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Research Article

Boundedness of Stochastic Delay Differential Systems with Impulsive Control and Impulsive Disturbance

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This paper considers the p-moment boundedness of nonlinear impulsive stochastic delay differential systems (ISDDSs). Using the Lyapunov-Razumikhin method and stochastic analysis techniques, we obtain sufficient conditions which guarantee the p-moment boundedness of ISDDSs. Two cases are considered, one is that the stochastic delay differential system (SDDS) may not be bounded, and how an impulsive strategy should be taken to make the SDDS be bounded. The other is that the SDDS is bounded, and an impulsive disturbance appears in this SDDS, then what restrictions on the impulsive disturbance should be adopted to maintain the boundedness of the SDDS. Our results provide sufficient criteria for these two cases. At last, two examples are given to illustrate the correctness of our results.

1. Introduction

Boundedness is an important property of a given system; for example, in the population models, the boundedness of a biological population is strongly connected with the persistence and extinction [1]. Another important application is on the stability; the practical stability actually is of a kind of boundedness [2]. Impulsive phenomena widely exist in the real world, and known, impulsive effects can change the properties of a given system; for example, given an unstable system, if a suitable impulsive strategy, including the impulsive strength and impulsive moments, is adopted, this system can be stabilized [3]. It is easy to understand that the impulsive effects can destroy the boundedness of a given system when the impulsive strength is large enough and the impulsive interval is small enough. Time delay is extensive in the engineering and applications and impulsive delay differential systems were considered in lots of papers [3–9]. The boundedness of impulsive delay differential systems has also been paid considerable attentions in the past decades. In [10], the authors presented sufficient conditions for uniform ultimate boundedness by virtue of the Lyapunov functional method. The boundedness of variable impulsive perturbations system was considered in [11] and the eventual boundedness was studied in [12]. Recently, the perturbing Lyapunov function method was also used in the study of boundedness [13].

Stochastic noise is ubiquitous [14–16] and stochastic delay differential systems (SDDSs) have been one of the focuses of scientific research for many years. Many properties of SDDSs have been studied and lots of papers were published; see [17, 18] and the references therein. Being the wide existence of stochastic delay and impulsive effects, it is a natural task to consider the stochastic delay differential systems with impulsive effects. These systems are described by impulsive stochastic delay differential systems (ISDDSs). In the past ten years, the stability of ISDDSs has attracted a lot of researchers, and a great deal of results on the stability of ISDDSs have been reported; see [19–24] and the references therein.

However, little attention has been paid to the boundedness of ISDDSs. In this paper, the boundedness of ISDDSs is considered under two cases. The first case is that the SDDSs may be unbounded, then what kind of impulsive strategy should be taken to make the system be bounded.

The second case is that the SDDSs are bounded, then this system can tolerate what kind of impulsive effect to maintain the boundedness.

In this paper, sufficient conditions are presented to guarantee the boundedness of ISDDSs; these conditions also admit the global existence of solutions for ISDDSs, which usually was a standard assumption in many papers [25–27]. Making use of the Lyapunov-Razumikhin method, we generalize the results of [10] to the stochastic situation. At last, two examples are given to illustrate the correctness of our results.

2. Preliminaries and Model Description

Let $(\Omega, F, \{F_t\}_{t\geqslant 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, and $\mathbb{N} = \{1, 2, \ldots\}$. If A is a vector or a matrix, its transpose is denoted by A^T. Consider $PC(\mathbb{J}; \mathbb{R}^n) = \{\varphi : \mathbb{J} \to \mathbb{R}^n, \varphi(s)$ is continuous for all but at most countable points $s \in \mathbb{J}$ and at these points, $\varphi(s^+)$ and $\varphi(s^-)$ exist and $\varphi(s^+) = \varphi(s)\}$, where $\mathbb{J} \subset \mathbb{R}$ is an interval and $\varphi(s^+)$ and $\varphi(s^-)$ denote the right-hand and left-hand limits of the function $\varphi(s)$ at time s, respectively. Consider $PC^{1,2} = \{\varphi(t,x) : \varphi(\cdot, x) \in PC$ and $\varphi(t,x) \in C^{1,2}$ if t is not at the uncontinuous points $s\}$. Let $PC^b_{F_0}([-\tau, 0]; \mathbb{R}^n)(PC^b_{F_t}([-\tau, 0]; \mathbb{R}^n))$ denote the family of all bounded $F_0(F_t)$-measurable, PC-valued random variables. Let $|\cdot|$ be the Euclidean norm in $\mathbb{R}^n$ and $\|\varphi\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0}|\varphi(t+\theta)|$.

Consider the following nonlinear impulsive stochastic delay differential system:

$$\begin{aligned} dx(t) &= f(t, x_t)\,dt + g(t, x_t)\,dB(t), \\ & \qquad t > t_0, \quad t \neq t_k, \quad k \in \mathbb{N}, \\ x(t_k) &= x(t_k^-) + I(t_k, x(t_k^-)), \quad k \in \mathbb{N}, \\ x(t_0 + s) &= \varphi(s), \quad s \in [-\tau, 0], \end{aligned} \tag{1}$$

where $x_t(s) = x(t+s), s \in [-\tau, 0]$, $f : \mathbb{R}^+ \times PC([-\tau, 0], \mathbb{R}^n) \to \mathbb{R}^n$, $g : \mathbb{R}^+ \times PC([-\tau, 0], \mathbb{R}^n) \to \mathbb{R}^{n\times m}$, $I : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ and satisfies global Lipschitz condition, τ represents the delay in system (1), impulsive moment t_k satisfies $0 < t_1 < t_2 < \cdots < t_n < \cdots$, and $t_k \to \infty$ as $k \to \infty$. $B(t)$ is an m-dimensenal Brownian motion and $\varphi(s) \in PC^b_{F_0}([-\tau, 0], \mathbb{R}^n)$.

