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Research Article

H_∞ Filtering for Networked Markovian Jump Systems with Multiple Stochastic Communication Delays

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This paper is concerned with the H_∞ filtering for a class of networked Markovian jump systems with multiple communication delays. Due to the existence of communication constraints, the measurement signal cannot arrive at the filter completely on time, and the stochastic communication delays are considered in the filter design. Firstly, a set of stochastic variables is introduced to model the occurrence probabilities of the delays. Then based on the stochastic system approach, a sufficient condition is obtained such that the filtering error system is stable in the mean-square sense and with a prescribed H_∞ disturbance attenuation level. The optimal filter gain parameters can be determined by solving a convex optimization problem. Finally, a simulation example is given to show the effectiveness of the proposed filter design method.

1. Introduction

Analysis and synthesis of the networked systems has received much research attention in the last decade due to the wide applications of networked systems into the industrials, such as process control system, chemical systems, and remote surgery. Despite the advantages brought by the networked systems, various challenging problems occur, such as packet dropouts, communication delays, and medium access constraints. In the last few years, the state estimation for networks systems has become a hot research topic. Basically, there are state estimation algorithms: one is the Kalman filtering and the other one is the H_∞ filtering. For example, the authors in [1] studied the remote state estimation for the networked systems with missing observations. They addressed this problem under the discrete Kalman filtering formulation and also gave the statistical convergence properties of the estimation error covariance, showing the existence of a critical value for the arrival rate of the observations. Many new results have also been reported based on the results in [1]; see [2–5] and the references therein. It is well known that the Kalman filter is only applicable to the systems with a certain well-posed model, and the external noise is required to be a stochastic process with known statistical property. In many applications, these requirements may not hold. In this scenario, an alternative filtering algorithm has been proposed, that is, the H_∞ filtering algorithm. The recent advances on the H_∞ filtering for networked systems with various networked issues are referred to in [6–10].

On another research front line, Markovian jump systems have attracted significant attention from control society for many decades. This is due to the fact that such systems are shown to be appropriate and convenient to model a large number of practical systems that are subjected to abrupt variations in their structures, owing to sudden environmental disturbances, abrupt variations in the operating point of a nonlinear plant, and so on. Many works have been reported on the analysis and H_∞ filtering for Markovian jump systems [11–16]. To mention a few, the H_∞ filtering for a class of Markovian jump systems with time-varying transition probabilities has been considered in [16]; the authors showed that the filter gain parameter can be determined by solving a set of linear matrix inequalities. Recently, the H_∞ filtering for Markovian jump systems with missing measurement and signal quantization is investigated in [17]; the corresponding filter synthesis problem is transformed into a convex optimization problem that can be efficiently solved by using standard software packages. It should be noted that the communication delay has not been considered in their design of the filter. The communication delay is an important issue that occurs in the networked control systems and it cannot be ignored. Though the H_∞ filtering for networked systems with stochastic delays was studied in [18], the authors assumed that the measurement signal is transmitted to the remote filter within a single packet. In reality, the networked systems may distribute in a large area, and incorporating the measurement into one packet is impossible. Hence, the H_∞ filtering for networked systems with multiple communication delays deserves investigation, which motivates the present study.

In this paper, the H_∞ filtering problem is addressed for a class of networked Markovian jump systems with multiple communication delays. Firstly, a set of stochastic variables are introduced to model the communication delays. Based on the Laypunov stability theory and the stochastic system approach, a sufficient condition is obtained such that the filtering error system is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level. Finally, a simulation study is given to show the effectiveness of the proposed design.

Notations. The notations used in this paper are standard. The R^n denotes the n-dimensional Euclidean space, and $l_2[0,+\infty)$ is the space of summable sequences. A^T represents the transpose of the matrix A.

