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Research Article

Periodic Switched Control of Dual-Rate Sampled-Data Systems

Dawei Zhang, Wei Bai, and Xinchun Jia

School of Mathematical Sciences, Shanxi University, Taiyuan 030006, China

Correspondence should be addressed to Dawei Zhang; zhangdaweisx@sxu.edu.cn

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This paper is concerned with the periodic switched control of a linear dual-rate sampled-data system. The state variables of the continuous-time plant are sampled by two types of sensors. The ratio of two sampling rates is assumed to be a rational number. Depending on whether the sampled-data of state variables at two sampling rates is available simultaneously or separately, a periodic switched controller is constructed. Applying an input delay approach, the closed-loop system is modeled as a switched system with subsystems having different input delays. Some delay-dependent criteria for the H_∞ performance of the switched system and the existence of the switched controller are derived by employing a Lyapunov-Krasovskii functional that includes information about two sampling periods. The dual-rate sampled-data control of a vehicle dynamic system is given to show that the proposed method is effective and it can achieve a better H_∞ control performance than the single-rate design method.

1. Introduction

Sampled-data control of continuous-time practical systems, especially complex industrial systems, offers several advantages such as flexibility, low cost, and increased reliability [1, 2]. In a sampled-data control system, where a continuous-time plant is controlled with a digital controller, the sampling rate is a critical design parameter. The choice of the sampling rate mainly depends on some factors like bandwidth and response time of closed-loop systems, physical limitations of sensors and actuators, and the effect of the noise. The sampling rate must be chosen as fast as possible to ensure the high precision and the fast response time of the system but as slow as possible to satisfy the hardware limits and to eliminate the effect of the noise on the control input. Taking these factors into account, the effective range of the sampling rate is determined.

In many industrial applications, it is impractical to sample all physical signals uniformly at one single-rate, which demands a multirate sampling scheme. For instance, for an industrial vehicle, laser sensors are used to measure the heave position and the heave velocity, and gyrometers are chosen to measure the angular velocity and the heading angle [3]. Due to the sensor restrictions and the control performance requirement, it is often necessary to sample the signals for different types of sensors at different sampling rates. The multirate sampling technique has received much attention since the early 1950s. Compared with the single-rate sampling scheme, the use of the multirate sampling technique is of two main benefits: (i) it may improve the performance such as improving the transient system behavior and enhancing the disturbance rejection property and (ii) it can provide a better tradeoff between the system performance and the implementation cost, which can be achieved by using analog-to-digital converters and digital-to-analog converters at different rates. Motivated by these benefits, much work has been done to deal with system modeling and identification, stability analysis, and controller synthesis of multirate sampled-data systems in the past few decades [3–12]. For example, in [4], a general framework of a multirate sampled-data control system is presented using nest operators and nest algebras, and an H_∞ suboptimal controller satisfying causality constraint is designed by the lifting technique, in which the outputs $y_i(t)$ $(i = 1, 2, \ldots, p)$ and the control inputs $u_i(t)$ $(i = 1, 2, \ldots, q)$ are paired with sampling periods $h_s = m_i h$ and holding periods $h_u = n_i h$, respectively, where $m_i \in \mathbb{N}^+$ $(i = 1, 2, \ldots, p)$, $n_i \in \mathbb{N}^+$ $(i = 1, 2, \ldots, q)$, $\mathbb{N}^+$ is the set of positive integers, and h is the base sampling period. Based on this framework, some particular cases of the multirate sampled-data systems are considered in [5–15].

More specifically, when the multirate sampled-data system involves a fast sampling rate $1/h_s$ and a slow control input rate $1/h_u$ (i.e., $n/h_u = 1/h_s$ ($n \in \mathbb{N}^+$)), a new multirate sampling method for acceleration control is proposed in [9]; on the other hand, when the multirate sampled-data system involves a slow sampling rate $1/h_s$ and a fast control input rate $1/h_u$ (i.e., $n/h_s = 1/h_u$), some estimation and/or control problems for several practical systems such as polymer reactors [10], visual servo control systems [11, 12], read-write arm of the hard disk drive [13, 14], and a pilot plant [15] are addressed. It should be mentioned that for the multirate sampling scheme with $h_u > h_s$, in [9], the control signal is calculated by output measurement at each sampling rate $1/h_s$, but only the one produced at the input rate $1/h_u$ is implemented. Although such a multirate scheme is effective in the realization of acceleration control in wide bandwidth, some of the output measurements may not be used to update the control actuation in time; for the multirate sampling scheme with $h_u < h_s$ in [3, 5–8, 10–15], the control signal is calculated recursively at each input rate $1/h_u$ only using the available data at each sampling rate $1/h_s$. Moreover, using the lifting technique, the multirate sampled-data system is converted into an equivalent discrete-time single-rate time-invariant system in [3, 5–8, 10–15]. However, the equivalent conversion is not readily applicable to the continuous-time systems with polytopic uncertainties [16–18]. In [16], an input delay approach is proposed to investigate the sampled-data stabilization of linear systems, which can be extended to deal with the sampled-data control for systems with polytopic uncertainties and the networked control systems [19–24]. Most of the existing results developed by using the input delay approach, such as [16–18], have been largely focused on the sampled-data control of single-rate sampled-data systems. However, there are few results available on the dual-rate or multirate sampled-data control of a continuous-time system using the input delay approach except [20, 25], which provides the main motivation of the current study. In [20], exponential stability and the induced L_2-gain of networked control systems are investigated, in which the sampled-data via dual-rate samplings are transmitted one after another by introducing a Round-Robin scheduling protocol. In [25], multirate sampled-data systems are modeled as systems with multiple input delays by reordering the updating instants, and some stability and stabilization conditions are established in terms of linear matrix inequalities. Without reordering sensor instants [20] or updating instants [25], this paper attempts to apply the input delay approach for a dual-rate sampled-data control system.

In this paper, we apply the input delay approach to investigate the periodic switched control of a linear continuous-time system system with two different sampling rates $1/h_1$ and $1/h_2$, where the sampling periods h_1 and h_2 satisfy $h_1 < h_2$ and $l_1h_1 = l_2h_2$, with l_i ($i = 1, 2$) being two positive integers, l_ih_i being the unique basic time period, and $\{l_1, l_2\}$ having no common factors greater than unity. Once the sampled-data of the state variable $x_1(kh_1)$ or $x_2(kh_2)$ is available, the control input is computed to update the system. Depending on whether the sampled-data of state variables at two sampling rates are available simultaneously or separately, a periodic switched controller with three switching modes is constructed to implement the sampled-data control. Using such a controller and the input delay approach, the resulting closed-loop system is modeled as a switched system with subsystems that have different input delays. A Lyapunov-Krasovskii functional that involves information about two sampling periods is constructed to derive some delay-dependent criteria for the H_∞ performance of the switched system and the existence of the periodic switched controller. Comparing with the existing results for multirate sampled-data systems based on lifting technique [3–15], the proposed results can be trivially extended to handle the multirate sampled-data systems or networked control systems with polytopic uncertainties. The effectiveness of the proposed method and its advantage over a single-rate sampled-data control is shown by performing the dual-rate sampled-data control of a vehicle system.

Notation. The superscript "T" stands for the transposition of a vector or a matrix. $\mathbb{R}^n$ is the n dimensional Euclidean space. $\mathbb{N}$ is the set of nonnegative integers and $\mathbb{N}^+$ is the set of positive integers. For symmetric matrices P and Q, $P \le Q$ (resp., $P < Q$) means that $P - Q$ is negative semidefinite matrix (resp., negative definite matrix). $\lambda_{\max}(P)$ is the maximum eigenvalue of a symmetric matrix P. We use an asterisk "$*$" to denote a term induced by symmetry and $\mathrm{diag}\{\cdots\}$ to denote the block-diagonal matrix. The space of square-integrable vector functions over $[t_0, \infty)$ is denoted by $\mathscr{L}_2[t_0, \infty)$.

2. Modeling of a Dual-Rate Sampled-Data System with a Switched Controller

Consider the linear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + E\omega(t), \\ z(t) &= Cx(t) + D\omega(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $z(t) \in \mathbb{R}^p$ are the state, the control input, and the controlled output, respectively; $\omega(t) \in \mathbb{R}^q$ is the external disturbance acting on system (1) and $\omega(t) \in \mathscr{L}_2[t_0, \infty)$; $x(t_0) = x_0$ is the initial state; A, B, C, D, and E are constant matrices of appropriate dimensions. It is assumed that all state variables of system (1) are sampled by two different types of sensors. Let $x(t) = [x_1^T(t)\ \ x_2^T(t)]^T$, where $x_1(t) = [x_{1,1}^T(t)\ \ x_{1,2}^T(t)\ \ \cdots\ \ x_{1,p_1}^T(t)]^T \in \mathbb{R}^{p_1}$, $x_2(t) = [x_{2,1}^T(t)\ \ x_{2,2}^T(t)\ \ \cdots\ \ x_{2,p_2}^T(t)]^T \in \mathbb{R}^{p_2}$, and $p_1 + p_2 = n$. Without loss of generality, we assume that $x_1(t)$ and $x_2(t)$ are paired with two sampling periods h_1 and h_2, respectively, and $l_1h_1 = l_2h_2$ ($h_1 \le h_2$), where $l_i \in \mathbb{N}^+$ ($i = 1, 2$) and $l_1 \ge l_2$. Then the sequences of sampled-data of the state variables $x_1(t)$ and $x_2(t)$ are $\{x_1(kh_1) : k \in \mathbb{N}\}$ and $\{x_2(kh_2) : k \in \mathbb{N}\}$.

In the proposed dual-rate sampling scheme, once $x_1(kh_1)$ or $x_2(kh_2)$ is available, the control signal is computed immediately for input update. By taking full advantage of $x_1(kh_1)$ and $x_2(kh_2)$ ($k \in \mathbb{N}$) in a real-time way, we construct the following switched controller, which is shown in Figure 1:

$$u(t^+) = F_{1,\sigma(t)}x_1(kh_1) + F_{2,\sigma(t)}x_2(kh_2), \quad \forall k \in \mathbb{N}, \tag{2}$$

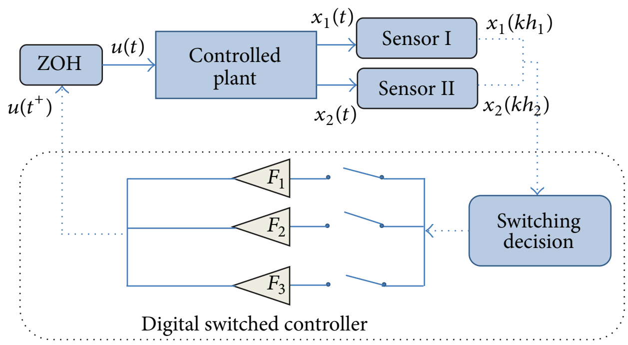

FIGURE 1: The multirate sampled-data system architecture using a digital switched controller.

where $F_{\sigma(t)} = [F_{1\sigma(t)} \;\; F_{2\sigma(t)}]$ are the control gain matrices to be determined and $\sigma(t) : [0,\infty) \to \mathbb{S} = \{1,2,3\}$ is the switching signal. Notice that there are three cases of available sampled-data of the state variables for control computation of the controller (2): (i) both $x_1(kh_1)$ and $x_2(kh_2)$ are available, (ii) only $x_1(kh_1)$ is available, and (iii) only $x_2(kh_2)$ is available. Define the switching rules as follows. Set $\sigma(t) = 1$ when $x_1(kh_1)$ and $x_2(kh_2)$ are available simultaneously, $\sigma(t) = 2$ when only $x_1(kh_1)$ is available, and $\sigma(t) = 3$ when only $x_2(kh_2)$ is available, respectively.

