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Research Article

Adaptive MIMO Supervisory Control Design Using Modeling Error

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This paper proposes an adaptive control scheme for nonlinear systems with significant nonminimum phase dynamics. The scheme is composed of an inner-level adaptive fuzzy PD control law and an outer-level supervisory control law. Importantly, the inner-level controller of the two-level scheme is designed based on a fuzzy model, which takes nonminimum phase phenomenon and modeling error explicitly into account. The scheme is both much simpler in design and more applicable to general nonlinear systems when compared with most existing nonlinear controllers. Effectiveness of the proposed control strategy is demonstrated by numerical simulation of the control of a five-degree-of-freedom aircraft system in the face of bursting disturbances.

1. Introduction

Many critical dynamic systems, such as aircraft, are nonminimum phase, MIMO, and highly nonlinear, which undergo significant disturbances and parameter variation during operation. To control these systems, robust control [1], optimal tuning of fuzzy controllers with output sensitivity function [2], adaptive control [3–5], and feedback linearization with discrete sliding-mode control [6] have attracted much attention from both academic and industrial communities due to their robustness to uncertainties. Recently, many interests have been focused on applying these techniques to flight control systems, such as [7, 8]. However, for systems with significant nonminimum phase phenomenon, direct application of these approaches tends to introduce unstable zero dynamics.

For instance, in [9], the nonminimum phase plants are approximated by minimum phase models. The research [10] applied the output regulation theory to solve the output tracking problem, but a set of partial differential equations must be solved. The control scheme of [11] is based on decomposing the aircraft dynamics into a minimum phase part and a nonminimum phase part. Inversion is used on the minimum phase part to obtain asymptotic tracking, while a robust linear control approach is used to stabilize the nonminimum phase part, which is linearized at equilibrium. As this strategy is based on local linearization of the nonminimum phase part, the result can only apply to simplified models.

By estimating parameters online, adaptive control can adapt to a controlled system with varying or unknown parameters. Nevertheless, in spite of the prosperous literature of adaptive control, practical application of these control strategies on MIMO systems has been restricted by the lack of assurance in closed-loop stability. Among them, the adaptive neural controller of [3] is too complex to implement, while the adaptive fuzzy terminal sliding-mode controller of [4, 5] is applicable only to robotic manipulators.

The proposed adaptive control scheme is inspired by [12], which was developed for SISO nonlinear systems based on the feedback linearization technique, with the distinction that the scheme is extended to nonminimum phase MIMO control systems.

The scheme is composed of an inner-level tracking control law and an outer-level supervisory control law.

The design procedure is hence divided into two parts. First, an adaptive fuzzy-model-based PD control scheme is designed at the inner level to achieve robust output tracking. Special care is taken for the nonminimum phase fuzzy subsets in the control law by restricting parameter magnitudes in the singular-value decomposition operation. Next, a supervisory controller is employed at the outer level to minimize both the approximation error between the fuzzy model and the plant and the effects of external disturbance. Effectiveness of the adaptive control scheme is demonstrated by simulation results of the fight control of a complete 5-DOF aircraft model.

2. Problem Formulation

System dynamics of the plant are firstly represented in a general MIMO state-space representation as

$$\dot{x} = F(x) + G(x) \cdot u + w_0, \qquad y = H \cdot x, \tag{1}$$

where $x \in \mathfrak{R}^{n\times 1}$ is the state vector, $u \in \mathfrak{R}^{m\times 1}$ is the control vector, $w_0 \in \mathfrak{R}^{n\times 1}$ is the disturbance vector, $y \in \mathfrak{R}^{N\times 1}$ is the output vector, and F, G are corresponding nonlinear matrices in state vectors with H being a constant matrix, all of compatible dimensions.

