

# Research Article

# Discrete Event-Triggered Robust Fault-Tolerant Control for Nonlinear Networked Control Systems with $\alpha$ -Safety Degree and Actuator Saturation

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Received 31 December 2014; Revised 30 April 2015; Accepted 4 May 2015

Academic Editor: Gisella Tomasini

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This paper deals with the discrete event-triggered robust fault-tolerant control problem for uncertain nonlinear networked control systems (NNCSs) with  $\alpha$ -safety degree. A discrete event-triggered communication scheme (DETCS) is initially proposed, and a closed-loop fault model is subsequently established for NNCSs with actuator saturation under the DETCS. Based on an appropriately constructed delay-dependent Lyapunov–Krasovskii function, sufficient conditions are derived to guarantee the asymptotic stability of NNCSs under two different event-triggered conditions and are established as the contractively invariant sets of fault tolerance with  $\alpha$ -safety degree. Furthermore, codesign methods between the robust fault-tolerant controller and event-triggered weight matrix are also proposed in terms of linear matrix inequality. The simulation shows that the resultant closed-loop fault NNCSs possesses a high safety margin, and an improved dynamic performance, as well as a reduced communication load. A comparative analysis of the two event-triggered conditions is discussed in the experiment section.

# **1. Introduction**

With the increasing scale and complexity of networked control systems (NCSs), high safety and reliability have ceased to be extravagant demands for NCSs. These features have become intrinsic properties of the modern NCSs. As an important technology for improving system safety and reliability, fault-tolerant control for NCSs has thus attracted increasing attention [1–4] and has made outstanding progress [5-8]. As is well known, the controlled plant more or less contains several nonlinear characteristics; that is, the vast majority of controlled plants are nonlinear plants. Given the unique attributes derived from the nonlinear system, which is inherently complex, and the network as system transmission medium, the fault-tolerant design for nonlinear networked control systems (NNCSs) become increasingly difficult and challenging and, as such, has become the focus of academic research [9-13]. Considering sensor failures, an augmented closed-loop system model based on a state observer was established in [11], and a sufficient condition

was derived to maintain asymptotic stability in the NNCSs. In consideration of actuator failures, the irrelevant augmented matrix was introduced into the Lyapunov function in [13], and the reliability control problem was studied for NNCSs with random time delay. In addition, with the goal of ensuring stable operation for NNCSs with failures, researchers have also carried out work on fault-tolerant control with other performance constraints [14-18], such as the pole assignment,  $\alpha$ -stability [19],  $H_{\infty}$  disturbance rejection, and generalized  $H_2$  performance index. In consideration of actuator faults, a time delay dependent condition with the robust stability was derived for NNCSs in [17], where the design method for the robust  $H_{\infty}$  fault-tolerant controller was also given under the terms of a cone complementarity linearization algorithm. Several performance indexes, such as  $\alpha$ -stability,  $H_{\infty}$  performance index, and  $H_2$  performance index, were introduced into the fault-tolerant design field for NNCSs [18], and some robust satisfactory fault-tolerant control problems were also systematically studied therein for uncertain NNCSs.

In the area of network communications, many of the achieved results are based on a periodic time-triggered communication scheme (PTTCS), where system data are transmitted within an equal period of time determined by a physical clock. The PTTCS possesses several outstanding advantages of simplicity and convenience in system analysis and design, but it also leads to many redundant data transmissions. Meanwhile, under this scheme, the controller design passively relies on the existing quality of service (QoS) for networks, and the codesign of control and network communication cannot be implemented by taking two things into consideration which are quality of control (QoC) for systems and QoS for networks. As a result, some scholars have recently presented a series of event-triggered communication schemes [20-25] to solve the existing problems in PTTCS. The discrete event-triggered communication scheme (DETCS) was first presented in [20], wherein system state information need only be detected in the discrete time point. A novel DETCS was proposed in [25], where the filtering problem was studied for NNCSs with networked induced time delay. Recently, a few scholars introduced the DETCS into fault-tolerant control for NCSs, which soon after proved to be highly interesting and valuable [26, 27]. A reliable control design for linear NCSs was studied under DETCS in [26], where a criterion for exponential stability was also obtained for NCSs with probabilistic sensor and actuator faults. In [27], a robust integrity design problem was studied for NCSs with actuator failures and time-varying delay under DETCS, and the codesign method between the robust fault-tolerant control and network communication was also presented.

From the aforementioned results, the following conclusions naturally follow. On the one hand, the methods in [14-18] cause the NNCSs to possess not only fault-tolerant abilities against certain failures but also some performance indexes. However, these methods do not consider network communication resource saving. On the other hand, although the methods in [26, 27] studied the codesign of faulttolerant control for NCSs and network communication, results therein are also limited to the stability of NCSs with failures and do not cover other performances. To date, no study has been involved in such research work for NNCSs as that in [26, 27]. In addition, none of the aforementioned references considered the actuator saturation problem, despite being an unavoidable problem in practice [28, 29], which leads to system performance degradation and instability of the closed-loop system. Especially in cases when the redundant actuator shares the responsibility for fault actuator, the actuator easily enters into the saturation region. Motivated by these problems and in consideration of actuator saturation constraints and actuator failures, we thus investigate the codesign problem between the robust faulttolerant control for NNCSs and network communication under the DETCS architecture.

This study has three main contributions:

(1) The DETCS is introduced into fault-tolerant control for NNCSs. The DETCS can transform the conventional delay-dependent state-feedback control law into a delay/event codependent state/state-error control law and serves as the basis of a closed-loop fault system model that we carefully established for NNCSs with actuator saturation.

- (2) Three new definitions are proposed by introducing the concepts of  $\alpha$ -stability, domain of attraction, and contractively invariant set into the field of fault-tolerant control research. The introduction of the  $\alpha$ -safety degree, in particular, can improve performance satisfaction in systems with failures.
- (3) Sufficient conditions and codesign methods are derived for the closed-loop fault NNCSs with actuator saturation under two different event-triggered conditions. The derived conditions and methods can make NNCSs with actuator failures possess  $\alpha$ -safety degree and low occupancy rate of network communication resource. The simulation indicates that the codesign methods can provide a certain trade-off in balancing the required communication and the desired performance.

Notations.  $\mathbb{R}^n$  represents the *n*-dimensional real vector space;  $\mathbb{R}^{m \times n}$  is the set of all  $(m \times n)$ -dimensional real matrices;  $A > 0 (\geq 0)$  indicates that the matrix is positive (nonnegative) definite; diag{ $\cdots$ } refers to the block-diagonal matrix; *I* is the identity matrix of appropriate dimension; and  $\mathbb{A}^T$  is the transpose of matrix *A*. In symmetric block matrices, "\*" is used as an ellipsis for terms induced by symmetry, and matrices, if not explicitly stated, are assumed to have appropriate dimensions.

#### 2. Problem Statement

2.1. Uncertain Closed-Loop Fault NNCSs under DETCS. The controlled plant with actuator saturation can be described according to the following if-then rule.

If  $\theta_1(t)$  is  $M_{i1}$ ... and  $\theta_q(t)$  is  $M_{iq}$ , then

$$\dot{x}(t) = \left(A_i + \Delta A_i\right) x(t) + \left(B_i + \Delta B_i\right) \operatorname{sat}\left(u(t)\right), \quad (1)$$

where i = 1, 2, ..., r and r is the number of if-then rules;  $M_{ij}$  (i = 1, 2, ..., r; j = 1, 2, ..., r) is the fuzzy set;  $\theta(t) = [\theta_1(t), ..., \theta_g(t)]^T$  denotes the premise variables;  $x(t) \in \mathbb{R}^n$ and  $u(t) \in \mathbb{R}^m$  denote the state vector and the control input, respectively; function  $\operatorname{sat}(\cdot): \mathbb{R}^m \to \mathbb{R}^m$  denotes the standard multivariable saturation function defined as  $\operatorname{sat}(u) = [\operatorname{sat}(u_1), \operatorname{sat}(u_2), ..., \operatorname{sat}(u_m)]^T$  and  $\operatorname{sat}(u_i) = \operatorname{sign}(u_i) \min\{1, |u_i|\}$ ; and  $A_i \in \mathbb{R}^{n \times n}$  and  $B_i \in \mathbb{R}^{n \times m}$  are the system matrix and the input matrix, respectively. In addition,  $\Delta A_i, \Delta B_i$  (i = 1, 2, ..., r), which are assumed to be norm bounded, denote the uncertainty of system parameters. They are time varying and satisfy

$$\left[\Delta A_{i}, \Delta B_{i}\right] = MF(t)\left[E_{ai}, E_{bi}\right], \qquad (2)$$

where M,  $E_{ai}$ , and  $E_{bi}$  are real constant matrices with appropriate dimensions; F(t) is an unknown time-varying continuous matrix function with real values, the elements of which are Lebesgue measurable; and F(t) satisfies  $F^{T}(t)F(t) \leq I$ .