Given a function $V \in PC^{1,2} : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$, the operator $\mathscr{L}$ of $V(t,x)$ with respect to system (1) is defined by

$$\begin{aligned} \mathscr{L}V(t,x) = V_t &+ V_x f(t, x_t) \\ &+ \frac{1}{2}\operatorname{trace}\left[g^T(t, x_t) V_{xx} g(t, x_t)\right], \end{aligned} \tag{2}$$

where

$$\begin{aligned} V_t &= \frac{\partial V(t,x)}{\partial t}, \\ V_x &= \left(\frac{\partial V(t,x)}{\partial x_1}, \frac{\partial V(t,x)}{\partial x_2}, \ldots, \frac{\partial V(t,x)}{\partial x_n}\right)^T, \\ V_{xx} &= \left(\frac{\partial^2 V(t,x)}{\partial x_i \partial x_j}\right)_{n\times n}. \end{aligned} \tag{3}$$

Definition 1. System (1) is said to be

(1) p-moment bounded if, for every $B_1 > 0$ and $t_0 \in \mathbb{R}_+$, there exists $B_2 = B_2(t_0, B_1)$ such that if $\varphi \in PC^b_{F_0}([-\tau, 0], \mathbb{R}^n)$ with $E\|\varphi\|^p_\tau \leqslant B_1$ and $x = x(t, t_0, \varphi)$ is a solution of (1), then $E|x(t, t_0, \varphi)|^p \leqslant B_2$ for all $t \geqslant t_0$;

(2) p-moment uniformly bounded if the system (1) is p-moment bounded and B_2 is independent of t_0;

(3) p-moment ultimately bounded if the system (1) is p-moment bounded and there exists a positive constant B such that for every $B_3 > 0$ and $t_0 \in \mathbb{R}^+$ there exists some $T = T(t_0, B_3) > 0$; if $\varphi \in PC^b_{F_0}([-\tau, 0], \mathbb{R}^n)$ with $E\|\varphi\|^p_\tau \leqslant B_3$, then $E|x(t, t_0, \varphi)|^p \leqslant B$ for $t \geqslant t_0 + T$;

(4) p-moment uniformly ultimately bounded, if the system (1) is p-moment ultimately bounded and T is independent of t_0.

3. Boundedness with Impulsive Control

In this section, we consider the first case: when the given SDDS may not be bounded, we adopt an impulsive strategy to get the boundedness. The main result is stated as follows.

Theorem 2. *Assume there exist a positive function $V(t,x) \in PC^{1,2}$ and positive constants $\rho, p, a, b, \gamma, \lambda$, where $0 < \lambda < 1$ and $1 - \lambda - \gamma\tau > 0$, such that*

(1) *$a|x|^p \leqslant V(t,x) \leqslant b|x|^p$ for any (t,x);*

(2) *for $t \neq t_k$, any $s \in [-\tau, 0]$, and $\phi(t) \in PC([-\tau, 0], \mathbb{R}^n)$, $\mathscr{L}V(t, \phi(0)) \leqslant \gamma V(t, \phi(0))$ whenever $V(t, \phi(0)) \geqslant \lambda V(t+s, \phi(s))$ and $|\phi(0)|^p \geqslant \rho$;*

(3) *$V(t_k, \phi(0) + I(t_k, \phi(0))) \leqslant \lambda V(t_k^-, \phi(0))$ for all $|\phi(0)|^p \geqslant \rho$;*

(4) *there exists a positive constant $\rho_1 \geqslant \rho$ such that if $|\phi(0)|^p \leqslant \rho$, then $|\phi(0) + I(t_k, \phi(0))|^p \leqslant \rho_1$;*

(5) *$\alpha = \sup_{k\in\mathbb{Z}}\{t_k - t_{k-1}\} < \infty$, $\alpha\gamma < 1 - \lambda$.*

Then the system (1) is p-moment uniformly ultimately bounded.

Proof. We separate the proof into two parts. First, we show the p-moment uniform boundedness and then we give the ultimate uniform boundedness.

Step 1. Let $B_1 > 0$. Without loss of generality, we assume $B_1 \geqslant \rho_1 \geqslant \rho$. Choose $B_2 = B_2(B_1)$ such that $bB_1 < \lambda a B_2$; then we can see $B_2 > B_1$.

Let $E\|\varphi\|_\tau^p < B_1$ and $t_0 \in [t_{l-1}, t_l)$ for some positive integer l. Suppose $x(t) = x(t, t_0, \varphi)$ is a solution of system (1) with initial value φ and its maximal interval of existence is $[t_0 - \tau, t_0 + \beta)$ for some positive constant β. We will show that, for any $t \in [t_0 - \tau, t_0 + \beta)$, $E|x(t)|^p \leqslant B_2$. By the way, if this statement is true, we know that the solution of system (1) is not explored in $[t_0, t_0 + \beta)$, and the global existence of the solution follows.

