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Research Article

Robust Adaptive Fuzzy Control for a Class of Uncertain MIMO Nonlinear Systems with Input Saturation

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This paper studies the robust adaptive fuzzy control design problem for a class of uncertain multiple-input and multiple-output (MIMO) nonlinear systems in the presence of actuator amplitude and rate saturation. In the control scheme, fuzzy logic systems are used to approximate unknown nonlinear systems. To compensate the effect of input saturations, an auxiliary system is constructed and the actuator saturations then can be augmented into the controller. The modified tracking error is introduced and used in fuzzy parameter update laws. Furthermore, in order to deal with fuzzy approximation errors for unknown nonlinear systems and external disturbances, a robust compensation control is designed. It is proved that the closed-loop system obtains H_∞ tracking performance through Lyapunov analysis. Steady and transient modified tracking errors are analyzed and the bound of modified tracking errors can be adjusted by tuning certain design parameters. The proposed control scheme is applicable to uncertain nonlinear systems not only with actuator amplitude saturation, but also with actuator amplitude and rate saturation. Detailed simulation results of a rigid body satellite attitude control system in the presence of parametric uncertainties, external disturbances, and control input constraints have been presented to illustrate the effectiveness of the proposed control scheme.

1. Introduction

In most practical control applications, such as those in robot manipulation and aerospace industry, the performance of the controller is directly related to the accuracy of the mathematical model and external disturbances. However, it is difficult to establish an appropriate mathematical model for a large number of nonlinear systems when the systems are complex and highly coupled nonlinear with structured uncertainties and external disturbances [1]. To tackle with this problem, fuzzy logic systems and neural networks have been extensively used in complex and ill-defined nonlinear systems due to their approximation ability of dealing with the nonlinear smooth functions [2]. Many adaptive fuzzy control and adaptive neural network control schemes have been developed for single-input and single-output (SISO) nonlinear systems [3–7], MIMO nonlinear systems [8–17], and SISO/MIMO nonlinear systems with immeasurable states [18–20], respectively. Generally, these adaptive fuzzy and neural network control approaches can achieve nice control performance without control saturation. If physical actuators saturation such as magnitude and rate constraints is considered, the adaptive intelligent control approaches mentioned above can not be implemented [21].

As we know, in many practical dynamic systems, physical actuators saturation on hardware indicates an inevitable constraint of the magnitude and rate limitations of the control signal. For example, due to physical limitation, momentum exchange devices or thrusters as actuator for the satellite attitude control system fail to render infinite control torque and thus the actuator can only provide limited control torques within a limited rate [22]. Control saturation is one of the most common nonsmooth nonlinearity that should be explicitly considered in the control design. The controllers that ignore actuator limitations may give rise to undesirable inaccuracy, severely degrade the performance of system, or even damage the stability of system [23]. Hence, the controller design subjected to the control saturation while simultaneously achieving higher performance is a very practical problem.

The design of tracking controllers for uncertain MIMO nonlinear systems with actuator constraints is a challenging problem. During the past decades, there have been extensive researches on the control of nonlinear systems with various constraints. Analysis and design of control systems with control saturation have been widely studied in [24–42]. Farrell et al. [24–26] have presented an adaptive backstepping approach for unknown nonlinear systems with known magnitude, rate, and bandwidth constraints on intermediate states or actuators without disturbance. To tackle with the physical saturation, an auxiliary system with the same order as that of the plant was constructed to compensate the effect of saturation. The control input saturation is investigated through online approximation based control for uncertain nonlinear systems in [26]. An adaptive control and the constrained adaptive control in combination with the backstepping technique are proposed in [29]. A direct adaptive fuzzy control approach for uncertain nonlinear systems with input saturation is presented in [30], in which a Nussbaum function is used to compensate for the nonlinear term arising from the input saturation. In [31], an adaptive fuzzy output feedback control algorithm for a class of output constrained uncertain nonlinear systems with input saturation is developed by employing a barrier Lyapunov function and an auxiliary system. Adaptive backstepping tracking control based on fuzzy neural networks is investigated for unknown chaotic systems in [32] and with control input constraints in [33]. Neural network based adaptive control schemes with external disturbances and actuator saturations are presented in [35], in which auxiliary systems are added to attenuate the effects of input saturation. It is apparent that the presence of input saturation constraint substantially increases the complexity of control system design for uncertain MIMO nonlinear systems. In the constrained adaptive control, the key problem is how to handle the constraint effect of the actuator's physical constraints. To this end, we introduce an auxiliary design system to handle the constraint effect in this paper. Based on the states of the auxiliary design system, constrained adaptive control is investigated for a class of uncertain MIMO nonlinear systems with input constraints using robust fuzzy control technique.

In this paper, a robust adaptive fuzzy tracking control scheme is presented to handle the external disturbances and actuator physical constraints for uncertain MIMO nonlinear systems. In control design, fuzzy logic systems are used to approximate unknown nonlinear systems. Note that input saturations are nonsmooth functions but the adaptive fuzzy control technique requires all functions differentiable [6]. To compensate the effect of input saturations, an auxiliary system is constructed and the actuator saturations then can be augmented into the controller. The modified tracking error is introduced and used in fuzzy parameter update laws. Besides, in order to deal with fuzzy approximation errors for unknown nonlinear systems and external disturbances, a robust compensation control is designed. It is proved that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded, and the closed-loop system obtains H_∞ tracking performance through Lyapunov analysis. The transient modified tracking errors performance is derived to be explicit functions of design parameters and thus bounds of modified tracking errors can be adjusted by tuning design parameters.

The rest of this paper is organized as follows. The description of the uncertain MIMO nonlinear system under consideration and necessary preliminaries are given in Section 2. In Section 3, the robust adaptive fuzzy control without control saturation is firstly designed. When the actuators have physical limitations, this approach may not be able to be successfully implemented. In order to solve this problem, a robust adaptive fuzzy control scheme with input saturation is investigated. The simulation results of satellite attitude control are presented to demonstrate the effectiveness of proposed controller in Section 4. Section 5 contains the conclusion.

2. Problem Formulation and Preliminary

2.1. Problem Formulation. Consider a class of uncertain MIMO affine nonlinear system

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{m} \mathbf{g}_i(\mathbf{x}) u_i + \mathbf{d}' \\ y_1 &= h_1(\mathbf{x}) \\ &\vdots \\ y_m &= h_m(\mathbf{x}),\end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_n] \in R^n$ is the state vector available for measurement; $\mathbf{u} = [u_1, \ldots, u_m] \in R^m$ is the control vector with $|u_i| \le u_{i\max}$, where $u_{i\max}$ denote the actuator amplitude; $\mathbf{y} = [y_1, \ldots, y_m] \in R^m$ is the output vector; $h_1(\mathbf{x}), \ldots, h_m(\mathbf{x})$ are smooth functions defined on the open set of $\mathbf{R}^n$. $\mathbf{f}(\mathbf{x})$, $\mathbf{g}_1(\mathbf{x}), \ldots, \mathbf{g}_m(\mathbf{x})$ are continuous unknown but smooth vector fields. $\mathbf{d}' = [d'_1, \ldots, d'_n]^T \in \mathbf{R}^n$ is external disturbance vector, where d'_1 is unknown but bounded.

Define that $L_{\mathbf{f}} h_i$ is the Lie derivative of h_i along vector field $\mathbf{f}(\mathbf{x})$ and $L_{\mathbf{f}}^k h_i$ is recursively as $L_{\mathbf{f}}^k h_i = L_{\mathbf{f}}(L_{\mathbf{f}}^{k-1} h_i)$. A multivariable uncertain nonlinear system of the form (1) has a vector relative degree $[r_1, r_2, \ldots, r_m]$ at a point $\mathbf{x}_0$ if

$$L_{\mathbf{g}_j} L_{\mathbf{f}}^k h_i(\mathbf{x}) = 0 \quad \forall 1 \le j \le m,\ k \le r_i - 1 \tag{2}$$

for all $\mathbf{x}$ in a neighborhood of $\mathbf{x}_0$.

Denote

$$\begin{aligned}\mathbf{G}(\mathbf{x}) &= \begin{bmatrix} L_{\mathbf{g}_1} L_{\mathbf{f}}^{r_1-1} h_1(\mathbf{x}) & \cdots & L_{\mathbf{g}_m} L_{\mathbf{f}}^{r_1-1} h_1(\mathbf{x}) \\ L_{\mathbf{g}_1} L_{\mathbf{f}}^{r_2-1} h_2(\mathbf{x}) & \cdots & L_{\mathbf{g}_m} L_{\mathbf{f}}^{r_2-1} h_2(\mathbf{x}) \\ \cdots & \ddots & \cdots \\ L_{\mathbf{g}_1} L_{\mathbf{f}}^{r_m-1} h_m(\mathbf{x}) & \cdots & L_{\mathbf{g}_m} L_{\mathbf{f}}^{r_m-1} h_m(\mathbf{x}) \end{bmatrix} \\ &= \begin{bmatrix} g_{11}(\mathbf{x}) & \cdots & g_{1m}(\mathbf{x}) \\ \vdots & \ddots & \vdots \\ g_{m1}(\mathbf{x}) & \cdots & g_{mm}(\mathbf{x}) \end{bmatrix}.\end{aligned} \tag{3}$$

Note that $\mathbf{G}(\mathbf{x})$ is nonsingular at $\mathbf{x} = \mathbf{x}_0$.