Equation (1) can be further represented in output vector y as

$$\begin{aligned}\dot{y} &= H \cdot F(x) + H \cdot G(x) \cdot u + H \cdot w_0 \\ &= f(x) + g(x) \cdot u + w \\ &= h_f \cdot A_f + \sum_{i=1}^{L} h_i \cdot B_i \cdot u + \left[f(x) - h_f \cdot A_f\right] \\ &\quad + \left[g(x) - \sum_{i=1}^{L} h_i \cdot B_i\right] \cdot u + w \\ &= h_f \cdot A_f + \sum_{i=1}^{L} h_i \cdot B_i \cdot u + \bar{e}_{\text{mod}} \\ &\triangleq h_f \cdot A_f + B_c \cdot u + \bar{e}_{\text{mod}},\end{aligned} \tag{2}$$

where

$$\begin{aligned} B_c &\triangleq \sum_{i=1}^{L} h_i \cdot B_i, \\ h_f &= \begin{bmatrix} h_{f1} & 0 & 0 & 0 \\ 0 & h_{f2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & h_{fn} \end{bmatrix}, \\ A_f &= \left[A_{f1} \;\; A_{f2} \;\; \cdots \;\; A_{fn}\right]^T, \\ f_1 &= \sum_{i=1}^{p} h_{1i} \cdot A_{1i} = h_{f1} \cdot A_{f1}, \\ f_2 &= \sum_{i=1}^{p} h_{2i} \cdot A_{2i} = h_{f2} \cdot A_{f2}, \ldots, \\ f_n &= \sum_{i=1}^{p} h_{ni} \cdot A_{ni} = h_{fn} \cdot A_{fn}, \\ \bar{e}_{\text{mod}} &= \left[f(x) - h_f \cdot A_f\right] + \left[g(x) - \sum_{i=1}^{L} h_i \cdot B_i\right] \cdot u \\ &\quad + w, \end{aligned} \tag{3}$$

and the external disturbance $w = H \cdot w_0$.

In the last representation, it is assumed that $g(x)$ is bounded and is away from singularity in a compact set. Furthermore, $f(x)$ and $g(x)$ are identified in fuzzy form as $h_f(y) \cdot A_f(t)$ and $\sum_{j=1}^{L} h_j(y) \cdot B_j$, respectively, where the fuzzy logic systems are universal approximations which can uniformly approximate nonlinear continuous functions to arbitrary accuracy [13–15].

3. Controller Design for the Nonminimum Phase Dynamics

Firstly, the tracking error is defined as

$$e(t) = -\bar{e}, \tag{4}$$

where $\bar{e} = (y_r - y)$ (y_r is reference input); we have that

$$\begin{aligned}\dot{e}(t) &= h_f \cdot A_f + \sum_{i=1}^{L} h_i \cdot B_i \cdot u + \bar{e}_{\text{mod}} - \dot{y}_r \\ &= h_f \cdot A_f + B_c \cdot u + \bar{e}_{\text{mod}} - \dot{y}_r,\end{aligned} \tag{5}$$

where u is a combination of two signals [16]:

$$u = u_F + u_S, \tag{6}$$

with

$$\begin{aligned} u_F &= (1 - I^*) \cdot B_c^{-1} \\ &\quad \cdot \left\{-h_f(y) \cdot A_f(t) + \dot{y}_r + K_P \cdot e(t) + K_D \cdot \dot{e}(t)\right\}, \end{aligned} \tag{7}$$

$$\begin{aligned} u_S &= -I^* \cdot \operatorname{sgn}(B_c \cdot P \cdot e(t)) \\ &\quad \cdot \left\{\left|B_c^{-1} \cdot \left[h_f(y) \cdot A_f(t) - \dot{y}_r\right]\right| + e_U\right\}. \end{aligned} \tag{8}$$