The following fuzzy system state equation is obtained using center-average defuzzifier, product inference, and singleton fuzzifier:

$$\dot{x}(t) = \sum_{i=1}^{r} u_i(\theta(t))$$

$$\cdot \left[ (A_i + \Delta A_i) x(t) + (B_i + \Delta B_i) \operatorname{sat}(u(t)) \right],$$
(3)

where

$$u_{i}(\theta(t)) = \frac{a_{i}(\theta(t))}{\sum_{i=1}^{r} a_{i}(\theta(t))} \ge 0,$$

$$\sum_{i=1}^{r} u_{i}(\theta(t)) = 1,$$

$$(i = 1, 2, \dots, r),$$
(4)

 $a_i(\theta(t)) = \prod_{s=1}^{g} M_{is}(\theta_s(t))$ , and  $M_{is}(\theta_s(t))$  is the degree of the membership of variable  $\theta_s(t)$  in fuzzy set  $M_{is}$ .

The communication scheme must be built with some constraints that can determine whether or not to send the state signal through the network to reduce the network resource waste and network congestion in PTTCS. Based on [20, 25], a new DETCS is built, as shown in Figure 1.

In contrast to traditional NCSs, the sample data need to pass the event generator before being transmitted by the network, as depicted in Figure 1. The function of the event generator is to determine whether or not to transmit the latest sample signal to the controller. The event-triggered condition is presented as follows:

$$[x(i_{k}h) - x(t_{k}h)]^{T} \Phi_{1} [x(i_{k}h) - x(t_{k}h)]$$

$$\leq x^{T} (i_{k}h) \Phi_{2} x(i_{k}h),$$
(5)

where  $\Phi_1$  and  $\Phi_2$  are symmetric positive definite matrices that are the weight matrices of the DETCS to be designed; *h* is the sampling period that drives the sensor clock;  $x(i_kh)$ and  $x(t_kh)$  denote the current sampled data and the latest transmission data, respectively, where  $i_kh = t_kh + lh$ , l = $1, 2, ..., d_k$ , and  $d_k = t_{k+1} - t_k - 1$ ;  $\{t_kh \mid t_k \in N\}$  is the release instant set of data transmission and  $\{t_0h, t_1h, t_2h, ...\}$  is the subset of the period sampling instant set  $\{0, h, 2h, ...\}$ ;  $t_{k+1}h$  $t_kh$  denotes the release period  $h_k$  given in terms of condition (5) at time  $t_kh$ . When  $x(i_kh)$  and  $x(t_kh)$  satisfy event-triggered condition (5), the event generator is not triggered, and data  $x(i_kh)$  is not transmitted.

To perform the following research work conveniently, another event-triggered condition, as in [23], is listed as follows:

$$[x(i_{k}h) - x(t_{k}h)]^{T} \Phi [x(i_{k}h) - x(t_{k}h)]$$

$$\leq \sigma x^{T} (i_{k}h) \Phi x (i_{k}h),$$
(6)

where  $\Phi$  is the symmetric positive definite matrix that is the weight matrix of the DETCS to be designed and  $\sigma$  is the bounded positive scalar that is the event-triggered parameter.



FIGURE 1: Diagram of NCSs structure based on DETCS.

Regardless of which condition we select from eventtriggered conditions (5) and (6), we make the following assumption and description. First, we suppose that the system state is completely measured and that the system adopts the static state-feedback controller. Considering the effect of network transmission time delay and calculation time delay, we set the comprehensive time delay as  $\tau_{t_k} = \tau_{t_k}^{sc} + \tau_{t_k}^{ca} + \tau_{t_k}^{c}$  at time  $t_k h$ , where  $\tau_{t_k}^{sc}$  and  $\tau_{t_k}^{ca}$  denote the transmission time delays from the sensor to the controller and from the controller to the actuator, respectively, and  $\tau_{t_k}^c$  denotes the calculation time delay. Meanwhile, considering the role of zero-order holder, when  $t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$ , we can express the control input as

$$u(t) = \sum_{j=1}^{r} u_j(\theta(t)) K_j x(t_k h), \qquad (7)$$

where  $K_j$  (j = 1, 2, ..., r) is the state-feedback control gain matrix.

On the basis of the aforementioned description, when  $x(t_kh)$  has reached the actuator but  $x(t_{k+1}h)$  has not, we define the keep interval as

$$\Omega = \left[ t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}} \right).$$
(8)

The keep interval is divided into several subintervals, such that

$$\Omega = \Delta_k^0 \cup \Delta_k^1 \cup \dots \cup \Delta_k^{d_k},\tag{9}$$

where  $\Delta_k^{l_k} = [i_k h + \tau_{i_k h}, i_k h + h + \tau_{i_k h+h}), i_k h = t_k h + l_k h$ , and  $l_k = 0, 1, 2, \dots, d_k, d_k = t_{k+1} - t_k - 1$ . To guarantee the effectiveness of the interval division, we suppose that  $\tau_{i_k h}$  is the virtual network transmission delay at sample instant  $i_k h$ .

When  $t \in \Delta_k^{l_k}$ , the function  $\tau(t)$  is defined as

$$\tau\left(t\right) = t - i_k h. \tag{10}$$

According to (9) and (10), the upper and lower bounds of time delay function  $\tau(t)$  are described as

$$\tau_1 < \tau_{i_{\nu}h} \le \tau(t) \le h + \tau_{i_{\nu}h+h} \le h + \overline{\tau} = \tau_2, \tag{11}$$

where  $\tau_1 = \min{\{\tau_{t_k}\}}, \tau_2 = h + \max{\{\tau_{t_k}\}} = h + \overline{\tau}$ , and  $\overline{\tau}$  is the upper bound of  $\tau_{t_k}$ .

When  $t \in \Delta_k^{l_k}$ , the state error  $e(i_k h)$  is defined as

$$e(i_kh) = x(i_kh) - x(t_kh).$$
<sup>(12)</sup>

When  $t \in \Delta_k^{t_k}$ , based on the combinations of (5), (10), and (12) and the combinations of (6), (10), and (12), we, respectively, obtain

$$e^{T}(i_{k}h)\Phi_{1}e(i_{k}h) \leq x^{T}(t-\tau(t))\Phi_{2}x(t-\tau(t)),$$
 (13)

$$e^{T}(i_{k}h)\Phi e(i_{k}h) \leq \sigma x^{T}(t-\tau(t))\Phi x(t-\tau(t)).$$
(14)

Based on the combination of (7), (10), and (12), u(t) is also written as

$$u(t) = \sum_{j=1}^{r} u_j(\theta(t)) K_j(x(t-\tau(t)) - e(i_k h)).$$
(15)

In consideration of general actuator failures [13], the model of control input with actuator failure is described as

$$u^{f}(t) = Lu(t).$$
<sup>(16)</sup>

Matrix *L* denotes the mode set of system actuator failures and describes the fault extent, where  $L = \text{diag}\{l_1, \ldots, l_m\}$ ,  $l_q \in [0, 1], q = 1, 2, \ldots, m; l_q = 0$  indicates that the *q*th system actuator is invalid;  $l_q \in (0, 1)$  implies that the *q*th system actuator is at fault to some extent; and  $l_q = 1$  denotes that the *q*th system actuator operates properly.

Through the combination of (1), (15), and (16), the nonlinear networked closed-loop fault systems (NNCFSs) model with actuator saturation constraints can be obtained based on the DETCS as follows:

$$\dot{x}(t) = \sum_{i=1}^{1} \sum_{j=1}^{1} \left[ \left( A_i + \Delta A_i \right) x(t) + \left( B_i + \Delta B_i \right) L \operatorname{sat} \left( K_j \left( x(t - \tau(t)) - e(i_k h) \right) \right) \right],$$
(17)

where  $t \in \Delta_k^{l_k}$  and the initial state x(t) is denoted by  $\Psi(t)$ , where  $t \in [-\tau_2, 0]$ . Meanwhile, set  $\Psi(0)$  as  $x_0$ , where  $\Psi(t)$  is a continuous function in the interval  $[-\tau_2, 0]$ .

*Remark 1.* The NNCSs model integrates many factors into a unified framework. These factors include the communication constraint condition, network time delay, actuator saturation, actuator failures, and the control law. The model lays a solid foundation for the following codesign of communication parameters and fault-tolerant controller for NNCSs.

2.2. Related Definition and Lemma. Before commencing the proof of the theorem, we present several related definitions and lemmas.