Using feedback linearization, the nonlinear system (1) can be transformed into the following form [43]:

$$\mathbf{y}^{(\mathbf{r})} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} + \mathbf{d}, \tag{4}$$

where

$$\mathbf{y}^{(\mathbf{r})} = \left[y_1^{(r_1)}, y_2^{(r_2)}, \ldots, y_m^{(r_m)}\right]^T, \tag{5}$$

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} L_{\mathbf{f}}^{r_1} h_1(\mathbf{x}) \\ L_{\mathbf{f}}^{r_2} h_2(\mathbf{x}) \\ \vdots \\ L_{\mathbf{f}}^{r_m} h_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_m(\mathbf{x}) \end{bmatrix} \tag{6}$$

$$\mathbf{d} = \begin{bmatrix} \sum_{i=1}^{r_1} L_{\dot{\mathbf{x}}}^{i-1} L_{d'} L_{\mathbf{f}}^{r_1-i} h_1(\mathbf{x}) \\ \sum_{i=1}^{r_2} L_{\dot{\mathbf{x}}}^{i-1} L_{d'} L_{\mathbf{f}}^{r_2-i} h_2(\mathbf{x}) \\ \vdots \\ \sum_{i=1}^{r_m} L_{\dot{\mathbf{x}}}^{i-1} L_{d'} L_{\mathbf{f}}^{r_m-i} h_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix} \tag{7}$$

$$\mathbf{x} = \left[y_1, \ldots, y_1^{(r_1)}, y_2, \ldots, y_2^{(r_2)}, \ldots, y_m, \ldots, y_m^{(r_m)}\right]. \tag{8}$$

In system (4), $\mathbf{d}$ still represents unknown bounded external disturbance vector. The relative degree of the system is assumed to be equal to the order of the system and is expressed as $\sum_{i=1}^{m} r_i = n$, which implies that the system does not have any zero dynamics.

Let the desired output trajectory be given by

$$\mathbf{y}_d = \left[y_{d1}, y_{d2}, \ldots, y_{dm}\right]^T. \tag{9}$$

This paper aims to develop a robust adaptive fuzzy tracking control scheme such that all closed-loop system signals asymptotically converge to a compact set in the presence of input saturation, system uncertainties, and external disturbances and ensure the system outputs $\mathbf{y}$ track the desired trajectory $\mathbf{y}_d$. For system (4) to be controllable, the following assumptions are made.

Assumption 1. The desired trajectory y_{di} and its nth order derivatives are known and bounded.

Assumption 2. For $\mathbf{x}$ in certain controllability region $\mathbf{U}_c \in R^n$, $\sigma(\mathbf{G}(\mathbf{x})) \neq 0$, where $\sigma(\mathbf{G}(\mathbf{x}))$ denotes the minimum singular value of the matrix $\mathbf{G}(\mathbf{x})$.

2.2. Fuzzy Logic Systems. In this paper, fuzzy logic systems are used to approximate unknown nonlinear functions $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$. Following, the approximation presentation of fuzzy logic systems is given. Without loss of generality, the unknown uncertainty is assumed as $f(\mathbf{x})$. A fuzzy logic system consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The fuzzy inference engine uses fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ to an output variable $y \in R$. Then, the ith fuzzy rule can be represented as [3]

$$R^l: \text{If } x_1 \text{ is } F_1^l \text{ and } \cdots \text{ and } x_n \text{ is } F_n^l, \text{ then } y \text{ is } G^l \quad (l = 1, 2, \ldots, r), \tag{10}$$

where F_i^l and G^l are fuzzy sets characterized with the fuzzy membership functions $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively, and r is the number of fuzzy rules.

Through singleton function, center average defuzzifier, and product inference, the fuzzy logic system can be expressed as follows:

$$y(\mathbf{x}) = \frac{\sum_{l=1}^{r} \overline{y}^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{r} \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}, \tag{11}$$

where $\overline{y}^l = \max_{y \in R} \mu_{G^l}(y)$.

By introducing the concept of fuzzy basis function vector, the final output of the fuzzy logic system can be expressed as

$$y(\mathbf{x}) = \boldsymbol{\theta}^T \boldsymbol{\xi}(\mathbf{x}), \tag{12}$$

where $\boldsymbol{\theta} = [\overline{y}^1, \ldots, \overline{y}^r]^T$ is the adjustable parameter vector and $\boldsymbol{\xi}(\mathbf{x}) = [\xi_1(\mathbf{x}), \ldots, \xi_r(\mathbf{x})]^T$ is the fuzzy basis function vector. The fuzzy basis function is defined as follows:

$$\xi_l(\mathbf{x}) = \frac{\sum_{l=1}^{r} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{r} \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}. \tag{13}$$

Lemma 3. *Let $f(\mathbf{x})$ be a continuous function defined on a compact set $\mathbf{U}_c$. Then, for any constants $\varepsilon > 0$, there exists a fuzzy logic system such as [2]*

$$\sup_{\mathbf{x} \in \mathbf{U}_c} \left| f(\mathbf{x}) - \boldsymbol{\theta}^T \boldsymbol{\xi}(\mathbf{x}) \right| \leq \varepsilon. \tag{14}$$

Define the optimal parameter vector $\boldsymbol{\theta}^*$ as

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta} \in \boldsymbol{\Omega}_f} \left[\sup_{\mathbf{x} \in \mathbf{U}_c} \left| f(\mathbf{x}) - \boldsymbol{\theta}^T \boldsymbol{\xi}(\mathbf{x}) \right| \right]. \tag{15}$$

Under the optimization parameter vector, the unknown function $f(\mathbf{x})$ can be written as

$$f(\mathbf{x}) = \boldsymbol{\theta}^{*T} \boldsymbol{\xi}(\mathbf{x}) + \varepsilon, \tag{16}$$

where ε is the fuzzy minimum approximation error.

In this work, $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are unknown vector and matrix, respectively, and the fuzzy logic systems are used to approximate the unknown functions. Let $f_i(\mathbf{x})$ $(i = 1, \ldots, m)$ and $g_{ij}(\mathbf{x})$ $(i, j = 1, \ldots, m)$ be approximated by fuzzy logic systems as follows:

$$\begin{aligned} \widehat{f}_i\left(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}\right) &= \boldsymbol{\theta}_{f_i}^T \boldsymbol{\xi}_{f_i}(\mathbf{x}) \quad i = 1, \ldots, m \\ \widehat{g}_{ij}\left(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}\right) &= \boldsymbol{\theta}_{g_{ij}}^T \boldsymbol{\xi}_{g_{ij}}(\mathbf{x}) \quad i, j = 1, \ldots, m, \end{aligned} \tag{17}$$

where $\boldsymbol{\theta}_{f_i} \in \mathbf{R}^{M_{f_i}}$ and $\boldsymbol{\theta}_{g_{ij}} \in \mathbf{R}^{M_{g_{ij}}}$ are parameter vectors, $\boldsymbol{\xi}_{f_i}(\mathbf{x}) \in \mathbf{R}^{M_{f_i}}$ and $\boldsymbol{\xi}_{g_{ij}}(\mathbf{x}) \in \mathbf{R}^{M_{g_{ij}}}$ are fuzzy basis function vectors, and M_{f_i} and $M_{g_{ij}}$ are the corresponding dimensions of the basis vectors.

Denote

$$\widehat{\mathbf{F}}(\mathbf{x} \mid \boldsymbol{\theta}_F) = \begin{bmatrix} \hat{f}_1(\mathbf{x} \mid \boldsymbol{\theta}_{f_1}) \\ \vdots \\ \hat{f}_m(\mathbf{x} \mid \boldsymbol{\theta}_{f_m}) \end{bmatrix}$$
$$\widehat{\mathbf{G}}(\mathbf{x} \mid \boldsymbol{\theta}_G) = \begin{bmatrix} \hat{g}_{11}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{11}}) & \cdots & \hat{g}_{1m}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{1m}}) \\ \vdots & \ddots & \vdots \\ \hat{g}_{m1}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{m1}}) & \cdots & \hat{g}_{mm}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{mm}}) \end{bmatrix}. \tag{18}$$

Then $\widehat{\mathbf{F}}(\mathbf{x} \mid \boldsymbol{\theta}_F)$ and $\widehat{\mathbf{G}}(\mathbf{x} \mid \boldsymbol{\theta}_G)$ can be used as the approximation of $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$, respectively.

Define the optimal approximation weight vectors for f_i $(i = 1, \ldots, m)$ and g_{ij} $(i, j = 1, \ldots, m)$ as follows:

$$\boldsymbol{\theta}_{f_i}^{*} = \arg \min_{\boldsymbol{\theta}_{f_i} \in \boldsymbol{\Omega}_{\mathbf{F}}} \left\{ \sup_{\mathbf{x} \in \mathbf{U}_c} \left| \hat{f}_i(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}) - f_i(\mathbf{x}) \right| \right\}$$
$$\boldsymbol{\theta}_{g_{ij}}^{*} = \arg \min_{\boldsymbol{\theta}_{g_{ij}} \in \boldsymbol{\Omega}_{\mathbf{G}}} \left\{ \sup_{\mathbf{x} \in \mathbf{U}_c} \left| \hat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}) - g_{ij}(\mathbf{x}) \right| \right\}, \tag{19}$$

where $\boldsymbol{\Omega}_{\mathbf{F}}$, $\boldsymbol{\Omega}_{\mathbf{G}}$, and $\mathbf{U}_c$ denote the sets of suitable bounds on $\boldsymbol{\theta}_{f_i}$, $\boldsymbol{\theta}_{g_{ij}}$, and $\mathbf{x}$ respectively. $\boldsymbol{\theta}_{f_i}^{*}$ $(i = 1, \ldots, m)$ and $\boldsymbol{\theta}_{g_{ij}}^{*}$ $(i, j = 1, \ldots, m)$ are constants vectors.

The unknown function f_i and g_{ij} can be expressed as

$$f_i(\mathbf{x}) = \boldsymbol{\theta}_{f_i}^{*T} \boldsymbol{\xi}_{f_i}(\mathbf{x}) + \varepsilon_{f_i} \quad i = 1, \ldots, m$$
$$g_{ij}(\mathbf{x}) = \boldsymbol{\theta}_{g_{ij}}^{*T} \boldsymbol{\xi}_{g_{ij}}(\mathbf{x}) + \varepsilon_{g_{ij}} \quad i, j = 1, \ldots, m, \tag{20}$$

where ε_{f_i} and $\varepsilon_{g_{ij}}$ are the smallest approximation errors of the fuzzy logic systems.

3. Adaptive Fuzzy Robust Control Designs

In this section, we first design the adaptive fuzzy approximation based control problem without control saturation and then consider the case where actuators have physical limitations, such as magnitude and rate constraints.