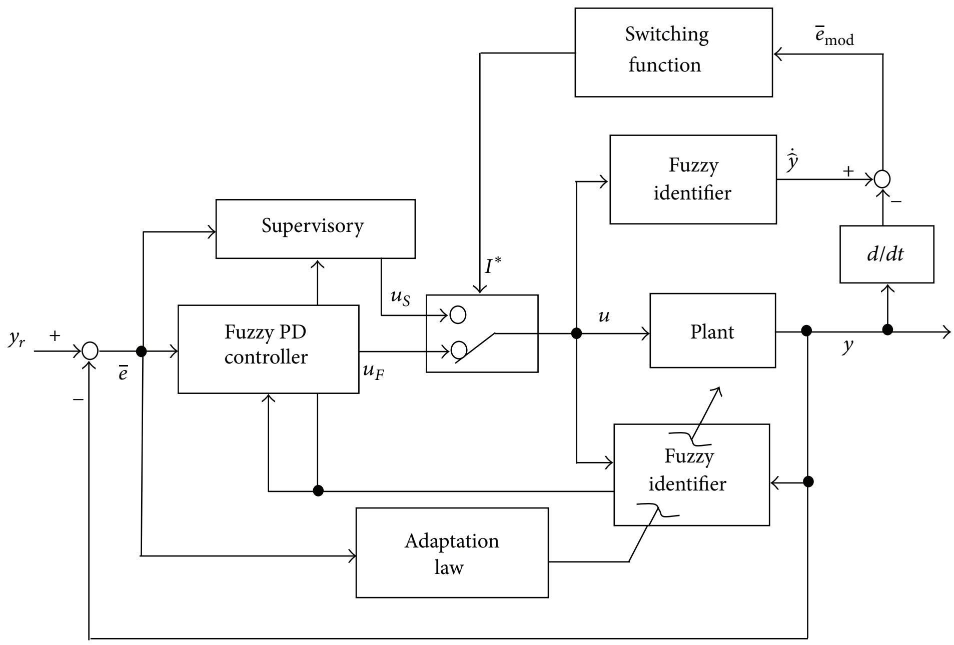

Figure 1: The proposed two-level switching control scheme.

In (7), the proportional gain of the inner fuzzy control law is designed as

$$K_P = \alpha - R^{-1} \cdot P. \tag{9}$$

The switching variable in both (7) and (8) is defined as

$$\begin{aligned} I^* &= 0, \quad \text{if } \left\| \bar{e}_{\mathrm{mod}} \right\| \le \bar{e}_U, \\ I^* &= 1, \quad \text{otherwise}, \end{aligned} \tag{10}$$

with

$$\begin{aligned} &\left| B_c^{-1} \cdot \bar{e}_{\mathrm{mod}} \right| \le e_U, \\ &\alpha = \begin{bmatrix} -\alpha_1 & 0 & 0 & 0 \\ 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & -\alpha_n \end{bmatrix}, \\ &P > 0, \\ &R > 0. \end{aligned} \tag{11}$$

A complete control scheme of the two-level architecture is shown in Figure 1.

To avoid encountering singularity of the control law, the singular-value decomposition of the matrix B_c is introduced as follows:

$$B_c = U \cdot S \cdot V^T, \tag{12}$$

where

$$S = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_n \end{bmatrix} \tag{13}$$

and σ_i is replaced by ε if $\sigma_i \le \varepsilon$, where ε is a small value.

Substituting the adaptive fuzzy PD controller u_F (7) into (5), we have

$$\dot{e}(t) = K_P \cdot e(t) + h_f(y) \cdot \widetilde{A}_f(t) + e_{\mathrm{mod}}, \tag{14}$$

after algebra manipulations, where

$$\widetilde{A}_f(t) = A_f^* - A_f(t), \tag{15}$$

and the modeling error

$$\begin{aligned} e_{\mathrm{mod}} = {} & \left[f(x) - h_f(y) \cdot A_f^* \right] \\ & + \left[g(x) - \sum_{i=1}^{L} h_i(y) \cdot B_i \right] \cdot u + w + K_D \\ & \cdot \dot{e}(t). \end{aligned} \tag{16}$$

In the following derivation, we need the following condition to be satisfied [17–20]:

$$J \le e(0)^T \cdot P \cdot e(0) + \gamma_f^{-1} \cdot \text{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right), \quad (17)$$

where

$$J = \int_0^{t_f} \left[e(t)^T \cdot \left(Q + P^T \cdot R^{-T} \cdot P\right) \cdot e(t) - \rho^2 \cdot e_{\text{mod}}^T \cdot e_{\text{mod}} \right] \cdot dt, \quad (18)$$

the weighting factor $\gamma_f > 0$, $\rho^2 \cdot I \ge R$, the matrix $Q > 0$, and $\|X\|_F = \sqrt{\text{trace}(X^T \cdot X)}$ is the Frobenius norm of matrix X. Derivation of this condition, (17), is given in the Appendix.