Due to the fact that $l_1h_1 = l_2h_2$, the interval of the control input is $[kl_1h_1, (k+1)l_1h_1)$ or equally $[kl_2h_2, (k+1)l_2h_2)$ $(k \in \mathbb{N})$. Define $m_i = \max\{k \mid kh_1 - ih_2 < 0,\ 0 < k < l_1,\ k \in \mathbb{N}\}$ $(i = 1,2,\dots,l_2)$ and $m_0 = 0$. Based on the switching rules of controller (2), we propose the following interval partition:

$$\begin{aligned}[kl_1h_1,(k+1)l_1h_1) = {} & \mathcal{I}_1 \cup \mathcal{I}_2 \cup \cdots \cup \mathcal{I}_{m_{\nu-1}+\nu} \\ & \cup \mathcal{I}_{m_{\nu-1}+\nu+1} \cup \cdots \cup \mathcal{I}_{m_\nu+\nu} \cup \cdots \\ & \cup \mathcal{I}_{m_{l_2-1}+l_2} \cup \cdots \cup \mathcal{I}_{m_{l_2}+l_2},\end{aligned} \tag{3}$$

where

$$\begin{aligned}&\mathcal{I}_{m_{\nu-1}+\nu} = [kl_1h_1 + (\nu-1)h_2, (kl_1 + m_{\nu-1} + 1)h_1),\\ &\mathcal{I}_{m_{\nu-1}+\nu+1}\\ &\quad = [(kl_1 + m_{\nu-1} + 1)h_1, (kl_1 + m_{\nu-1} + 2)h_1),\\ &\vdots,\\ &\mathcal{I}_{m_\nu+\nu} = [(kl_1 + m_\nu)h_1, kl_1h_1 + \nu h_2),\\ &\qquad\qquad \nu = 1,2,\dots,l_2.\end{aligned} \tag{4}$$

To clearly show the partition process, a special case for the interval partition of $[kl_1h_1, (k+1)l_1h_1)$ with $l_1 = 8$ and $l_2 = 3$ is depicted by Figure 2.

Considering interval partition (3) and the availability of $x_1(kh_1)$ and $x_2(kh_2)$, one can see that switching controller (2) is periodically activated on the time instants $\{kl_1h_1 + (m_{i-1}+1)h_1,\ kl_1h_1 + (i-1)h_2,\ i = 1,\dots,l_2\}_{k=0}^{\infty}$. Then the control input can be described by

$$u(t) = \begin{cases} F_{11}x_1(kl_1h_1) + F_{21}x_2(kl_1h_1), & t \in \mathcal{I}_1,\ \sigma(t) = 1\\ F_{12}x_1((kl_1+1)h_1) + F_{22}x_2(kl_1h_1), & t \in \mathcal{I}_2,\ \sigma(t) = 2\\ \vdots & \\ F_{12}x_1((kl_1+m_1)h_1) + F_{22}x_2(kl_1h_1), & t \in \mathcal{I}_{m_1+1},\ \sigma(t) = 2\\ F_{13}x_1((kl_1+m_1)h_1) + F_{23}x_2(kl_1h_1 + h_2), & t \in \mathcal{I}_{m_1+2},\ \sigma(t) = 3\\ F_{12}x_1((kl_1+m_1+1)h_1) + F_{22}x_2(kl_1h_1 + h_2), & t \in \mathcal{I}_{m_1+3},\ \sigma(t) = 2\\ \vdots & \\ F_{12}x_1((kl_1+m_\nu)h_1) + F_{22}x_2(kl_1h_1 + (\nu-1)h_2), & t \in \mathcal{I}_{m_\nu+\nu},\ \sigma(t) = 2\\ F_{13}x_1((kl_1+m_\nu)h_1) + F_{23}x_2(kl_1h_1 + \nu h_2), & t \in \mathcal{I}_{m_\nu+\nu+1},\ \sigma(t) = 3\\ F_{12}x_1((kl_1+m_\nu+1)h_1) + F_{22}x_2(kl_1h_1 + (\nu-1)h_2), & t \in \mathcal{I}_{m_\nu+\nu+2},\ \sigma(t) = 2\\ \vdots & \\ F_{12}x_1((kl_1+m_{l_2})h_1) + F_{22}x_2(kl_1h_1 + (l_2-1)h_2), & t \in \mathcal{I}_{m_{l_2}+l_2},\ \sigma(t) = 2. \end{cases} \tag{5}$$

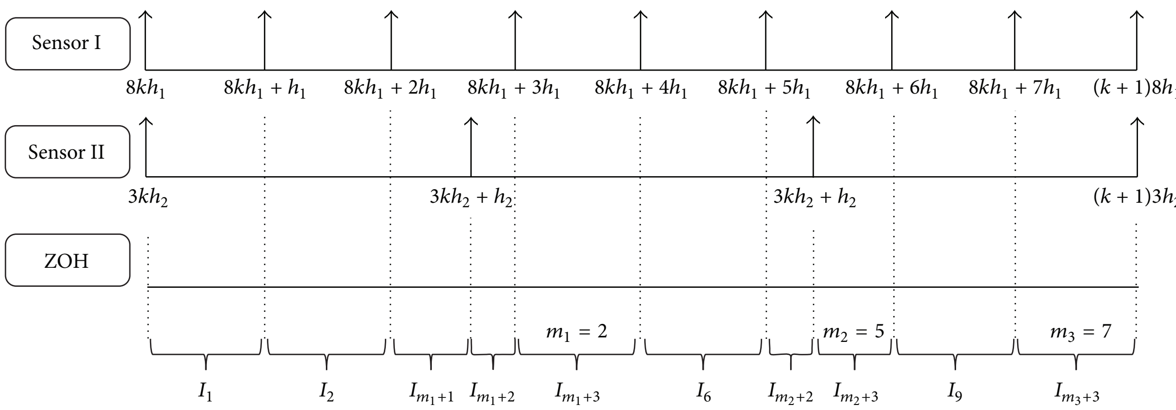

FIGURE 2: A special case for the interval partition of $[kl_1h_1,(k+1)l_1h_1)$ with $l_1=8$ and $l_2=3$.

Define the following input delays on the subintervals in (3):

$$
\begin{aligned}
&t\in\mathcal{I}_{m_{\nu-1}+\nu}:\\
&\tau^{(1)}_{m_{\nu-1}+\nu}(t)=t-\left(kl_1+m_{\nu-1}\right)h_1,\\
&\tau^{(2)}_{m_{\nu-1}+\nu}(t)=t-kl_1h_1-(\nu-1)h_2\\
&t\in\mathcal{I}_{m_{\nu-1}+\nu+1}:\\
&\tau^{(1)}_{m_{\nu-1}+\nu+1}(t)=t-\left(kl_1+m_{\nu-1}+1\right)h_1,\\
&\tau^{(2)}_{m_{\nu-1}+\nu+1}(t)=t-kl_1h_1-(\nu-1)h_2\\
&\vdots\\
&t\in\mathcal{I}_{m_\nu+\nu}:\\
&\tau^{(1)}_{m_\nu+\nu}(t)=t-\left(kl_1+m_\nu\right)h_1,\\
&\tau^{(2)}_{m_\nu+\nu}(t)=t-kl_1h_1-(\nu-1)h_2,
\end{aligned}
\tag{6}
$$

where $\nu=1,2,\ldots,l_2$. It follows that

$$
\begin{aligned}
&\sigma(t)=1:\quad 0\le\tau_1(t)<h_1,\ t\in\mathcal{I}_1\\
&\sigma(t)=2:\\
&0\le\tau^{(1)}_i(t)<h_1,\ 0\le\tau^{(1)}_i(t)<\tau^{(2)}_i(t)<h_2,\ t\in\mathcal{I}_i,\ i=m_{\nu-1}+\nu+1,\ldots,m_\nu+\nu,\ \nu=1,2,\ldots,l_2\\
&\sigma(t)=3:\quad 0\le\tau^{(2)}_{m_{\nu-1}+\nu}(t)<\tau^{(1)}_{m_{\nu-1}+\nu}(t)<h_1,\ t\in\mathcal{I}_{m_{\nu-1}+\nu},\ \nu=2,\ldots,l_2.
\end{aligned}
\tag{7}
$$

Using (1), (5), and (6), the resulting closed-loop system can be described by

$$
\begin{aligned}
&\Sigma^{(1)}_1:\ \dot{x}(t)=Ax(t)+BF_1x\left(t-\tau_1(t)\right)+E\omega(t),\quad t\in\mathcal{I}_1,\ \sigma(t)=1,\\
&\Sigma^{(2)}_{2\nu}:\ \dot{x}(t)=Ax(t)+BF_2I_{p_1}x\left(t-\tau^{(1)}_i(t)\right)+BF_2I_{p_2}x\left(t-\tau^{(2)}_i(t)\right)+E\omega(t),\\
&\qquad t\in\bigcup_{m_{\nu-1}+\nu+1}^{m_\nu+\nu}\mathcal{I}_i,\ i=m_{\nu-1}+\nu+1,\ldots,m_\nu+\nu,\ \nu=1,2,\ldots,l_2,\ \sigma(t)=2,\\
&\Sigma^{(3)}_{2\nu-1}:\ \dot{x}(t)=Ax(t)+BF_3I_{p_1}x\left(t-\tau^{(1)}_{m_{\nu-1}+\nu}(t)\right)+BF_3I_{p_2}x\left(t-\tau^{(2)}_{m_{\nu-1}+\nu}(t)\right)+E\omega(t),\\
&\qquad t\in\mathcal{I}_{m_{\nu-1}+\nu},\ \nu=2,\ldots,l_2,\ \sigma(t)=3,
\end{aligned}
\tag{8}
$$

where $I_{p_1} = \operatorname{diag}\{I_{p_1\times p_1}, 0\}$, $I_{p_2} = \operatorname{diag}\{0, I_{p_2\times p_2}\}$ and $I_{p_1\times p_1}$ and $I_{p_2\times p_2}$ denote $p_1 \times p_1$ and $p_2 \times p_2$ identity matrices, respectively.

The purpose of this paper is to investigate the periodic switched control such that system (8) is exponentially stable with a prescribed H_∞ performance, which means that

(i) system (8) with $\omega(t) = 0$ is exponentially stable; that is, there exist constants $\beta > 0$ and $\lambda > 0$ such that $\|x(t)\|^2 \leq \beta\|x_0\|_{c1}e^{-\lambda t}$ for $t \geq 0$, where $\|x_t\|_{c1} = \sup_{-h_2\leq s\leq 0}\{\|x(t+s)\|, \|\dot{x}(t+s)\|\}$ [23, 26];

(ii) $\int_0^\infty e^{-\alpha s} z^T(s)z(s)ds \leq \gamma^2 \int_0^\infty \omega^T(s)\omega(s)ds$ can be ensured for all nonzero $\omega(t) \in \mathscr{L}_2[0,\infty)$ under the zero initial condition $x(t) = \phi(t) = 0$ $(t \in [-h_2, 0])$, where α and γ are positive constants.

To end this section, we introduce the following lemma.