Definition 2. If the  $\alpha$  of  $\alpha$ -stability is defined as the system stability margin for a system without failure, then the  $\alpha$  of  $\alpha$ -stability can be extended as the system safety margin for a system with any possible actuator failures in mode set *L*; the system safety margin can also be abbreviated as  $\alpha$ -safety degree. The definition indicates that all the closed-loop poles of system  $s_i$  (i = 1, 2, ..., n) satisfy  $\operatorname{Re}(s_i) < -\alpha$  and  $\alpha > 0$  for the system with any possible actuator failures in mode set *L*. *Definition 3.* In the process of state transformation, if the following conditions are satisfied for the system with any possible actuator failures in mode set L,

- (1) the system possesses  $\alpha$ -safety degree,
- (2) the state trajectory whose initial state is from any point of set R<sup>n</sup> will converge to the equilibrium point; namely

$$\varphi_{a1} = \left\{ x_0 \in \mathbb{R}^n : \lim_{t \to \infty} \psi(t, x_0) = 0, \forall L \right\};$$
(18)

then  $\wp_{a1}$  is defined as fault-tolerant domain of attraction with  $\alpha$ -safety degree, where  $\psi(t, x_0)$  is the corresponding state trajectory.

*Definition 4.* In the process of state transformation, if the following conditions are satisfied for system with any possible actuator failures in mode set *L*,

- (1) the system possesses  $\alpha$ -safety degree,
- the state trajectory, the initial state of which is from any point of set *φ*<sub>α2</sub>, remains inside the set *φ*<sub>α2</sub>,

$$x_0 \in \varphi_{\alpha 2} \Longrightarrow x(t) \in \varphi_{\alpha 2}, \quad \forall t \ge 0, L.$$
 (19)

(3) the state trajectory, the initial state of which is from any point of set 
 *ρ*<sub>α2</sub> \ {0}, converges to the equilibrium point,

$$x_0 \in \wp_{\alpha 2} \setminus \{0\} \Longrightarrow \lim_{t \to \infty} \psi(t, x_0) = 0, \quad \forall L,$$
 (20)

then  $\varphi_{\alpha 2}$  is the contractively invariant set of fault tolerance with  $\alpha$ -safety degree, where  $\psi(t, x_0)$  is the corresponding state trajectory.

The contractively invariant set of fault tolerance with  $\alpha$ safety degree is within the fault-tolerant domain of attraction with  $\alpha$ -safety degree. In general, obtaining the corresponding fault-tolerant domain of attraction is difficult; thus, the fault-tolerant domain of attraction with  $\alpha$ -safety degree can be estimated in terms of the corresponding contractively invariant set of fault tolerance.

If  $\ell(F) = \{x_0 \in \mathbb{R}^n : |f_l x| \le 1, l = 1, 2, ..., m\}$ , where matrix  $F \in \mathbb{R}^{m \times n}$  and  $f_l$  denotes the *l*th row of matrix *F*, then  $\ell(F)$  is defined as the region where the feedback control  $u = \operatorname{sat}(Fx)$  is linear for *x*, as indicated in [30].

Based on an ellipsoid estimation of the domain of attraction,  $P \in \mathbb{R}^{n \times n}$  is a positive definite matrix. For  $\rho > 0$ , the ellipsoid is defined as  $\varepsilon(P, \rho) = \{x \in \mathbb{R}^n, x^T P x \le \rho\}$ , where  $\varepsilon(p)$  denotes  $\varepsilon(p, 1)$ .

**Lemma 5** (see [31]). *Given two feedback matrices*  $K \in \mathbb{R}^{m \times n}$ and  $F \in \mathbb{R}^{m \times n}$ , if  $x \in \ell(F)$ , then

sat 
$$(Kx) \in co \{ \Upsilon_i Kx + \Upsilon_i^- Fx : i = 1, 2, ..., 2^m \},$$
 (21)

where  $co\{\cdot\}$  denotes the convex hull of the linear feedback control group  $\Upsilon_i Kx + \Upsilon_i^- Fx$ ,  $\Upsilon_i \in \Upsilon$ ,  $i = 1, ..., 2^m$ ;  $\Upsilon$  denotes the set of  $m \times m$  diagonal matrices whose diagonal elements are either 1 or 0; and  $2^m$  elements exist in Y. If we suppose that each element of Y is labeled as  $Y_i$ , for  $i = 1, 2, ..., 2^m$ , then  $Y = \{Y_i : i \in [1, 2^m]\}$ . Define  $Y_i^- = I - Y_i$ ; clearly, if  $Y_i \in Y, Y_i^$ is also an element of Y.

**Lemma 6** (see [32]). For any constant matrices  $Z \in \mathbb{R}^{n \times n}$ ,  $Z = Z^T > 0$ , scalar  $\delta > 0$ , and vector function  $x : [0, \delta] \to \mathbb{R}^m$ , such that the integrations in the following are well defined:

$$\delta \int_{0}^{\delta} x^{T}(s) Zx(s) ds$$

$$\geq \left( \int_{0}^{\delta} x^{T}(s) ds \right)^{T} Z\left( \int_{0}^{\delta} x(s) ds \right).$$
(22)

**Lemma 7** (see [33]). For any constant matrix  $Z \in \mathbb{R}^{n \times n}$ ,  $Z = Z^T > 0$ , scalars  $\tau_1 \leq \tau(t) \leq \tau_2$ , and vector function  $\dot{x} : [-\tau_2, -\tau_1] \rightarrow \mathbb{R}^n$ , such that the following integration is well defined:

$$-(\tau_{2} - \tau_{1}) \int_{t-\tau_{2}}^{t-\tau_{1}} \dot{x}^{T}(v) Z\dot{x}(v) dv$$

$$\leq \sum_{i=1}^{2} \zeta_{i}^{T}(t) [1 + \pi_{i}] \begin{bmatrix} -Z & Z \\ Z & -Z \end{bmatrix} \zeta_{i}(t),$$
(23)

where  $\zeta_1(t) = \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \end{bmatrix}$ ,  $\zeta_2(t) = \begin{bmatrix} x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}$ ,

$$\varepsilon = \frac{\tau_2 - \tau(t)}{\tau(t) - \tau_1},$$

$$\begin{cases}
\pi_1 = -1, \ \pi_2 = 0, \quad \tau(t) = \tau_1, \\
\pi_1 = \varepsilon, \ \pi_2 = \frac{1}{\varepsilon}, \quad \tau_1 < \tau(t) < \tau_2, \\
\pi_1 = 0, \ \pi_2 = -1, \quad \tau(t) = \tau_2.
\end{cases}$$
(24)

**Lemma 8** (see [33]). For given matrices  $\Pi \in \mathbb{R}^{m \times m}$ ,  $\zeta(t) \in \mathbb{R}^{m}$ ,  $\Omega_1 \in \mathbb{R}^{m \times m}$ ,  $\Omega_2 \in \mathbb{R}^{m \times m}$ ,  $\Omega_1 \leq 0$ , and  $\Omega_2 \leq 0$ , scalars  $\tau_1 \leq \tau(t) \leq \tau_2$ ; if

$$\zeta^{T}(t) \left[\Pi + 3\Omega_{1} + \Omega_{2}\right] \zeta(t) \leq 0,$$

$$\zeta^{T}(t) \left[\Pi + \Omega_{1} + 3\Omega_{2}\right] \zeta(t) \leq 0$$
(25)

then

$$\zeta^{T}(t) \left\{ \Pi + [1 + \pi_{1}] \Omega_{1} + [1 + \pi_{2}] \Omega_{2} \right\} \zeta(t) \le 0, \qquad (26)$$

where  $\pi_1$  and  $\pi_2$  are defined in Lemma 7. If  $\tau(t) = \tau_1$ , then  $\zeta^T(t)\Omega_1\zeta(t) \equiv 0$ ; if  $\tau(t) = \tau_2$ , then  $\zeta^T(t)\Omega_2\zeta(t) \equiv 0$ .

**Lemma 9** (see [34]). Given matrices Y, M, and E with appropriate dimensions and  $Y = Y^T$ , then

$$Y + MF(t) E + E^{T}F^{T}(t) M^{T} < 0, \quad \forall F : F^{T}F \le I$$
 (27)

*holds if and only if, for some scalar*  $\varepsilon > 0$ *,* 

$$Y + \varepsilon M M^T + \varepsilon^{-1} E^T E < 0.$$
<sup>(28)</sup>

#### 3. Main Results

3.1. Goal of Codesign between Network Communication and the Robust Fault-Tolerant Control for Uncertain NNCSs. Based on event-triggered condition (5) or (6) under the DETCS, when we consider the actuator saturation constraints and actuator failures, the goal of codesign between network communication and the robust fault-tolerant control for uncertain NNCSs with  $\alpha$ -safety degree is to seek the statefeedback controller gain  $K_j$  (j = 1, 2, ..., r) and discrete event-triggered weight matrices,  $\Phi_1$  and  $\Phi_2$ , or  $\Phi$ , which can ensure that the NNCFSs (17) satisfies the following conditions.

- With regard to the allowable uncertainty of parameters, NNCFSs (17) possesses α-safety degree.
- (2) Based on the premise of satisfying the preceding condition, the occupancy rate of network communication resource is ensured to be as low as possible.

*3.2. Condition of Invariant Set.* We initially use event-triggered condition (5) to expand the work in related research.