Hence, we have that

$$\begin{aligned} \dot{\widetilde{A}}_f(t) &= -\dot{A}_f = -\gamma_f \cdot h_f(y)^T \cdot P \cdot e(t), \\ \left[\alpha^T \cdot P + P \cdot \alpha + Q - P^T \cdot R^{-T} \cdot P\right] &= -\rho^{-2} \cdot P^T \cdot P. \end{aligned} \quad (19)$$

Furthermore, to guarantee boundedness of A_f, the parameter update laws must be modified as follows:

$$\dot{A}_f = \begin{cases} \gamma_f \cdot h_f(y)^T \cdot P \cdot e(t), & \text{if } \|A_f\| < M_f \text{ or } \left(\|A_f\| = M_f,\ \dot{A}_f^T \cdot A_f \le 0\right), \\ F\left(\gamma_f \cdot h_f(y)^T \cdot P \cdot e(t)\right), & \text{otherwise}, \end{cases} \quad (20)$$

where M_f is a positive design parameter and the projection function $F(\cdot)$ is defined as

$$\begin{aligned} F\left(\gamma_f \cdot h_f(y)^T \cdot P \cdot e(t)\right) &= \gamma_f \cdot h_f(y)^T \cdot P \cdot e(t) - \gamma_f \\ &\cdot \frac{A_f \cdot A_f^T \cdot h_f(y)^T \cdot P \cdot e(t)}{\|A_f\|^2}. \end{aligned} \quad (21)$$

Next, the supervisor control law of (8) is designed by the following Lyapunov candidate:

$$V = e(t)^T \cdot P \cdot e(t). \quad (22)$$

Its time derivative, $\dot{V}$, can be obtained as

$$\begin{aligned} \dot{V} &= \left[h_f \cdot A_f + B_c \cdot u_S + \bar{e}_{\text{mod}} - \dot{y}_r\right]^T \cdot P \cdot e(t) \\ &\quad + e(t)^T \cdot P \cdot \left[h_f \cdot A_f + B_c \cdot u_S + \bar{e}_{\text{mod}} - \dot{y}_r\right] \\ &= 2e(t)^T \cdot P \cdot B_c \cdot u_S + 2e(t)^T \cdot P \\ &\quad \cdot \left[h_f \cdot A_f + \bar{e}_{\text{mod}} - \dot{y}_r\right]. \end{aligned} \quad (23)$$

Substituting (8) into (23) yields

$$\begin{aligned} \dot{V} &\le 2e(t)^T \cdot P \cdot B_c \cdot u_S + 2\left|e(t)^T \cdot P \cdot B_c\right| \\ &\quad \cdot \left|B_c^{-1} \cdot \left(h_f \cdot A_f + \bar{e}_{\text{mod}} - \dot{y}_r\right)\right| \le 0. \end{aligned} \quad (24)$$

Hence, we can infer that if the supervisory control signal (8) is injected into fuzzy system (2), time derivative of the Lyapunov candidate $\dot{V} \le 0$ and system (2) is UUB stable.

4. Numerical Simulation

In this section, the proposed control strategy is applied on a five-degree-of-freedom aircraft system described in [21] for performance evaluation. We consider the angle of attack α and the roll angle ϕ as outputs to be tracked. Tracking of angle of attack is directly related to tracking of normal acceleration [21], which plays an important role in many practical maneuvers.