Lemma 1. *For any constant matrices $R \in \mathbb{R}^{n\times n}$, $S_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2)$, $\mathfrak{R}_1 = \begin{bmatrix} R & S_1^T \\ S_1 & R \end{bmatrix} \geq 0$, $\mathfrak{R}_2 = \begin{bmatrix} R & S_2^T \\ S_2 & R \end{bmatrix} \geq 0$, and scalars τ_i, $\tau_i(t)$ $(i = 1, 2)$ satisfying $0 \leq \tau_1 \leq \tau_1(t) < \tau_2(t) \leq \tau_2$ and a vector function $\dot{x} : [-\tau_2, -\tau_1] \to \mathbb{R}^n$ such that the integration concerned is well defined, the following inequality holds:*

$$-(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) R\dot{x}(s)\, ds \leq -\sum_{i=1}^{7}\mathscr{E}_i, \tag{9}$$

where

$$\begin{aligned}
\mathscr{E}_1 &= e_3^T(t) Re_3(t), \\
\mathscr{E}_2 &= e_2^T(t) Re_2(t), \\
\mathscr{E}_3 &= e_1^T(t) Re_1(t), \\
\mathscr{E}_4 &= e_1^T(t) S_1^T e_2(t), \\
\mathscr{E}_5 &= e_2^T(t) S_1 e_1(t), \\
\mathscr{E}_6 &= e_2^T(t) S_2^T e_3(t), \\
\mathscr{E}_7 &= e_3^T(t) S_2 e_2(t), \\
e_1(t) &= x(t-\tau_1) - x(t-\tau_1(t)), \\
e_2(t) &= x(t-\tau_1(t)) - x(t-\tau_2(t)), \\
e_3(t) &= x(t-\tau_2(t)) - x(t-\tau_2).
\end{aligned} \tag{10}$$

Proof. If $\tau_1(t) \equiv \tau_1$ (resp., $\tau_2(t) \equiv \tau_2$), inequality (9) reduces to be the one in Lemma 1 [27]. If $0 \leq \tau_1 \leq \tau_1(t) < \tau_2(t) \leq \tau_2$, using Jensen inequality to obtain

$$\begin{aligned}
&-(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) R\dot{x}(s)\, ds \\
&\quad \leq -\left(1 + \frac{\alpha_1}{\alpha_2}\right)\mathscr{E}_1 - \left(1 + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_4}{\alpha_3}\right)\mathscr{E}_2 \\
&\qquad -\left(1 + \frac{\alpha_3}{\alpha_4}\right)\mathscr{E}_3,
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\alpha_1 &= \frac{\tau_2(t) - \tau_1}{\tau_2 - \tau_1}, \\
\alpha_2 &= \frac{\tau_2 - \tau_2(t)}{\tau_2 - \tau_1}, \\
\alpha_3 &= \frac{\tau_2 - \tau_1(t)}{\tau_2 - \tau_1}, \\
\alpha_4 &= \frac{\tau_1(t) - \tau_1}{\tau_2 - \tau_1},
\end{aligned} \tag{12}$$

for any $\mathfrak{R}_i \geq 0$ $(i = 1, 2)$, the following inequalities hold:

$$\begin{aligned}
&\begin{bmatrix} \sqrt{\frac{\alpha_3}{\alpha_4}} e_1(t) \\ -\sqrt{\frac{\alpha_4}{\alpha_3}} e_2(t) \end{bmatrix}^T \mathfrak{R}_1 \begin{bmatrix} \sqrt{\frac{\alpha_3}{\alpha_4}} e_1(t) \\ -\sqrt{\frac{\alpha_4}{\alpha_3}} e_2(t) \end{bmatrix} \geq 0, \\
&\begin{bmatrix} \sqrt{\frac{\alpha_2}{\alpha_1}} e_2(t) \\ -\sqrt{\frac{\alpha_1}{\alpha_2}} e_3(t) \end{bmatrix}^T \mathfrak{R}_2 \begin{bmatrix} \sqrt{\frac{\alpha_2}{\alpha_1}} e_2(t) \\ -\sqrt{\frac{\alpha_1}{\alpha_2}} e_3(t) \end{bmatrix} \geq 0.
\end{aligned} \tag{13}$$

It follows from (13) that

$$\frac{\alpha_3}{\alpha_4}\mathscr{E}_3 + \frac{\alpha_4}{\alpha_3}\mathscr{E}_2 \geq \mathscr{E}_4 + \mathscr{E}_5, \tag{14}$$

$$\frac{\alpha_2}{\alpha_1}\mathscr{E}_2 + \frac{\alpha_1}{\alpha_2}\mathscr{E}_1 \geq \mathscr{E}_6 + \mathscr{E}_7. \tag{15}$$

Combining (14) and (15), we obtain inequality (9). This completes the proof. □

3. H_∞ Performance Analysis and Periodic Switched Controller Design

In this section, we first derive a delay-dependent criterion such that system (8) is exponentially stable with prescribed H_∞ performance by using the Lyapunov-Krasovskii functional method. Based on the derived performance criterion, we establish the delay-dependent criterion on the existence of a switched controller for system (8).

Proposition 2. *Given $\gamma > 0$, $h_2 > h_1 > 0$, $l_1 > l_2 > 0$, $0 \leq a_1 \leq a_2 \leq a_3 \leq ((l_2 - 2)a_1 + (l_2 + 3)a_2)/(l_2 + 1)$, $\mu \geq 1$ satisfying $\ln(\mu) < [(a_1(l_2 - 2) + a_2(l_2 + 3) - a_3(l_2 + 1))/l_2](h_2/2)$, and the gain matrices F_i $(i = 1, 2, 3)$, system (8) is exponentially stable with a given H_∞ performance γ for any switching signal with $\ln(\mu) < [(a_1(l_2 - 2) + a_2(l_2 + 3) - a_3(l_2 + 1))/l_2](h_2/2)$, if there exist matrices S_1, S_2, $S_{i,1}$, and $S_{i,2}$ $(i = 2, 3)$ and symmetric matrices $P_j > 0$, $Q_{ij} > 0$, and $R_{ij} > 0$ $(i = 1, 2;\ j = 1, 2, 3)$ such that $P_2 \leq \mu P_1$, $P_1 \leq \mu P_2$, $P_2 \leq \mu P_3$, $P_3 \leq \mu P_2$, $Q_{i2} \leq \mu Q_{i1}$,*

$Q_{i1} \le \mu Q_{i2}$, $Q_{i2} \le \mu Q_{i3}$, $Q_{i3} \le \mu Q_{i2}$, $R_{i2} \le \mu R_{i1}$, $R_{i1} \le \mu R_{i2}$, $R_{i2} \le \mu R_{i3}$, $R_{i3} \le \mu R_{i2}$ $(i = 1, 2)$, *and*

$$\begin{bmatrix} R_{11} & S_1^T \\ S_1 & R_{11} \end{bmatrix} \ge 0, \tag{16}$$

$$\begin{bmatrix} h_1 R_{12} - h_{12} R_{22} & S_2^T \\ S_2 & h_1 R_{12} - h_{12} R_{22} \end{bmatrix} \ge 0, \tag{17}$$

$$\begin{bmatrix} R_{1i} & S_{i,1}^T \\ S_{i,1} & R_{1i} \end{bmatrix} \ge 0, \quad (i = 2, 3) \tag{18}$$

$$\begin{bmatrix} R_{1i} & S_{i,2}^T \\ S_{i,2} & R_{1i} \end{bmatrix} \ge 0, \quad (i = 2, 3) \tag{19}$$

$$\Xi^{(1)} < 0, \quad t \in \mathscr{I}_1, \tag{20}$$

$$\Xi^{(i1)} < 0, \quad t \in \mathscr{I}_i, \ i = m_{\nu-1} + \nu + 1, \ldots, m_\nu + \nu, \ \nu = 1, 2, \ldots, l_2, \tag{21}$$

$$\Xi^{(i2)} < 0, \quad t \in \mathscr{I}_i, \ i = m_{\nu-1} + \nu, \ \nu = 2, \ldots, l_2, \tag{22}$$

where

$$\Xi^{(1)} = \begin{bmatrix} \Xi_{11}^{(1)} & \Xi_{12}^{(1)} & \Xi_{13}^{(1)} & 0 & P_1 E + C^T D \\ * & \Xi_{22}^{(1)} & \Xi_{23}^{(1)} & 0 & 0 \\ * & * & \Xi_{33}^{(1)} & \Xi_{34}^{(1)} & 0 \\ * & * & * & \Xi_{44}^{(1)} & 0 \\ * & * & * & * & -\gamma^2 I + D^T D \end{bmatrix} + \Psi_1^T \left(h_1^2 R_{11} + h_{12}^2 R_{21} \right) \Psi_1,$$

$$h_{12} = h_2 - h_1,$$

$$\Xi_{11}^{(1)} = A^T P_1 + P_1 A + a_1 P_1 + Q_{11} - \sigma_{11} R_{11} + C^T C,$$

$$\Xi_{12}^{(1)} = P_1 B F_1 + \sigma_{11} \left(R_{11} - S_1^T \right),$$

$$\Xi_{13}^{(1)} = \sigma_{11} S_1^T,$$

$$\Xi_{22}^{(1)} = -\sigma_{11} \left(2R_{11} - S_1^T - S_1 \right),$$

$$\Xi_{23}^{(1)} = \sigma_{11} \left(R_{11} - S_1^T \right),$$

$$\Xi_{33}^{(1)} = -\sigma_{11} R_{11} - \sigma_{21} R_{21} + \sigma_{11} \left(Q_{21} - Q_{11} \right),$$

$$\Xi_{34}^{(1)} = \sigma_{21} R_{21},$$

$$\Xi_{44}^{(1)} = -\sigma_{21} \left(R_{21} + Q_{21} \right),$$

$$\Psi_1 = \begin{bmatrix} A & BF_1 & 0 & 0 & E \end{bmatrix},$$

$$\sigma_{11} = e^{-a_1 h_1},$$

$$\sigma_{21} = e^{-a_1 h_2},$$

$$\Xi^{(i1)} = \begin{bmatrix} \Xi_{11}^{(i1)} & \Xi_{12}^{(i1)} & \Xi_{13}^{(i1)} & \Xi_{14}^{(i1)} & 0 & \Xi_{16}^{(i1)} \\ * & \Xi_{22}^{(i1)} & \Xi_{23}^{(i1)} & \Xi_{24}^{(i1)} & \Xi_{25}^{(i1)} & 0 \\ * & * & \Xi_{33}^{(i1)} & 0 & \Xi_{35}^{(i1)} & 0 \\ * & * & * & \Xi_{44}^{(i1)} & 0 & 0 \\ * & * & * & * & \Xi_{55}^{(i1)} & 0 \\ * & * & * & * & * & \Xi_{66}^{(i1)} \end{bmatrix} + \Psi_2^T \left(h_1^2 R_{12} + h_{12}^2 R_{22} \right) \Psi_2,$$

$$\Xi_{11}^{(i1)} = A^T P_2 + P_2 A + a_2 P_2 + Q_{12} + C^T C - \sigma_{22} R_{22} - \sigma_{12} \left(h_1 R_{12} - h_{12} R_{22} \right),$$

$$\Xi_{12}^{(i1)} = P_2 B F_{12} + \sigma_{12} \left(h_1 R_{12} - h_{12} R_{22} - S_2^T \right) + \sigma_{22} \left(R_{22} - S_{2,1}^T \right),$$

$$\Xi_{13}^{(i1)} = P_2 B F_{22} + \sigma_{22} S_{2,1}^T,$$

$$\Xi_{14}^{(i1)} = \sigma_{12} S_2^T,$$

$$\Xi_{22}^{(i1)} = \sigma_{12} \left(-2h_1 R_{12} + 2h_{12} R_{22} + S_2^T + S_2 \right) + \sigma_{22} \left(-2R_{22} + S_{2,1}^T + S_{2,1} \right),$$

$$\Xi_{16}^{(i1)} = P_2 E + C^T D,$$

$$\Xi_{23}^{(i1)} = \sigma_{22} \left(R_{22} - S_{2,1}^T - S_{2,2}^T \right),$$

$$\Xi_{24}^{(i1)} = \sigma_{12} \left(h_1 R_{12} - h_{12} R_{22} - S_2^T \right),$$

$$\Xi_{25}^{(i1)} = \sigma_{22} S_{2,2}^T,$$

$$\Xi_{33}^{(i1)} = \sigma_{22} \left(-2R_{22} + S_{2,2}^T + S_{2,2} \right),$$

$$\Xi_{35}^{(i1)} = \sigma_{22} \left(R_{22} - S_{2,2}^T \right),$$

$$\Xi_{44}^{(i1)} = \sigma_{12} \left(-h_1 R_{12} + h_{12} R_{22} \right) + e^{-a_2 h_1} \left(Q_{22} - Q_{12} \right),$$