**Theorem 10.** We consider the following parameters: under the event-triggered condition (5) in the DETCS, in consideration of system (17), for the given constants  $\tau_1$ ,  $\tau_2$ ,  $\overline{\tau}$ , h, and  $\alpha$  and the given matrices  $K_j$  (j = 1, 2, ..., r),  $\Phi_1$ , and  $\Phi_2$ , exist some matrices,  $P = P^T > 0$ ,  $Z_1 = Z_1^T > 0$ ,  $Z_2 = Z_2^T > 0$ ,  $Z_3 = Z_3^T > 0$ ,  $Q_1 > 0$ , and  $Q_2 > 0$ . If these parameters satisfy the following matrix inequalities ( $\varepsilon = 0, 1$ ) and  $\varepsilon(P) \subset \ell(F)$  for any possible actuator failures in mode set L and any acceptable uncertainty of system parameters,

$$\Sigma_{1} = \begin{bmatrix} \overline{\Xi}_{11}^{ij}(\varepsilon) & \overline{\Xi}_{12}^{ij} \\ * & \overline{\Xi}_{22} \end{bmatrix} < 0,$$
(29)

then NNCFSs (17) with actuator saturation keeps asymptotically stable in the domain of attraction  $\varepsilon(P)$  and possesses  $\alpha$ -safety degree. That is, (15) denotes the robust fault-tolerant control law which can make NNCFSs (17) possess  $\alpha$ -safety degree and low occupancy rate of network resource, where

$$\begin{split} \overline{\Xi}_{11}^{ij}\left(\varepsilon\right) &= \overline{\Xi}_{11}^{ij} + 2\left(1-\varepsilon\right)\overline{\Omega}_{1} + \overline{\Omega}_{1} + 2\varepsilon\overline{\Omega}_{2} + \overline{\Omega}_{2}, \\ \overline{\Xi}_{11}^{ij} &= \begin{bmatrix} \overline{\Gamma}_{11}^{ij} & \overline{\Gamma}_{12} & \overline{\Gamma}_{13}^{ij} & \overline{\Gamma}_{14} & \overline{\Gamma}_{15}^{ij} \\ * & \overline{\Gamma}_{22} & 0 & 0 & 0 \\ * & * & \overline{\Gamma}_{33} & 0 & 0 \\ * & * & * & \overline{\Gamma}_{44} & 0 \\ * & * & * & * & \overline{\Gamma}_{55} \end{bmatrix}, \\ \overline{\Gamma}_{11}^{ij} &= P\overline{A} + \overline{A}^{T}P + Q_{1} - Z_{1} - Z_{2}, \\ \overline{\Gamma}_{12} &= Z_{1}, \\ \overline{\Gamma}_{13}^{ij} &= P\overline{B}\left\{\gamma_{q}K_{j} + \gamma_{q}^{-}F_{j}\right\}, \end{split}$$

*Proof.* To ensure that system (17) possesses  $\alpha$ -safety degree, the state transformation  $x(t) = \exp(-\alpha t)\eta(t)$  must be

(30)

introduced into the proof. Based on Lemma 5, when  $\varepsilon(P) \subset$  $\ell(F)$ , then

$$\dot{\eta}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} u_i(\theta(t)) u_j(\theta(t)) \left[ \overline{A}\eta(t) + \overline{B} \cos\left(\gamma_q K_j + \gamma_q^- F_j\right) \eta(t - \tau(t)) - \overline{B} \cos\left(\gamma_q K_j + \gamma_q^- F_j\right) e_\alpha(i_k h) \right],$$
(31)

where  $\overline{A} = A_i + \Delta A_i + \alpha I$ ,  $\overline{B} = \exp(\alpha \tau(t))(B_i + \Delta B_i)L$ , and  $\exp(\alpha t)e(i_kh) = \exp(\alpha \tau(t))e_{\alpha}(i_kh).$ 

According to Definition 2, if system (31) is asymptotically stable, then system (17) possesses  $\alpha$ -safety degree.

Construct the following Lyapunov-Krasovskii function of system (31) for  $t \in \Delta_k^{l_k}$ 

$$V(\eta(t)) = \eta^{T}(t) P\eta(t) + \int_{t-\tau_{1}}^{t} \eta^{T}(s) Q_{1}\eta(s) ds$$
  
+  $\int_{t-\tau_{2}}^{t-\tau_{1}} \eta^{T}(s) Q_{2}\eta(s) ds$   
+  $\tau_{1} \int_{-\tau_{1}}^{0} \int_{t+\theta}^{t} \dot{\eta}^{T}(s) Z_{1}\dot{\eta}(s) ds d\theta$  (32)  
+  $\tau_{2} \int_{-\tau_{2}}^{0} \int_{t+\theta}^{t} \dot{\eta}^{T}(s) Z_{2}\dot{\eta}(s) ds d\theta$   
+  $(\tau_{2} - \tau_{1}) \int_{-\tau_{2}}^{-\tau_{1}} \int_{t+\theta}^{t} \dot{\eta}^{T}(s) Z_{3}\dot{\eta}(s) ds d\theta,$ 

where  $P^T = P > 0$ ,  $Q_1 > 0$ ,  $Q_2 > 0$ ,  $Z_1 = Z_1^T > 0$ ,  $Z_2 = Z_2^T > 0$ , and  $Z_3 = Z_3^T > 0$ . Taking the difference of V(t) along the trajectory of (31),

we obtain

$$\begin{split} \dot{V}(\eta(t)) &= 2\eta^{T}(t) P\dot{\eta}(t) + \eta^{T}(t) Q_{1}\eta(t) \\ &- \eta^{T}(t - \tau_{1}) Q_{1}\eta(t - \tau_{1}) \\ &+ \eta^{T}(t - \tau_{1}) Q_{2}\eta(t - \tau_{1}) \\ &- \eta^{T}(t - \tau_{2}) Q_{2}\eta(t - \tau_{2}) \\ &+ \tau_{1}^{2}\dot{\eta}^{T}(t) Z_{1}\dot{\eta}(t) \\ &- \tau_{1} \int_{t - \tau_{1}}^{t} \dot{\eta}^{T}(s) Z_{1}\dot{\eta}(s) ds \\ &+ \tau_{2}^{2}\dot{\eta}^{T}(t) Z_{2}\dot{\eta}(t) \\ &- \tau_{2} \int_{t - \tau_{2}}^{t} \dot{\eta}^{T}(s) Z_{2}\dot{\eta}(s) ds \\ &+ (\tau_{2} - \tau_{1})^{2} \dot{\eta}^{T}(t) Z_{3}\dot{\eta}(t) \\ &- (\tau_{2} - \tau_{1}) \int_{t - \tau_{2}}^{t - \tau_{1}} \dot{\eta}^{T}(s) Z_{3}\dot{\eta}(s) ds \end{split}$$

$$+ e_{\alpha}^{T}(i_{k}h) \Phi_{1}e_{\alpha}(i_{k}h)$$
$$- e_{\alpha}^{T}(i_{k}h) \Phi_{1}e_{\alpha}(i_{k}h).$$
(33)

According to Lemma 5, we obtain

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$$2\eta^{T}(t) P\dot{\eta}(t) \leq \max_{q \in [1,...,2^{m}]} \left\{ \sum_{i=1}^{r} \sum_{j=1}^{r} u_{i}(\theta(t)) u_{j}(\theta(t)) \left[ 2\eta^{T}(t) P\overline{A}\eta(t) + 2\eta^{T}(t) P\overline{B} \left( \gamma_{q}K_{j} + \gamma_{q}^{-}F_{j} \right) \eta(t - \tau(t)) - 2\eta^{T}(t) P\overline{B} \left( \gamma_{q}K_{j} + \gamma_{q}^{-}F_{j} \right) e_{\alpha}(i_{k}h) \right] \right\}.$$

$$(34)$$

According to Lemma 6, we obtain

$$-\tau_{1} \int_{t-\tau_{1}}^{t} \dot{\eta}^{T}(s) Z_{1} \dot{\eta}(s) ds$$

$$\leq \begin{bmatrix} \eta(t) \\ \eta(t-\tau_{1}) \end{bmatrix}^{T} \begin{bmatrix} -Z_{1} & Z_{1} \\ * & -Z_{1} \end{bmatrix} \begin{bmatrix} \eta(t) \\ \eta(t-\tau_{1}) \end{bmatrix},$$

$$-\tau_{2} \int_{t-\tau_{2}}^{t} \dot{\eta}^{T}(s) Z_{2} \dot{\eta}(s) ds$$

$$\leq \begin{bmatrix} \eta(t) \\ \eta(t-\tau_{2}) \end{bmatrix}^{T} \begin{bmatrix} -Z_{2} & Z_{2} \\ * & -Z_{2} \end{bmatrix} \begin{bmatrix} \eta(t) \\ \eta(t-\tau_{2}) \end{bmatrix}.$$
(35)