3.1. Adaptive Fuzzy Robust Control. The tracking errors are defined as

$$e_i = y_{di} - y_i \quad i = 1, 2, \ldots, m. \tag{21}$$

Define u_{li} $(i = 1, \ldots, m)$ as follows:

$$u_{li} = y_{di}^{(r_i)} + \sum_{j=1}^{r_i} k_{ij} e_i^{(j-1)} \quad i = 1, \ldots, m, \tag{22}$$

where $k_{i1}, \ldots, k_{ir_i}$ are parameters to be chosen such that the roots of the equation $s^{r_i} + k_{ir_i} s^{r_i - 1} + \cdots + k_{i1} = 0$ in the open left-half complex plane.

If $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are known, external disturbances are ignored; then, according to nonlinear dynamic inversion control techniques, the control law is given by

$$\mathbf{u} = \mathbf{G}^{-1}(\mathbf{x}) \left[-\mathbf{F}(\mathbf{x}) + \mathbf{u}_l \right], \tag{23}$$

where $\mathbf{u}_l = [u_{l1}, \ldots, u_{lm}]^T$.

Because $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are unknown, $d_i \neq 0$, the control law (23) cannot be implemented in practice. Using the approximation of $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$, and considering external disturbances, the controller is modified as follows:

$$\mathbf{u} = \widehat{\mathbf{G}}^{-1}(\mathbf{x} \mid \boldsymbol{\theta}_G) \left[-\widehat{\mathbf{F}}(\mathbf{x} \mid \boldsymbol{\theta}_F) + \mathbf{u}_l + \mathbf{u}_d \right], \tag{24}$$

where $\mathbf{u}_d$ is the robust compensation term, which is used to attenuate the effect of external disturbances and approximation errors.

Substituting (24) into system (4) yields

$$\mathbf{y}^{(r)} = \mathbf{F}(\mathbf{x}) - \widehat{\mathbf{F}}(\mathbf{x} \mid \boldsymbol{\theta}_F) + \left[\mathbf{G}(\mathbf{x}) - \widehat{\mathbf{G}}(\mathbf{x} \mid \boldsymbol{\theta}_G) \right] \mathbf{u} + \mathbf{u}_l + \mathbf{u}_d + \mathbf{d}. \tag{25}$$

Considering (22), the ith subsystem of (25) can be rewritten as

$$\dot{\mathbf{e}}_i = \mathbf{A}_i \mathbf{e}_i + \mathbf{B}_i \left\{ \left[\hat{f}_i(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}) - f_i(\mathbf{x}) \right] + \sum_{j=1}^{m} \left[\hat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}) - g_{ij}(\mathbf{x}) \right] u_j \right\} - \mathbf{B}_i u_{di} - \mathbf{B}_i d_i, \tag{26}$$

where u_{di} is the ith element of $\mathbf{u}_d$ and

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_{i1} & -k_{i2} & -k_{i3} & \cdots & -k_{ir_i} \end{bmatrix} \qquad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$
$$\mathbf{e}_i = \begin{bmatrix} e_i \\ \dot{e}_i \\ \vdots \\ e_i^{(r_i-2)} \\ e_i^{(r_i-1)} \end{bmatrix}. \tag{27}$$

Define the minimum approximation error as

$$\omega_i' = \left[\hat{f}_i(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}^{*}) - f_i(\mathbf{x}) \right] + \sum_{j=1}^{m} \left[\hat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}^{*}) - g_{ij}(\mathbf{x}) \right] u_j. \tag{28}$$

According to fuzzy system theory, the following assumption is reasonable.

Assumption 4. The minimum approximation error is square integrable, that is, $\int_0^T \boldsymbol{\omega}_i'^T \boldsymbol{\omega}_i' \, dt < \infty$.

Using the definition (28) and the optimal approximation for $f_i(\mathbf{x})$ and $g_{ij}(\mathbf{x})$, (26) can be rewritten in the following form:

$$\begin{aligned}\dot{\mathbf{e}}_i = \mathbf{A}_i\mathbf{e}_i + \mathbf{B}_i\left[\widetilde{\boldsymbol{\theta}}_{f_i}^T\boldsymbol{\xi}_{f_i}(\mathbf{x}) + \sum_{j=1}^{m}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T\boldsymbol{\xi}_{g_{ij}}(\mathbf{x})\,u_j\right] \\ - \mathbf{B}_i u_{di} + \mathbf{B}_i\omega_i,\end{aligned} \tag{29}$$

where $\widetilde{\boldsymbol{\theta}}_{f_i} = \boldsymbol{\theta}_{f_i} - \boldsymbol{\theta}_{f_i}^*$, $\widetilde{\boldsymbol{\theta}}_{g_{ij}} = \boldsymbol{\theta}_{g_{ij}} - \boldsymbol{\theta}_{g_{ij}}^*$, and $\omega_i = \omega_i' - d_i$.

Theorem 5. *Consider the uncertain MIMO nonlinear system presented by (4) with external disturbance. If the controller is chosen as (24), the parameters update laws and u_{di} are adopted as follows:*

$$\dot{\boldsymbol{\theta}}_{f_i} = -\gamma_{f_i}\boldsymbol{\xi}_{f_i}(\mathbf{x})\,\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i \quad i = 1,\ldots,m \tag{30}$$

$$\dot{\boldsymbol{\theta}}_{g_{ij}} = -\gamma_{g_{ij}}\boldsymbol{\xi}_{g_{ij}}(\mathbf{x})\,\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i u_j \quad i, j = 1,\ldots,m \tag{31}$$

$$u_{di} = \frac{1}{2\rho_i^2}\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i \quad i = 1,\ldots,m. \tag{32}$$

Then the following H_∞ tracking performance can be obtained

$$\begin{aligned}\int_0^T \mathbf{e}^T\mathbf{Q}\mathbf{e}\,dt \le \mathbf{e}^T(0)\,\mathbf{P}\mathbf{e}(0) + \sum_{i=1}^{m}\frac{1}{\gamma_{f_i}}\widetilde{\boldsymbol{\theta}}_{f_i}^T(0)\,\widetilde{\boldsymbol{\theta}}_{f_i}(0) \\ + \sum_{i,j=1}^{m}\frac{1}{\gamma_{g_{ij}}}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T(0)\,\widetilde{\boldsymbol{\theta}}_{g_{ij}}(0) + \sum_{i=1}^{m}\rho_i^2\int_0^T\omega_i^2 dt,\end{aligned} \tag{33}$$

where γ_{f_i} $(i = 1,\ldots,m)$ and $\gamma_{g_{ij}}$ $(i, j = 1,\ldots,m)$ are positive adaptive scalar, ρ_i $(i = 1,\ldots,m)$ are positive parameters representing for prescribed disturbance attenuation levels, $\mathbf{e} = [\mathbf{e}_1^T,\ldots,\mathbf{e}_m^T]^T$, $\mathbf{Q} = \mathrm{diag}[\mathbf{Q}_1,\ldots,\mathbf{Q}_m]$, and $\mathbf{Q}_i \in \mathbf{R}^{m\times m}$ $(i = 1,\ldots,m)$ are arbitrary symmetric positive definite matrices, and $\mathbf{P} = \mathrm{diag}[\mathbf{P}_1,\ldots,\mathbf{P}_m]$ and $\mathbf{P}_i \in \mathbf{R}^{m\times m}$ $(i = 1,\ldots,m)$ are symmetric positive definite solution of the following equations:

$$\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_i = -\mathbf{Q}_i, \tag{34}$$

where $\mathbf{Q}_i \in \mathbf{R}^{m\times m}$ $(i = 1,\ldots,m)$ are arbitrary symmetric positive definite matrices.

Proof. For the ith subsystem of (4), consider the following Lyapunov function candidate

$$V_i = \frac{1}{2}\mathbf{e}_i^T\mathbf{P}_i\mathbf{e}_i + \frac{1}{2\gamma_{f_i}}\widetilde{\boldsymbol{\theta}}_{f_i}^T\widetilde{\boldsymbol{\theta}}_{f_i} + \frac{1}{2}\sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T\widetilde{\boldsymbol{\theta}}_{g_{ij}}. \tag{35}$$

Differentiating (35) and considering (29) yield

$$\begin{aligned}\dot{V}_i &= \frac{1}{2}\dot{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{e}_i + \frac{1}{2}\mathbf{e}_i^T\mathbf{P}_i\dot{\mathbf{e}}_i + \frac{1}{\gamma_{f_i}}\dot{\widetilde{\boldsymbol{\theta}}}_{f_i}^T\widetilde{\boldsymbol{\theta}}_{f_i} + \sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\dot{\widetilde{\boldsymbol{\theta}}}_{g_{ij}}^T\widetilde{\boldsymbol{\theta}}_{g_{ij}} \\ &= \frac{1}{2}\mathbf{e}_i^T\left(\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_i\right)\mathbf{e}_i + \frac{1}{2}\left(-u_{di} + \omega_i\right)\left(\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i + \mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\right) \\ &\quad + \frac{1}{\gamma_{f_i}}\left[\gamma_{f_i}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\boldsymbol{\xi}_{f_i}^T(\mathbf{x}) + \dot{\widetilde{\boldsymbol{\theta}}}_{f_i}^T\right]\widetilde{\boldsymbol{\theta}}_{f_i} \\ &\quad + \sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\left[\gamma_{g_{ij}}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\boldsymbol{\xi}_{g_{ij}}^T(\mathbf{x})\,u_j + \dot{\widetilde{\boldsymbol{\theta}}}_{g_{ij}}^T\right]\widetilde{\boldsymbol{\theta}}_{g_{ij}}.\end{aligned} \tag{36}$$

Substituting (30)–(32) into (36), we obtain

$$\begin{aligned}\dot{V}_i &= -\frac{1}{2}\mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i + \frac{1}{2}\omega_i\left(\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i + \mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\right) - \frac{1}{2\rho_i^2}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i\mathbf{B}_i^T\mathbf{P}_i\mathbf{e}_i \\ &= -\frac{1}{2}\mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i + \frac{1}{2}\rho_i^2\omega_i^2 - \frac{1}{2}\left(\frac{1}{\rho_i}\mathbf{e}_i^T\mathbf{P}_i\mathbf{B}_i - \rho_i\omega_i\right)^2 \\ &\le -\frac{1}{2}\mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i + \frac{1}{2}\rho_i^2\omega_i^2.\end{aligned} \tag{37}$$