$$\Xi_{55}^{(i1)} = -\sigma_{22} R_{22} - e^{-a_2 h_2} Q_{22},$$

$$\Xi_{66}^{(i1)} = -\gamma^2 I + D^T D,$$

$$\Psi_2 = \begin{bmatrix} A & BF_{12} & BF_{22} & 0 & 0 & E \end{bmatrix},$$

$$\sigma_{12} = \frac{e^{-a_2 h_1}}{h_1},$$

$$\sigma_{22} = \frac{e^{-a_2 h_2} h_{12}}{h_2},$$

$$\Xi^{(i2)} = \begin{bmatrix} \Xi_{11}^{(i2)} & \Xi_{12}^{(i2)} & \Xi_{13}^{(i2)} & 0 & 0 & \Xi_{16}^{(i2)} \\ * & \Xi_{22}^{(i2)} & \Xi_{23}^{(i2)} & \Xi_{24}^{(i2)} & 0 & 0 \\ * & * & \Xi_{33}^{(i2)} & \Xi_{34}^{(i2)} & 0 & 0 \\ * & * & * & \Xi_{44}^{(i2)} & \Xi_{45}^{(i2)} & 0 \\ * & * & * & * & \Xi_{55}^{(i2)} & 0 \\ * & * & * & * & * & \Xi_{66}^{(i2)} \end{bmatrix}$$
$$+ \Psi_3^T \left(h_1^2 R_{13} + h_{12}^2 R_{23}\right) \Psi_3,$$
$$\Xi_{11}^{(i2)} = A^T P_3 + P_3 A + a_3 P_3 + Q_{13} + C^T C - \sigma_{13} R_{13},$$
$$\Xi_{12}^{(i2)} = P_3 B F_{13} + \sigma_{13} S_{3,1}^T,$$
$$\Xi_{13}^{(i2)} = P_3 B F_{23} + \sigma_{13} \left(R_{13} - S_{3,1}^T\right),$$
$$\Xi_{16}^{(i2)} = E + \overline{P}_3 C^T D,$$
$$\Xi_{22}^{(i2)} = -\sigma_{13} \left(2R_{13} - S_{3,2}^T - S_{3,2}\right),$$
$$\Xi_{23}^{(i2)} = \sigma_{13} \left(R_{13} - S_{3,1} - S_{3,2}\right),$$
$$\Xi_{24}^{(i2)} = \sigma_{13} \left(R_{13} - S_{3,2}^T\right),$$
$$\Xi_{33}^{(i2)} = \sigma_{13} \left(-2R_{13} + S_{3,1}^T + S_{3,1}\right),$$
$$\Xi_{34}^{(i2)} = \sigma_{13} S_{3,2}^T,$$
$$\Xi_{44}^{(i2)} = -\sigma_{13} R_{13} - \sigma_{23} R_{23} + \sigma_{13} \left(Q_{23} - Q_{13}\right),$$
$$\Xi_{45}^{(i2)} = \sigma_{23} R_{23},$$
$$\Xi_{55}^{(i2)} = -\sigma_{23} \left(R_{23} + Q_{23}\right),$$
$$\Xi_{66}^{(i2)} = -\gamma^2 I + D^T D,$$
$$\Psi_3 = \begin{bmatrix} A & BF_{13} & BF_{23} & 0 & 0 & E \end{bmatrix},$$
$$\sigma_{13} = e^{-a_3 h_1},$$
$$\sigma_{23} = e^{-a_3 h_2}. \tag{23}$$

Moreover, the parameters in the exponential stability and H_∞ performance are given by

$$\alpha = \frac{\left[a_1 \left(l_2 - 2\right) + a_2 \left(l_2 + 3\right) - a_3 \left(l_2 + 1\right)\right]}{l_2},$$
$$\beta = \left(\frac{b}{a}\right)^{1/2}, \tag{24}$$
$$\lambda = \frac{\left[a_1 \left(l_2 - 2\right) + a_2 \left(l_2 + 3\right) - a_3 \left(l_2 + 1\right)\right]}{\left(2l_2\right)} - \frac{\ln \mu}{h_2}$$

with

$$a = \min \left\{\lambda_{\min}\left(P_1\right), \lambda_{\min}\left(P_2\right), \lambda_{\min}\left(P_3\right)\right\},$$
$$b = \lambda_{\max}\left(P_1\right) + \frac{\left(1 - e^{-a_1 h_2}\right)}{a_1 b_q} + \frac{h_1\left(1 - e^{-a_1 h_1}\right)}{a_1^2 b_r} + \frac{h_{12}\left(e^{-a_1 h_1} - e^{-a_1 h_2}\right)}{a_1^2 b_r}, \tag{25}$$
$$b_q = \max\left\{\lambda_{\max}\left(Q_{11}\right), \lambda_{\max}\left(Q_{21}\right)\right\},$$
$$b_r = \max\left\{\lambda_{\max}\left(R_{11}\right), \lambda_{\max}\left(R_{21}\right)\right\}.$$

Proof. See Appendix A. □

We now state and establish the delay-dependent criterion for the switched controller design.

Proposition 3. *Given $\gamma > 0$, $h_2 > h_1 > 0$, $l_1 > l_2 > 0$, $\nu > 0$, $0 \le a_1 \le a_2 \le a_3 \le ((l_2 - 2)a_1 + (l_2 + 3)a_2)/(l_2 + 1)$, and $\mu \ge 1$ satisfying $\ln(\mu) < [(a_1(l_2-2)+a_2(l_2+3)-a_3(l_2+1))/l_2](h_2/2)$, system (8) is exponentially stable with a given H_∞ performance γ for any switching signal with $\ln(\mu) < [(a_1(l_2-2)+a_2(l_2+3)-a_3(l_2+1))/l_2](h_2/2)$, if there exist matrices $\overline{S}_1$, $\overline{S}_2$, $\overline{S}_{i,1}$, $\overline{S}_{i,2}$ $(i = 2, 3)$, $\overline{F}_1$, $\overline{F}_{12}$, $\overline{F}_{22}$, $\overline{F}_{13}$, and $\overline{F}_{23}$ and symmetric matrices $\overline{P}_j > 0$, $\overline{Q}_{ij} > 0$, and $\overline{R}_{ij} > 0$ $(i = 1, 2; j = 1, 2, 3)$ such that $\overline{P}_2 \le \mu\overline{P}_1$, $\overline{P}_1 \le \mu\overline{P}_2$, $\overline{P}_2 \le \mu\overline{P}_3$, $\overline{P}_3 \le \mu\overline{P}_2$, $\overline{Q}_{i2} \le \mu\overline{Q}_{i1}$, $\overline{Q}_{i1} \le \mu\overline{Q}_{i2}$, $\overline{Q}_{i2} \le \mu\overline{Q}_{i3}$, $\overline{Q}_{i3} \le \mu\overline{Q}_{i2}$, $\overline{R}_{i2} \le \mu\overline{R}_{i1}$, $\overline{R}_{i1} \le \mu\overline{R}_{i2}$, $\overline{R}_{i2} \le \mu\overline{R}_{i3}$, $\overline{R}_{i3} \le \mu\overline{R}_{i2}$ $(i = 1, 2)$, and*

$$\begin{bmatrix} \overline{R}_{11} & \overline{S}_1^T \\ \overline{S}_1 & \overline{R}_{11} \end{bmatrix} \ge 0, \tag{26}$$
$$\begin{bmatrix} h_1\overline{R}_{12} - h_{12}\overline{R}_{22} & \overline{S}_2^T \\ \overline{S}_2 & h_1\overline{R}_{12} - h_{12}\overline{R}_{22} \end{bmatrix} \ge 0, \tag{27}$$
$$\begin{bmatrix} \overline{R}_{1i} & \overline{S}_{i,1}^T \\ \overline{S}_{i,1} & \overline{R}_{1i} \end{bmatrix} \ge 0, \quad (i = 2, 3), \tag{28}$$
$$\begin{bmatrix} \overline{R}_{1i} & \overline{S}_{i,2}^T \\ \overline{S}_{i,2} & \overline{R}_{1i} \end{bmatrix} \ge 0, \quad (i = 2, 3), \tag{29}$$
$$\overline{\Xi}^{(1)} = \begin{bmatrix} \overline{\Pi}_{11}^{(1)} & \overline{\Pi}_{12}^{(1)} \\ * & \overline{\Pi}_{22}^{(1)} \end{bmatrix} < 0, \quad t \in \mathcal{I}_1, \tag{30}$$
$$\overline{\Xi}^{(i1)} = \begin{bmatrix} \overline{\Pi}_{11}^{(i1)} & \overline{\Pi}_{12}^{(i1)} \\ * & \overline{\Pi}_{22}^{(i1)} \end{bmatrix} < 0, \tag{31}$$
$$t \in \mathcal{I}_i, \; i = m_{\nu-1} + \nu + 1, \ldots, m_\nu + \nu, \; \nu = 1, 2, \ldots, l_2,$$

$$\overline{\Xi}^{(i2)} = \begin{bmatrix} \overline{\Pi}_{11}^{(i2)} & \overline{\Pi}_{12}^{(i2)} \\ * & \overline{\Pi}_{22}^{(i2)} \end{bmatrix} < 0, \tag{32}$$

$$t \in \mathcal{I}_i, \ i = m_{\nu-1} + \nu, \ \nu = 2, \ldots, l_2,$$

where

$$\overline{\Pi}_{11}^{(1)} = \begin{bmatrix} \overline{\Xi}_{11}^{(1)} & \overline{\Xi}_{12}^{(1)} & \overline{\Xi}_{13}^{(1)} & 0 & E + \overline{P}_1 C^T D \\ * & \overline{\Xi}_{22}^{(1)} & \overline{\Xi}_{23}^{(1)} & 0 & 0 \\ * & * & \overline{\Xi}_{33}^{(1)} & \overline{\Xi}_{34}^{(1)} & 0 \\ * & * & * & \overline{\Xi}_{44}^{(1)} & 0 \\ * & * & * & * & -\gamma^2 I + D^T D \end{bmatrix},$$

$$\overline{\Pi}_{12}^{(1)} = \begin{bmatrix} h_1 \overline{P}_1 A^T & h_{12} \overline{P}_1 A^T & \overline{P}_1 C^T \\ h_1 \overline{F}_1^T B^T & h_{12} \overline{F}_1^T B^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_1 E^T & h_{12} E^T & 0 \end{bmatrix},$$

$$\overline{\Pi}_{22}^{(1)} = \begin{bmatrix} \nu^2 \overline{R}_{11} - 2\nu \overline{P}_1 & 0 & 0 \\ * & \nu^2 \overline{R}_{21} - 2\nu \overline{P}_1 & 0 \\ * & * & -I \end{bmatrix},$$

$$\overline{\Xi}_{11}^{(1)} = \overline{P}_1 A^T + A \overline{P}_1 + a_1 \overline{P}_1 + \overline{Q}_{11} - \sigma_{11} \overline{R}_{11},$$

$$\overline{\Xi}_{12}^{(1)} = B \overline{F}_1 + \sigma_{11} \left(\overline{R}_{11} - \overline{S}_1^T \right),$$

$$\overline{\Xi}_{13}^{(1)} = \sigma_{11} \overline{S}_1^T,$$

$$\overline{\Xi}_{22}^{(1)} = -\sigma_{11} \left(2\overline{R}_{11} - \overline{S}_1^T - \overline{S}_1 \right),$$

$$\overline{\Xi}_{23}^{(1)} = \sigma_{11} \left(\overline{R}_{11} - \overline{S}_1^T \right),$$

$$\overline{\Xi}_{33}^{(1)} = -\sigma_{11} \overline{R}_{11} - \sigma_{21} \overline{R}_{21} + \sigma_{11} \left(\overline{Q}_{21} - \overline{Q}_{11} \right),$$

$$\overline{\Xi}_{34}^{(1)} = \sigma_{21} \overline{R}_{21},$$

$$\overline{\Xi}_{44}^{(1)} = -\sigma_{21} \left(\overline{R}_{21} + \overline{Q}_{21} \right),$$

$$\sigma_{11} = e^{-a_1 h_1},$$

$$\sigma_{21} = e^{-a_1 h_2},$$

$$\overline{\Pi}_{11}^{(i1)} = \begin{bmatrix} \overline{\Xi}_{11}^{(i1)} & \overline{\Xi}_{12}^{(i1)} & \overline{\Xi}_{13}^{(i1)} & \overline{\Xi}_{14}^{(i1)} & 0 & \overline{\Xi}_{16}^{(i1)} \\ * & \overline{\Xi}_{22}^{(i1)} & \overline{\Xi}_{23}^{(i1)} & \overline{\Xi}_{24}^{(i1)} & \overline{\Xi}_{25}^{(i1)} & 0 \\ * & * & \overline{\Xi}_{33}^{(i1)} & 0 & \overline{\Xi}_{35}^{(i1)} & 0 \\ * & * & * & \overline{\Xi}_{44}^{(i1)} & 0 & 0 \\ * & * & * & * & \overline{\Xi}_{55}^{(i1)} & 0 \\ * & * & * & * & * & \overline{\Xi}_{66}^{(i1)} \end{bmatrix},$$

$$\overline{\Pi}_{12}^{(i1)} = \begin{bmatrix} h_1 \overline{P}_2 A^T & h_{12} \overline{P}_2 A^T & \overline{P}_2 C^T \\ h_1 \overline{F}_{12}^T B^T & h_{12} \overline{F}_{12}^T B^T & 0 \\ h_1 \overline{F}_{22}^T B^T & h_{12} \overline{F}_{22}^T B^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_1 E^T & h_{12} E^T & 0 \end{bmatrix},$$