2. Problem Formulation

The structure of the considered filtering system is shown in Figure 1. The physical plant under the consideration is

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}w(k),$$
$$z(k) = L_{\sigma(k)}x(k), \tag{1}$$

where $x(k) \in R^{n_x}$ is the state, $z(k) \in R^{n_z}$ is the signal to be estimated, and $w(k) \in R^{n_w}$ is the unknown disturbance belonging to $l_2[0,+\infty)$. $A_{\sigma(k)}$, $B_{\sigma(k)}$, and $L_{\sigma(k)}$ are the constant matrices with appropriate dimensions. $\sigma(k) \in \Gamma = \{1,2,\ldots,S\}$ is a discrete-time homogeneous Markov chain with following probabilistic transfer matrix $\Pi = [\sigma_{ij}]$:

$$\sigma_{ij} = \text{Prob}\{\sigma(k+1) = j \mid \sigma(k) = i\} \geq 0,$$
$$\sum_{j=1}^{S}\sigma_{ij} = 1, \quad \forall i,j \in \Gamma. \tag{2}$$

Suppose there are m distributed sensors for the system and the pth sensor produces its measurement as

$$y_p(k) = C_p x(k) + D_p w(k) \quad (p = 1,2,\ldots,m), \tag{3}$$

where $y_p(k) \in R^{n_p}$; C_p and D_p are the known constant matrix with appropriate dimensions. Due to the existence of the communication delay, the filter may not be able to use the current measurement signal to update its state. In this paper, we assume that the largest delay in the pth channel is N_p time step. Then, at each time instant k, the filter input, $\overline{y}_p(t_k)$, will be the most recent member of the transmitted subset of $\{y_p(k), y_p(k-1),\ldots,y_p(k-N_p)\}$.

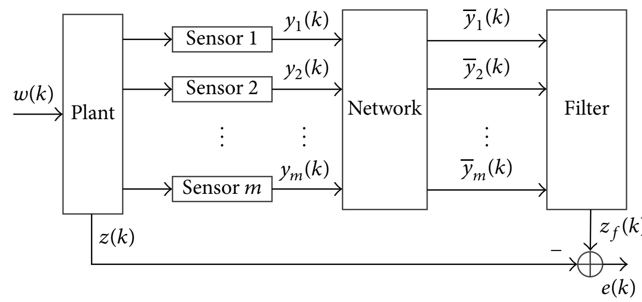

Figure 1: The structure of networked systems.

In the real networked systems, the delay may occur in a stochastic way; hence, a set of stochastic variables, $\alpha_{p,s}(k) \in \{0,1\}$, $s = 0,1,\ldots,N_p$, is introduced such that $\alpha_{p,s}(k) = 1$ if $y_p(k-s)$ is the value used as the filter input at k. Due to the fact that at each time instant only one case may happen in our system we have $\sum_{s=0}^{N_p}\alpha_{p,s}(k) = 1$. In this paper, the probabilities $\mathbb{E}\{\alpha_{p,s}(k) = 1\} = \overline{\alpha}_{p,s}$ are assumed to be known, which may be obtained by some stochastic analysis method. Clearly, $\sum_{s=0}^{N_p}\overline{\alpha}_{p,s} = 1$. The input of the filter can be described as

$$\overline{y}_p(k) = \alpha_{p,0}(k)\,y_p(k) + \alpha_{p,1}(k)\,y_p(k-1) + \cdots$$
$$+ \alpha_{p,N_p}(k)\,y_p\left(k-N_p\right). \tag{4}$$

Let $N = \max_{p=1,2,\ldots,m}\{N_p\}$,

$$X(k) = \left[x^T(k) \;\; x^T(k-1) \;\; \cdots \;\; x^T(k-N)\right]^T,$$
$$W(k) = \left[w^T(k) \;\; w^T(k-1) \;\; \cdots \;\; w^T(k-N)\right]^T. \tag{5}$$

We have

$$\overline{y}_p(k) = \sum_{s=0}^{N_p}\alpha_{p,s}(k)\left\{C_pE_{sp}X(k) + D_pH_{sp}W(k)\right\}, \tag{6}$$

where $E_{sp} = [0 \;\cdots\; 0 \;\; I \;\; 0 \;\cdots\; 0]$, and $H_{sp} = [0 \;\cdots\; 0 \;\; I \;\; 0 \;\cdots\; 0]$, in which all elements are zeros except that the $(s+1)$th block is an identity matrix.