Let $b = 3$ be the reference length (m), $\bar{c} = 2$ the mean aerodynamic chord (m), $g = 9.8$ the gravitational acceleration (m/s^2), $I = 50$ the moment of inertia (kg-m^2), p the roll angle rate, q the pitch angle rate, r the yaw angle rate, $Q = 80$ the dynamic pressure (kg/m^2), $S = 5$ the reference wing area (m^2), $V = 100$ the aircraft velocity (m/s), θ the pitch angle, $\delta_a = 0$ the aileron deflection, δ_e the elevator deflection, δ_r the rudder deflection, and $m = 100$ the mass of aircraft (kg); the aircraft dynamics can be written as [21]

$$\begin{aligned} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} &= \begin{bmatrix} f_p \\ f_q \\ f_r \end{bmatrix} + \begin{bmatrix} L_{\delta a} & 0 & L_{\delta r} \\ 0 & M_{\delta e} & 0 \\ N_{\delta a} & 0 & N_{\delta r} \end{bmatrix} \cdot \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix}, \\ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\varphi} \\ \dot{\theta} \end{bmatrix} &= \begin{bmatrix} f_\alpha \\ f_\beta \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -t_\beta \cdot c_\alpha & 1 & -t_\beta \cdot s_\alpha \\ s_\alpha & 0 & -c_\alpha \\ 1 & t_\theta \cdot s_\varphi & t_\theta \cdot c_\varphi \\ 0 & c_\varphi & -s_\varphi \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\ &\quad + w, \end{aligned} \quad (25)$$

where

$$
\begin{aligned}
\phi &= y_1, \\
\alpha &= y_2, \\
L_{\delta a} &= I \cdot Q \cdot S \cdot b \cdot C_{l\delta a}, \\
L_{\delta r} &= I \cdot Q \cdot S \cdot b \cdot C_{l\delta r}, \\
M_{\delta e} &= I \cdot Q \cdot S \cdot \bar{c} \cdot C_{m\delta e}, \\
N_{\delta a} &= I \cdot Q \cdot S \cdot b \cdot C_{n\delta a}, \\
N_{\delta r} &= I \cdot Q \cdot S \cdot b \cdot C_{n\delta r}, \\
f_p &= \frac{Q \cdot S \cdot I \cdot b^2 \cdot C_{lp} \cdot p}{2V}, \\
f_q &= \frac{Q \cdot S \cdot I \cdot \bar{c}^2 \cdot C_{mq} \cdot q}{2V}, \\
f_r &= \frac{Q \cdot S \cdot I \cdot b^2 \cdot C_{nr} \cdot r}{2V}, \\
f_\alpha &= \frac{-Q \cdot S \cdot C_{L\alpha} \cdot \alpha + m \cdot g \cdot \left(c_\theta \cdot c_\phi \cdot c_\alpha + s_\theta \cdot s_\alpha\right)}{m \cdot V \cdot c_\beta}, \\
f_\beta &= \frac{Q \cdot S \cdot C_{Y\beta} \cdot \beta + m \cdot g \cdot \left[s_\theta \cdot c_\alpha \cdot s_\beta + c_\theta \cdot s_\phi \cdot c_\beta - c_\theta \cdot c_\phi \cdot s_\alpha \cdot s_\beta\right]}{m \cdot V}, \\
t_\beta &= \tan(\beta), \\
s_\beta &= \sin(\beta), \\
c_\beta &= \cos(\beta), \\
s_\theta &= \sin(\theta), \\
c_\theta &= \cos(\theta), \\
t_\theta &= \tan(\theta), \\
c_\alpha &= \cos(\alpha), \\
s_\alpha &= \sin(\alpha), \\
s_\phi &= \sin(\phi), \\
c_\phi &= \cos(\phi).
\end{aligned}
\tag{26}
$$

In the following simulation, we assume

$$
\begin{aligned}
C_{l\delta a} &= -10^{-4}, \\
C_{l\delta r} &= 10^{-2}, \\
C_{m\delta e} &= -1.6 \times 10^{-4}, \\
C_{n\delta a} &= 10^{-2}, \\
C_{n\delta r} &= -10^{-4}, \\
C_{lp} &= -3.8 \times 10^{-2}, \\
C_{mq} &= -0.9 \times 10^{-2},
\end{aligned}
$$

$$C_{nr} = -4.5 \times 10^{-3},$$
$$C_{L\alpha} = 2.8 \times 10^{-1},$$
$$C_{Y\beta} = -2.8. \quad (27)$$