$$\overline{\Pi}_{22}^{(i1)} = \begin{bmatrix} \nu^2 \overline{R}_{12} - 2\nu \overline{P}_2 & 0 & 0 \\ * & \nu^2 \overline{R}_{22} - 2\nu \overline{P}_2 & 0 \\ * & * & -I \end{bmatrix},$$

$$\overline{\Xi}_{11}^{(i1)} = \overline{P}_2 A^T + A \overline{P}_2 + a_2 \overline{P}_2 + \overline{Q}_{12} - \sigma_{22} \overline{R}_{22} - \sigma_{12} \left(h_1 \overline{R}_{12} - h_{12} \overline{R}_{22} \right), \quad h_{12} = h_2 - h_1,$$

$$\overline{\Xi}_{12}^{(i1)} = B \overline{F}_{12} + \sigma_{12} \left(h_1 \overline{R}_{12} - h_{12} \overline{R}_{22} - \overline{S}_2^T \right) + \sigma_{22} \left(\overline{R}_{22} - \overline{S}_{2,1}^T \right),$$

$$\overline{\Xi}_{13}^{(i1)} = B \overline{F}_{22} + \sigma_{22} \overline{S}_{2,1}^T,$$

$$\overline{\Xi}_{14}^{(i1)} = \sigma_{12} \overline{S}_2^T,$$

$$\overline{\Xi}_{16}^{(i1)} = E + \overline{P}_2 C^T D,$$

$$\overline{\Xi}_{22}^{(i1)} = \sigma_{12} \left(-2h_1 \overline{R}_{12} + 2h_{12} \overline{R}_{22} + \overline{S}_2^T + \overline{S}_2 \right) + \sigma_{22} \left(-2\overline{R}_{22} + \overline{S}_{2,1}^T + \overline{S}_{2,1} \right),$$

$$\overline{\Xi}_{23}^{(i1)} = \sigma_{22} \left(\overline{R}_{22} - \overline{S}_{2,1}^T - \overline{S}_{2,2}^T \right),$$

$$\overline{\Xi}_{24}^{(i1)} = \sigma_{12} \left(h_1 \overline{R}_{12} - h_{12} \overline{R}_{22} - \overline{S}_2^T \right),$$

$$\overline{\Xi}_{25}^{(i1)} = \sigma_{22} \overline{S}_{2,2}^T,$$

$$\overline{\Xi}_{33}^{(i1)} = \sigma_{22} \left(-2\overline{R}_{22} + \overline{S}_{2,2}^T + \overline{S}_{2,2} \right),$$

$$\overline{\Xi}_{35}^{(i1)} = \sigma_{22} \left(\overline{R}_{22} - \overline{S}_{2,2}^T \right),$$

$$\overline{\Xi}_{44}^{(i1)} = \sigma_{12} \left(-h_1 \overline{R}_{12} + h_{12} \overline{R}_{22} \right) + e^{-a_2 h_1} \left(\overline{Q}_{22} - \overline{Q}_{12} \right),$$

$$\overline{\Xi}_{55}^{(i1)} = -\sigma_{22} \overline{R}_{22} - e^{-a_2 h_2} \overline{Q}_{22},$$

$$\overline{\Xi}_{66}^{(i1)} = -\gamma^2 I + D^T D,$$

$$\sigma_{12} = \frac{e^{-a_2 h_1}}{h_1},$$

$$\sigma_{22} = \frac{e^{-a_2 h_2} h_{12}}{h_2},$$

$$
\begin{aligned}
\overline{\Pi}_{11}^{(i2)} &= \begin{bmatrix} \overline{\Xi}_{11}^{(i2)} & \overline{\Xi}_{12}^{(i2)} & \overline{\Xi}_{13}^{(i2)} & 0 & 0 & \overline{\Xi}_{16}^{(i2)} \\ * & \overline{\Xi}_{22}^{(i2)} & \overline{\Xi}_{23}^{(i2)} & \overline{\Xi}_{24}^{(i2)} & 0 & 0 \\ * & * & \overline{\Xi}_{33}^{(i2)} & \overline{\Xi}_{34}^{(i2)} & 0 & 0 \\ * & * & * & \overline{\Xi}_{44}^{(i2)} & \overline{\Xi}_{45}^{(i2)} & 0 \\ * & * & * & * & \overline{\Xi}_{55}^{(i2)} & 0 \\ * & * & * & * & * & \overline{\Xi}_{66}^{(i2)} \end{bmatrix}, \\
\overline{\Pi}_{12}^{(i2)} &= \begin{bmatrix} h_1\overline{P}_3A^T & h_{12}\overline{P}_3A^T & \overline{P}_3C^T \\ h_1\overline{F}_{13}^TB^T & h_{12}\overline{F}_{13}^TB^T & 0 \\ h_1\overline{F}_{23}^TB^T & h_{12}\overline{F}_{23}^TB^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_1E^T & h_{12}E^T & 0 \end{bmatrix}, \\
\overline{\Pi}_{22}^{(i2)} &= \begin{bmatrix} \nu^2\overline{R}_{13} - 2\nu\overline{P}_3 & 0 & 0 \\ * & \nu^2\overline{R}_{23} - 2\nu\overline{P}_3 & 0 \\ * & * & -I \end{bmatrix}, \\
\overline{\Xi}_{11}^{(i2)} &= \overline{P}_3A^T + A\overline{P}_3 + a_3\overline{P}_3 + \overline{Q}_{13} - \sigma_{13}\overline{R}_{13}, \\
\overline{\Xi}_{12}^{(i2)} &= B\overline{F}_{13} + \sigma_{13}\overline{S}_{3,1}^T, \\
\overline{\Xi}_{13}^{(i2)} &= B\overline{F}_{23} + \sigma_{13}\left(\overline{R}_{13} - \overline{S}_{3,1}^T\right), \\
\overline{\Xi}_{16}^{(i2)} &= E + \overline{P}_3C^TD, \\
\overline{\Xi}_{22}^{(i2)} &= -\sigma_{13}\left(2\overline{R}_{13} - \overline{S}_{3,2}^T - \overline{S}_{3,2}\right), \\
\overline{\Xi}_{23}^{(i2)} &= \sigma_{13}\left(\overline{R}_{13} - \overline{S}_{3,1} - \overline{S}_{3,2}\right), \\
\overline{\Xi}_{24}^{(i2)} &= \sigma_{13}\left(\overline{R}_{13} - \overline{S}_{3,2}^T\right), \\
\overline{\Xi}_{33}^{(i2)} &= \sigma_{13}\left(-2\overline{R}_{13} + \overline{S}_{3,1}^T + \overline{S}_{3,1}\right), \\
\overline{\Xi}_{34}^{(i2)} &= \sigma_{13}\overline{S}_{3,2}^T, \\
\overline{\Xi}_{44}^{(i2)} &= -\sigma_{13}\overline{R}_{13} - \sigma_{23}\overline{R}_{23} + \sigma_{13}\left(\overline{Q}_{23} - \overline{Q}_{13}\right), \\
\overline{\Xi}_{45}^{(i2)} &= \sigma_{23}\overline{R}_{23}, \\
\overline{\Xi}_{55}^{(i2)} &= -\sigma_{23}\left(\overline{R}_{23} + \overline{Q}_{23}\right), \\
\overline{\Xi}_{66}^{(i2)} &= -\gamma^2I + D^TD, \\
\sigma_{13} &= e^{-a_3h_1}, \\
\sigma_{23} &= e^{-a_3h_2}.
\end{aligned} \tag{33}
$$

Moreover, the control gains of controller (2) are obtained by $F_1 = \overline{F}_1\overline{P}_1^{-1}$, $F_2 = \overline{F}_2\overline{P}_2^{-1}$, *and* $F_3 = \overline{F}_3\overline{P}_3^{-1}$, *where* $\overline{F}_2 = \overline{F}_{12} + \overline{F}_{22}$ *and* $\overline{F}_3 = \overline{F}_{13} + \overline{F}_{23}$. *The parameters in the exponential stability and* H_∞ *performance are given by*

$$
\begin{aligned}
\alpha &= \frac{\left[a_1(l_2-2) + a_2(l_2+3) - a_3(l_2+1)\right]}{l_2}, \\
\beta &= \left(\frac{\overline{b}}{\overline{a}}\right)^{1/2}, \\
\lambda &= \frac{\left[a_1(l_2-2) + a_2(l_2+3) - a_3(l_2+1)\right]}{(2l_2)} - \frac{\ln\mu}{h_2}
\end{aligned} \tag{34}
$$

with

$$
\begin{aligned}
\overline{a} &= \min\left\{\lambda_{\min}\left(\overline{P}_1^{-1}\right), \lambda_{\min}\left(\overline{P}_2^{-1}\right), \lambda_{\min}\left(\overline{P}_3^{-1}\right)\right\}, \\
\overline{b} &= \lambda_{\max}\left(\overline{P}_1^{-1}\right) + \frac{\left(1 - e^{-a_1h_2}\right)}{a_1\overline{b}_q} + \frac{h_1\left(1 - e^{-a_1h_1}\right)}{a_1^2\overline{b}_r} \\
&\quad + \frac{h_{12}\left(e^{-a_1h_1} - e^{-a_1h_2}\right)}{a_1^2\overline{b}_r}, \\
\overline{b}_q &= \max\left\{\lambda_{\max}\left(\overline{P}_1^{-1}\overline{Q}_{11}\overline{P}_1^{-1}\right), \lambda_{\max}\left(\overline{P}_1^{-1}\overline{Q}_{21}\overline{P}_1^{-1}\right)\right\}, \\
\overline{b}_r &= \max\left\{\lambda_{\max}\left(\overline{P}_1^{-1}\overline{R}_{11}\overline{P}_1^{-1}\right), \lambda_{\max}\left(\overline{P}_1^{-1}\overline{R}_{21}\overline{P}_1^{-1}\right)\right\}.
\end{aligned} \tag{35}
$$

Proof. See Appendix B. □

Remark 4. Suppose that the system matrices $\Omega := [A \;\; B]$ are not exactly known and they reside in the uncertain polytope [16, 17, 26]. Consider

$$
\Omega \in \left\{\sum_{i=1}^{M}\mu_i\Omega_i,\ 0 \le \mu_i \le 1,\ \sum_{i=1}^{M}\mu_i = 1\right\}, \tag{36}
$$

where the M vertices of the polytope are described by $\Omega_i := [A_i \;\; B_i]$. For the H_∞ performance analysis and the periodic switched controller design of the dual-rate system with polytopic uncertainties, one can solve the LMIs in Propositions 2 and 3 for all the M vertices Ω_i by applying the same decision matrices, respectively.

For comparison purpose, a criterion for the existence of a sampled-data controller for system (1) via single-rate sampling is developed and given by the following corollary.

Corollary 5. *Given* $h > 0$, $a > 0$, *and* $\nu > 0$, *system (1) with a single-rate sampled-data controller is exponentially stable with a given* H_∞ *performance* γ, *where the exponential decay rate*

$\alpha = a$, if there exist a matrix $\overline{S}$ and symmetric matrices $\overline{P} > 0$, $\overline{Q} > 0$, and $\overline{R} > 0$ such that

$$\overline{\Xi} = \begin{bmatrix} \overline{\Xi}_{11} & \overline{\Xi}_{12} & \sigma\overline{S}^T & \overline{\Xi}_{14} & h\overline{P}A^T & \overline{P}C^T \\ * & \overline{\Xi}_{22} & \overline{\Xi}_{23} & 0 & h\overline{F}^T B^T & 0 \\ * & * & \overline{\Xi}_{33} & 0 & 0 & 0 \\ * & * & * & \overline{\Xi}_{44} & hE^T & 0 \\ * & * & * & * & \overline{\Xi}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \quad (37)$$

where

$$\begin{aligned} \overline{\Xi}_{11} &= \overline{P}A^T + A\overline{P} + a\overline{P} + \overline{Q} - \sigma\overline{R}, \\ \overline{\Xi}_{12} &= B\overline{F} + \sigma\left(\overline{R} - \overline{S}^T\right), \\ \overline{\Xi}_{14} &= E + \overline{P}C^T D, \\ \overline{\Xi}_{22} &= \sigma\left(-2\overline{R} + \overline{S}^T + \overline{S}\right), \\ \overline{\Xi}_{23} &= \sigma\left(\overline{R} - \overline{S}^T\right), \\ \overline{\Xi}_{33} &= -\sigma\left(\overline{R} + \overline{Q}\right), \\ \overline{\Xi}_{44} &= -\gamma^2 I + D^T D, \\ \overline{\Xi}_{55} &= v^2\overline{R} - 2v\overline{P}, \\ \sigma &= e^{-ah}. \end{aligned} \quad (38)$$

Moreover, the control gain is obtained as $F = \overline{F}\,\overline{P}^{-1}$.