According to Lemma 7, we have

$$-\left(\tau_{2}-\tau_{1}\right)\int_{t-\tau_{2}}^{t-\tau_{1}}\dot{\eta}^{T}\left(s\right)Z_{3}\dot{\eta}\left(s\right)ds\leq\left(1+\pi_{1}\right)$$

$$\left[ \begin{array}{c} \eta\left(t-\tau_{1}\right)\\ \eta\left(t-\tau\left(t\right)\right) \end{array} \right]^{T} \left[ \begin{array}{c} -Z_{3} & Z_{3}\\ Z_{3} & -Z_{3} \end{array} \right] \left[ \begin{array}{c} \eta\left(t-\tau_{1}\right)\\ \eta\left(t-\tau\left(t\right)\right) \end{array} \right]$$
$$+ \left(1+\pi_{2}\right) \left[ \begin{array}{c} \eta\left(t-\tau\left(t\right)\right)\\ \eta\left(t-\tau_{2}\right) \end{array} \right]^{T} \\\\ \cdot \left[ \begin{array}{c} -Z_{3} & Z_{3}\\ Z_{3} & -Z_{3} \end{array} \right] \left[ \begin{array}{c} \eta\left(t-\tau\left(t\right)\right)\\ \eta\left(t-\tau_{2}\right) \end{array} \right] .$$
(36)

When  $t \in \Delta_k^{l_k}$ , according to (13) and  $e_{\alpha}(i_k h) = \exp(\alpha(t - \tau(t)))e(i_k h)$ , we have

$$e_{\alpha}^{T}\left(i_{k}h\right)\Phi_{1}e_{\alpha}\left(i_{k}h\right) \leq \eta^{T}\left(t-\tau\left(t\right)\right)\Phi_{2}\eta\left(t-\tau\left(t\right)\right).$$
(37)

Combining (34) with (37), we have

$$\dot{V}(\eta(t)) \leq \max_{q \in [1, \dots, 2^{m}]} \left\{ \sum_{i=1}^{r} \sum_{j=1}^{r} u_{i}(\theta(t)) u_{j}(\theta(t)) \left[ \xi^{T}(t) \left( \overline{\Xi}_{11}^{ij} + (1+\pi_{1}) \overline{\Omega}_{1} + (1+\pi_{2}) \overline{\Omega}_{2} - \overline{\Xi}_{12}^{ij} \overline{\Xi}_{22}^{-1} \left( \overline{\Xi}_{12}^{ij} \right)^{T} \right) \xi(t) \right] \right\}, \quad (38)$$

where

$$\begin{aligned} \xi^{T}(t) \\ &= \left[ \eta^{T}(t) \quad \eta^{T}(t-\tau_{1}) \quad \eta^{T}(t-\tau(t)) \quad \eta^{T}(t-\tau_{2}) \quad e_{\alpha}^{T}(i_{k}h) \right], \\ &= \left[ \overrightarrow{\Gamma}_{11}^{ij} \quad \overrightarrow{\Gamma}_{12} \quad \overrightarrow{\Gamma}_{13}^{ij} \quad \overrightarrow{\Gamma}_{14} \quad \overrightarrow{\Gamma}_{15}^{ij} \\ &* \quad \overrightarrow{\Gamma}_{22} \quad 0 \quad 0 \quad 0 \\ &* \quad * \quad \overrightarrow{\Gamma}_{33} \quad 0 \quad 0 \\ &* \quad * \quad * \quad \overrightarrow{\Gamma}_{33} \quad 0 \quad 0 \\ &* \quad * \quad * \quad * \quad \overrightarrow{\Gamma}_{55} \end{array} \right], \end{aligned}$$
(39)

where  $\overline{\Xi}_{11}^{ij}$ ,  $\overline{\Xi}_{12}^{ij}$ , and  $\overline{\Xi}_{22}^{-1}$  are the same as the corresponding element of Theorem 10.

If the following inequality is satisfied,

$$\overline{\Xi}_{11}^{ij} + (1 + \pi_1) \overline{\Omega}_1 + (1 + \pi_2) \overline{\Omega}_2 - \overline{\Xi}_{12}^{ij} \overline{\Xi}_{22}^{-1} \left(\overline{\Xi}_{12}^{ij}\right)^T$$

$$< 0, \qquad (40)$$

then system (31) is asymptotically stable, in accordance with the Lyapunov stable theory; that is, uncertain NNCFSs (17) possesses  $\alpha$ -safety degree.

According to Lemma 8, inequality (40) is equivalent to

$$\begin{split} \overline{\Xi}_{11}^{ij} + 3\overline{\Omega}_1 + \overline{\Omega}_2 - \overline{\Xi}_{12}^{ij}\overline{\Xi}_{22}^{-1} \left(\overline{\Xi}_{12}^{ij}\right)^T < 0, \\ \overline{\Xi}_{11}^{ij} + \overline{\Omega}_1 + 3\overline{\Omega}_2 - \overline{\Xi}_{12}^{ij}\overline{\Xi}_{22}^{-1} \left(\overline{\Xi}_{12}^{ij}\right)^T < 0; \end{split}$$
(41)

that is,

$$\overline{\Xi}_{11}^{ij}(\varepsilon) - \overline{\Xi}_{12}^{ij}\overline{\Xi}_{22}^{-1} \left(\overline{\Xi}_{12}^{ij}\right)^T < 0, \quad \varepsilon = 0, 1,$$
(42)

where  $\overline{\Xi}_{11}^{ij}(\varepsilon) = \overline{\Xi}_{11}^{ij} + 2(1-\varepsilon)\overline{\Omega}_1 + \overline{\Omega}_1 + 2\varepsilon\overline{\Omega}_2 + \overline{\Omega}_2$ . We then obtain (29) by applying the Schur complement.

Therefore, if (29) and  $\varepsilon(P) \subset \ell(F)$  are satisfied, then NNCFSs (17) possesses  $\alpha$ -safety degree; moreover, the ellipsoid  $\varepsilon(P)$  is the invariant set for system (17). That is, feedback control

law (15) can make NNCFSs (17) with actuator saturation remain inside the domain of attraction  $\varepsilon(P)$  and possess  $\alpha$ safety degree based on the DETCS. Furthermore, the system possesses as little occupancy of network resource as possible. The proof is hereby completed.

*Remark 11.* The information in the inequalities of Theorem 10 is of four classes. The first class is the two event-triggered weight matrices  $\Phi_1$  and  $\Phi_2$  in the DETCS, which can limit the quantity of network communication resource. The second class is the upper and lower bounds of networked time delays  $\tau_1$  and  $\tau_2$ , which can denote the property of the network. The third class is the system safety degree  $\alpha$ , which can reflect system performance. The fourth class is the fault-tolerant controller gain *K*, which can make the system possess  $\alpha$ safety degree. The inner relation for codesign between the robust fault-tolerant controller and event-triggered matrices for NNCSs is thus established.

3.3. Codesign Method. If we know the related parameters of system (1), the upper and lower bounds of the networked time delay, and the given  $\alpha$ -safety degree, then the robust fault-tolerant controller gain matrix  $K_j$  (j = 1, 2, ..., r) and the event-triggered weight matrices,  $\Phi_1$  and  $\Phi_2$ , may be obtained according to Theorem 12 in terms of linear matrix inequality. As is well known, linear matrix inequality is an effective and convenient way to solve controller [35].

**Theorem 12.** One considers the following parameters: under the event-triggered condition (5) in the DETCS, in consideration of system (17), for the given constants  $\tau_1$ ,  $\tau_2$ ,  $\overline{\tau}$ , h, and  $\alpha$ , exist some matrices,  $R_1 > 0$ ,  $R_2 > 0$ ,  $\widetilde{K}_j$  (j = 1, 2, ..., r),  $X = X^T > 0$ ,  $V_1 = V_1^T > 0$ ,  $V_2 = V_2^T > 0$ ,  $R_3 = R_3^T > 0$ ,  $R_4 = R_4^T > 0$ , and  $R_5 = R_5^T > 0$ , and a positive real scalar  $\varepsilon_{ij} > 0$  (i, j = 1, 2, ..., r). If these parameters satisfy the following linear matrix inequalities ( $\varepsilon = 0, 1$ ) for any possible actuator failures in mode set L and any acceptable uncertainty of system parameters

$$\Sigma_{2} = \begin{bmatrix} \widetilde{\Xi}_{11}^{ij} (\varepsilon) & \widetilde{\Xi}_{12}^{ij} & \widetilde{\Xi}_{13}^{ij} \\ * & \widetilde{\Xi}_{22} & \widetilde{\Xi}_{23} \\ * & * & \widetilde{\Xi}_{33}^{ij} \end{bmatrix} < 0,$$
(43)  
1  $\tilde{f}_{1}$ ]

$$\begin{bmatrix} 1 & f_l \\ * & X \end{bmatrix} \ge 0, \quad l \in [1, m],$$
(44)

then there is a feedback control law which can make the state trajectories of NNCFSs (17) with actuator saturation remain inside the ellipsoid  $\varepsilon(P)$  and can make the system possess  $\alpha$ safety degree. Furthermore, we can possibly obtain the robust fault-tolerant controller gain  $K_j$  and the event-triggered weight matrices  $\Phi_1$  and  $\Phi_2$  through  $K_j = \widetilde{K}_j X^{-1}$ ,  $\Phi_1 = V_1^{-1}$ , and  $\Phi_2 = V_2^{-1}$ , where