Let $\alpha_p(k) = [\alpha_{p,0}(k), \alpha_{p,1}(k),\ldots,\alpha_{p,N_p}(k)]$. Then we have

$$\alpha_p(k) \in \begin{cases} \{1,0,0,\ldots,0\} \\ \{0,1,0,\ldots,0\} \\ \vdots \\ \{0,0,0,\ldots,1\}. \end{cases} \tag{7}$$

Define $\alpha(k) = [\alpha_1(k), \alpha_2(k), \ldots, \alpha_m(k)]$. The total number of possible realizations of $\alpha(k)$ is $N = N_1 \times N_2 \times \cdots \times N_m$. Let

$$\overline{E}_{\alpha(k)} = \left[E^T_{1,\alpha_1(k)} \; E^T_{2,\alpha_2(k)} \; \cdots \; E^T_{m,\alpha_m(k)}\right]^T, \qquad \overline{H}_{\alpha(k)} = \left[H^T_{1,\alpha_1(k)} \; H^T_{2,\alpha_2(k)} \; \cdots \; H^T_{m,\alpha_m(k)}\right]^T. \tag{8}$$

One particular realization of $\alpha(k)$ means one sequence of $\alpha_p(k)$, which specifies one particular case of $(\overline{E}_{\alpha(k)}, \overline{H}_{\alpha(k)})$. For presentation simplicity, we now define a new variable $\beta_l(k) \in \{1, 0\}$, $l \in \Gamma$ to model each possible case for all the measurement signals, where $\Gamma = \{1, 2, \ldots, \overline{N}\}$, and $\overline{N}$ is the possible numbers. For example, $\beta_1(k) = 1$ if and only if $\alpha_p(k) = [1, 0, \ldots, 0]$, $p = 1, 2, \ldots, m$ and $\beta_2(k) = 1$ if and only if $\alpha_p(k) = [1, 0, \ldots, 0]$, $p = 1, 2, \ldots, m-1$, $\alpha_m(k) = [0, 1, \ldots, 0]$ and so on. It is easy to see that $\sum_{l=1}^{\overline{N}} \beta_l(k) = 1$ and $\sum_{l=1}^{\overline{N}} \overline{\beta}_l = 1$.

Based on the above discussions, the filter input signal is

$$\overline{y}(k) = \sum_{l=1}^{\overline{N}} \beta_l(k) \left\{\overline{C}\,\overline{E}_l X(k) + \overline{D}\,\overline{H}_l W(k)\right\}, \tag{9}$$

where $\overline{C} = \operatorname{diag}\{C_1, C_2, \ldots, C_m\}$ and $\overline{D} = \operatorname{diag}\{D_1, D_2, \ldots, D_m\}$. We propose the following filter:

$$x_f(k+1) = A_f x_f(k) + B_f \overline{y}(k), \qquad z_f(k) = C_f x_f(k), \tag{10}$$

where $x_f(k) \in R^{n_x}$ is the state of the filter and $z_f(k) \in R^{n_z}$ is the estimate of $z(k)$. A_f, B_f, and C_f are the filter parameters to be designed.

In order to derive the filtering error system, we rewrite state equation in (1) and the estimation equation in (9) as

$$X(k+1) = \overline{A}_{\sigma(k)} X(k) + \overline{B}_{\sigma(k)} W(k), \qquad z(k) = \overline{L}_{\sigma(k)} X(k), \tag{11}$$

where

$$\overline{A}_{\sigma(k)} = \begin{bmatrix} A_{\sigma(k)} & 0 & 0 & \cdots & 0 \\ I_n & 0 & 0 & \cdots & 0 \\ 0 & I_n & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_n & 0 \end{bmatrix},$$

$$\overline{B}_{\sigma(k)} = \begin{bmatrix} B_{\sigma(k)} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \tag{12}$$

$$\overline{L}_{\sigma(k)} = \left[L_{\sigma(k)} \; 0 \; 0 \; \cdots \; 0\right].$$