Remark 6. For the case $l_1 = Nl_2$ ($N > 1$ and $N \in \mathbb{N}$), periodic switched controller (2) reduces to a switched controller with switching modes $\sigma(t) = 1$ and $\sigma(t) = 2$. Correspondingly, Proposition 3 with $\overline{P}_3 = 0$, $\overline{Q}_{i3} = 0$, $\overline{S}_{3,i} = 0$, and $\overline{R}_{i3} = 0$ ($i = 1, 2$) reduces to a sampled-data H_∞ control design result for system (1) via two sampling rates $1/h_1$ and $1/Nh_1$. Particularly, when $N = 2$, the design result can be reduced to the one in [28].

Remark 7. In this paper, we obtain some results on periodic switched control of dual-rate sampled-data systems. It should be pointed out that these results can be extended to multirate sampled-data systems although the process is very tedious. The corresponding results are omitted.

Remark 8. As shown in [20, 29], analysis and synthesis of research on networked control systems with multiple samplings are promising and significant issues. Compared with [20], this paper is distinguished based on the following evidences: (i) The dual-rate sampling schemes in this paper and [20] are different, resulting in two essentially different switched system models. In [20], by introducing a Round-Robin scheduling protocol, the sampled-data via two different sampling rates is available one after another. In this paper, however, the sampled-data is available in three cases (both $x_1(kh_1)$ and $x_2(kh_2)$, only $x_1(kh_1)$, and/or only $x_2(kh_2)$). In other words, under the sampling relationship $l_1h_1 = l_2h_2$ in this paper, the case that $x_1(kh_1)$ and $x_2(kh_2)$ are available simultaneously must occur, but this surely does not occur in [20]. (ii) The problems addressed in this paper and [20] are different. Stability and L_2-gain analysis carried out for dual-rate networked control systems is considered in [20], while this paper focuses on controller design for a dual-rate sampled-data system without considering the effect of network-induced delays. It should be mentioned that the constant constraint on network-induced delays in [20] does not significantly change the nature of the proposed results. (iii) The methodologies of performance analysis in this paper and [20] are different. In [20], a common discontinuous Lyapunov-Krasovskii functional is constructed to derive some delay-dependent stability and L_2-gain criteria. However, in this paper, a switched continuous Lyapunov-Krasovskii functional is proposed to establish delay-dependent criteria for the H_∞ performance and switched controller design by fully utilizing the relation of multiple input delays and the information about two sampling periods.

4. The Dual-Rate Sampled-Data Control of a Vehicle Dynamic System

In this section, we consider the dual-rate sampled-data control of a vehicle dynamic system described by the following state-space representation [30]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + E\omega(t), \\ z(t) &= Cx(t) + D\omega(t), \end{aligned} \quad (39)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 & 0 & 22.3 \\ 0 & -4.2796 & -19.4355 & 0 \\ 0 & 1.4391 & -4.2743 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 25.0655 \\ 17.7548 \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \\ D &= 0.1, \\ E &= \begin{bmatrix} 0 & 0.1 & 0.1 & 0 \end{bmatrix}^T, \end{aligned} \quad (40)$$

$x(t) = [y \;\; \phi \;\; r \;\; \psi]^T$, y is the inertial lateral displacement of the vehicle mass center, ϕ is lateral velocity in the vehicle body axis system, r is the angular velocity, and ψ is the vehicle heading angle.

Table 1: Solvability comparison between Proposition 3 with $3h_1 = 2h_2$ and Corollary 5 with $h = h_2$.

h_2	0.24 s	0.27 s	0.30 s	0.36 s
Proposition 3 (dual-rate)	Feasible	Feasible	Feasible	Feasible
Corollary 5 (single-rate)	Feasible	Feasible	Infeasible	Infeasible

The state variables y and ϕ are measured by using laser sensors at a sampling rate $1/h_1$, and the state variables r and ψ are measured by using gyrometers at a sampling rate $1/h_2$, where h_i $(i = 1, 2)$ are two sampling periods.

We now show the effectiveness of the proposed dual-rate sampling design result. Set $a_1 = 0.75$, $a_2 = 0.85$, $a_3 = 0.9$, $\nu = 0.25$, $l_1 = 3$, $l_2 = 2$, $\mu = 1.01$, $h_1 = 0.16$ s, and $h_2 = 0.24$ s. Using Proposition 3, we can obtain the minimum H_∞ performance $\gamma^d_{\min} = 0.1539$, an exponential decay rate $\alpha = 0.775$, and corresponding control gain matrices $F_1 = [-0.0141 \ \ -0.0487 \ \ -0.0532 \ \ -0.7625]$, $F_2 = [-0.0142 \ \ -0.0492 \ \ -0.0539 \ \ -0.7672]$, and $F_3 = [-0.0142 \ \ -0.0490 \ \ -0.0546 \ \ -0.7686]$, which means that switched controller (2) with the above F_i $(i = 1, 2, 3)$ can stabilize system (39) and achieve an H_∞ performance $\gamma^d_{\min} = 0.1539$ and $\alpha = 0.775$ for system (39) with an external disturbance. Using Corollary 5, the minimum H_∞ performance $\gamma^s_{\min} = 0.2317$, the exponential decay rate $a = \alpha = 0.775$, and the control gain is $F = [-0.0092 \ \ -0.0252 \ \ -0.0386 \ \ -0.6028]$. Clearly, Proposition 3 can be used to search for a better H_∞ control performance than Corollary 5. For comparison purpose, we choose different delay bounds of h_2 for both dual-rate sampling case and single-rate sampling case. The solvability of Proposition 3 and Corollary 5 is shown in Table 1, from which one can see that the dual-rate sampling design result can provide a larger sampling period than the single-rate design result.

Then we compare the proposed dual-rate sampling design result and the single-rate design result in simulation. Choose the initial state $x_0 = [-2 \ \ 0 \ \ 1 \ \ 0]^T$ and the external disturbance $\omega(t) = 2e^{-0.3t}\sin(2.7t + 0.05)$. Using the switched controller with F_i $(i = 1, 2, 3)$, we can depict the state responses of system (39) by Figure 3 and the corresponding control input by Figure 4, respectively. Under the zero initial condition, $\mathscr{L}_2$-norm $\|z_d(t)\|_2$ using the switched controller and $\|z_s(t)\|_2$ using the single-rate sampled-data controller are shown in Figure 5. From Figures 3–5, one can conclude that the dual-rate sampled-data can stabilize the system (39) and achieve a better H_∞ performance than the single-rate sampled-data controller.

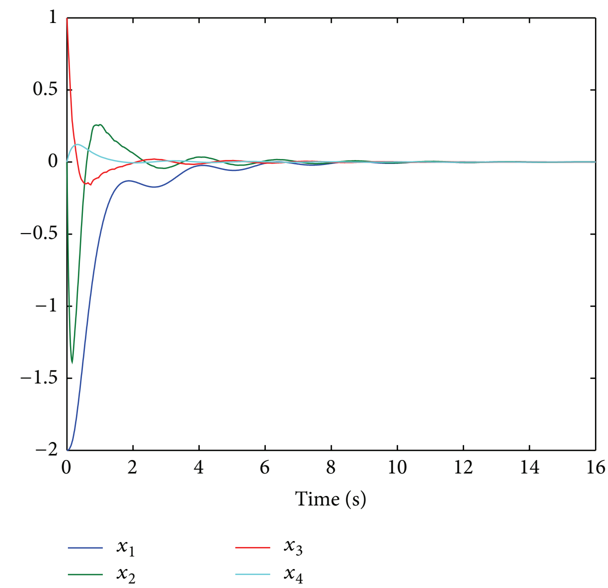

Figure 3: The state responses of system (39) using the switched controller.

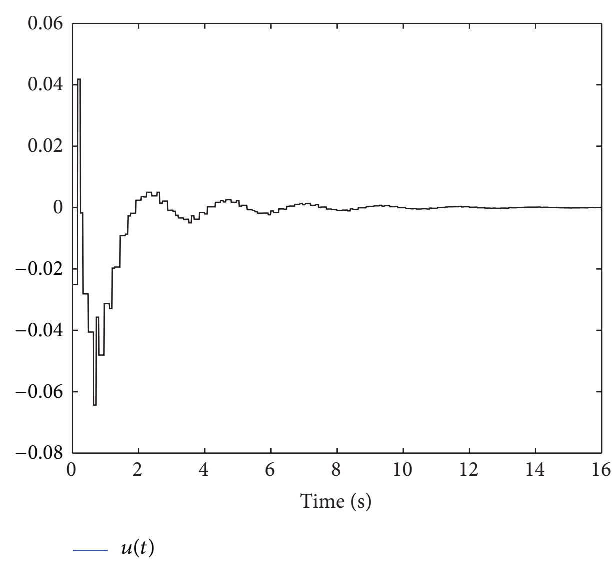

Figure 4: The control input of system (39) using the switched controller.

5. Conclusion

This paper has dealt with the H_∞ sampled-data control for a multirate system with two sampling rates by using an input delay approach. A periodic switched controller has been proposed to implement the sampled-data control such that the sampled-data of state variables with different sampling rates can be used real-timely. The resulting closed-loop system has been modeled as a switched system, where the subsystems have different input delays. Some delay-dependent criteria for the H_∞ performance of the switched system and the existence of the switched controller have been established by using the Lyapunov-Krasovskii functional method. By comparing with single-rate design methods, the effectiveness of the proposed design method has been illustrated by an example.

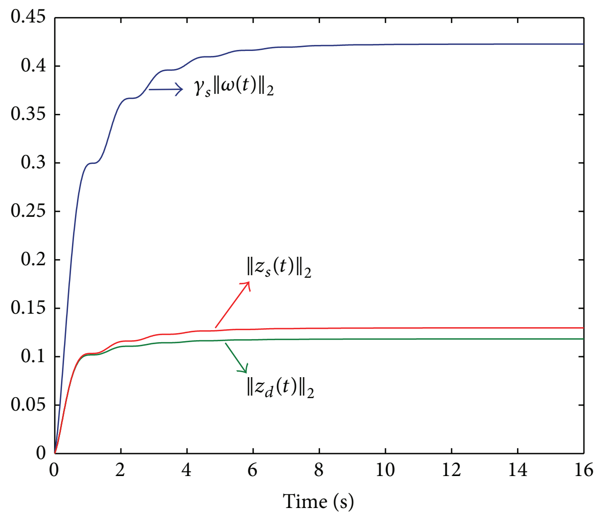

Figure 5: The comparison of $\|e^{-\alpha t/2}z(t)\|_2 \le \gamma\|\omega(t)\|_2$ under the zero initial condition by two different methods.

Appendices

A. Proof of Proposition 2

Proof. Construct the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V_{\sigma(t)}(t) &= x^T(t) P_{\sigma(t)} x(t) \\ &+ \int_{t-h_1}^{t} x^T(s) e^{a_{\sigma(t)}(s-t)} Q_{1\sigma(t)} x(s)\, ds \\ &+ \int_{t-h_2}^{t-h_1} x^T(s) e^{a_{\sigma(t)}(s-t)} Q_{2\sigma(t)} x(s)\, ds \\ &+ h_1 \int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) e^{a_{\sigma(t)}(s-t)} R_{1\sigma(t)} \dot{x}(s)\, ds\, d\theta \\ &+ h_{12} \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s) e^{a_{\sigma(t)}(s-t)} R_{2\sigma(t)} \dot{x}(s)\, ds\, d\theta, \end{aligned} \tag{A.1}$$

where $P_{\sigma(t)} > 0$, $Q_{i\sigma(t)} > 0$, and $R_{i\sigma(t)} > 0$ $(i = 1, 2, \sigma(t) = 1, 2, 3)$.