Integrating both sides of the above inequality from 0 to T yields

$$\frac{1}{2}\int_0^T \mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i\,dt \le V_i(0) - V_i(T) + \frac{\rho_i^2}{2}\int_0^T\omega_i^2 dt. \tag{38}$$

Since $V_i(T)$ is nonnegative, according to the definition of V_i, the following inequality is obtained:

$$\begin{aligned}\int_0^T \mathbf{e}_i^T\mathbf{Q}_i\mathbf{e}_i\,dt \le \mathbf{e}_i^T(0)\,\mathbf{P}_i\mathbf{e}_i(0) + \frac{1}{\gamma_{f_i}}\widetilde{\boldsymbol{\theta}}_{f_i}^T(0)\,\widetilde{\boldsymbol{\theta}}_{f_i}(0) \\ + \sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T(0)\,\widetilde{\boldsymbol{\theta}}_{g_{ij}}(0) + \rho_i^2\int_0^T\omega_i^2 dt.\end{aligned} \tag{39}$$

Considering the Lyapunov function $V = \sum_{i=1}^{m}V_i$, it is easy to obtain the H_∞ tracking performance index (55). This completes the proof. □

The controller proposed by (24) is able to guarantee the Lyapunov stability of the closed-loop system and attenuate the effect of system uncertainties and external disturbances. However, no saturation on actuators is taken into account, which is rather important for practical applications (including satellite systems). Assume that the control input $\mathbf{u}$ is constraint by saturation functions; it can be readily shown that the control law (24), the parameters update laws (30), (31), and the robust compensation term (32) with saturation limits cannot guarantee the stability of the closed-loop system.

It is expected that during saturation the magnitude of the tracking error will increase, since the control signal is not being achieved. This tracking error is not the result of function approximation error; therefore, we need to be careful so that the approximator does not cause "unlearning" during the period when the actuators are saturated. Clearly, the parameters update laws (30) and (31) depend on the tracking error $\mathbf{e}_i$, thus if the tracking error increases due to saturation, the parameters update laws may cause a significant change in the weights in response to the increase in tracking error. Next, we develop a robust fuzzy adaptive control scheme to address the input saturation problem.

3.2. Adaptive Fuzzy Control with Input Saturation. When the actuators have physical constraints such as the magnitude and rate limitations, the above approach may not be able to be successfully implemented. Considering the magnitude and rate limitations on the actuator, controller (24) is modified as

$$\begin{aligned} \mathbf{u}_c &= \widehat{\mathbf{G}}^{-1}(\mathbf{x} \mid \boldsymbol{\theta}_G)\left[-\widehat{\mathbf{F}}(\mathbf{x} \mid \boldsymbol{\theta}_F) + \mathbf{u}_l + \mathbf{u}_{aw} + \mathbf{u}_d\right] \\ \mathbf{u} &= S_{MR}(\mathbf{u}_c), \end{aligned} \tag{40}$$

where $\mathbf{u}_c$ is obtained by certainty equivalence principle; $\mathbf{u}_{aw}$ is the auxiliary control term which is used to compensate the effect of input saturation; $S_{MR}(\cdot)$ is a function including the magnitude and rate constraints which can produce a limited output.

The state space representation of each component of $\mathbf{u}_c$ is

$$\begin{aligned} \begin{bmatrix} \dot{r}_{i1} \\ \dot{r}_{i2} \end{bmatrix} &= \begin{bmatrix} r_{i2} \\ S_R(\omega_i(S_M(u_{ci}) - r_{i1})) \end{bmatrix} \\ \begin{bmatrix} u_i \\ \dot{u}_i \end{bmatrix} &= \begin{bmatrix} r_{i1} \\ r_{i2} \end{bmatrix}, \end{aligned} \tag{41}$$

where u_{ci} is the ith element of $\mathbf{u}_c$, and u_i is the ith element of $\mathbf{u}$; ω_i is the natural frequency; and $S_M(\cdot)$ and $S_R(\cdot)$ are the saturation functions corresponding to magnitude and rate, respectively. The function $S_M(\cdot)$ is defined as

$$S_M(\cdot) = \begin{cases} M & \text{if } x \geq M \\ x & \text{if } |x| < M \\ -M & \text{if } x \leq -M \end{cases} \tag{42}$$

and $S_R(\cdot)$ has the same definition.

To compensate the effects of input limitations, an auxiliary system is introduced as follows [35]:

$$\begin{aligned} \dot{\xi}_{i1} &= \xi_{i2} - \lambda_{i1}\xi_{i1} \cdots \dot{\xi}_{i,r_i-1} = \xi_{ir_i} - \lambda_{i,r_i-1}\xi_{i,r_i-1} \\ \dot{\xi}_{ir_i} &= -\lambda_{ir_i}\xi_{ir_i} + \sum_{j=1}^{m}\widehat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}})(u_j - u_{cj}) \quad i = 1, 2, \ldots, m, \end{aligned} \tag{43}$$

where λ_{ij} $(i = 1, \ldots, m;\ j = 1, \ldots, r_i)$ are positive design parameters.

Define the modified tracking error as

$$\bar{e}_i = y_{di} - y_i - \xi_{i1} \quad i = 1, 2, \ldots, m. \tag{44}$$

From (4), (40), and (43), we obtain

$$\begin{aligned} &\bar{e}_i^{(r_i)} + \sum_{j=1}^{r_i} k_{ij}\bar{e}_i^{(r_i-1)} \\ &= \widehat{f}_i(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}) - f_i(\mathbf{x}) + \sum_{j=1}^{m}\left[\widehat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}) - g_{ij}(\mathbf{x})\right]u_j \\ &\quad - u_{awi} - u_{di} - d_i - \xi_{i1}^{(r_i)} - \sum_{j=1}^{r_i} k_{ij}\xi_{i1}^{(j-1)} \\ &\quad + \sum_{j=1}^{m}\widehat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}})(u_j - u_{cj}), \end{aligned} \tag{45}$$

where u_{awi} is the ith element of $\mathbf{u}_{aw}$.

From (43), the term $\xi_{i1}^{(r_i)}$ can be expressed as

$$\xi_{i1}^{(r_i)} = -\sum_{j=1}^{r_i}\lambda_{ij}\xi_{ij}^{(r_i-j)} + \sum_{j=1}^{m}\widehat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}})(u_j - u_{cj}). \tag{46}$$

Let

$$u_{awi} = \sum_{j=1}^{r_i}\left(\lambda_{ij}\xi_{ij}^{(r_i-j)} - k_{ij}\xi_{ij}^{(j-1)}\right). \tag{47}$$

With (46) and (47), (45) becomes

$$\begin{aligned} &\bar{e}_i^{(r_i)} + \sum_{j=1}^{r_i} k_{ij}\bar{e}_i^{(r_i-1)} \\ &= \widehat{f}_i(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}) - f_i(\mathbf{x}) + \sum_{j=1}^{m}\left[\widehat{g}_{ij}(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}) - g_{ij}(\mathbf{x})\right]u_j \\ &\quad - u_{di} - d_i. \end{aligned} \tag{48}$$

Define $\bar{\mathbf{e}}_i = [\bar{e}_i, \dot{\bar{e}}_i, \ldots, \bar{e}_i^{(r_i-1)}]^T$ and using the optimal approximation for $f_i(\mathbf{x})$ and $g_{ij}(\mathbf{x})$, (48) can be rewritten as

$$\dot{\bar{\mathbf{e}}}_i = \mathbf{A}_i\bar{\mathbf{e}}_i + \mathbf{B}_i\left[\widetilde{\boldsymbol{\theta}}_{f_i}^T\boldsymbol{\xi}_{f_i}(\mathbf{x}) + \sum_{j=1}^{m}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T\boldsymbol{\xi}_{g_{ij}}(\mathbf{x})u_j\right] - \mathbf{B}_i u_{di} + \mathbf{B}_i\omega_i. \tag{49}$$

Theorem 6. *Consider the uncertain MIMO nonlinear system presented by (4) with external disturbance and input saturation, if the controller is chosen as (40), the auxiliary control* *term* $\mathbf{u}_{aw}$*, the parameters update laws, and* u_{di} *are adopted as follows:*

$$\dot{\xi}_{i1} = \xi_{i2} - \lambda_{i1}\xi_{i1} \cdots \dot{\xi}_{i,r_i-1} = \xi_{ir_i} - \lambda_{i,r_i-1}\xi_{i,r_i-1}$$
$$\dot{\xi}_{ir_i} = -\lambda_{ir_i}\xi_{ir_i} + \sum_{j=1}^{m}\widehat{g}_{ij}\left(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}\right)\left(u_j - u_{cj}\right) \quad i = 1, 2, \ldots, m \tag{50}$$

$$\mathbf{u}_{aw} = \left[\sum_{j=1}^{r_1}\left(\lambda_{1j}\xi_{1j}^{(r_1-j)} - k_{1j}\xi_{1j}^{(j-1)}\right) \cdots \sum_{j=1}^{r_m}\left(\lambda_{mj}\xi_{mj}^{(r_m-j)} - k_{mj}\xi_{mj}^{(j-1)}\right)\right]^T \tag{51}$$

$$\dot{\boldsymbol{\theta}}_{f_i} = -\gamma_{f_i}\boldsymbol{\xi}_{f_i}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i \quad i = 1, \ldots, m \tag{52}$$

$$\dot{\boldsymbol{\theta}}_{g_{ij}} = -\gamma_{g_{ij}}\boldsymbol{\xi}_{g_{ij}}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i u_j \quad i, j = 1, \ldots, m \tag{53}$$

$$u_{di} = \frac{1}{2\rho_i^2}\mathbf{B}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i \quad i = 1, \ldots, m. \tag{54}$$