For the inner fuzzy control law, we select the following membership functions:

$$\begin{aligned}
\mu_{F_1^1}(y_1) &= \exp\left[-\frac{1}{2}\left(\frac{y_1 - c_{11}}{d_{11}}\right)^2\right],\\
\mu_{F_1^2}(y_1) &= \exp\left[-\frac{1}{2}\left(\frac{y_1 - c_{12}}{d_{12}}\right)^2\right],\\
\mu_{F_1^3}(y_1) &= \exp\left[-\frac{1}{2}\left(\frac{y_1 - c_{13}}{d_{13}}\right)^2\right],\\
\mu_{F_1^4}(y_1) &= \exp\left[-\frac{1}{2}\left(\frac{y_1 - c_{14}}{d_{14}}\right)^2\right],\\
\mu_{F_2^1}(y_2) &= \exp\left[-\frac{1}{2}\left(\frac{y_2 - c_{21}}{d_{21}}\right)^2\right],\\
\mu_{F_2^2}(y_2) &= \exp\left[-\frac{1}{2}\left(\frac{y_2 - c_{22}}{d_{22}}\right)^2\right],\\
\mu_{F_2^3}(y_2) &= \exp\left[-\frac{1}{2}\left(\frac{y_2 - c_{23}}{d_{23}}\right)^2\right],\\
\mu_{F_2^4}(y_2) &= \exp\left[-\frac{1}{2}\left(\frac{y_2 - c_{24}}{d_{24}}\right)^2\right],
\end{aligned} \quad (28)$$

where

$$\begin{aligned}
c_{11} &= c_{21} = 0,\\
c_{12} &= c_{22} = 0.4,\\
c_{13} &= c_{23} = 0.8,\\
c_{14} &= c_{24} = 1.2,\\
d_{11} &= d_{21} = d_{12} = d_{22} = d_{13} = d_{23} = d_{14} = d_{24} = 0.4.
\end{aligned} \quad (29)$$

Furthermore, 8 fuzzy rules of the following form comprise the fuzzy rule base:

$R^{(1)}$: if y_1 is F_1^j, then $f_1 = A_{1j}$ for $j = 1, 2, 3, 4$ and $l = 1, 2, 3, 4$.

$R^{(2)}$: if y_2 is F_2^j, then $f_2 = A_{2j}$ for $j = 1, 2, 3, 4$ and $l = 5, 6, 7, 8$.

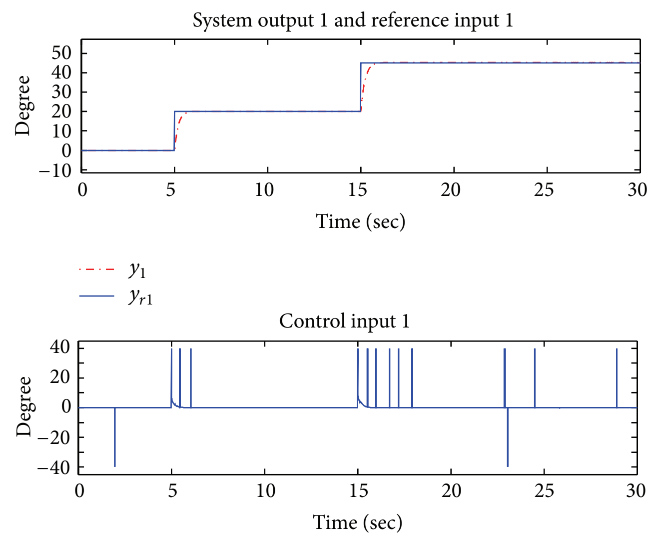

FIGURE 2: Time history of system output y_1 (the angle of attack, α), reference y_{r1}, and control input u_1.