When $\sigma(t) = 1$, taking the time derivative of $V_1(t)$, we have

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t) P_1 \dot{x}(t) + a_1 x^T(t) P_1 x(t) \\ &\quad + x^T(t) Q_{11} x(t) \\ &\quad + e^{-a_1 h_1} x^T(t-h_1)(Q_{21} - Q_{11}) x(t-h_1) \\ &\quad - e^{-a_1 h_2} x^T(t-h_2) Q_{21} x(t-h_2) \\ &\quad + \dot{x}^T(t)\left(h_1^2 R_{11} + h_{12}^2 R_{21}\right)\dot{x}(t) - a_1 V_1(t) \\ &\quad - h_1 \int_{t-h_1}^{t} \dot{x}^T(s) e^{a_1(s-t)} R_{11} \dot{x}(s)\, ds \\ &\quad - h_{12} \int_{t-h_2}^{t-h_1} \dot{x}^T(s) e^{a_1(s-t)} R_{21} \dot{x}(s)\, ds \\ &\le -a_1 V_1(t) + 2x^T(t) P_1 \dot{x}(t) \\ &\quad + x^T(t)(a_1 P_1 + Q_{11}) x(t) \\ &\quad + \dot{x}^T(t)\left(h_1^2 R_{11} + h_{12}^2 R_{21}\right)\dot{x}(t) \\ &\quad + e^{-a_1 h_1} x^T(t-h_1)(Q_{21} - Q_{11}) x(t-h_1) \\ &\quad - e^{-a_1 h_2} x^T(t-h_2) Q_{21} x(t-h_2) \\ &\quad - h_1 e^{-a_1 h_1} \int_{t-h_1}^{t} \dot{x}^T(s) R_{11} \dot{x}(s)\, ds \\ &\quad - h_{12} e^{-a_1 h_2} \int_{t-h_2}^{t-h_1} \dot{x}^T(s) R_{21} \dot{x}(s)\, ds. \end{aligned} \tag{A.2}$$

For system (8) with $\omega(t) \equiv 0$, by using Lemma 1 and Jensen's inequality to (A.2), it can be derived that $\dot{V}_1(t) \le -a_1 V_1(t) + \tilde{\xi}_1^T(t)\tilde{\Xi}^{(1)}\tilde{\xi}_1(t)$, where

$$\begin{aligned} \tilde{\xi}_1^T(t) &= \left[x^T(t) \;\; x^T(t-\tau_1(t)) \;\; x^T(t-h_1) \;\; x^T(t-h_2)\right], \\ \tilde{\Xi}^{(1)} &= \begin{bmatrix} \Xi_{11}^{(1)} & \Xi_{12}^{(1)} & \Xi_{13}^{(1)} & 0 \\ * & \Xi_{22}^{(1)} & \Xi_{23}^{(1)} & 0 \\ * & * & \Xi_{33}^{(1)} & \Xi_{34}^{(1)} \\ * & * & * & \Xi_{44}^{(1)} \end{bmatrix} + \left[A \;\; BF_1 \;\; 0 \;\; 0\right]^T \\ &\quad \cdot \left(h_1^2 R_{11} + h_{12}^2 R_{21}\right)\left[A \;\; BF_1 \;\; 0 \;\; 0\right]. \end{aligned} \tag{A.3}$$

If the inequality $\tilde{\Xi}^{(1)} < 0$ holds, then we obtain $\dot{V}_1(t) \le -a_1 V_1(t)$. On the other hand, $\tilde{\Xi}^{(1)} < 0$ is implied by (20).

For the cases $\sigma(t) = 2$ and $\sigma(t) = 3$, similar to the above process, one can obtain from (21) and (22) that $\dot{V}_2(t) \le -a_2 V_2(t)$ and $\dot{V}_3(t) \le -a_3 V_3(t)$, respectively.

On different subintervals $\mathcal{I}_i$ $(i = 1, 2, \ldots, m_{l_2} + l_2)$, it can be obtained that

$$\begin{aligned}
&V_1(t) \le e^{-a_1(t-kl_1h_1)} V_1(kl_1h_1), \quad t \in \mathcal{I}_1,\\
&V_2(t) \le e^{-a_2(t-(kl_1+m_{\nu-1}+1)h_1)} V_2((kl_1 + m_{\nu-1} + 1)h_1),\\
&\qquad t \in \bigcup_{m_{\nu-1}+\nu+1}^{m_\nu+\nu} \mathcal{I}_i, \ \nu = 1, 2, \ldots, l_2, \\
&V_3(t) \le e^{-a_3(t-kl_1h_1-(\nu-1)h_2)} V_3(kl_1h_1 + (\nu-1)h_2),\\
&\qquad t \in \mathcal{I}_{m_{\nu-1}+\nu}, \ \nu = 2, 3, \ldots, l_2.
\end{aligned} \tag{A.4}$$

For $P_2 \le \mu P_1$, $P_1 \le \mu P_2$, $P_2 \le \mu P_3$, $P_3 \le \mu P_2$, $Q_{i2} \le \mu Q_{i1}$, $Q_{i1} \le \mu Q_{i2}$, $Q_{i2} \le \mu Q_{i3}$, $Q_{i3} \le \mu Q_{i2}$, $R_{i2} \le \mu R_{i1}$, $R_{i1} \le \mu R_{i2}$, $R_{i2} \le \mu R_{i3}$, and $R_{i3} \le \mu R_{i2}$ $(i = 1, 2)$, one has

$$\begin{aligned}
V_1(t) &\le \mu e^{(a_2-a_1)h_2} V_2(t),\\
V_2(t) &\le \mu e^{(a_3-a_2)h_2} V_3(t),\\
V_2(t) &\le \mu V_1(t),\\
V_3(t) &\le \mu V_2(t).
\end{aligned} \tag{A.5}$$

Defining $\upsilon_1 = \mu e^{(a_2-a_1)h_2}$ and $\upsilon_2 = \mu e^{(a_3-a_2)h_2}$, we obtain

$$\begin{aligned}
&V_{\sigma(t^+)} \le \upsilon_1 V_{\sigma(t^-)}, \quad t = kl_1h_1,\\
&V_{\sigma(t^+)} \le \mu V_{\sigma(t^-)}, \quad t = kl_1h_1 + h_1, \ kl_1h_1 + ih_2,\\
&V_{\sigma(t^+)} \le \upsilon_2 V_{\sigma(t^-)}, \quad t = kl_1h_1 + (m_i + 1)h_1,
\end{aligned} \tag{A.6}$$

where $i = 1, 2, \ldots, l_2 - 1$, $k = 0, 1, 2, \ldots$.

For $t \in [kl_1h_1, kl_1h_1 + h_1)$, using (A.4) and (A.6), we have

$$\begin{aligned}
V_1(t) &\le \upsilon_1 e^{-a_1(t-kl_1h_1)} V_2(kl_1h_1 - 0)\\
&\le \upsilon_1\upsilon_2 e^{-a_1(t-kl_1h_1)} e^{-a_2(l_1h_1-(m_{l_2-1}+1)h_1)} V_3((k-1)\\
&\quad \cdot l_1h_1 + (m_{l_2-1}+1)h_1 - 0)\\
&\le \upsilon_1\upsilon_2\mu e^{-a_1(t-kl_1h_1)} V_2((k-1)l_1h_1 + (l_2-1)h_1\\
&\quad - 0) e^{-a_2(l_1h_1-(m_{l_2-1}+1)h_1)-a_3((m_{l_2-1}+1)h_1-(l_2-1)h_2)} \le \cdots\\
&\le \upsilon_1\upsilon_2^{l_2-1}\mu^{l_2} e^{-a_1(t-kl_1h_1)} e^{\overline{a}} V_1((k-1)l_1h_1) \le \cdots\\
&\le (\upsilon_1\upsilon_2^{l_2-1}\mu^{l_2})^k e^{-a_1(t-kl_1h_1)} e^{k\overline{a}} V_1(0),
\end{aligned} \tag{A.7}$$

where

$$\begin{aligned}
\overline{a} = &-a_2((l_1 - m_{l_2-1} - 1)h_1)\\
&- a_3((m_{l_2-1}+1)h_1 - (l_2-1)h_2)\\
&- a_2((l_2-1)h_2 - (m_{l_2-2}+1)h_1)\\
&- a_3((m_{l_2-2}+1)h_1 - (l_2-2)h_2) - \cdots\\
&- a_2(h_2 - h_1) - a_1h_1.
\end{aligned} \tag{A.8}$$

Using $kl_1h_1 \le t$ to (A.7), one can see that

$$\begin{aligned}
&V_1(t)\\
&\le \left[\mu^{2l_2} e^{(a_2-a_1)h_2+(l_2-1)(a_3-a_2)h_2}\right]^k e^{-a_1(t-kl_1h_1)} e^{k\overline{a}} V_1(0)\\
&\le \left[\mu^{2l_2} e^{(a_2-a_1)h_2+(l_2-1)(a_3-a_2)h_2+\overline{a}}\right]^k e^{-a_1(t-kl_1h_1)} V_1(0)\\
&\le \left[\mu^{2/h_2} e^{[a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2)]/l_2}\right]^t V_1(0).
\end{aligned} \tag{A.9}$$

For $t \in [kl_1h_1 + h_1, kl_1h_1 + h_2)$, it is clear that

$$\begin{aligned}
V_2(t) &\le (\upsilon_1\upsilon_2^{l_2-1}\mu^{l_2})^k e^{-a_2(t-kl_1h_1-h_1)} e^{k\overline{a}} \mu e^{-a_1h_1} V_1(0)\\
&\le \left[\mu^{2l_2} e^{(a_2-a_1)h_2+(l_2-1)(a_3-a_2)h_2}\right]^k\\
&\quad \cdot e^{-a_2(t-kl_1h_1-h_1)} e^{k\overline{a}} \mu e^{-a_1h_1} V_1(0)\\
&\le \left[\mu^{2/h_2} e^{[a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2)]/l_2}\right]^{kl_1h_1+h_1}\\
&\quad \cdot e^{-a_2(t-kl_1h_1-h_1)} V_1(0)\\
&\le \left[\mu^{2/h_2} e^{[a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2)]/l_2}\right]^t V_1(0).
\end{aligned} \tag{A.10}$$

For $t \in [kl_1h_1 + h_2, (kl_1 + m_1 + 1)h_1)$, it can be seen that

$$\begin{aligned}
V_3(t) &\le (\upsilon_1\upsilon_2^{l_2-1}\mu^{l_2})^k\\
&\quad \cdot e^{-a_3(t-kl_1h_1-h_2)} e^{k\overline{a}} e^{-a_2(h_2-h_1)-a_1h_1} \mu^2 V_1(0)\\
&\le (\mu^{2l_2} e^{(a_2-a_1)h_2+(l_2-1)(a_3-a_2)h_2})^k\\
&\quad \cdot e^{-a_3(t-kl_1h_1-h_2)} e^{k\overline{a}} e^{-a_2(h_2-h_1)-a_1h_1} \mu^2 V_1(0)\\
&\le (\mu^{2/h_2} e^{[a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2)]/l_2})^{kl_1h_1+h_2}\\
&\quad \cdot e^{-a_3(t-kl_1h_1-h_2)} V_1(0)\\
&\le (\mu^{2/h_2} e^{[a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2)]/l_2})^t V_1(0).
\end{aligned} \tag{A.11}$$

For $t \in [kl_1h_1 + (l_2-1)h_2, (kl_1 + m_{l_2-1} + 1)h_1)$, one has

$$\begin{aligned} V_3(t) &\le \left(\upsilon_1\upsilon_2^{l_2-1}\mu^{l_2}\right)^k e^{-a_3(t-kl_1h_1-(l_2-1)h_2)} e^{k\bar{a}}\upsilon_2^{l_2-2}\mu^{l_2} e^{-a_2((l_2-1)h_2-(m_{l_2-2}+1)h_1)-\cdots-a_2(h_2-h_1)-a_1h_1} V_1(0) \\ &\le \left(\mu^{2/h_2} e^{(a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2)]/l_2}\right)^{kl_1h_1+(l_2-1)h_2} e^{-a_3(t-kl_1h_1-(l_2-1)h_2)} V_1(0) \\ &\le \left(\mu^{2/h_2} e^{(a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2))/l_2}\right)^t V_1(0). \end{aligned} \tag{A.12}$$

For $t \in [(kl_1 + m_{l_2-1} + 1)h_1, kl_1h_1 + l_2h_2)$, one has

$$\begin{aligned} V_2(t) &\le \left(\upsilon_1\upsilon_2^{l_2-1}\mu^{l_2}\right)^k e^{-a_2(t-kl_1h_1-(m_{l_2-1}+1)h_1)} e^{k\bar{a}}\upsilon_2^{l_2-1}\mu^{l_2} e^{-a_3((m_{l_2-1}+1)h_1-(l_2-1)h_2)-\cdots-a_2(h_2-h_1)-a_1h_1} V_1(0) \\ &\le \left(\mu^{2/h_2} e^{(a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2))/l_2}\right)^{kl_1h_1+(m_{l_2-1}+1)h_1} e^{-a_2(t-kl_1h_1-(m_{l_2-1}+1)h_1)} V_1(0) \\ &\le \left(\mu^{2/h_2} e^{(a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2))/l_2}\right)^t V_1(0). \end{aligned} \tag{A.13}$$

Denote $\varepsilon = \mu^{2/h_2} e^{(a_3(l_2+1)-a_2(l_2+3)-a_1(l_2-2))/l_2}$. If $0 < \varepsilon < 1$ is satisfied, then one obtains $\lambda = -(1/2)\ln(\varepsilon)$. Integrating all cases (A.9)–(A.13), we have $V_{\sigma(t)}(t) < e^{-2\lambda t}V_1(0)$, which means that system (8) is exponentially stable for system (8) with $\omega(t) \equiv 0$, where λ and β are given in Proposition 2.