Define $\eta(k) = [X^T(k) \;\; x_f^T(k)]^T$ and the estimation error as $e(k) = z(k) - z_f(k)$. Then the filtering error system is given by

$$\eta(k+1) = \widetilde{A}_{\sigma(k)} \eta(k) + \widetilde{B}_{\sigma(k)} W(k) + \sum_{l=1}^{\overline{N}} \left(\beta_l(k) - \overline{\beta}_l\right) \left[\widehat{A}_l \eta(k) + \widehat{B}_l W(k)\right], \qquad e(k) = \widetilde{L}_{\sigma(k)} \eta(k), \tag{13}$$

where

$$\widetilde{A}_{\sigma(k)} = \begin{bmatrix} \overline{A}_{\sigma(k)} & 0 \\ B_f \overline{C}\,\overline{E} & A_f \end{bmatrix}, \qquad \widetilde{B}_{\sigma(k)} = \begin{bmatrix} \overline{B}_{\sigma(k)} \\ B_f \overline{D}\,\overline{H} \end{bmatrix},$$

$$\widetilde{L}_{\sigma(k)} = \left[\overline{L}_{\sigma(k)} \;\; -C_f\right],$$

$$\widehat{A}_l = \begin{bmatrix} 0 & 0 \\ B_f \overline{C}\,\overline{E}_l & 0 \end{bmatrix}, \qquad \widehat{B}_l = \begin{bmatrix} 0 \\ B_f \overline{D}\,\overline{H}_l \end{bmatrix}, \tag{14}$$

$$\overline{E} = \sum_{l=1}^{\overline{N}} \overline{\beta}_l \overline{E}_l, \qquad \overline{H} = \sum_{l=1}^{\overline{N}} \overline{\beta}_l \overline{H}_l.$$

System (13) is a complex stochastic system; we need the following definitions before presenting our main results.

Definition 1 (see [12]). System (13), with $w(k) = 0$, is said to be stochastically stable if the following inequality

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \left\|\eta(k)\right\|^2 \mid \varphi(0)\right\} < \infty \tag{15}$$

holds for any initial condition $\varphi(0) = \{\eta(t_0), \sigma(0)\}$.

Definition 2. For a given scalar $\gamma > 0$, system (13) is said to be asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance γ, if it is asymptotically stable and under zero initial condition, $\sum_{s=0}^{+\infty} \mathbb{E}\{e^T(s)e(s)\} \le \sum_{s=0}^{+\infty} \gamma^2 w^T(s) w(s)$ holds for all nonzero $w(k) \in l_2[0, \infty)$.

3. Main Results

A sufficient condition is firstly presented to guarantee the stability of the filtering error system.

Theorem 3. *For given scalars $\overline{\beta}_i$ and τ, if there exists a positive-definite matrix P_i such that the following inequalities*

$$\begin{bmatrix} \Xi_1 & \Xi_2 & \sqrt{\overline{\beta}_1}\Phi_1 & \cdots & \sqrt{\overline{\beta}_N}\Phi_N & \Xi_3 \\ * & -\overline{P}_i & 0 & 0 & 0 & 0 \\ * & * & -\overline{P}_i & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & -\overline{P}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{16}$$

hold for all $i \in \Gamma$, then the filtering error system is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level $\gamma = \tau\sqrt{N+1}$, where

$$\Xi_1 = \begin{bmatrix} -P_i & 0 \\ 0 & -\tau^2 I \end{bmatrix}, \qquad \Xi_2 = \begin{bmatrix} \widetilde{A}_i & \widetilde{B}_i \end{bmatrix}^T,$$
$$\Phi_l = \begin{bmatrix} \widehat{A}_l & \widehat{B}_l \end{bmatrix}^T, \qquad \Xi_3 = \begin{bmatrix} \widetilde{L}_i & 0 \end{bmatrix}^T, \tag{17}$$
$$\overline{P}_i = \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right)^{-1}.$$

Proof. We first consider the stability of the filtering error system (13) with $w(k) = 0$. To do this, we choose the following Lyapunov function for system (13):

$$V(k) = \eta^T(k) P_{\sigma(k)} \eta(k). \tag{18}$$

Then, one sees that

$$\begin{aligned}
\mathbb{E}\{V(k+1) - V(k)\} &= \left[\widetilde{A}_i \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widetilde{A}_i \eta(k)\right] - \eta^T(k) P_i \eta(k) \\
&\quad + \mathbb{E}\left\{\left[\sum_{l=1}^{\overline{N}} \phi_l(k) \widehat{A}_l \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \cdot \left[\sum_{l=1}^{\overline{N}} \phi_l(k) \widehat{A}_l \eta(k)\right]\right\},
\end{aligned} \tag{19}$$