Then the following H_∞ tracking performance can be obtained:

$$\int_0^T \overline{\mathbf{e}}^T\mathbf{Q}\overline{\mathbf{e}}\,dt \le \overline{\mathbf{e}}^T(0)\mathbf{P}\overline{\mathbf{e}}(0) + \sum_{i=1}^{m}\frac{1}{\gamma_{f_i}}\widetilde{\boldsymbol{\theta}}_{f_i}^T(0)\widetilde{\boldsymbol{\theta}}_{f_i}(0) + \sum_{i,j=1}^{m}\frac{1}{\gamma_{g_{ij}}}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T(0)\widetilde{\boldsymbol{\theta}}_{g_{ij}}(0) + \sum_{i=1}^{m}\rho_i^2\int_0^T\omega_i^2\,dt, \tag{55}$$

where γ_{f_i} $(i = 1, \ldots, m)$ and $\gamma_{g_{ij}}$ $(i, j = 1, \ldots, m)$ are positive adaptive scalar, ρ_i $(i = 1, \ldots, m)$ are positive parameters representing for prescribed disturbance attenuation levels, $\overline{\mathbf{e}} = [\overline{\mathbf{e}}_1^T, \ldots, \overline{\mathbf{e}}_m^T]^T$, $\mathbf{Q} = \operatorname{diag}[\mathbf{Q}_1, \ldots, \mathbf{Q}_m]$, and $\mathbf{Q}_i \in \mathbf{R}^{m\times m}$ $(i = 1, \ldots, m)$ are arbitrary symmetric positive definite matrices, and $\mathbf{P} = \operatorname{diag}[\mathbf{P}_1, \ldots, \mathbf{P}_m]$ and $\mathbf{P}_i \in \mathbf{R}^{m\times m}$ $(i = 1, \ldots, m)$ are symmetric positive definite solution of the following equations:

$$\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_i = -\mathbf{Q}_i. \tag{56}$$

Proof. For the ith subsystem of (4), consider the following Lyapunov function candidate:

$$V_i = \frac{1}{2}\overline{\mathbf{e}}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i + \frac{1}{2\gamma_{f_i}}\widetilde{\boldsymbol{\theta}}_{f_i}^T\widetilde{\boldsymbol{\theta}}_{f_i} + \frac{1}{2}\sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T\widetilde{\boldsymbol{\theta}}_{g_{ij}}. \tag{57}$$

Differentiating (57) and considering (49) yield

$$\begin{aligned}\dot{V}_i &= \frac{1}{2}\dot{\overline{\mathbf{e}}}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i + \frac{1}{2}\overline{\mathbf{e}}_i^T\mathbf{P}_i\dot{\overline{\mathbf{e}}}_i + \frac{1}{\gamma_{f_i}}\dot{\widetilde{\boldsymbol{\theta}}}_{f_i}^T\widetilde{\boldsymbol{\theta}}_{f_i} + \sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\dot{\widetilde{\boldsymbol{\theta}}}_{g_{ij}}^T\widetilde{\boldsymbol{\theta}}_{g_{ij}} \\ &= \frac{1}{2}\overline{\mathbf{e}}_i^T\left(\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_i\right)\overline{\mathbf{e}}_i + \frac{1}{2}\left(-u_{di} + \omega_i\right)\left(\mathbf{B}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i + \overline{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i\right) \\ &\quad + \frac{1}{\gamma_{f_i}}\left[\gamma_{f_i}\overline{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i\boldsymbol{\xi}_{f_i}^T(\mathbf{x}) + \dot{\widetilde{\boldsymbol{\theta}}}_{f_i}^T\right]\widetilde{\boldsymbol{\theta}}_{f_i} \\ &\quad + \sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\left[\gamma_{g_{ij}}\overline{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i\boldsymbol{\xi}_{g_{ij}}^T(\mathbf{x})u_j + \dot{\widetilde{\boldsymbol{\theta}}}_{g_{ij}}^T\right]\widetilde{\boldsymbol{\theta}}_{g_{ij}}.\end{aligned} \tag{58}$$

Substituting (52)–(54) into (58), we obtain

$$\begin{aligned}\dot{V}_i &= -\frac{1}{2}\overline{\mathbf{e}}_i^T\mathbf{Q}_i\overline{\mathbf{e}}_i + \frac{1}{2}\omega_i\left(\mathbf{B}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i + \overline{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i\right) - \frac{1}{2\rho_i^2}\overline{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i\mathbf{B}_i^T\mathbf{P}_i\overline{\mathbf{e}}_i \\ &= -\frac{1}{2}\overline{\mathbf{e}}_i^T\mathbf{Q}_i\overline{\mathbf{e}}_i + \frac{1}{2}\rho_i^2\omega_i^2 - \frac{1}{2}\left(\frac{1}{\rho_i}\overline{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i - \rho_i\omega_i\right)^2 \\ &\le -\frac{1}{2}\overline{\mathbf{e}}_i^T\mathbf{Q}_i\overline{\mathbf{e}}_i + \frac{1}{2}\rho_i^2\omega_i^2.\end{aligned} \tag{59}$$

Integrating both sides of the above inequality from 0 to T yields

$$\frac{1}{2}\int_0^T\overline{\mathbf{e}}_i^T\mathbf{Q}_i\overline{\mathbf{e}}_i\,dt \le V_i(0) - V_i(T) + \frac{\rho_i^2}{2}\int_0^T\omega_i^2\,dt. \tag{60}$$

Since $V_i(T)$ is nonnegative, according to the definition of V_i, the following inequality is obtained:

$$\int_0^T\overline{\mathbf{e}}_i^T\mathbf{Q}_i\overline{\mathbf{e}}_i\,dt \le \overline{\mathbf{e}}_i^T(0)\mathbf{P}_i\overline{\mathbf{e}}_i(0) + \frac{1}{\gamma_{f_i}}\widetilde{\boldsymbol{\theta}}_{f_i}^T(0)\widetilde{\boldsymbol{\theta}}_{f_i}(0) + \sum_{i=1}^{m}\frac{1}{\gamma_{g_{ij}}}\widetilde{\boldsymbol{\theta}}_{g_{ij}}^T(0)\widetilde{\boldsymbol{\theta}}_{g_{ij}}(0) + \rho_i^2\int_0^T\omega_i^2\,dt. \tag{61}$$

Considering the Lyapunov function $V = \sum_{i=1}^{m}V_i$, it is easy to obtain the H_∞ tracking performance index (55). This completes the proof. □

From the above analysis, it is concluded that, in the case of no control saturations, the signals ξ_{ij} $(i = 1, \ldots, m;\ j = 1, \ldots, r_i)$ remain zeros and the control law becomes the same as the standard robust adaptive control law described in the previous section. In the presence of control saturations, ξ_{ij} is nonzero, thus giving rise to a modified tracking error $\overline{e}_i = y_{di} - y_i - \xi_{i1}$, which is used in fuzzy parameter update laws. The auxiliary control term $\mathbf{u}_{aw}$ in (40) will be used to compensate the effects of control saturations. Note that the fuzzy parameter update laws (52) and (53) are similar to the corresponding update laws (30) and (31) derived in the standard fuzzy approximation based control problem with tracking error e_i being replaced by the modified tracking error $\overline{e}_i$. The use of the modified tracking error in the fuzzy update laws is crucial in preventing actuator constraints in online approximation schemes.

Corollary 7. *For the ith subsystem of (4), it is assumed that $\int_0^T d_i^2 dt < \infty$. If the control law (40), the auxiliary control term (51), and the parameter update laws (52)–(54) are adopted, then the following statements hold.*

(i) *The closed loop system is stable, and the signals $\bar{\mathbf{e}}_i$, $\boldsymbol{\theta}_{f_i}$, $\boldsymbol{\theta}_{g_{ij}}$, and u_i are bounded.*

(ii) *The steady modified tracking error satisfies $\lim_{t\to\infty}\bar{e}_i = 0$, and the bound of the transient modified tracking error will be given as follows:*

$$\|\bar{e}_i\|^2 \leq \frac{2\left[\left(1/\gamma_{f_i}\right)\tilde{\boldsymbol{\theta}}_{f_i}^T(\mathbf{0})\tilde{\boldsymbol{\theta}}_{f_i}(\mathbf{0}) + \sum_{i=1}^{m}\left(1/\gamma_{g_{ij}}\right)\tilde{\boldsymbol{\theta}}_{g_{ij}}^T(\mathbf{0})\tilde{\boldsymbol{\theta}}_{g_{ij}}(\mathbf{0})\right]}{\lambda_{\min}(\mathbf{Q}_i)} + \frac{2\bar{\mathbf{e}}_i^T(\mathbf{0})\mathbf{P}_i\bar{\mathbf{e}}_i(\mathbf{0}) + \rho_i^2\int_0^T \omega_i^2 dt}{\lambda_{\min}(\mathbf{Q}_i)}. \tag{62}$$

Proof. From (59), it can be obtained that

$$\dot{V}_i \leq -c_i V_i + \mu_i, \tag{63}$$

where $c_i = \min\{\lambda_v, 1/\gamma_{f_i}, 1/\gamma_{g_{ij}}\}$, $\lambda_v = \min\{\inf\lambda_{\min}(\mathbf{Q}_i)/\sup\lambda_{\max}(\mathbf{Q}_i)\}$, $M_i = \max(\boldsymbol{\theta}_{f_i})$, $M_{ij} = \max(\boldsymbol{\theta}_{g_{ij}})$, $\mu_i = M_i^2/2\gamma_{f_i} + \sum_{j=1}^m(1/2\gamma_{g_{ij}})M_{ij}^2 + (1/2)\rho_i^2\overline{\omega}_i^2$, $\overline{\omega}_i = \sup\|\omega_i\|$, and $\lambda_{\min}(\mathbf{Q}_i)$ and $\lambda_{\max}(\mathbf{Q}_i)$ are the minimum and maximum eigenvalue of $\mathbf{Q}_i$, respectively.

From inequality (63), all the signals $\bar{\mathbf{e}}_i$, $\boldsymbol{\theta}_{f_i}$, $\boldsymbol{\theta}_{g_{ij}}$, and u_i are bounded by using Barbalat's lemma [8].