Now, we are in a position to show the H_∞ performance for system (8) with nonzero $\omega(t) \in \mathscr{L}_2[0,\infty)$ under the zero initial condition.

Using Lemma 1 and Jensen's inequality to (A.2), we have

$$\begin{aligned} &\dot{V}_1(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\ &\quad \le -a_1V_1(t) + \xi_1^T(t)\,\Xi^{(1)}\xi_1(t), \end{aligned} \tag{A.14}$$

where $\xi_1^T(t) = [x^T(t)\ \ x^T(t-\tau_1(t))\ \ x^T(t-h_1)\ \ x^T(t-h_2)\ \ \omega^T(t)]$.

Similarly, we obtain

$$\begin{aligned} &\dot{V}_2(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\ &\quad \le -a_2V_2(t) + \xi_{i1}^T(t)\,\Xi^{(i1)}\xi_{i1}(t), \\ &\dot{V}_3(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\ &\quad \le -a_3V_3(t) + \xi_{i2}^T(t)\,\Xi^{(i2)}\xi_{i2}(t), \end{aligned} \tag{A.15}$$

where $\xi_{ij}^T(t) = [x^T(t)\ \ x^T(t-\tau_i^{(1)}(t))\ \ x^T(t-\tau_i^{(2)}(t))\ \ x^T(t-h_1)\ \ x^T(t-h_2)\ \ \omega^T(t)]$ $(j = 1,2)$.

It follows from (20)–(22) and (A.14)-(A.15) that

$$\begin{aligned} &\dot{V}_{\sigma(t)}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\ &\quad \le -a_{\sigma(t)}V_{\sigma(t)}(t). \end{aligned} \tag{A.16}$$

Let $\mathscr{I}_i = [t_i, t_{i+1})$. When $t \in \mathscr{I}_i$, integrating both sides of (A.16) from t_i to t, we have

$$\begin{aligned} V_{\sigma(t_i^+)}(t) &\le e^{-a_{\sigma(t_i^+)}(t-t_i)}V_{\sigma(t_i^+)}(t_i^+) \\ &\quad - \int_{t_i}^t e^{-a_{\sigma(t_i^+)}(t-s)}\Delta(s)\,ds, \end{aligned} \tag{A.17}$$

where $\Delta(s) = z^T(s)z(s) - \gamma^2\omega^T(s)\omega(s)$.

Combining (A.6) and (A.17), we obtain

$$\begin{aligned} V_{\sigma(t_i^+)}(t) &\le e^{-a_{\sigma(t_i^+)}(t-t_i)}V_{\sigma(t_i^+)}(t_i^+) \\ &\quad - \int_{t_i}^t e^{-a_{\sigma(t_i^+)}(t-s)}\Delta(s)\,ds \\ &\le \hat{\upsilon}e^{-a_{\sigma(t_i^+)}(t-t_i)}V_{\sigma(t_i^-)}(t_i^-) \\ &\quad - \int_{t_i}^t e^{-a_{\sigma(t_i^+)}(t-s)}\Delta(s)\,ds \le \cdots \\ &\le e^{-2\lambda t}V_1(0) - \int_0^t e^{-2\lambda(t-s)}\Delta(s)\,ds, \end{aligned} \tag{A.18}$$

where

$$\hat{\upsilon} = \begin{cases} \upsilon_1, & t_i = kl_1h_1, \\ \mu, & t_i = kl_1h_1 + h_1,\ kl_1h_1 + jh_2,\ j = 1,2,\ldots,l_2-1, \\ \upsilon_2, & t_i = kl_1h_1 + (m_j+1)h_1. \end{cases} \tag{A.19}$$

For $t \in [0,\infty)$, it can be seen that

$$V_{\sigma(t)}(t) \le e^{-2\lambda t}V_1(0) - \int_0^t e^{-2\lambda(t-s)}\Delta(s)\,ds. \tag{A.20}$$

Multiplying both sides of (A.20) by μ^{-2t/h_2} yields

$$\begin{aligned}\mu^{-2t/h_2}V_{\sigma(t)}(t)+\int_0^t e^{-2\lambda(t-s)-(2t/h_2)\ln\mu}z^T(s)z(s)\,ds\\ \leq e^{-2\lambda t-(2t/h_2)\ln\mu}V_1(0)\\ +\int_0^t e^{-2\lambda(t-s)-(2t/h_2)\ln\mu}\gamma^2\omega^T(s)\omega(s)\,ds\\ \leq e^{-2\lambda t-(2t/h_2)\ln\mu}V_1(0)+\int_0^t\gamma^2\omega^T(s)\omega(s)\,ds.\end{aligned} \tag{A.21}$$

Taking zero initial condition and $V_{\sigma(t)}(t)>0$ into consideration, we have

$$\begin{aligned}\int_0^t e^{-2\lambda(t-s)-(2t/h_2)\ln\mu}z^T(s)z(s)\,ds\\ <\int_0^t\gamma^2\omega^T(s)\omega(s)\,ds.\end{aligned} \tag{A.22}$$

Notice that $-2\lambda(t-s)-(2t/h_2)\ln\mu=-\alpha t+2\lambda s>-\alpha t$, where $\alpha=(a_1(l_2-2)+a_2(l_2+3)-a_3(l_2+1))/l_2$. So it is easy to obtain

$$\int_0^t e^{-\alpha t}z^T(s)z(s)\,ds\leq\gamma^2\int_0^t\omega^T(s)\omega(s)\,ds. \tag{A.23}$$

Integrating both sides of (A.23) from $t=0$ to $t=\infty$ leads to

$$\int_0^\infty e^{-\alpha s}z^T(s)z(s)\,ds\leq\gamma^2\int_0^\infty\omega^T(s)\omega(s)\,ds. \tag{A.24}$$

This completes the proof. □

B. Proof of Proposition 3

Proof. By Schur complement, (20) is equivalent to

$$\widehat{\Xi}^{(1)}=\begin{bmatrix}\widehat{\Pi}_{11}^{(1)} & \widehat{\Pi}_{12}^{(1)}\\ * & \widehat{\Pi}_{22}^{(1)}\end{bmatrix}<0, \tag{B.1}$$

where

$$\begin{aligned}\widehat{\Pi}_{11}^{(1)}&=\begin{bmatrix}\widehat{\Xi}_{11}^{(1)} & \Xi_{12}^{(1)} & 0 & 0 & P_1E+C^TD\\ * & \Xi_{22}^{(1)} & \Xi_{23}^{(1)} & 0 & 0\\ * & * & \Xi_{33}^{(1)} & \Xi_{34}^{(1)} & 0\\ * & * & * & \Xi_{44}^{(1)} & 0\\ * & * & * & * & -\gamma^2I+D^TD\end{bmatrix},\\ \widehat{\Pi}_{12}^{(1)}&=\begin{bmatrix}h_1A^T & h_{12}A^T & C^T\\ h_1F_1^TB^T & h_{12}F_1^TB^T & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ h_1E^T & h_{12}E^T & 0\end{bmatrix},\\ \widehat{\Pi}_{22}^{(1)}&=\begin{bmatrix}R_{11}^{-1} & 0 & 0\\ * & R_{21}^{-1} & 0\\ * & * & -I\end{bmatrix},\\ \widehat{\Xi}_{11}^{(1)}&=A^TP_1+P_1A+a_1P_1+Q_{11}-\sigma_{11}R_{11}.\end{aligned} \tag{B.2}$$

Define $\Delta_1=\operatorname{diag}\{P_1^{-1},P_1^{-1},P_1^{-1},P_1^{-1},I,I,I,I\}$. Performing a congruence transformation to (B.1) by Δ_1, together with the changes of matrix variables defined by $\overline{P}_1=P_1^{-1}$, $\overline{Q}_{i1}=\overline{P}_1Q_{i1}\overline{P}_1$ $(i=1,2)$, $\overline{R}_{i1}=\overline{P}_1R_{i1}\overline{P}_1$ $(i=1,2)$, $\overline{S}_1=\overline{P}_1S_1\overline{P}_1$, $\overline{F}_1=F_1\overline{P}_1$, and using $-\overline{P}_1\overline{R}_{i1}^{-1}\overline{P}_1\leq\nu^2\overline{R}_{i1}-2\nu\overline{P}_1$ $(i=1,2)$, we can obtain (30).

Using a similar method to (21) and (22), we can obtain (31) and (32), respectively, where $\overline{P}_j=P_j^{-1}$, $\overline{S}_2=\overline{P}_2S_2\overline{P}_2$, $\overline{Q}_{ij}=\overline{P}_jQ_{ij}\overline{P}_j$, $\overline{R}_{ij}=\overline{P}_jR_{ij}\overline{P}_j$, $\overline{S}_{2,1}=\overline{P}_2S_{2,1}\overline{P}_2$, $\overline{S}_{2,2}=\overline{P}_2S_{2,2}\overline{P}_2$, $\overline{S}_{3,1}=\overline{P}_3S_{3,1}\overline{P}_3$, $\overline{S}_{3,2}=\overline{P}_3S_{3,2}\overline{P}_3$, $\overline{F}_{1j}=F_jI_{p_1}\overline{P}_j$, and $\overline{F}_{2j}=F_jI_{p_2}\overline{P}_j$ $(i=1,2;\ j=1,2,3)$. Before and after multiplying the inequalities $P_1\leq\mu P_2$, $P_2\leq\mu P_1$, $P_3\leq\mu P_2$, and $P_2\leq\mu P_3$ by P_2^{-1}, P_1^{-1}, P_2^{-1}, and P_3^{-1}, respectively, we have $\overline{P}_2\leq\mu\overline{P}_1$, $\overline{P}_1\leq\mu\overline{P}_2$, $\overline{P}_2\leq\mu\overline{P}_3$, and $\overline{P}_3\leq\mu\overline{P}_2$. Similarly, we obtain $\overline{Q}_{i2}\leq\mu\overline{Q}_{i1}$, $\overline{Q}_{i1}\leq\mu\overline{Q}_{i2}$, $\overline{Q}_{i2}\leq\mu\overline{Q}_{i3}$, $\overline{Q}_{i3}\leq\mu\overline{Q}_{i2}$, $\overline{R}_{i2}\leq\mu\overline{R}_{i1}$, $\overline{R}_{i1}\leq\mu\overline{R}_{i2}$, $\overline{R}_{i2}\leq\mu\overline{R}_{i3}$, $\overline{R}_{i3}\leq\mu\overline{R}_{i2}$ $(i=1,2)$, and the equalities (26)–(29). The proof is completed. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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