where $\phi_i(k) = \beta_i(k) - \overline{\beta}_i$. On the other hand, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left[\sum_{l=1}^{\overline{N}} \phi_l(k) \widehat{A}_l \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\sum_{l=1}^{\overline{N}} \phi_l(k) \widehat{A}_l \eta(k)\right]\right\} \\
&\quad = \mathbb{E}\left\{\sum_{l=1}^{\overline{N}} (\phi_l(k))^2 \left[\widehat{A}_l \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widehat{A}_l \eta(k)\right]\right. \\
&\qquad + \sum_{l=1}^{N} \sum_{t=1, t\neq l}^{N} \phi_l(k) \phi_t(k) \left[\widehat{A}_l \eta(k)\right]^T \\
&\qquad \left. \cdot \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widehat{A}_t \eta(k)\right]\right\} \\
&\quad \leq \sum_{l=1}^{\overline{N}} \overline{\beta}_l \left[\widehat{A}_l \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widehat{A}_l \eta(k)\right].
\end{aligned} \tag{20}$$

Hence,

$$\begin{aligned}
\mathbb{E}\{V(k+1) - V(k)\} &\leq \left[\widetilde{A}_i \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widetilde{A}_i \eta(k)\right] - \eta^T(k) P_i \eta(k) \\
&\quad + \sum_{l=1}^{\overline{N}} \overline{\beta}_l \left[\widehat{A}_l \eta(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widehat{A}_l \eta(k)\right].
\end{aligned} \tag{21}$$

It is seen that the right hand side of (21) is negative under (16) and then $\mathbb{E}[V(k+1) \mid \varphi(k)] < V(k)$. It is not difficult to find a scalar $0 < \mu < 1$, such that $\mathbb{E}[V(k+1) \mid \varphi(k)] < \mu V(k)$ holds; then by deduction, we have $\mathbb{E}[V(k) \mid \varphi(0)] < \mu^k V(0)$, and, consequently,

$$\begin{aligned}
\mathbb{E}\left[\sum_{k=0}^{h} V(k) \mid \varphi(0)\right] &\leq \left(1 + \mu + \cdots + \mu^h\right) V(k_0) \\
&= \frac{1 - \mu^{h+1}}{1 - \mu} V(k_0).
\end{aligned} \tag{22}$$

Let $h \to \infty$; the following inequality is obtained:

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \|\eta(k)\|^2 \mid \varphi(0)\right] \leq \frac{1}{(1-\mu)\overline{\lambda}} V(k_0) < \infty, \tag{23}$$

where $\overline{\lambda} = \min_{i\in\Gamma} \lambda_{\min}(P_i)$. According to **Definition 1**, system (13) is stochastically stable.

Now we consider the H_∞ performance of the filtering error system (13). It follows from the above analysis method that

$$\begin{aligned}
&\mathbb{E}\left\{V(k+1) - V(k) + e^T(k) e(k) - \tau^2 W^T(k) W(k)\right\} \\
&\quad = \left[\widetilde{A}_i \eta(k) + \widetilde{B}_i W(k)\right]^T \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\widetilde{A}_i \eta(k) + \widetilde{B}_i W(k)\right] \\
&\qquad - \eta^T(k) P_i \eta(k) \\
&\qquad + \mathbb{E}\left\{\left[\sum_{l=1}^{\overline{N}} \left(\beta_l(k) - \overline{\beta}_l\right)\left(\widehat{A}_l \eta(k) + \widehat{B}_l W(k)\right)\right]^T\right. \\
&\qquad \times \left(\sum_{j=1}^{s} \sigma_{ij} P_j\right) \left[\sum_{l=1}^{\overline{N}} \left(\beta_l(k) - \overline{\beta}_l\right)\right. \\
&\qquad \left.\left. \cdot \left(\widehat{A}_l \eta(k) + \widehat{B}_l W(k)\right)\right]\right\}
\end{aligned}$$

$$+\left[\widetilde{L}_i\eta(k)\right]^T\left[\widetilde{L}_i\eta(k)\right]-\tau^2W^T(k)W(k)$$
$$\leq \overline{\eta}^T(k)\left[\Xi_1+\Xi_2\left(\sum_{j=1}^{s}\sigma_{ij}P_j\right)\Xi_2^T\right.$$
$$\left.+\sum_{i=1}^{N}\overline{\beta}_i\Phi_i\left(\sum_{j=1}^{s}\sigma_{ij}P_j\right)\Phi_i^T+\Xi_3\Xi_3^T\right]\overline{\eta}(k), \tag{24}$$

