C_j$, then

$$p\left(C_j = 1\right) = \sum_{i=k-m}^{i=k+n} p\left(C_i\right) \times p_{ij} = 1 - \left[\left(1 - x_{k-m}\right) \times \left(1 - x_{k-m+1}\right) \times \cdots \times \left(1 - x_k\right) \times \cdots \times \left(1 - x_{k+n}\right) \times 1\right]. \tag{12}$$

(2) Evaluation Process. The calculation process of the new evaluation method is shown in Figure 3. According to the evaluation process, the evaluation indicators are derived, which indicate the importance of each element factor. Based on the results, we can efficiently identify the weakest element factor.

3. Numerical Example

Figure 4 shows an FTA model of risk evaluation for dangerous goods transport process, where I_i represents an element factor. The corresponding importance of each factor (I_i) is given by experts. Based on the modified Buckley Decision Making method, the fuzzy probabilities of all element factors are calibrated (see Table 4).

Table 4: Fuzzy probabilities of element factors.

Element factor	Fuzzy probability
I_1	(0.021 07, 0.028 52, 0.038 31, 0.053 23)
I_2	(0.018 97, 0.026 64, 0.035 25, 0.048 03)
I_3	(0.024 65, 0.033 06, 0.043 22, 0.061 38)
I_4	(0.002 50, 0.003 34, 0.004 07, 0.005 90)
I_5	(0.023 28, 0.031 16, 0.041 59, 0.057 45)
I_6	(0.006 82, 0.009 68, 0.013 09, 0.019 67)
I_7	(0.034 91, 0.045 97, 0.056 38, 0.078 31)
I_8	(0.036 44, 0.048 75, 0.059 69, 0.079 29)
I_9	(0.003 82, 0.005 28, 0.007 30, 0.010 42)

According to Section 2.2, the BN is established (see Figure 5).

As illustrated in Figure 5, the joint probability distribution function is derived in

$$\begin{aligned} p(t) &= p\left(t, S_1, \ldots, S_4, I_1, \ldots, I_9\right) \\ &= p\left(t \mid S_1, S_2, I_9\right) p\left(I_9\right) p\left(S_1 \mid I_1, I_2\right) p\left(I_1\right) p\left(I_2\right) \\ &\quad \cdot p\left(S_2 \mid S_3, S_4\right) p\left(S_3 \mid I_3, I_4, I_5\right) p\left(I_3\right) p\left(I_4\right) \\ &\quad \cdot p\left(I_5\right) p\left(S_4 \mid I_6, I_7, I_8\right) p\left(I_6\right) p\left(I_7\right) p\left(I_8\right). \end{aligned} \tag{13}$$

Based on Section 2.4, the evaluation value $p(t) = (0.008\,00, 0.012\,71, 0.019\,33, 0.032\,95)$ and three importance indexes are calculated as follows (see Figures 6–8).

According to Figure 6, the structure importance of each element factor is sorted to be

$$Q_9^{\mathrm{St}} > Q_3^{\mathrm{St}} = Q_4^{\mathrm{St}} = Q_5^{\mathrm{St}} = Q_6^{\mathrm{St}} = Q_7^{\mathrm{St}} = Q_8^{\mathrm{St}} > Q_1^{\mathrm{St}} = Q_2^{\mathrm{St}}. \tag{14}$$

According to Figure 7, the probability importance of each element factor is sorted to be

$$Q_2^{\mathrm{Pr}} > Q_1^{\mathrm{Pr}} > Q_9^{\mathrm{Pr}} > Q_3^{\mathrm{Pr}} > Q_8^{\mathrm{Pr}} > Q_5^{\mathrm{Pr}} > Q_7^{\mathrm{Pr}} > Q_6^{\mathrm{Pr}} > Q_4^{\mathrm{Pr}}. \tag{15}$$

According to Figure 8, the criticality importance of each element factor is sorted to be

$$Q_2^{\mathrm{Cr}} > Q_1^{\mathrm{Cr}} > Q_8^{\mathrm{Cr}} > Q_7^{\mathrm{Cr}} > Q_3^{\mathrm{Cr}} > Q_5^{\mathrm{Cr}} > Q_9^{\mathrm{Cr}} > Q_6^{\mathrm{Cr}} > Q_4^{\mathrm{Cr}}. \tag{16}$$