For the sake of contradiction, suppose $E|x(t)|^p \geqslant B_2$ for some $t \in [t_0, t_0 + \beta)$. Then there exists $\hat{t} = \inf\{t \in [t_0 - \tau, t_0 + \beta) \mid E|x(t)|^p > B_2\}$. Note that $E|x(t)|^p \leqslant E\|\varphi\|_\tau^p \leqslant B_1 < B_2$ for $t \in [t_0 - \tau, t_0]$; we see that $\hat{t} \in (t_0, t_0 + \beta)$ and $E|x(t)|^p \leqslant B_2$ for $t \in [t_0 - \tau, \hat{t})$ and $E|x(\hat{t})|^p \geqslant B_2$.

Write $V(t, x(t)) = V(t)$. For $t \in [t_0 - \tau, t_0]$, we have $EV(t) \leqslant bE|x(t)|^p \leqslant bE\|\varphi\|_\tau^p \leqslant bB_1 < \lambda aB_2 < aB_2$, and $EV(\hat{t}) \geqslant aE|x(\hat{t})|^p \geqslant aB_2$. Define $t^* = \inf\{t \in [t_0, \hat{t}] \mid EV(t) \geqslant aB_2\}$ and then $t^* \in (t_0, \hat{t}]$ and $EV(t) < aB_2$ for $t \in [t_0 - \tau, t^*)$ and $EV(t^*) \geqslant aB_2$.

We claim that $t^* \neq t_k$ for any $k \in \mathbb{N}$ and then $EV(t^*) = aB_2$.

If it is not true, suppose $t^* = t_k$ for some k. If $E|x(t_k^-)|^p \geqslant \rho$, then $aB_2 \leqslant EV(t_k) \leqslant \lambda EV(t_k^-) < \lambda aB_2 < aB_2$, which is a contradiction. If $E|x(t_k^-)|^p < \rho$, then $E|x(t_k)|^p = E|x(t_k^-) + I(t_k, x(t_k^-))|^p < \rho_1 < B_1$. Then $aB_2 < EV(t_k) < bB_1 < \lambda aB_2 < aB_2$, which is a contradiction.

Now we will proceed under two cases.

Case 1. Consider $t_{l-1} \leqslant t_0 < t^* < t_l$.

Let $\bar{t} = \sup\{t \in [t_0, t^*] \mid EV(t) \leqslant \lambda aB_2\}$. Since $EV(t_0) < bB_1 < \lambda aB_2$, $EV(t^*) = aB_2 > \lambda aB_2$, and $EV(t)$ is continuous on $[t_0, t^*]$, then $\bar{t} \in (t_0, t^*)$ and $EV(\bar{t}) = \lambda aB_2$ and, when $t \in [\bar{t}, t^*]$, $EV(t) \geqslant \lambda aB_2$. Hence, for $t \in [\bar{t}, t^*]$ and $s \in [-\tau, 0]$, we have

$$\begin{gathered} \lambda EV(t+s) \leqslant \lambda aB_2 \leqslant EV(t), \\ bB_1 \leqslant \lambda aB_2 \leqslant EV(t) \leqslant bE|x(t)|^p, \end{gathered} \tag{4}$$

and we can get

$$E|x(t)|^p \geqslant B_1 \geqslant \rho. \tag{5}$$

Then, by virtue of condition (2), for $t \in [\bar{t}, t^*]$,

$$\begin{aligned} E\mathscr{L}V(t) &\leqslant \gamma EV(t), \\ EV(t^*) - EV(\bar{t}) &= \int_{\bar{t}}^{t^*} E\mathscr{L}V(s)\,ds \\ &\leqslant \int_{\bar{t}}^{t^*} \gamma EV(s)\,ds < \gamma\alpha aB_2. \end{aligned} \tag{6}$$

However,

$$EV(t^*) - EV(\bar{t}) = aB_2 - \lambda aB_2 = (1-\lambda)aB_2 > \gamma\alpha aB_2, \tag{7}$$

which is contradiction. Then we get, in this case,

$$E|x(t)|^p \leqslant B_2. \tag{8}$$

Case 2. Consider $t_k < t^* < t_{k+1}$ for some $k \geqslant l$.

Note that $EV(t_k) \leqslant \lambda aB_2$. This inequality can be obtained by the following reason: if $E|x(t_k^-)|^p \geqslant \rho$, then $EV(t_k) \leqslant \lambda EV(t_k^-) \leqslant \lambda aB_2$. If $E|x(t_k^-)|^p < \rho$, we get $E|x(t_k)|^p < \rho_1 < B_1$, and then

$$EV(t_k) < bB_1 < \lambda aB_2. \tag{9}$$

Define $\bar{t} = \sup\{t \in [t_k, t^*] \mid EV(t) \leqslant \lambda aB_2\}$, and then $\bar{t} \in [t_k, t^*)$, $EV(\bar{t}) = \lambda aB_2$, and $EV(t) \geqslant \lambda aB_2$ for $t \in [\bar{t}, t^*]$. The same argument as the one in Case 1 yields a contradiction. Therefore, in this case, we have, for any $t \in [t_0 - \tau, \infty)$,

$$E|x(t)|^p \leqslant B_2. \tag{10}$$

Now we get that, under conditions (1) to condition (5), the solutions of (1) are p-moment uniformly bounded. That is, if $E\|\varphi\|_\tau^p \leqslant \rho_1$, there exists a constant $B > 0$, such that $E|x(t, t_0, \varphi)|^p \leqslant B$ for all $t \geqslant t_0 - \tau$, and, from the proof, we have $b\rho_1 < \lambda aB$.