where $\overline{\eta}(k) = [\eta^T(k)\ \ W^T(k)]^T$. By using the schur complement [19], it is easy to see

$$\mathbb{E}\left\{V(k+1)-V(k)+e^T(k)e(k)-\tau^2W^T(k)W(k)\right\}<0. \tag{25}$$

Summing both sides of (21) from $k=0$ to $k=T$ gives

$$\mathbb{E}\left\{V(k+1)-V(0)+\sum_{k=0}^{T}\left\{e^T(k)e(k)-\tau^2W^T(k)W(k)\right\}\right\}<0, \tag{26}$$

which implies, by the zero initial condition and positiveness of $V(k+1)$, that

$$\mathbb{E}\left\{\sum_{k=0}^{T}\left\{e^T(k)e(k)-\tau^2W^T(k)W(k)\right\}\right\}<0. \tag{27}$$

Let $T\to+\infty$; then

$$\sum_{k=0}^{+\infty}\mathbb{E}\left\{e^T(k)e(k)\right\}\leq\tau^2W^T(k)W(k)=\gamma^2w^T(k)w(k). \tag{28}$$

Hence, system (13) achieves a prescribed H_∞ performance level as well. This completes the proof. □

It should be pointed out that Theorem 3 cannot be used to determine the filter gain directly due to the coexistence of P and P^{-1}. In the following theorem, we present the filter gain design.

Theorem 4. *For given scalars $\overline{\beta}_i$ and τ, if there exist a positive-definite matrix P_i and a matrix G with appropriate dimensions, such that the following inequalities*

$$\begin{bmatrix} \Xi_1 & \overline{\Xi}_2 & \sqrt{\overline{\beta}_1}\overline{\Phi}_1 & \cdots & \sqrt{\overline{\beta}_{\overline{N}}}\overline{\Phi}_{\overline{N}} & \Xi_3 \\ * & \widehat{P}_i & 0 & 0 & 0 & 0 \\ * & * & \widehat{P}_i & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \widehat{P}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix}<0 \tag{29}$$

hold, then the filtering problem is solvable. Moreover, the filter gains are determined by $A_f=G_3^{-T}A_F$, $B_f=G_3^{-T}B_F$, $C_f=C_F$, where

$$\overline{\Xi}_2=\begin{bmatrix}\overline{\Xi}_2^1\\ \overline{\Xi}_2^2\end{bmatrix},\qquad \overline{\Phi}_i=\begin{bmatrix}\overline{\Phi}_i^1\\ \overline{\Phi}_i^2\end{bmatrix},\qquad \overline{\Xi}_3=\begin{bmatrix}\overline{\Xi}_3^1\\ 0\end{bmatrix},$$
$$\widehat{P}_i=\left(\sum_{j=1}^{s}\sigma_{ij}P_j\right)-G-G^T, \tag{30}$$

with

$$\overline{\Xi}_2^1=\begin{bmatrix}\overline{A}_i^TG_1+E^TB_F^TM & \overline{A}_i^TG_2+E^TB_F^T\\ A_F^TM & A_F^T\end{bmatrix},$$
$$\overline{\Xi}_2^2=\left[\overline{B}_i^TG_1+\overline{H}^TB_F^TM\ \ \overline{B}_i^TG_2+\overline{H}^TB_F^T\right],$$
$$\overline{\Phi}_i^1=\begin{bmatrix}\overline{E}_i^T\overline{C}^TB_F^TM & \overline{E}_i^T\overline{C}^TB_F^T\\ 0 & 0\end{bmatrix},$$
$$\overline{\Phi}_i^2=\left[\overline{H}_i^T\overline{D}^TB_F^TM\ \ \overline{H}_i^T\overline{D}^TB_F^T\right], \tag{31}$$
$$\overline{\Xi}_3^1=\begin{bmatrix}\overline{L}^T\\ -C_F^T\end{bmatrix},\qquad P_i=\begin{bmatrix}P_{1i} & P_{2i}\\ * & P_{3i}\end{bmatrix},$$
$$G=\begin{bmatrix}G_1 & G_2\\ G_3M & G_3\end{bmatrix},\qquad M=\left[I\ \cdots\ I\right].$$