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$$\widetilde{\Xi}_{11}^{ij}\left(\varepsilon\right)=\widetilde{\Xi}_{11}^{ij}+2\left(1-\varepsilon\right)\widetilde{\Omega}_{1}+\widetilde{\Omega}_{1}+2\varepsilon\widetilde{\Omega}_{2}+\widetilde{\Omega}_{2},$$

$$\begin{split} \widetilde{\Xi}_{11}^{ij} &= \begin{bmatrix} \widetilde{\Gamma}_{11}^{ij} & \widetilde{\Gamma}_{12} & \widetilde{\Gamma}_{13}^{ij} & \widetilde{\Gamma}_{14} & \widetilde{\Gamma}_{15}^{ij} \\ * & \widetilde{\Gamma}_{22} & 0 & 0 & 0 \\ * & * & \widetilde{\Gamma}_{33} & 0 & 0 \\ * & * & * & \widetilde{\Gamma}_{44} & 0 \\ * & * & * & \widetilde{\Gamma}_{44} & 0 \\ * & * & * & * & \widetilde{\Gamma}_{55} \end{bmatrix}, \\ \widetilde{\Gamma}_{11}^{ij} &= X \left( A_i + aI \right)^T + \left( A_i + aI \right) X - 2X - R_1 + R_3 \\ & + R_4, \\ \widetilde{\Gamma}_{12} = 2X - R_3, \\ \widetilde{\Gamma}_{13}^{ij} = \exp \left( a\tau \left( t \right) \right) B_i L \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}, \\ \widetilde{\Gamma}_{14} = 2X - R_4, \\ \widetilde{\Gamma}_{15}^{ij} = -\exp \left( a\tau \left( t \right) \right) B_i L \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}, \\ \widetilde{\Gamma}_{22} = -2X + R_1 - R_2 + R_3, \\ \widetilde{\Gamma}_{33} = 2X - V_2, \\ \widetilde{\Gamma}_{44} = -4X + R_2 + R_4, \\ \widetilde{\Gamma}_{55} = -2X + V_1, \\ \widetilde{\Xi}_{12}^{ij} &= \begin{bmatrix} \widetilde{\Upsilon}_{11} & \widetilde{\Upsilon}_{12} & \widetilde{\Upsilon}_{13} \\ 0 & 0 & 0 \\ \widetilde{\Upsilon}_{31} & \widetilde{\Upsilon}_{32} & \widetilde{\Upsilon}_{33} \\ 0 & 0 & 0 \\ \widetilde{\Upsilon}_{51} & \widetilde{\Upsilon}_{52} & \widetilde{\Upsilon}_{53} \end{bmatrix}, \\ \widetilde{\Upsilon}_{11} = \tau_1 X \left( A_i + aI \right)^T, \\ \widetilde{\Upsilon}_{12} = \tau_2 X \left( A_i + aI \right)^T, \\ \widetilde{\Upsilon}_{31} = (\tau_2 - \tau_1) X \left( A_i + aI \right)^T, \\ \widetilde{\Upsilon}_{32} = \tau_2 \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{32} = \tau_2 \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{51} = -\tau_1 \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{52} = -\tau_2 \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{52} = -\tau_2 \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a\tau \left( t \right) \right) \left\{ \gamma_q \widetilde{K}_j + \gamma_q^- \widetilde{F}_j \right\}^T L^T B_i^T, \\ \widetilde{\Upsilon}_{53} = - \left( \tau_2 - \tau_1 \right) \exp \left( a$$

*Proof.* Substituting  $\overline{A} = A_i + \Delta A_i + \alpha I$ ,  $\overline{B} = \exp(\alpha \tau(t))(B_i + \Delta B_i)L$ , and (2) into (29), according to Lemma 9, we have

$$\Sigma_1 = \Sigma_3 + \varepsilon_{ij}^{-1} \Psi \Psi^T + \varepsilon_{ij} \Psi_* \Psi_*^T.$$
(46)

(45)

The related expressions of  $\Sigma_3$ ,  $\Psi$ , and  $\Psi_*$  are omitted due to the limited space.

According to the Schur complement, we obtain

$$\Sigma_{4} = \begin{bmatrix} \Xi_{11}^{ij'}(\varepsilon) & \Xi_{12}^{ij'} & \Xi_{13}^{ij'} \\ * & \Xi_{22}' & \Xi_{23}' \\ * & * & \Xi_{33}^{ij'} \end{bmatrix},$$
(47)

where the related expressions of  $\Xi_{11}^{ij'}(\varepsilon)$ ,  $\Xi_{12}^{ij'}$ ,  $\Xi_{13}^{ij'}$ ,  $\Xi_{22}^{\prime}$ ,  $\Xi_{23}^{\prime}$ , and  $\Xi_{33}^{ij'}$  are also omitted. To solve matrix inequality (47) conveniently, transforming matrix inequality (47) into the corresponding linear matrix inequality through congruent transformation is necessary. Before and after multiplying (47) with  $J_1 = \text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, I, I\}$ , we have

$$\Sigma_{5} = \begin{bmatrix} \Xi_{11}^{ij} * (\varepsilon) & \Xi_{12}^{ij} * & \Xi_{13}^{ij} * \\ * & \Xi_{22}^{*} & \Xi_{23}^{*} \\ * & * & \Xi_{33}^{ij} * \end{bmatrix},$$
(48)

where the related expressions of  $\Xi_{11}^{ij}^{ij}(\varepsilon)$ ,  $\Xi_{12}^{ij}^{ij}$ ,  $\Xi_{13}^{ij}^{ij}$ ,  $\Xi_{22}^{*}$ ,  $\Xi_{23}^{*}$ , and  $\Xi_{33}^{ij}^{ij}$  are also omitted.

We define  $P^{-1} = X$ ,  $\tilde{K}_j = K_j X$ ,  $\tilde{F}_j = F_j X$ ,  $Q_1^{-1} = R_1$ ,  $Q_2^{-1} = R_2$ ,  $Z_1^{-1} = R_3$ ,  $Z_2^{-1} = R_4$ ,  $Z_3^{-1} = R_5$ ,  $\Phi_1^{-1} = V_1$ , and  $\Phi_2^{-1} = V_2$ .

For the matrix N with appropriate dimension, if  $(N^{-1} - P^{-1})N(N^{-1} - P^{-1}) \ge 0$  for  $N^{-1} > 0$ , then  $-P^{-1}NP^{-1} \le N^{-1} - 2P^{-1}$ . Therefore, we obtain

$$\begin{aligned} &-P^{-1}Q_{1}P^{-1} \leq -2P^{-1} + Q_{1}^{-1} = -2X + R_{1}, \\ &-P^{-1}Q_{2}P^{-1} \leq -2P^{-1} + Q_{2}^{-1} = -2X + R_{2}, \\ &-P^{-1}Z_{1}P^{-1} \leq -2P^{-1} + Z_{1}^{-1} = -2X + R_{3}, \\ &-P^{-1}Z_{2}P^{-1} \leq -2P^{-1} + Z_{2}^{-1} = -2X + R_{4}, \\ &-P^{-1}Z_{3}P^{-1} \leq -2P^{-1} + Z_{3}^{-1} = -2X + R_{5}, \\ &-P^{-1}\Phi_{1}P^{-1} \leq -2P^{-1} + \Phi_{1}^{-1} = -2X + V_{1}, \\ &-P^{-1}\Phi_{2}P^{-1} \leq -2P^{-1} + \Phi_{2}^{-1} = -2X + V_{2}. \end{aligned}$$

According to (49), we obtain linear matrix inequality (43). Performing transformation for the linear domain condition of the feedback control system with actuator saturation, we obtain

$$\varepsilon(P) \subset \ell(F) \iff |f_l X| \le 1, \quad \forall x \in \varepsilon(P),$$
 (50)

where  $f_l$  is the *l*th row of matrix *F* for  $l \in [1, m]$ . Consider

$$\iff f_l P^{-1} f_l^T \le 1. \tag{51}$$

Furthermore, applying the Schur complement, we have

$$\longleftrightarrow \begin{bmatrix} 1 & f_l P^{-1} \\ * & P^{-1} \end{bmatrix} \ge 0, \quad l \in [1, m].$$
 (52)

As previously defined,  $P^{-1} = X$  and  $F_j X = \tilde{F}_j$ , (52) is equivalent to (44). When the system parameters satisfy (43) and (44), control law (15) can make the state trajectories of NNCFSs (17) with actuator saturation remain inside the ellipsoid  $\varepsilon(P)$  and possess  $\alpha$ -safety degree. Meanwhile, obtaining the robust fault-tolerant controller gain matrix and the event-triggered weight matrices is practicable through  $K = \tilde{K}X^{-1}$ ,  $\Phi_1 = V_1^{-1}$ , and  $\Phi_2 = V_2^{-1}$ . The proof is hereby completed.