Step 2. Now, let $B_3 > 0$ and assume, without loss of generality, that $B_3 > B$. Then, from the proof of uniform boundedness, there exists some $B_2 = B_2(B_3) > B_3$ for which if $E\|\varphi\|_\tau^p \leqslant B_3$, then $E|x(t)|^p \leqslant B_2$ for $t \geqslant t_0 - \tau$.

Take a constant d satisfying $0 < d \leqslant (1-\lambda-\gamma\tau)aB/(1-\gamma\tau)$; it is easy to verify that $0 < d < (1-\lambda)aB$. Let $N = N(B_3)$ be the smallest positive integer for which $bB_2 < aB + Nd$ and $T = T(B_3) = \alpha + (\tau + \alpha)(N-1)$. Given a solution $x(t) = x(t, t_0, \varphi)$ where $E\|\varphi\|_\tau^p \leqslant B_3$ and $t_0 \in [t_{l-1}, t_l)$, we will show $E|x(t)|^p \leqslant B$ for $t \geqslant t_0 + T$.

Given a constant A satisfying $aB \leqslant A - d \leqslant bB_2$ and $j > l$, we will show that if $EV(t) \leqslant A$ for $t \in [t_j - \tau, t_j)$, then $EV(t) \leqslant A - d$ for $t \geqslant t_j$.

For the sake of contradiction, suppose that there exists some $t \geqslant t_j$ for which $EV(t) > A - d$ and define

$$t^* = \inf\left\{t \geqslant t_j \mid EV(t) > A - d\right\}, \tag{11}$$

and we suppose $t^* \in [t_k, t_{k+1})$ for some $k \in \mathbb{N}$. We can get $EV(t) \leqslant A - d$ for $t \in [t_j - \tau, t^*)$ and $EV(t^*) \geqslant A - d$.

We claim that $EV(t_k) \leqslant \lambda A$. The fact follows that if $E|x(t_k^-)|^p \geqslant \rho$, then $EV(t_k) \leqslant \lambda EV(t_k^-) \leqslant \lambda A$. If $E|x(t_k^-)|^p < \rho$ and we have $E|x(t_k)|^p \leqslant \rho_1$, then $EV(t_k) \leqslant b\rho < bB \leqslant \lambda aB \leqslant \lambda A$.

Now, since $aB \leqslant A$, we have $\lambda A = A - (1-\lambda)A < A - (1-\lambda)aB < A - d$ and $EV(t_k) < A - d$. This implies that $t^* \neq t_k$; that is, $t^* \in (t_k, t_{k+1})$ and $EV(t^*) = A - d$ since $EV(t)$ is continuous at t^*. Also, for $t \in [t_k, t^*]$, we have $EV(t) \leqslant A - d$. Define

$$\bar{t} = \sup\left\{t \in [t_k, t^*] \mid EV(t) \leqslant \lambda(A-d)\right\}. \tag{12}$$

Since $EV(t^*) = A - d > \lambda A > \lambda(A-d)$, we have $\bar{t} \in [t_k, t^*)$ and $EV(\bar{t}) = \lambda(A-d)$ and $EV(t) \geqslant \lambda(A-d)$ for $t \in [\bar{t}, t^*]$. Then, if $t \in [\bar{t}, t^*]$ and $s \in [-\tau, 0]$,

$$\begin{gathered} \lambda EV(t+s) \leqslant \lambda(A-d) < EV(t), \\ bE|x(t)|^p > EV(t) > \lambda(A-d) > \lambda aB > b\rho, \end{gathered} \tag{13}$$

which yields $E|x(t)|^p > \rho$. Then, in light of condition (2),

$$E\mathscr{L}V(t) \leqslant \gamma EV(t). \quad (14)$$

In terms of Itô formula,

$$\begin{aligned} EV(t^*) - EV(\bar{t}) &= \int_{\bar{t}}^{t^*} E\mathscr{L}V(s)\,ds \\ &\leqslant \int_{\bar{t}}^{t^*} \gamma EV(s)\,ds \leqslant \gamma\alpha(A-d). \end{aligned} \quad (15)$$

But

$$EV(t^*) - EV(\bar{t}) = A - d - \lambda(A-d) > \gamma\alpha(A-d), \quad (16)$$

and this contradiction proves that $EV(t) < A - d$ for all $t \geqslant t_j$. □

Now we define a sequence $t_{k^{(i)}} \in \{t_k, k = l, l+1, \ldots\}$, satisfying $t_{k^{(1)}} = t_l$ and $t_{k^{(i)}-1} - \tau \leqslant t_{k^{(i-1)}} \leqslant t_{k^{(i)}} - \tau$, and then we have $t_{k^{(i)}} \leqslant t_{k^{(i)}-1} + \alpha \leqslant t_{k^{(i-1)}} + \tau + \alpha$. By induction, we get $t_{k^{(N)}} \leqslant t_0 + \alpha + (\tau+\alpha)(N-1) = t_0 + T$. We know that when $t \in [t_0 - \tau, t_l)$, that is, $t \in [t_0 - \tau, t_{k^{(1)}})$, $EV(t) \leqslant bB_2$; then by induction we get $EV(t) \leqslant bB_2 - Nd$ for $t \in [t_{k^{(N)}}, \infty)$ and then $EV(t) \leqslant aB$ for $t \in [t_0 + T, \infty)$. Using condition (1), we get that $aE|x(t)|^p \leqslant EV(t) \leqslant aB$; that is,

$$E|x(t)|^p \leqslant B. \quad (17)$$

Remark 3. Condition (2) means the system without impulse may be unbounded. If the impulsive effects satisfy condition (3) to condition (5), then this system can be bounded.