Proof. By pre- and postmultiplying (16) with $\text{diag}\{I,G^T,\underbrace{G^T,\cdots G^T}_{N},I\}$ and its transpose, respectively, (16) is equivalent to

$$\begin{bmatrix} \Xi_1 & \Xi_2G & \sqrt{\overline{\beta}_1}\Phi_1G & \cdots & \sqrt{\overline{\beta}_{\overline{N}}}\Phi_{\overline{N}}G & \Xi_3 \\ * & \widetilde{P}_i & 0 & 0 & 0 & 0 \\ * & * & \widetilde{P}_i & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \widetilde{P}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix}<0, \tag{32}$$

where $\widetilde{P}_i = -G^T(\sum_{j=1}^{s}\sigma_{ij}P_j)^{-1}G$. On the other hand, it is easy to see that $-G^T(\sum_{j=1}^{s}\sigma_{ij}P_j)^{-1}G^T \leq (\sum_{j=1}^{s}\sigma_{ij}P_j) - G - G^T$ always holds for any matrix G. Then, (32) holds if

$$\begin{bmatrix} \Xi_1 & \Xi_2 G & \sqrt{\beta_1}\Phi_1 G & \cdots & \sqrt{\beta_{\overline{N}}}\Xi_{\overline{N}} G & \Xi_3 \\ * & \widehat{P}_i & 0 & 0 & 0 & 0 \\ * & * & \widehat{P}_i & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \widehat{P}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0. \quad (33)$$

The above inequality is the same as (29) with $P_i = \left[\begin{smallmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{smallmatrix}\right]$, $G = \left[\begin{smallmatrix} G_1 & G_2 \\ G_3 M & G_3 \end{smallmatrix}\right]$, $A_F = G_3^T A_f$, $B_F = G_3^T B_f$, and $C_F = C_f$. This completes the proof. □

Remark 5. In order to obtain the minimum H_∞ performance γ^*, one can solve the following optimization problem

$$\begin{aligned} &\min \quad \nu \\ &\text{subject to} \quad (29) \text{ with } \nu = \tau^2 \end{aligned} \quad (34)$$

and find the minimum H_∞ performance γ^* by $\gamma^* = \sqrt{\nu^*(N+1)}$.

4. Simulation Example

Consider a satellite yaw angles control system with noise perturbation, which is given by the following state-space representations [20]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_1 & 0 \\ 0 & 0 & 0 & J_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \dot{\delta}_1(t) \\ \dot{\delta}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix} \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \delta_1(t) \\ \delta_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ 0.1 \end{bmatrix} w(t). \quad (35)$$

The satellite yaw angles control system consists of two rigid bodies jointed by a flexible link. This link is modeled as a spring with torque constant k and viscous damping f. $\theta_1(t)$ and $\theta_2(t)$ are the yaw angles for the main body and the instrumentation module of the satellite. Moreover, $\delta_1(t) = \dot{\theta}_1(t)$ and $\delta_2(t) = \dot{\theta}_2(t)$. $u(t)$ is the control torque, while J_1 and J_2 are the moments of the main body and the instrumentation module, respectively.

Choosing $J_1 = J_2 = 1$, $k = 0.3$, $f = 0.004$ and setting the sampling period $T = 0.1$ s, then with the controller $u(k) = 10^3\,[-0.1591 \;\; -5.9343 \;\; -0.0172 \;\; -2.8604]\,x(k)$, we obtain the following discrete-time satellite yaw angles closed-loop system:

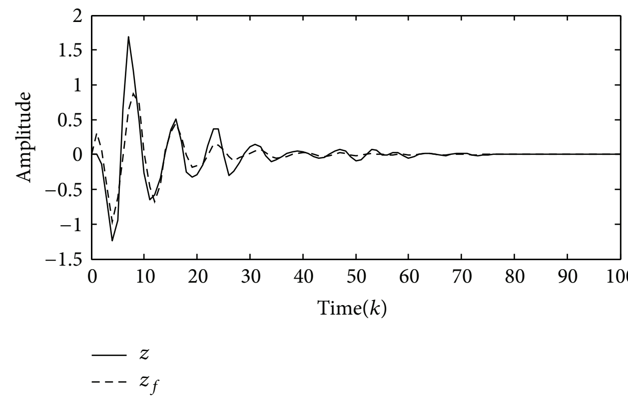

Figure 2: Trajectories of $z(k)$ and $z_f(k)$.