On the other hand, from (59), it can be obtained that

$$\dot{V}_i \leq -\frac{1}{2}\lambda_{\min}(\mathbf{Q}_i)\|\bar{\mathbf{e}}_i\|^2 + \frac{1}{2}\rho_i^2\omega_i^2. \tag{64}$$

From the above inequality, one obtains

$$\|\bar{e}_i\|^2 \leq \|\bar{\mathbf{e}}_i\|^2 \leq \frac{-2\dot{V}_i + \rho_i^2\omega_i^2}{\lambda_{\min}(Q)}. \tag{65}$$

Hence,

$$\|\bar{e}_i\|^2 = \int_0^T \bar{e}_i^2 dt \leq \frac{2\left[V_i(0) - V_i(T)\right] + \rho_i^2\int_0^T\omega_i^2 dt}{\lambda_{\min}(\mathbf{Q}_i)} \leq \frac{2V_i(0) + \rho_i^2\int_0^T\omega_i^2 dt}{\lambda_{\min}(\mathbf{Q}_i)}. \tag{66}$$

According to the definition of V_i, (62) is obtained. Assumption 4 implies $\int_0^T \omega_i'^2 dt < \infty$; then $\int_0^T \omega_i^2 dt = \int_0^T(\omega_i' - d_i)^2 dt < \infty$. Therefore, $\omega_i \in L_2$. From (66) we can conclude that $\bar{e}_i \in L_2$. Using Barbalat's lemma, it follows that $\lim_{t\to\infty}\bar{e}_i(t) = 0$. This completes the proof. □

Remark 8. According to Theorem 6, the ith subsystem of (4) achieves a H^∞ tracking performance with a prescribed disturbance attenuation level ρ_i; that is, the L_2 gain from ω_i to the modified tracking error $\bar{e}_i$ is equal or less than ρ_i.

Remark 9. As indicated from Corollary 7, the steady modified tracking error $\bar{e}_i$ will converge to zero. The bound of the transient modified tracking error $\bar{e}_i$ is an explicit function of the design parameters and the external disturbance and fuzzy approximation error ω_i. The bound can be decreased by choosing the initial estimates $\boldsymbol{\theta}_{f_i}(\mathbf{0})$, $\boldsymbol{\theta}_{g_{ij}}(\mathbf{0})$ closing to the true values $\boldsymbol{\theta}_{f_i}^*$, $\boldsymbol{\theta}_{g_{ij}}^*$. The effects of parameter initial estimate errors on the transient tracking performance can be reduced by increasing the adaptive gain values γ_{f_i}, $\gamma_{g_{ij}}$, and $\lambda_{\min}(\mathbf{Q}_i)$. Furthermore, the effect of external disturbance and fuzzy approximation error ω_i on the transient tracking performance can be reduced by decreasing ρ_i and increasing $\lambda_{\min}(\mathbf{Q}_i)$. Small ρ_i implies high disturbance attenuation level.

4. Simulation Examples

The attitude tracking control problem of a rigid body satellite system is simulated in this section to illustrate the effectiveness of the robust adaptive fuzzy controllers proposed in this paper. The mathematical model of the satellite attitude system can be reformulated to the general form of uncertain nonlinear MIMO system as follows [42, 44]:

$$\ddot{\mathbf{y}} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} + \mathbf{d}, \tag{67}$$

where $\mathbf{x} = [\mathbf{q}, \boldsymbol{\omega}]^T$ is the state vector, where $\mathbf{q} = [q_0, q_1, q_2, q_3]^T$ is the attitude quaternion in the body-fixed reference frame relative to the inertial frame satisfying $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$, here q_0 is chosen as $q_0 = \sqrt{1 - q_1^2 - q_2^2 - q_3^2}$; $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ is the angular velocity of the body-fixed reference relative to the inertial frame. $\mathbf{y} = [q_1, q_2, q_3]^T$ is the output vector, $\mathbf{u} = [u_1, u_2, u_3]^T$ is the control torque vector, $\mathbf{d}' \in \mathbf{R}^3$ denotes the bounded external disturbance torques, and $\mathbf{d} \triangleq \mathbf{G}(\mathbf{x})\mathbf{d}'$. $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are expressed as follows:

$$f_1(\mathbf{x}) = -\frac{1}{4}q_1\left(\omega_x^2 + \omega_y^2 + \omega_z^2\right) + \frac{I_x - I_y}{2I_z}q_2\omega_x\omega_y + \frac{I_x - I_z}{2I_y}q_3\omega_x\omega_z + \frac{I_y - I_z}{2I_x}q_0\omega_y\omega_z$$

$$f_2(\mathbf{x}) = -\frac{1}{4}q_2\left(\omega_x^2 + \omega_y^2 + \omega_z^2\right) + \frac{I_y - I_x}{2I_z}q_1\omega_x\omega_y + \frac{I_z - I_x}{2I_y}q_0\omega_x\omega_z + \frac{I_y - I_z}{2I_x}q_3\omega_y\omega_z$$

$$f_3(\mathbf{x}) = -\frac{1}{4}q_3\left(\omega_x^2 + \omega_y^2 + \omega_z^2\right) + \frac{I_x - I_y}{2I_z}q_0\omega_x\omega_y + \frac{I_z - I_x}{2I_y}q_1\omega_x\omega_z + \frac{I_z - I_y}{2I_x}q_2\omega_y\omega_z$$

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} \frac{q_0}{2I_x} & -\frac{q_3}{2I_y} & \frac{q_2}{2I_z} \\ \frac{q_3}{2I_x} & \frac{q_0}{2I_y} & -\frac{q_1}{2I_z} \\ -\frac{q_2}{2I_x} & \frac{q_1}{2I_y} & \frac{q_0}{2I_z} \end{bmatrix}, \tag{68}$$

where I_x, I_y, and I_z are the principal central moments of inertia of the satellite.

Within this simulation, the nonlinear functions $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are assumed completely unknown, that is, the fuzzy adaptive controllers do not require the knowledge of the system's model. In fact, the dynamic model of the satellite attitude system is only required for simulation purpose.

Since the components of $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are assumed unknown, three fuzzy logic systems in the form of (12) are used to approximate the elements of $\mathbf{F}(\mathbf{x})$, and nine are used to approximate the elements of $\mathbf{G}(\mathbf{x})$. The fuzzy logic systems used to describe $\mathbf{F}(\mathbf{x})$ have $q_1, q_2, q_3, \omega_1, \omega_2$, and ω_3 as inputs, and the ones used to describe $\mathbf{G}(\mathbf{x})$ have q_1, q_2, and q_3 as inputs. For each state variable $\mathbf{x} = [q_1, q_2, q_3, \omega_1, \omega_2, \omega_3]^T$, we define seven Gaussian membership functions as

$$\begin{aligned}
\mu_{F_i^1}(x_i) &= \frac{1}{1 + \exp\left(-5\left(x_i + 0.6\right)\right)}, \\
\mu_{F_i^2}(x_i) &= \exp\left(-0.5\left(x_i + 0.4\right)^2\right) \\
\mu_{F_i^3}(x_i) &= \exp\left(-0.5\left(x_i + 0.2\right)^2\right), \\
\mu_{F_i^4}(x_i) &= \exp\left(-0.5{x_i}^2\right) \\
\mu_{F_i^5}(x_i) &= \exp\left(-0.5\left(x_i - 0.2\right)^2\right), \\
\mu_{F_i^6}(x_i) &= \exp\left(-0.5\left(x_i - 0.4\right)^2\right) \\
\mu_{F_i^7}(x_i) &= \frac{1}{1 + \exp\left(-5\left(x_i - 0.6\right)\right)}, \\
& i = 1, 2, 3, 4, 5, 6.
\end{aligned} \tag{69}$$

The fuzzy basis function vectors are chosen as follows:

$$\boldsymbol{\xi}_{f_i} = \left[\frac{\prod_{i=1}^{6}\mu_{F_i^1}(x_i)}{\sum_{j=1}^{7}\prod_{i=1}^{6}\mu_{F_i^j}(x_i)}, \ldots, \frac{\prod_{i=1}^{6}\mu_{F_i^7}(x_i)}{\sum_{j=1}^{7}\prod_{i=1}^{6}\mu_{F_i^j}(x_i)}\right]^T \quad i = 1, 2, 3$$

$$\boldsymbol{\xi}_{g_{ij}} = \left[\frac{\prod_{i=1}^{3}\mu_{F_i^1}(x_i)}{\sum_{j=1}^{7}\prod_{i=1}^{3}\mu_{F_i^j}(x_i)}, \ldots, \frac{\prod_{i=1}^{3}\mu_{F_i^7}(x_i)}{\sum_{j=1}^{7}\prod_{i=1}^{3}\mu_{F_i^j}(x_i)}\right]^T \quad i, j = 1, 2, 3. \tag{70}$$

Then we obtain fuzzy logic systems

$$\begin{aligned}
\hat{f}_i\left(\mathbf{x} \mid \boldsymbol{\theta}_{f_i}\right) &= \boldsymbol{\theta}_{f_i}^T\boldsymbol{\xi}_{f_i}(\mathbf{x}) \quad i = 1, 2, 3 \\
\hat{g}_{ij}\left(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}\right) &= \boldsymbol{\theta}_{g_{ij}}^T\boldsymbol{\xi}_{g_{ij}}(\mathbf{x}) \quad i, j = 1, 2, 3.
\end{aligned} \tag{71}$$

Using (71) approximate the unknown functions f_i and g_{ij}, respectively.

For the given coefficients $k_{i1} = 0.25$ $(i = 1, 2, 3)$ and $k_{i2} = 0.5$ $(i = 1, 2, 3)$, we have

$$\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ -0.25 & -0.5 \end{bmatrix} \qquad \mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{72}$$

Selecting $\mathbf{Q}_i = \mathrm{diag}[0.5, 0.5]$ and solving (34), we obtain

$$\mathbf{P}_i = \begin{bmatrix} 1.625 & 0.5 \\ 0.5 & 3 \end{bmatrix} \quad i = 1, 2, 3. \tag{73}$$

The parameters of the adaptive fuzzy controllers are chosen as $\gamma_{f_i} = 1 \times 10^{-3}$, $\gamma_{g_{ij}} = 1 \times 10^{-3}$, and $\rho_i = 0.5$, $i, j = 1, 2, 3$.