4. Boundedness with Impulsive Disturbance

In this section, we consider the case that the SDDS is bounded, and when the impulsive disturbance appears in the SDDS, then what restrictions should be added to the disturbance to maintain the boundedness. The result is stated as follows.

Theorem 4. *Assume that there exist a positive function $V(t,x)$ and positive constants $a, b, c, p, \lambda_1, \lambda_2, \gamma$, where $1 \leqslant \lambda_1 < \lambda_2$, such that*

(1) $a|x|^p \leqslant V(t,x) \leqslant b|x|^p$ *for any* (t,x);

(2) *for* $t \neq t_k$, *any* $s \in [-\tau, 0]$, *and* $\phi(s) \in PC([-\tau,0],\mathbb{R}^n)$, $\mathscr{L}V(t,\phi(0)) \leqslant -\gamma V(t,\phi(0))$ *whenever* $\lambda_2 V(t,\phi(0)) \geqslant V(t+s,\phi(s))$ *and* $|\phi(0)|^p \geqslant \rho$;

(3) $V(t_k, \phi(0) + I(t_k,\phi(0))) \leqslant \lambda_1 V(t_k^-, \phi(0))$ *for all* $|\phi(0)|^p \geqslant \rho$;

(4) *there exists a positive constant* $\rho_1 \geqslant \rho$ *such that if* $|\phi(0)|^p \leqslant \rho$, *then* $|\phi(0) + I(\tau_k,\phi(0))|^p \leqslant \rho_1$;

(5) *there exist positive constants* μ *and* α, *such that* $\mu \leqslant t_k - t_{k-1} \leqslant \alpha$ *and* $\mu\gamma > \lambda_2 - 1$.

Then, the system (1) *is p-moment uniformly ultimately bounded.*

Proof. Step 1. Let $B_1 > 0$; without loss of generality, we assume $B_1 \geqslant \rho_1$. Choose $B_2 = B_2(B_1)$, such that $\lambda_2 bB_1 < aB_2$, and then we get $B_2 > B_1$. Let $E\|\varphi\|_\tau^p \leqslant B_1$ and assume $t_0 \in [t_{l-1}, t_l)$; moreover, we assume that (1) has a maximal interval of existence, $[t_0 - \tau, t_0 + \beta)$.

We will prove that $E|x(t)|^p \leqslant B_2$ for $t \in [t_0, t_0 + \beta)$. This will show that $\beta = \infty$ and that solutions of (1) are uniformly bounded.

For the sake of contradiction, we suppose that $E|x(t)|^p > B_2$ for some $t \in [t_0, t_0 + \beta)$. Let $\hat{t} = \inf\{t \in [t_0, t_0+\beta) \mid E|x(t)|^p > B_2\}$. Note that $E|x(t)|^p \leqslant E\|\varphi\|_\tau^p < B_1 < B_2$ for $t \in [t_0 - \tau, t_0]$, and we get $\hat{t} \in (t_0, t_0 + \beta)$, $E|x(t)|^p \leqslant B_2$ for $t \in [t_0 - \tau, \hat{t})$ and $E|x(\hat{t})|^p \geqslant B_2$.

For $t \in [t_0 - \tau, t_0]$, we have $EV(t) \leqslant bE|x(t)|^p \leqslant bE\|\varphi\|_\tau^p \leqslant bB_1$ and then $EV(t) \leqslant \lambda_2 EV(t) \leqslant \lambda_2 bB_1 < aB_2$. Particularly, $EV(t_0) \leqslant \lambda_2 EV(t_0) < aB_2$ and $EV(\hat{t}) \geqslant aE|x(\hat{t})|^p \geqslant aB_2$.

Define $t^* = \inf\{t \in [t_0, \hat{t}] \mid EV(t) \geqslant aB_2\}$ and then $t^* \in (t_0, \hat{t}]$, $EV(t^*) \geqslant aB_2$, and $EV(t) < aB_2$ for $t \in [t_0 - \tau, t^*)$.

Now we will proceed under two cases.

Case 1. Consider $t_{l-1} \leqslant t_0 < t^* < t_l$.