*Remark 13.* If we select a different value for the system safety degree  $\alpha$ , codesign between the robust fault-tolerant controller and event-triggered weight matrix can be obtained under a different  $\alpha$ -safety degree according to Theorems 10 and 12. When  $\alpha = 0$ , Theorems 10 and 12 will degenerate to the robust integrity design criterion for NNCFSs (17) with actuator saturation under the DETCS.

*Remark 14.* When  $\Phi_2 = 0$ , Theorems 10 and 12 will degenerate to the robust fault-tolerant control problem in the PTTCS. If we cease to consider the actuator saturation constraints and set  $\Phi_2$  as 0, the similar results in [18] can be obtained in terms

of Theorems 10 and 12. We have demonstrated that the result in [18] is only a special case of the proposed codesign method.

Similarly, based on event-triggered condition (6), if we study the analogous robust fault-tolerant design problem for NNCFSs (17) with actuator saturation, then Theorems 15 and 16 are presented as follows.

**Theorem 15.** One considers the following parameters: under event-triggered condition (6) in the DETCS, in consideration of system (17), for the given constants,  $\tau_1$ ,  $\tau_2$ ,  $\overline{\tau}$ , h,  $\sigma$ , and  $\alpha$ , and the given matrices,  $K_j$  (j = 1, 2, ..., r) and  $\Phi$ , exist some matrices,  $P = P^T > 0$ ,  $Z_1 = Z_1^T > 0$ ,  $Z_2 = Z_2^T > 0$ ,  $Z_3 = Z_3^T >$ 0,  $Q_1 > 0$ , and  $Q_2 > 0$ . If these parameters satisfy the following matrix inequalities ( $\varepsilon = 0, 1$ ) and  $\varepsilon(P) \subset \ell(F)$  for any possible actuator failures in mode set L and any acceptable uncertainty of system parameters

$$\Sigma_{6} = \begin{bmatrix} \widehat{\Xi}_{11}^{ij}(\varepsilon) & \widehat{\Xi}_{12}^{ij} \\ * & \widehat{\Xi}_{22} \end{bmatrix} < 0,$$
(53)

then NNCFSs (17) with actuator saturation keeps asymptotically stable in the domain of attraction  $\varepsilon(P)$  and possesses  $\alpha$ safety degree. That is, (15) is the robust fault-tolerant control law which can make NNCFSs (17) possess  $\alpha$ -safety degree and low occupancy rate of network resource, where

$$\widehat{\Xi}_{11}^{ij}(\varepsilon) = \widehat{\Xi}_{11}^{ij} + 2(1-\varepsilon)\widehat{\Omega}_{1} + \widehat{\Omega}_{1} + 2\varepsilon\widehat{\Omega}_{2} + \widehat{\Omega}_{2},$$

$$\widehat{\Xi}_{11}^{ij} = \begin{bmatrix}
\widehat{\Xi}_{11}^{ij} \quad \widehat{\Gamma}_{12} \quad \widehat{\Gamma}_{13}^{ij} \quad \widehat{\Gamma}_{14} \quad \widehat{\Gamma}_{15}^{ij} \\
\ast \quad \widehat{\Gamma}_{22} \quad 0 \quad 0 \quad 0 \\
\ast \quad \ast \quad \widehat{\Gamma}_{33} \quad 0 \quad 0 \\
\ast \quad \ast \quad \ast \quad \widehat{\Gamma}_{44} \quad 0 \\
\ast \quad \ast \quad \ast \quad \ast \quad \widehat{\Gamma}_{55}
\end{bmatrix},$$
(54)

 $\widehat{\Gamma}_{33} = \sigma \Phi$ ,  $\widehat{\Gamma}_{55} = -\Phi$ , and the remaining elements are the same as the corresponding element of Theorem 10.

**Theorem 16.** One considers the following parameters: under event-triggered condition (6) in the DETCS, in consideration of system (17), for the given constants,  $\tau_1, \tau_2, \overline{\tau}, h, \sigma$ , and  $\alpha$ , exist some matrices,  $R_1 > 0$ ,  $R_2 > 0$ ,  $\widetilde{K}_j$  (j = 1, 2, ..., r),  $X = X^T >$  $0, V = V^T > 0$ ,  $R_3 = R_3^T > 0$ ,  $R_4 = R_4^T > 0$ , and  $R_5 =$  $R_5^T > 0$ , and positive real scalar  $\varepsilon_{ij} > 0$  (i, j = 1, 2, ..., r). If these parameters satisfy the following linear matrix inequalities  $(\varepsilon = 0, 1)$  for any possible actuator failures in mode set L and any acceptable uncertainty of system parameters

$$\Sigma_{7} = \begin{bmatrix} \widehat{\Xi}_{11}^{ij} (\varepsilon) & \widehat{\Xi}_{12}^{ij} & \widehat{\Xi}_{13}^{ij} \\ * & \widehat{\Xi}_{22} & \widehat{\Xi}_{23} \\ * & * & \widehat{\Xi}_{33}^{ij} \end{bmatrix} < 0, \\ \begin{bmatrix} 1 & \tilde{f}_{l} \\ * & X \end{bmatrix} \ge 0, \quad l \in [1, m],$$
(55)

then there is feedback control law which can make the state trajectories of NNCFSs (17) with actuator saturation remain

inside the ellipsoid  $\varepsilon(P)$  and can make the system possess  $\alpha$ -safety degree. Furthermore, we may possibly obtain the robust fault-tolerant controller gain  $K_j$  and the event-triggered weight matrix  $\Phi$  through  $K_j = \widetilde{K}_j X^{-1}$ ,  $\Phi = V^{-1}$ , where

$$\widehat{\Xi}_{11}^{ij}(\varepsilon) = \widehat{\Xi}_{11}^{ij} + 2(1-\varepsilon)\widehat{\Omega}_1 + \widehat{\Omega}_1 + 2\varepsilon\widehat{\Omega}_2 + \widehat{\Omega}_2,$$

$$\widehat{\Xi}_{11}^{ij} = \begin{bmatrix}
\widehat{\Gamma}_{11}^{ij} \quad \widehat{\Gamma}_{12} \quad \widehat{\Gamma}_{13}^{ij} \quad \widehat{\Gamma}_{14} \quad \widehat{\Gamma}_{15}^{ij} \\
* \quad \widehat{\Gamma}_{22} \quad 0 \quad 0 \quad 0 \\
* \quad * \quad \widehat{\Gamma}_{33} \quad 0 \quad 0 \\
* \quad * \quad * \quad \widehat{\Gamma}_{44} \quad 0 \\
* \quad * \quad * \quad * \quad \widehat{\Gamma}_{55}
\end{bmatrix},$$
(56)

 $\widehat{\Gamma}_{33} = \sigma(2X - V), \ \widehat{\Gamma}_{55} = -2X + V$ , and the remaining elements are the same as the corresponding element of Theorem 12.

We omit the special proof detail for Theorems 15 and 16 due to the limited space.

*Remark 17.* When  $\sigma = 0$ , Theorems 15 and 16 will degenerate to the robust fault-tolerant control problem in the PTTCS. If we cease to consider actuator saturation constraints and set  $\sigma$  as 0, similar results in [18] can be obtained in terms of Theorems 15 and 16.

#### 4. Simulation Experiment and Result Analysis

4.1. Simulation Experiment. Given the uncertain NNCSs model in [16], if we select the fuzzy membership function as  $M_1(x_2) = \sin^2 x_2$  and  $M_2(x_2) = \cos^2 x_2$ , the system model is expressed as the following T-S fuzzy system of two rules.

Rule (i) is as follows: if  $x_2$  is  $M_i$ , then

$$\dot{x}(t) = (A_i + \Delta A_i) x(t) + (B_i + \Delta B_i) \operatorname{sat} (u(t))$$

$$i = 1, 2,$$
(57)

where

$$A_{1} = \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix},$$

$$A_{2} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix},$$

$$B_{1} = \begin{bmatrix} 1 & 0 \\ 0 & -0.5 \end{bmatrix},$$

$$B_{2} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}.$$
(58)

Matrices  $\Delta A_i$  and  $\Delta B_i$  (i = 1, 2) satisfy (2). Additionally, we set

$$M = \begin{bmatrix} 0.31 & 0.1 \\ 0 & 0 \end{bmatrix},$$

$$F(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix},$$

$$E_{ai} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix},$$

$$E_{bi} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix},$$

$$i = 1, 2.$$
(59)

As obtained through simple calculation, the eigenvalues of system matrices  $A_1$  and  $A_2$  are -3.4142, -0.5858, -2.4142, and 0.4142; therefore, the system is unstable without control. For actuator normal or failures, matrix L is defined as follows:  $L_0 = \text{diag}\{1,1\}$ ;  $L_1 = \text{diag}\{0,1\}$ ;  $L_2 = \text{diag}\{1,0\}$ ; and  $L_3 = \text{diag}\{0.8, 0.5\}$ . Setting  $x_0 = [1, -1]^T$ , h = 0.1 s,  $\overline{\tau} = 0.1$  s,  $\tau_1 = 0.07$  s,  $\tau_2 = 0.2$  s,  $\alpha = 0.1$ ,  $\varepsilon_{ij} = 1$  ( $i = 1, 2, \dots, r$ ;  $j = 1, 2, \dots, r$ ), and the lower and upper bounds of actuator saturation to [-0.5, 0.5], we perform the following simulation of two cases according to Theorems 12 and 16.