The initial attitude quaternion is $\mathbf{q}(0) = [0.9014, 0.25, 0.25, 0.25]^T$, and the initial value of the angular velocity is $\boldsymbol{\omega}(0) = [0.005 \ 0.005 \ 0.005]^T$ rad/s. The parameters of system are given as $I_x = 187.5\,\mathrm{kg{\cdot}m^2}$, $I_y = 468.5\,\mathrm{kg{\cdot}m^2}$, and $I_z = 468.5\,\mathrm{kg{\cdot}m^2}$. The external disturbance torque vector is given by $\mathbf{d}' = [0.3\sin(0.05t), 0.3\cos(0.05t), -0.3\sin(0.05t)]^T$ N·m. The initial values of the parameters $\boldsymbol{\theta}_{f_i}$ and $\boldsymbol{\theta}_{g_{ij}}$ are set to random values uniformly distributed between $[0, 1]$. The desired output trajectory is chosen as $\mathbf{y}_d = [0.5\sin(0.005t), 0.5\sin(0.005t), 0.5\sin(0.005t)]^T$. The control objective is to force the system output $\mathbf{y}$ to track the desired output trajectory $\mathbf{y}_d$.

4.1. Without Input Saturation. In this section, the tracking control problem is simulated to demonstrate the effectiveness of robust adaptive fuzzy controller (24) proposed in Section 3.1.

Using the control law (24) and (30)–(32), simulation results are presented in Figures 1 and 2. Figure 1 shows the curves of the system outputs and its reference trajectories, which indicates that the robust adaptive fuzzy controller achieves a good performance in tracking control problem and the effects of fuzzy approximation errors and external disturbances on tracking errors are effectively attenuated. Figure 2 shows that the control inputs can be carried out feasibly without any constraints. The control signals are obtained by certainty equivalence principle. Therefore, they do not satisfy the control input limitations naturally.

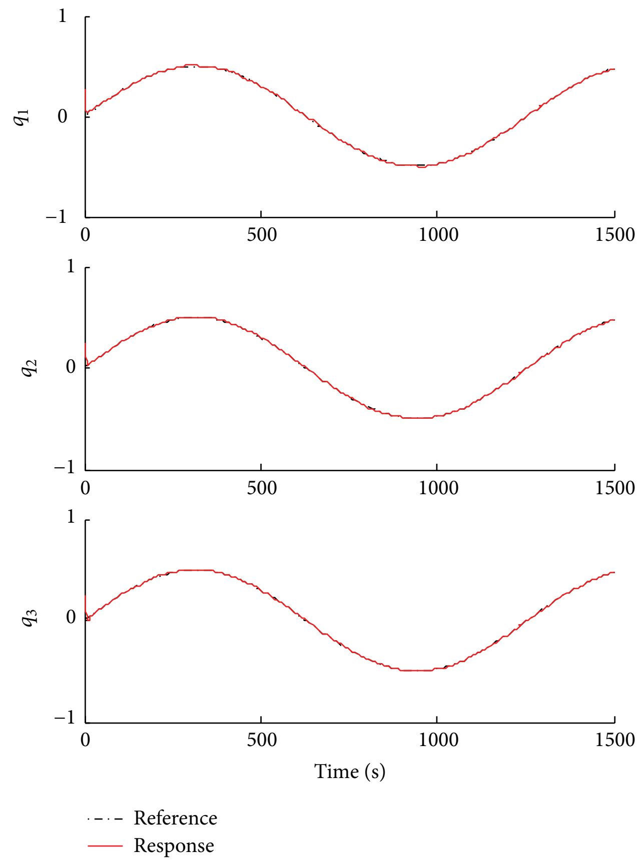

Figure 1: Trajectories of outputs without saturation.

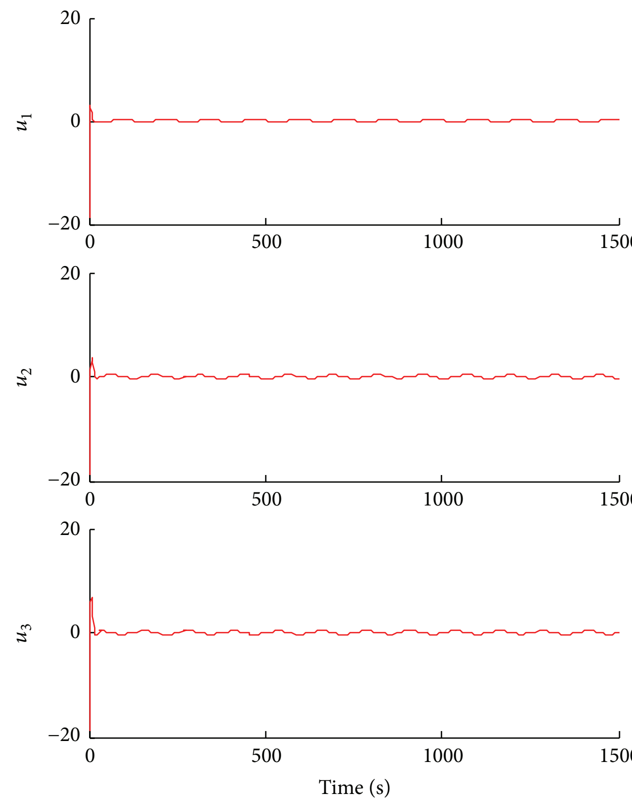

Figure 2: Trajectories of control inputs without saturation.

4.2. Actuator Amplitude Saturation. In order to demonstrate that the proposed adaptive control scheme (40) can work effectively under actuator amplitude saturation, numerical simulations have been performed and presented in this section.

Consider satellite attitude model (67) with the same disturbances and system initial conditions mentioned above. The control input vector $\mathbf{u} = [u_1, u_2, u_3]^T$ has amplitude limits $|u_i| \leq 1$ N·m, $i = 1, 2, 3$. The auxiliary system is constructed as follows:

$$\begin{aligned}\dot{\xi}_{i1} &= \xi_{i2} - \lambda_{i1}\xi_{i1}\\ \dot{\xi}_{i2} &= -\lambda_{i2}\xi_{i2} + \sum_{j=1}^{3}\widehat{g}_{ij}\left(\mathbf{x} \mid \boldsymbol{\theta}_{g_{ij}}\right)\left(u_j - u_{cj}\right) \quad i = 1, 2, 3,\end{aligned} \tag{74}$$

where $\lambda_{ij} = 1$ ($i = 1, 2, 3$; $j = 1, 2$) and $\xi_{ij}(0) = 0$ ($i = 1, 2, 3$; $j = 1, 2$).

Using the control law (40), the auxiliary control term (51), the parameters update laws (52)-(53), and the robust compensation term (54), we can get the simulations in Figures 3 and 4.

The system outputs and its reference trajectories are shown in Figure 3, which indicate that the outputs track their reference trajectories well in spite of the external disturbances, system uncertainties, and actuator amplitude saturation. Figure 4 shows the trajectories of the control

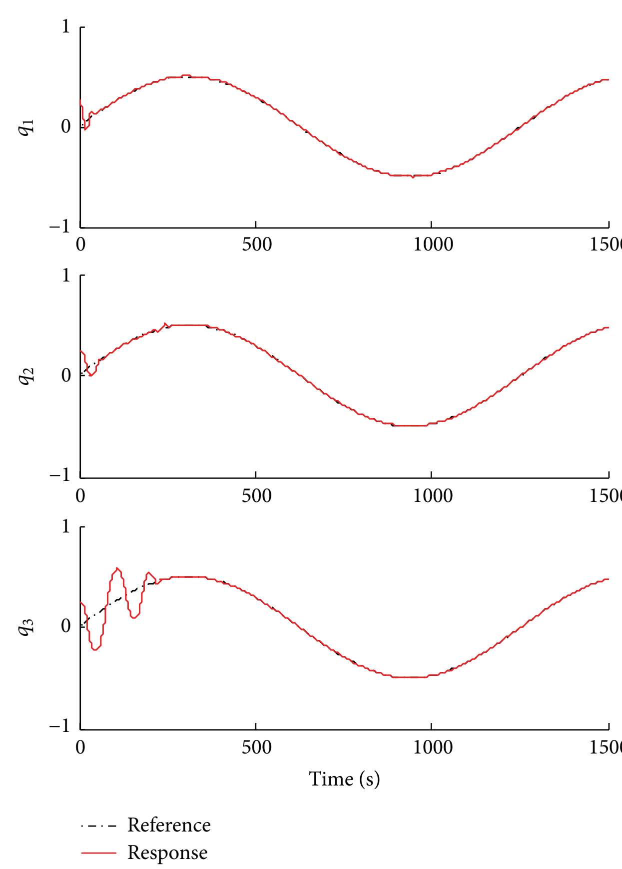

Figure 3: Trajectories of outputs with amplitude saturation.

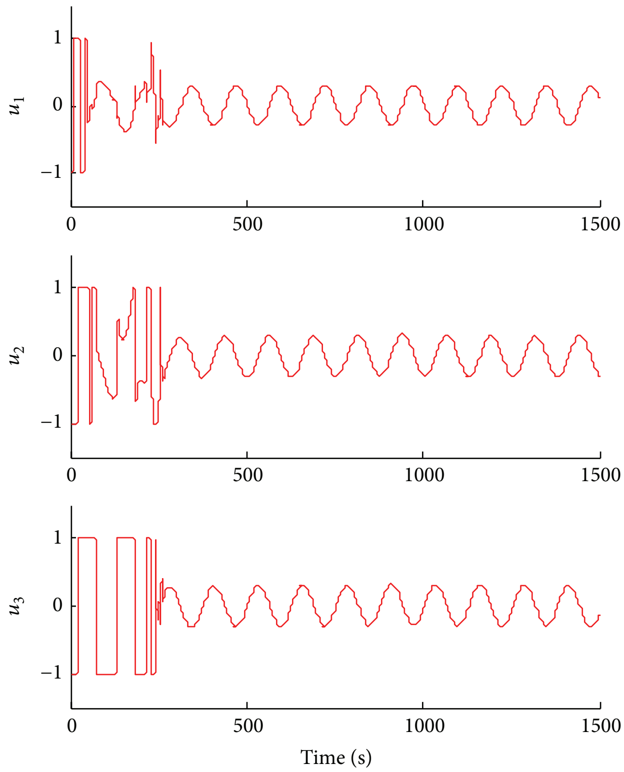

FIGURE 4: Trajectories of control inputs with amplitude saturation.

inputs with actuator amplitude saturation. From these simulations, it is obvious that proposed control scheme (40) can achieve a good tracking performance when the actuator amplitude limits are considered.