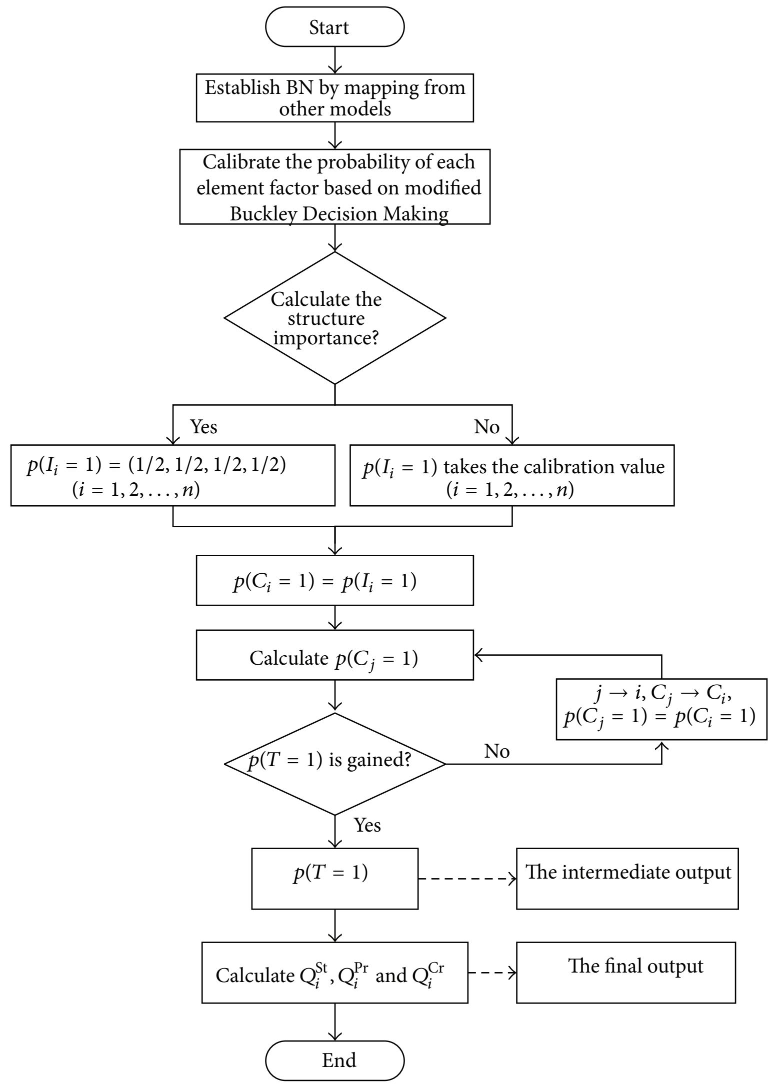


Figure 3: Calculation process of the new evaluation method.

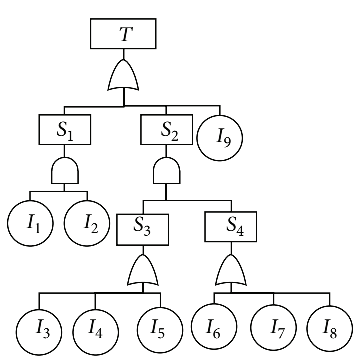
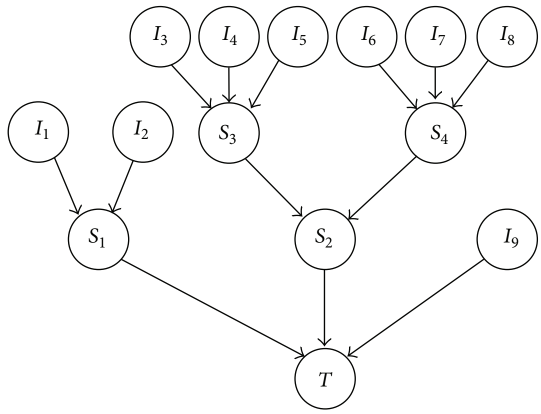


Figure 4: Fault tree.

As shown in (14), most structure importance of element factors is equal. Therefore, it is not proper to indicate the influence extent of different factors using only structure importance. By a comparison of (14), (15), and (16), we find that Q_9^{St} and Q_9^{Pr} are relatively large while Q_9^{Cr} is relatively small. It indicates that I_9 has great effects on $p(t)$, but it is difficult to reduce the influence of I_9 on $p(t)$ by taking measures. In addition, $\forall i \neq 1, 2$, $Q_1^{Pr}, Q_2^{Pr} > Q_i^{Pr}$ and $Q_1^{Cr}, Q_2^{Cr} > Q_i^{Cr}$. It can be concluded that I_1 and I_2 have larger effects on $p(t)$ than the other factors, and it can ameliorate the value of $p(t)$ efficiently by reducing the occurrence probabilities of I_1 and I_2. Therefore, in the actual productions, it is significantly important to monitor the procedures of I_1 and I_2 to ensure safety.