Under this case, we have $EV(t^*) = aB_2$ because of the continuity of $V(t)$ on (t_k, t_{k+1}) and $\lambda_2 EV(t^*) = \lambda_2 aB_2 > aB_2$. Define $\bar{t} = \sup\{t \in [t_0, t^*] \mid \lambda_2 EV(t) \leqslant aB_2\}$ and then $\bar{t} \neq t^*$, $\lambda_2 EV(\bar{t}) = aB_2$, and $\lambda_2 EV(t) \geqslant aB_2$ for $t \in [\bar{t}, t^*]$. Therefor, for any $t \in [\bar{t}, t^*]$ and $s \in [-\tau, 0]$, we have $EV(t+s) \leqslant aB_2 < \lambda_2 EV(t)$ and $\lambda_2 bB_1 < aB_2 < \lambda_2 EV(t)$, which yields $EV(t) > bB_1$, and then we have $E|x(t)|^p > B_1 \geqslant \rho$. Using condition (2), we have, when $t \in [\bar{t}, t^*]$,

$$E\mathscr{L}V(t) \leqslant -\gamma EV(t). \quad (18)$$

By virtue of Itô formula, we have

$$EV(t^*) - EV(\bar{t}) = \int_{\bar{t}}^{t^*} E\mathscr{L}V(s)\,ds \leqslant \int_{\bar{t}}^{t^*} -\gamma EV(s)\,ds \leqslant 0. \quad (19)$$

However,

$$EV(t^*) = aB_2 > \frac{aB_2}{\lambda_2} = EV(\bar{t}). \quad (20)$$

This contradiction gives

$$E|x(t)|^p \leqslant B_2 \quad \text{for } t \in [t_0, t_0 + \beta). \quad (21)$$

Case 2. Consider $t_k \leqslant t^* < t_{k+1}$ for some $k \geqslant l$.

We first show $\lambda_2 EV(t_k^-) \leqslant aB_2$. We have two situations to contemplate: $k = l$ and $k > l$.

If $k = l$, we suppose $\lambda_2 EV(t_l^-) > aB_2$. Define $\bar{t} = \sup\{t \in [t_0, t_l) \mid \lambda_2 EV(t) \leqslant aB_2\}$ and then $\bar{t} \in (t_0, t_l)$ and $\lambda_2 EV(\bar{t}) = aB_2$. In light of the definition of $\bar{t}$, we have, for $t \in [\bar{t}, t_l)$ and $s \in [-\tau, 0]$,

$$\lambda_2 EV(t) \geqslant aB_2 \geqslant EV(t+s), \quad (22)$$

and, for $t \in [\bar{t}, t_l)$,

$$E|x(t)|^p \geqslant B_1 \geqslant \rho. \quad (23)$$

By virtue of condition (2), an analogous calculation of $EV(t_l^-) - EV(\bar{t})$ yields $EV(t_l^-) \leqslant EV(\bar{t})$; then we get

$$aB_2 < \lambda_2 EV(t_l^-) \leqslant \lambda_2 EV(\bar{t}) = aB_2. \quad (24)$$

If $k > l$, we suppose $\lambda_2 EV(t_k^-) > aB_2$. We will proceed under two subcases.

Subcase 1. Consider $\lambda_2 EV(t) > aB_2$ for all $t \in [t_{k-1}, t_k)$.

Under this situation, we have $\lambda_2 EV(t) > aB_2 \geqslant EV(t+s)$ and $E|x(t)|^p \geqslant \rho$ for all $t \in [t_{k-1}, t_k)$ and $s \in [-\tau, 0]$. In terms of condition (2), an analogous discussion as done in Case 1 gives

$$\begin{aligned} EV(t_k^-) - EV(t_{k-1}) &= \int_{tk-1}^{t_k^-} E\mathscr{L}V(s)\,ds \\ &\leqslant \int_{tk-1}^{t_k^-} -\gamma EV(s)\,ds \leqslant -\gamma\mu\frac{aB_2}{\lambda_2}. \end{aligned} \quad (25)$$

However, by virtue of condition (5),

$$\begin{aligned} EV(t_k^-) - EV(t_{k-1}) &\geqslant \frac{aB_2}{\lambda_2} - aB_2 = \left(\frac{1}{\lambda_2} - 1\right) aB_2 \\ &> -\gamma\mu\frac{aB_2}{\lambda_2}. \end{aligned} \quad (26)$$

This contradiction implies

$$\lambda_2 EV(t_k^-) \leqslant aB_2 \quad \text{for } t_k \leqslant t^* < t_{k+1},\ k \geqslant l. \quad (27)$$

Subcase 2. Consider $\lambda_2 EV(t) \leqslant aB_2$ for some $t \in [t_{k-1}, t_k)$.

Define $\bar{t} = \sup\{t \in [t_{k-1}, t_k) \mid \lambda_2 EV(t) \leqslant aB_2\}$ and then $\bar{t} \in [t_{k-1}, t_k)$ and $\lambda_2 EV(\bar{t}) = aB_2$. Using the definition of $\bar{t}$, we get, for $t \in [\bar{t}, t_k)$ and $s \in [-\tau, 0]$, $\lambda_2 EV(t) \geqslant aB_2 \geqslant EV(t+s)$. Since $\lambda_2 EV(t) \geqslant aB_2$, using the fact $\rho_1 \geqslant \rho, \lambda_2 bB_1 < aB_2$ and $b|x|^p \geqslant V(t,x)$, we can get $E|x(t)|^p \geqslant \rho$. By virtue of condition (2), we get, for $t \in [\bar{t}, t_k)$,

$$E\mathscr{L}V(t) \leqslant -\gamma EV(t). \quad (28)$$

An analogous discussion as done in the case $k = l$ gives $EV(\bar{t}) \geqslant EV(t_k^-)$. Then we have

$$aB_2 < \lambda_2 EV(t_k^-) \leqslant \lambda_2 EV(\bar{t}) = aB_2. \quad (29)$$

This contradiction gives

$$\lambda_2 EV(t_k^-) \leqslant aB_2 \quad \text{for } t_k \leqslant t^* < t_{k+1},\ k \geqslant l. \quad (30)$$

Now we claim $EV(t_k) < aB_2$. If $E|x(t_k^-)|^p \geqslant \rho$, we get $EV(t_k) \leqslant \lambda_1 EV(t_k^-) < \lambda_2 EV(t_k^-) < aB_2$. If $E|x(t_k^-)|^p < \rho$, we get $EV(t_k) \leqslant b\rho_1 < bB_1 < \lambda_2 bB_1 < aB_2$. That is, the following inequality holds:

$$EV(t_k) < aB_2. \quad (31)$$

Since $EV(t^*) \geqslant aB_2$, we have $t^* \neq t_k$ and $EV(t^*) = aB_2$.