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.2035 & -29.6580 & 0.0142 & -14.2960 \\ 0.0012 & 0.9871 & 0.0000 & 0.0944 \\ -15.9256 & -592.9813 & -0.7168 & -285.8330 \\ 0.0188 & -0.4451 & 0.0007 & 0.7980 \end{bmatrix} x(k) \\ &+ \begin{bmatrix} 0.000005189 \\ 0.01049 \\ 0.0001569 \\ 0.009843 \end{bmatrix} w(k), \end{aligned} \quad (36)$$

where $x(k) = [\theta_1(k) \;\; \theta_2(k) \;\; \delta_1(k) \;\; \delta_2(k)]^T$.

In real control systems, controller failure may occur. In this example, we only consider a partial failure case; that is, the real controller is taken as $u_r(k) = \rho(k)u(k)$ with $\rho(k) \in \{0.5, 1\}$. The binary variable $\rho(k)$ is assumed to be a two-state Markovian process. The probabilistic transfer matrix is taken as $\Pi = \left[\begin{smallmatrix} 0.6 & 0.4 \\ 0.6 & 0.4 \end{smallmatrix}\right]$, which will be generated stochastically in the simulation setup. Hence, such a satellite yaw angles control system with partial controller failure can be modeled as the discrete-time Markovian jump system (1).

Suppose the remote filter is designed to estimate the signal $z(k) = Lx(k)$ by the measurement $y_p(k) = C_p x(k) + D_p w(k)$ of the satellite system, where $C_1 = [1 \;\; 0 \;\; 0 \;\; 0]$, $C_2 = [0 \;\; 1 \;\; 0 \;\; 0]$, $D_1 = 0.3$, $D_2 = 0.2$, and $L = [1 \;\; 1 \;\; 0 \;\; 0]$. The two measurements y_1 and y_2 are transmitted to the remote filter through communication channels with stochastic communication delays. The maximal time delays are assumed to be one time step, and the occurrence probabilities are $\overline{\alpha}_{1,0} = 0.2$, $\overline{\alpha}_{1,1} = 0.8$, $\overline{\alpha}_{2,0} = 0.4$, and $\overline{\alpha}_{2,1} = 0.6$. By solving the optimization problem (22), we have $\gamma^* = 3.1916$.

In the simulation setup, we choose the zero initial conditions, and the noise signal is taken as $w(k) = \exp(-0.1k) * \sin(0.8k)$. One-sample trajectories of $z(k)$ and $z_f(k)$ are shown in Figure 2. The trajectory of the filtering

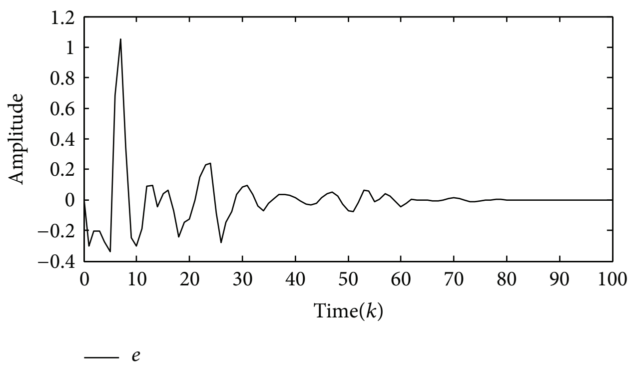

Figure 3: Trajectory of $e(k)$.

error is shown in Figure 3. By simple calculation, we have $\sqrt{\sum_{k=0}^{100} e^T(k)e(k)/\sum_{k=0}^{100} w^T(k)w(k)} = 1.1622 < \gamma^*$.

5. Conclusions

In this paper, the H_∞ filtering for a class of networked Markovian jump system with multiple stochastic communication delays has been investigated. A new stochastic delay model is proposed and formulated. Based on the stochastic system approach, a sufficient condition has been obtained such that the filtering error system is asymptotically stable in the mean-square sense and achieves a prescribed H_∞ performance level. A simulation study of the satellite yaw control system is given to show the effectiveness of our results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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