Then, we obtain the following initial system parameters:

$$\begin{aligned}
A_{11} &= -0.1338,\\
A_{12} &= 0.5183,\\
A_{13} &= -1.0891,\\
A_{14} &= 1.546,\\
A_{21} &= -1.0328,\\
A_{22} &= 1.669,\\
A_{23} &= -0.0957,\\
A_{24} &= -0.3342,\\
B_c &= \begin{bmatrix} 16.5381 & -0.4429 \\ 1.1439 & 0.4489 \end{bmatrix}.
\end{aligned} \quad (30)$$

Finally, we design the following control gains:

$$\begin{aligned}
K_P &= \begin{bmatrix} -0.52 & 0 \\ 0 & -51 \end{bmatrix},\\
K_D &= \begin{bmatrix} -0.05 & 0 \\ 0 & -4.9 \end{bmatrix}.
\end{aligned} \quad (31)$$

The tracking performances of α and ϕ, together with the reference (or command), are presented in Figures 2 and 3. These figures show the responses with several step reference inputs. The disturbance is $w = [1, 1, 1, 1]^T \cdot \delta(t-2)$, a burst at $t = 2$ s.

From the simulation results, it is clear that the output tracks the desired command asymptotically with small transient errors, and the zero dynamics remain stable for all the simulated interval.

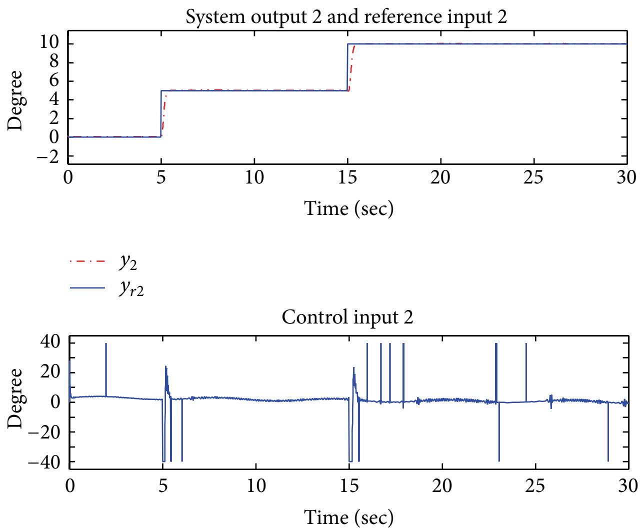

FIGURE 3: Time history of system output y_2 (the roll angle, ϕ), reference y_{r2}, and control input u_2.

5. Conclusion

We propose a two-level adaptive control scheme for nonlinear systems, such as the aircraft, which are MIMO and suffer from nonminimum phase phenomena. The control scheme is composed of an inner-level adaptive fuzzy PD control law and an outer-level supervisory control law. Importantly, the outer-level controller of the two-level scheme is designed based on a fuzzy model taking nonminimum phase phenomena and modeling error explicitly into account. Special care is taken of the nonminimum phase fuzzy subsets by restricting the magnitude of parameters in the singular-value decomposition operation.

The control strategy is much simpler and applicable to general MIMO, nonlinear, and nonminimum phase systems when compared with [3–5]. Simulation results of the application of the proposed control scheme on a five-degree-of-freedom nonlinear aircraft model verify its effectiveness.

Appendix

Derivation of the Condition of (17)