If $\lambda_2 EV(t^*) \geqslant aB_2$ for all $t \in [t_k, t^*]$, then let $\bar{t} = t_k$ and we have $EV(\bar{t}) < aB_2$. Otherwise, let $\bar{t} = \sup\{t \in [t_k, t^*) \mid \lambda_2 EV(t) \leqslant aB_2\}$, and we have $EV(\bar{t}) < \lambda_2 EV(\bar{t}) = aB_2$. Since $EV(t^*) = aB_2$, we get $\bar{t} \in [t_k, t^*)$. Moreover, for $t \in [\bar{t}, t^*]$, we have $\lambda_2 EV(t) \geqslant aB_2 > EV(t+s)$ and, by virtue of $\lambda_2 bB_1 < aB_2 < \lambda_2 EV(t)$, we obtain $EV(t) > bB_1$ and then $E|x(t)|^p > B_1 > \rho$. In terms of condition (2) and Itô formula, we can obtain $EV(\bar{t}) \geqslant EV(t^*)$. But $EV(\bar{t}) < aB_2 = EV(t^*)$, which is a contradiction and yields

$$E|x(t)|^p \leqslant B_2 \quad \text{for } t \in [t_0, t_0 + \beta). \quad (32)$$

Now we get that, under condition (1) to condition (5), the solutions of (1) are p-moment uniformly bounded. Then we know that if $E\|\varphi\|_\tau^p \leqslant \rho_1$, there exists a constant $B > 0$, such that $E|x(t, t_0, \varphi)|^p \leqslant B$ for all $t \geqslant t_0 - \tau$, and, from the above proof, we have $\lambda_2 b\rho_1 < aB$.

Step 2. Now, let $B_3 > 0$ and assume, without loss of generality, that $B_3 > B$. Then, from the proof of uniform boundedness, there exists a constant $B_2 = B_2(B_3) > B_3$ for which if $E\|\varphi\|_\tau^p \leqslant B_3$, then $E|x(t)|^p \leqslant B_2$ for $t \geqslant t_0 - \tau$.

Take a constant d satisfying $0 < d \leqslant \min\{aB - b\rho_1, ((\lambda_2 - \lambda_1)/\lambda_2)aB\}$, $N = \min\{n > ((bB_2 - aB)/d)\}$, and $T = \alpha + (2N-1)(\alpha + \tau)$.

Let $x(t) = x(t, t_0, \varphi)$ be a solution of (1) with $E\|\varphi\|_\tau^p \leqslant B_3$, $t_0 \in [t_{l-1}, t_l)$. We will show $E|x(t)|^p \leqslant B$ for $t \geqslant t_0 + T$.

Given a positive number A satisfying $aB \leqslant A \leqslant bB_2$ and $j \geqslant l$, we will show that if $EV(t) \leqslant A$ for $t \in [t_j - \tau, t_j)$ and $\lambda_2 EV(t_j^-) \leqslant A$, then $EV(t) \leqslant A$ for $t \geqslant t_j$ and $\lambda_2 EV(t_{j+1}^-) \leqslant A$.

For the sake of contradiction, suppose that there exists a constant $t \in [t_j, t_{j+1})$ for which $EV(t) > A$ and define

$$t^* = \inf\left\{t \in [t_j, t_{j+1}) \mid EV(t) \geqslant A\right\}. \quad (33)$$

Note that $EV(t_j) < A$, and we have that if $E|x(t_j^-)|^p \geqslant \rho$, then $EV(t_k) \leqslant \lambda_1 EV(t_j^-) < \lambda_2 EV(t_j^-) \leqslant A$. If $E|x(t_k^-)|^p < \rho$, we have $EV(t_j) \leqslant b\rho_1 < \lambda_2 b\rho_1 < aB \leqslant A$. Then we get $t^* \neq t_j$, $EV(t^*) = A$, and $EV(t) \leqslant A$ for $t \in (t_j, t_{j+1}]$.

If $\lambda_2 EV(t) > A$ for all $t \in [t_j, t_{j+1})$, we let $\bar{t} = t_j$, and then $EV(\bar{t}) = EV(t_j) < A$. Otherwise, let $\bar{t} = \sup\{t \in [t_j, t^*] \mid \lambda_2 EV(t) \leqslant A\}$, and we get $EV(\bar{t}) \leqslant \lambda_2 EV(\bar{t}) = A$. Since $\lambda_2 EV(t^*) = \lambda_2 A > A$, $\bar{t} \neq t^*$. For $t \in [\bar{t}, t^*]$ and $s \in [-\tau, 0]$, we have $\lambda_2 EV(t) \geqslant A \geqslant EV(t+s)$. Moreover, for $t \in [\bar{t}, t^*]$,

$$\lambda_2 EV(t) \geqslant A \geqslant aB > \lambda_2 b\rho_1, \quad (34)$$

and we get $E|x(t)|^p \geqslant \rho_1 \geqslant \rho$. By virtue of condition (2) and Itô formula, we can get $EV(\bar{t}) \geqslant EV(t^*)$. However, $EV(t^*) = A > EV(\bar{t})$.