4. Conclusion

Based on the combination of modified Buckley Decision Making and Bayesian Network, we present a new evaluation method in this paper, which can be widely used in the field of risk assessment, project management, cause analysis, and so forth. By using modifier formula $\mathbf{K}' = 9/(10 - \mathbf{K})$, 9/9~9/1 contrast scaling is successfully introduced into the Buckley



$P(S_1 = 1 \mid I_1 = 1, I_2 = 1) = 1$
$P(S_1 = 1 \mid \text{else}) = 0$
$P(S_3 = 1 \mid I_3 = 0, I_4 = 0, I_5 = 0) = 0$
$P(S_3 = 1 \mid \text{else}) = 1$
$P(S_4 = 1 \mid I_6 = 0, I_7 = 0, I_8 = 0) = 0$
$P(S_4 = 1 \mid \text{else}) = 1$
$P(S_2 = 1 \mid S_3 = 1, S_4 = 1) = 1$
$P(S_2 = 1 \mid \text{else}) = 0$
$P(T = 1 \mid S_1 = 0, S_2 = 0, I_9 = 0) = 0$
$P(T = 1 \mid \text{else}) = 1$

FIGURE 5: Bayesian Network.

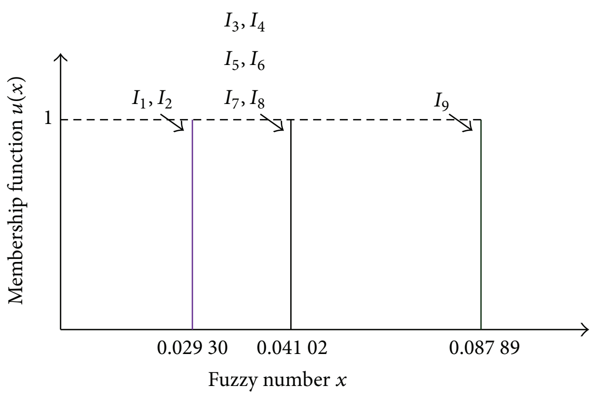


FIGURE 6: Structure importance.

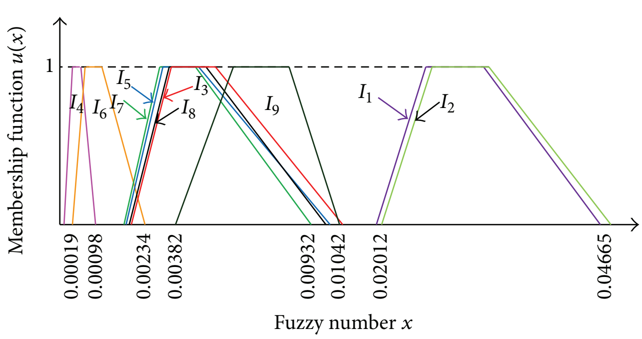


FIGURE 7: Probability importance. Note: x was only marked-out partial numbers because of limited margin.

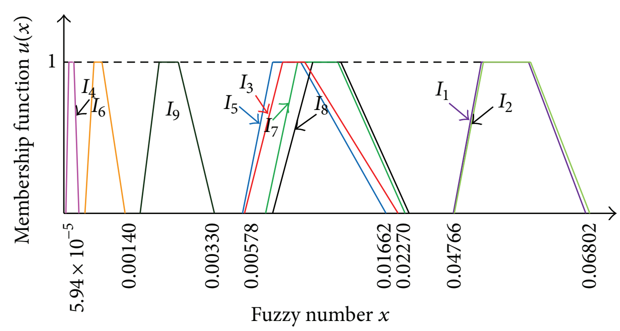


FIGURE 8: Criticality importance. Note: x was only marked-out partial numbers because of limited margin.

matrix. In addition, the elements in Buckley matrix are trapezoidal fuzzy number, which can well represent the fuzzy comparisons between element factors ideally. According to the modified Buckley Decision Making method, we can calibrate the probability of each element factor more logically. As has been said, the probabilities of factors are fundamental to evaluation. Therefore, the application of this core technology makes our evaluation method more reliable and objective. Based on the bidirectional reasoning mechanism of BN, the top-event occurrence rate (or evaluation value) and influence indexes of element factors are gained by the forward and backward calculation. Then, according to the sequence of these influence indexes, the impact of each element factor can be represented. Therefore, we can identify the weakest element factor efficiently. In addition, the application of CPTs in BN can make the diagram calculation of AHP, FAHP, FDM, and FTA into a logical table calculation, which is beneficial to the intelligence of calculating techniques.

There are many interesting directions in which we can extend our work. Ongoing and future research that we are pursuing are to construct a discrete time dynamic Bayesian Network model, combining the modified Buckley Decision Making method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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