Consider

$$
\begin{aligned}
J &= \int_0^{t_f} \left[e(t)^T \cdot \left(Q + P^T \cdot R^{-T} \cdot P\right) \cdot e(t) - \rho^2 \cdot e_{\text{mod}}^T \cdot e_{\text{mod}}\right] \cdot dt = e(0)^T \cdot P \cdot e(0) - e(t_f)^T \cdot P \cdot e(t_f) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right) - \gamma_f^{-1} \\
&\quad \cdot \operatorname{trace}\left(\widetilde{A}_f(t_f)^T \cdot \widetilde{A}_f(t_f)\right) + \int_0^{t_f} \left[e(t)^T \cdot \left(Q + P^T \cdot R^{-T} \cdot P\right) \cdot e(t) - \rho^2 \cdot e_{\text{mod}}^T \cdot e_{\text{mod}} + \dot{e}(t)^T \cdot P \cdot e(t) + e(t)^T \cdot P \cdot \dot{e}(t) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\dot{\widetilde{A}}_f(t)^T \cdot \widetilde{A}_f(t)\right) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(t)^T \cdot \dot{\widetilde{A}}_f(t)\right)\right] dt \\
&\le e(0)^T \cdot P \cdot e(0) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right) + \int_0^{t_f} \left\{e(t)^T \cdot \left(Q + P^T \cdot R^{-T} \cdot P\right) \cdot e(t) - \rho^2 \cdot e_{\text{mod}}^T \cdot e_{\text{mod}} + \left[\alpha \cdot e(t) + h_f(y) \cdot \widetilde{A}_f(t) - R^{-1} \cdot P \cdot e(t) + e_{\text{mod}}\right]^T \cdot P \cdot e(t) \right. \\
&\quad \left. + e(t)^T \cdot P \cdot \left[\alpha \cdot e(t) + h_f(y) \cdot \widetilde{A}_f(t) - R^{-1} \cdot P \cdot e(t) + e_{\text{mod}}\right] + \gamma_f^{-1} \cdot \operatorname{trace}\left(\dot{\widetilde{A}}_f(t)^T \cdot \widetilde{A}_f(t)\right) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(t)^T \cdot \dot{\widetilde{A}}_f(t)\right)\right\} dt = e(0)^T \cdot P \cdot e(0) + \gamma_f^{-1} \\
&\quad \cdot \operatorname{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right) + \int_0^{t_f} \left\{e(t)^T \cdot \left[\alpha^T \cdot P + P \cdot \alpha + Q + P^T \cdot R^{-T} \cdot P\right] \cdot e(t) - \rho^2 \cdot e_{\text{mod}}^T \cdot e_{\text{mod}} + \left[h_f(y) \cdot \widetilde{A}_f(t) - R^{-1} \cdot P \cdot e(t) + e_{\text{mod}}\right]^T \cdot P \cdot e(t) \right. \\
&\quad \left. + e(t)^T \cdot P \cdot \left[h_f(y) \cdot \widetilde{A}_f(t) - R^{-1} \cdot P \cdot e(t) + e_{\text{mod}}\right] + \gamma_f^{-1} \cdot \operatorname{trace}\left(\dot{\widetilde{A}}_f(t)^T \cdot \widetilde{A}_f(t)\right) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(t)^T \cdot \dot{\widetilde{A}}_f(t)\right)\right\} dt < e(0)^T \cdot P \cdot e(0) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right) \\
&\quad + \int_0^{t_f} \left\{-\rho^{-2} e(t)^T \cdot P^T \cdot P \cdot e(t) - \rho^2 \cdot e_{\text{mod}}^T \cdot e_{\text{mod}} + e_{\text{mod}}^T \cdot P \cdot e(t) + e(t)^T \cdot P \cdot e_{\text{mod}} + \left[h_f(y) \cdot \widetilde{A}_f(t)\right]^T \cdot P \cdot e(t) + e(t)^T \cdot P \cdot h_f(y) \cdot \widetilde{A}_f(t) \right. \\
&\quad \left. + \gamma_f^{-1} \cdot \operatorname{trace}\left(\dot{\widetilde{A}}_f(t)^T \cdot \widetilde{A}_f(t)\right) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(t)^T \cdot \dot{\widetilde{A}}_f(t)\right)\right\} dt = e(0)^T \cdot P \cdot e(0) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right) \\
&\quad - \int_0^{t_f} \left[\rho \cdot e_{\text{mod}} - \rho^{-1} \cdot P \cdot e(t)\right]^T \cdot \left[\rho \cdot e_{\text{mod}} - \rho^{-1} \cdot P \cdot e(t)\right] dt \le e(0)^T \cdot P \cdot e(0) + \gamma_f^{-1} \cdot \operatorname{trace}\left(\widetilde{A}_f(0)^T \cdot \widetilde{A}_f(0)\right).
\end{aligned}
\tag{A.1}
$$

This completes the proof.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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