*Case 1.* On the basis of event-triggered condition (5) in the DETCS, we can obtain the state-feedback controller gains,  $K_1$  and  $K_2$ , and the event-triggered weight matrices,  $\Phi_1$  and  $\Phi_2$ , by solving the linear matrix inequalities (43) and (44) in Theorem 12:

$$K_{1} = \begin{bmatrix} -0.5584 & -1.2979 \\ -0.2683 & -0.4379 \end{bmatrix},$$

$$K_{2} = \begin{bmatrix} -0.4494 & -1.0454 \\ -0.2165 & -0.3534 \end{bmatrix},$$

$$\Phi_{1} = \begin{bmatrix} 0.0059 & 0.0005 \\ 0.0005 & 0.0070 \end{bmatrix},$$

$$\Phi_{2} = \begin{bmatrix} 0.0056 & -0.0007 \\ -0.0007 & 0.0044 \end{bmatrix}.$$
(60)

Under the aforementioned actuator failures in mode set *L* and when we adopt the controllers,  $K_1$  and  $K_2$ , and the event-triggered weight matrices,  $\Phi_1$  and  $\Phi_2$ , as above, the control input and state response curves for NNCFSs (17) are as shown in Figures 2 and 3.

*Case 2.* On the basis of event-triggered condition (6) in the DETCS and setting  $\sigma = 0.05$ , we can obtain the state-feedback controller gains,  $K_1$  and  $K_2$ , and the event-triggered weight

matrix  $\Phi$  by solving the linear matrix inequalities (55) in Theorem 16:

$$K_{1} = \begin{bmatrix} -1.6508 & -3.7723 \\ -0.6621 & -1.3506 \end{bmatrix},$$

$$K_{2} = \begin{bmatrix} -1.3094 & -2.9913 \\ -0.5236 & -1.0682 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0.0038 & -0.0010 \\ -0.0010 & 0.0019 \end{bmatrix}.$$
(61)

Under the aforementioned actuator failures in mode set *L* and when we adopt the controllers,  $K_1$  and  $K_2$ , and the event-triggered weight matrix  $\Phi$  as above, the control input and state response curves for NNCFSs (17) are as shown in Figures 4 and 5.

Similarly, on the basis of event-triggered condition (6) in the DETCS and setting  $\sigma = 0.85$ , we can obtain the statefeedback controller gains,  $K_1$  and  $K_2$ , and the event-triggered weight matrix  $\Phi$  by solving the linear matrix inequalities (55) in Theorem 16:

$$K_{1} = \begin{bmatrix} -0.8893 & -2.0586 \\ -0.4476 & -0.6893 \end{bmatrix},$$

$$K_{2} = \begin{bmatrix} -0.6963 & -1.6121 \\ -0.3506 & -0.5398 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0.0074 & -0.0007 \\ -0.0007 & 0.0063 \end{bmatrix}.$$
(62)

Under the aforementioned actuator failures in mode set *L* and when we adopt the controllers,  $K_1$  and  $K_2$ , and the event-triggered weight matrix  $\Phi$  as above, the control input and state response curves for NNCFSs (17) are as shown in Figures 6 and 7.

Under event-triggered condition (5) or (6) in the DETCS and with simulation time  $t_s = 30$  s, the release instant and release interval of data transmission are as shown in Figure 8.

*4.2. Analysis of Simulation Results.* From Figures 2 to 8, we derive the following analysis:

- (1) Even though the control input has entered into the region of actuator saturation constraint, we can still adopt the codesign methods to achieve the design goal for NNCFSs (17). As shown in Figures 2 to 7, regardless of the condition we select from event-triggered conditions (5) and (6), NNCFSs (17) can keep asymptotic stability and satisfactory dynamic performance. For practical applications, system safety degree and dynamic performance can be improved by appropriately increasing the safety degree  $\alpha$ .
- (2) Observing Figure 8, we can conclude that, for the given  $t_s = 30$  s, whether we use event-triggered condition (5) or (6) in the DETCS, the quantity of data transmission significantly decreases. The DETCS









FIGURE 5: Responses of state *x* in Case 2 ( $\sigma = 0.05$ ).





FIGURE 7: Responses of state *x* in Case 2 ( $\sigma = 0.85$ ).



FIGURE 8: Release time and release interval of data transmission.

TABLE 1: A comparison of data transmission under different communication schemes and event-triggered conditions.

Communication scheme	σ	п	$r_{e/t}$	$\overline{h}$ (s)	$h_{\rm max}$ (s)
PTTCS	_	300	100%	0.1	0.1
DETCS					
Condition (5)	_	90	30.0%	0.345	1.5
	0.05	229	76.3%	0.149	1.3
	0.25	167	55.7%	0.199	1.8
Condition (6)	0.45	137	45.7%	0.249	2.1
	0.65	112	37.3%	0.291	2.2
	0.85	96	32%	0.366	3.1

*n* denotes the triggering times of event generator;  $r_{e/t}$  denotes the ratio between the quantity of data transmission in the DETCS and the corresponding quantity in the PTTCS;  $\bar{h}$  denotes the average release period in DETCS;  $h_{\rm max}$  denotes the maximum value of release period in DETCS.

driven by control demand is indicated to occupy less network resources than the PTTCS driven by a physical clock. Furthermore, event-triggered condition (5) can save more network communication resources than condition (6).

(3) As detailed in Table 1 and as indicated by the simulation experiment, data transmission circumstances naturally follow under different communication schemes and different event-triggered conditions in the DETCS.

Some conclusions can be summarized as follows. On the one hand, the data transmission cases shown in Figure 8, under event-triggered condition (6), represent only some special circumstances as listed under Table 1; and, in these cases, parameter  $\sigma$  is defined as a certain value for event-triggered condition (6). Meanwhile, with the increasing event-triggered parameter  $\sigma$ , data transmission quantity *n* 

and data transmission ratio  $r_{e/t}$  become increasingly small, and average release period  $\overline{h}$  and maximum release period  $h_{\text{max}}$  become increasingly large. However, if we excessively increase  $\sigma$  to save more network communication resources, system dynamic performance will deteriorate, as shown in Figures 5 and 7. Therefore, when selecting the value for  $\sigma$ , attention should be focused on the compromise between system performance and the occupancy ratio of network communication.

On the other hand, in contrast to event-triggered condition (6), event-triggered condition (5), wherein we need not preset the event-triggered parameter  $\sigma$ , can save more network communication resources. In the simulation example, even though we set the event-triggered parameter  $\sigma$  to 0.85, condition (5) can save 2% more of network communication resources than condition (6). This result is due to the fact that  $\Phi_1$  and  $\Phi_2$  have more decision variables than  $\Phi$  and  $\sigma$ , which can provide a wide degree of freedom for codesign. Therefore, the results of condition (5) will be more optimal than those of other conditions and can thus save much network communication resources.

# 5. Conclusions

Considering the uncertain NNCSs with time-varying delay, we study the discrete event-triggered robust fault-tolerant control problem for NNCSs with  $\alpha$ -safety degree and actuator saturation. Based on the Lyapunov stable theory and linear matrix inequality technology, some sufficient conditions, which can maintain asymptotic stability and  $\alpha$ -safety degree for the system, are derived under two event-triggered conditions for NNCSs. In our theorem proofs, we adopt the improved Jesson inequality and do not introduce any free weighting matrix, which can effectively reduce calculation complexity. Furthermore, the codesign methods between the robust fault-tolerant controller and event-triggered weight matrix are also presented in terms of linear matrix inequality. The simulation verifies that the codesign methods can make the closed-loop fault NNCSs possess good dynamic performance and save network communication resources effectively; that is, the results achieve the goal by taking two things into consideration-the system QoC and the network QoS. In addition, the performance of two eventtriggered conditions is also discussed, where condition (5) is found to save more communication resources than condition (6). In consideration of the time-varying data transmission period, next efforts will be directed at the codesign problem between satisfactory fault-tolerant control for NNCSs and network communication. These future efforts will simultaneously integrate  $\alpha$ -stability,  $H_{\infty}$  performance, generalized  $H_2$ performance, and network communication resource saving into a unified framework. Furthermore, the simulation experiment will be based on a real network background under professional environment [36].

#### **Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

### Acknowledgments

This work was supported and funded by the National Natural Science of China (Grants nos. 61364011 and 61463030) and the Provincial Natural Science Foundation of Gansu in China (Grant no. 1308RJZA148).

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