4.3. Actuator Amplitude and Rate Saturation. For further analysis, actuator amplitude and rate saturations are considered. The control input vector $\mathbf{u} = [u_1, u_2, u_3]^T$ has the amplitude and rate limitation $|u_i| \leq 1$, $|\dot{u}_i| \leq 2$, $i = 1, 2, 3$. The other initial parameters are the same as mentioned in Section 4.2.

According to (41), the dynamics of control inputs are expressed as follows:

$$\dot{u}_i = S_1\left(\omega_i S_1\left(u_{ci}\right) - u_i\right) \quad i = 1, 2, 3, \tag{75}$$

where $\omega_i = 20.5$ $(i = 1, 2, 3)$.

Using control law (40), the actuator amplitude and rate constraints (41), the auxiliary control term (51), the parameters update laws (52)-(53), and the robust compensation term (54), simulation results are presented in Figures 5–8. Figure 5 shows the curves of outputs and their reference trajectories, which indicate that a good tracking control performance is still achieved under actuator amplitude and rate constraint conditions. The control signals and their derivatives are given in Figures 6 and 7. It is observed that the control signals satisfy the amplitude and rate limitations. That is, the proposed robust control scheme for the satellite attitude control system can prevent the control signals from reaching amplitude and

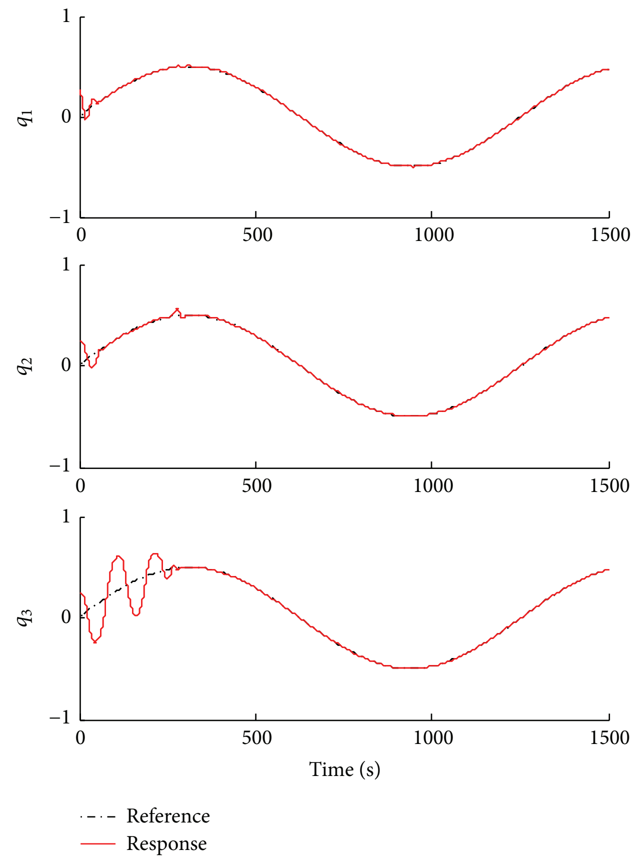

FIGURE 5: Trajectories of outputs with amplitude and rate saturation.

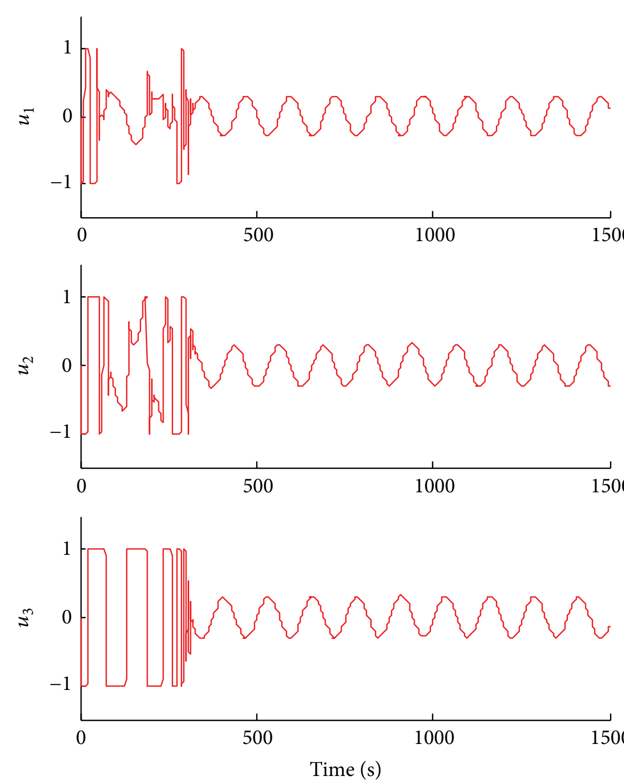

FIGURE 6: Trajectories of control inputs with amplitude and rate saturation.

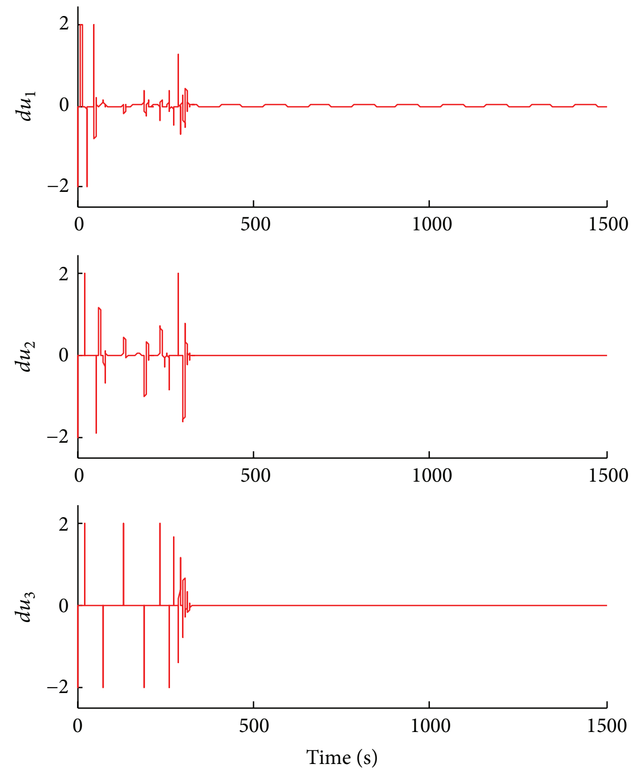

Figure 7: Derivatives of control inputs.

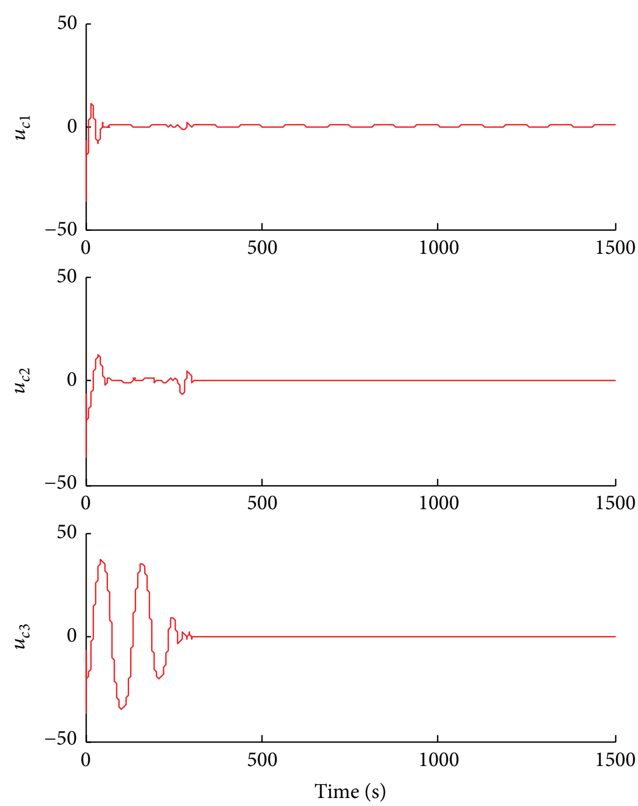

Figure 8: u_{c1}, u_{c2}, and u_{c3}.

rate saturation limits. Figure 8 shows the signals u_{ci} ($i = 1, 2, 3$). Obviously, they do not satisfy the control input limitations.

From the aforementioned simulations, it is demonstrated that the robust adaptive fuzzy tracking controller proposed in this paper not only can generate control inputs that satisfy actuator amplitude and rate saturations but also can effectively attenuate the effects of approximation error and extern disturbance on tracking errors. Thus, the proposed robust adaptive control scheme is valid for satellite attitude control system with actuator amplitude and rate saturation.

5. Conclusions

In this work, a robust fuzzy tracking control approach has been presented for a class of uncertain nonlinear MIMO systems in the presence of input saturation and external disturbances. Fuzzy logic systems are used to approximate the unknown system nonlinear terms. An auxiliary system is constructed to compensate the effects of actuator saturations, and then the actuator saturations can be augmented into the controller. The modified tracking error is introduced and used in fuzzy parameter update laws. A robust compensation control is designed to attenuate the effects of external disturbances and fuzzy approximation errors. The stability properties and tracking performance of the closed-loop system are obtained through Lyapunov analysis. Steady and transient modified tracking errors are analyzed and the bound of modified tracking errors can be adjusted by tuning certain design parameters. The proposed control scheme is applicable to uncertain nonlinear systems not only with actuator amplitude saturation, but also with actuator amplitude and rate saturation. The simulation results of satellite attitude control system are presented to demonstrate the effectiveness of proposed controller. In this paper, the control system relies on the output derivatives, up to $r_i - 1$ order as in (22), which is also practically restrictive for high order systems. Future research will be concentrated on an observer-based robust adaptive fuzzy control of uncertain nonlinear systems with actuator saturations based on the results of [18–20] and this paper.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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