Now we have proven $EV(t) \leqslant A$ for $t \in [t_j, t_{j+1})$, and we are on the position to show $\lambda_2 EV(t_{j+1}^-) \leqslant A$. This will follow in the same way as the arguments used in the proof of uniform boundedness, where we show $\lambda_2 EV(t_k^-) \leqslant aB_2$ for the case $k > l$; we just need to replace k by $j+1$ and ab_2 by A.

By induction, we get that if $EV(t) \leqslant A$ for $t \in [t_j - \tau, t_j)$ and $\lambda_2 EV(t_j^-) \leqslant A$, then $EV(t) \leqslant A$ for all $t \geqslant t_j$ and $\lambda_2 EV(t_k^-) \leqslant A$ for $k \geqslant j+1$.

Next, we will show $EV(t) \leqslant A - d$ for $t \in [t_{j+1}, t_{j+2})$, if $EV(t) \leqslant A$ for all $t \geqslant t_j$ and $\lambda_2 EV(t_k^-) \leqslant A$, $k \geqslant j$.

We first show $EV(t_{j+1}) \leqslant A - d$. This can be easily verified under two situations: iIf $E|x(t_{j+1}^-)|^p \leqslant \rho$, we have $EV(t_{j+1}) \leqslant b\rho_1 \leqslant aB - d \leqslant A - d$; if $E|x(t_{j+1}^-)|^p > \rho$, $EV(t_{j+1}) < \lambda_1 EV(t_{j+1}) = (\lambda_1/\lambda_2)\lambda_2 EV(t_{j+1}^-) \leqslant (\lambda_1/\lambda_2)A < A - d$.

In order to verify $EV(t) \leqslant A - d$ for all $t \in [t_{j+1}, t_{j+2})$, suppose that $EV(t) > A - d$ for some $t \in [t_{j+1}, t_{j+2})$. Let $t^* = \inf\{t \in [t_{j+1}, t_{j+2}) \mid EV(t) \geqslant A - d\}$; we know $t^* \neq t_{j+1}$ and then $EV(t^*) = A - d$ and $\lambda_2 EV(t^*) = \lambda_2(A - d) > A$.

If $\lambda_2 EV(t) > A$ for all $t \in [t_{j+1}, t^*]$, let $\bar{t} = t_{j+1}$, $EV(\bar{t}) = EV(t_{j+1}) < A - d$.

If $\lambda_2 EV(t) > A$ for some $t \in (t_{j+1}, t^*]$, let $\bar{t} = \sup\{t \in [t_{j+1}, t^*] \mid \lambda_2 EV(t) \leqslant A\}$ and we know $\bar{t} \neq t^*$, $EV(\bar{t}) = A/\lambda_2$.

For $t \in [\bar{t}, t^*]$ and $s \in [-\tau, 0]$, $\lambda_2 EV(t) \geqslant A > A - d > EV(t + s)$ and $EV(t) \geqslant A/\lambda_2 > aB/\lambda_2 > b\rho_1$, and we get $E|x(t)|^p > \rho_1 \geqslant \rho$. In terms of condition (2) and Itô formula, we can get $EV(t^*) < EV(\bar{t})$. However, $EV(t^*) = A - d > EV(\bar{t})$, which yields

$$EV(t) \leqslant A - d. \tag{35}$$

Applying our results to successive intervals of the form $[t_k, t_{k+1})$ for $k \geqslant j + 1$, we can get $EV(t) \leqslant A - d$ for $t \geqslant t_{j+1}$.

Now we need a fact $\lambda_2 EV(t_{j+2}^-) \leqslant A - d$. This can be verified just as we did in the proof of uniform boundedness, where we show $\lambda_2 EV(t_k^-) \leqslant aB_2$ for the case $k > l$.

Take $t_{k^{(i)}} \in \{t_j, j = l, l+1, \ldots\}$ satisfying $t_{k^{(i-1)}} + \tau \leqslant t_{k^{(i)}} \leqslant t_{k^{(i-1)}+1} + \tau$. Take $A = bB_2$, when $t \geqslant t_{k^{(2N)}}$, and we get $EV(t) \leqslant bB_2 - Nd < aB$. Since $t_{k^{(2N)}} \leqslant t_{k^{(1)}} + (2N-1)(\alpha + \tau) \leqslant t_0 + \alpha + (2N-1)(\alpha + \tau) = t_0 + T$, we have $EV(t) \leqslant aB$ when $t > t_0 + T$. By virtue of condition (1), $E|x(t)|^p \leqslant B$ for $t \geqslant t_0 + T$, which completes the proof. □

Remark 5. Theorem 4 considers that a bounded system without impulse can tolerate what kind of impulsive effects to hold the boundedness. It is not surprising that condition (3) to condition (5) should be satisfied: the interval of impulsive moments (μ) should be large and impulsive strength (λ_1) should be small.

5. Examples

In this section, we present two examples to illustrate our results.

Example 1. Consider the following impulsive stochastic delay differential system:

$$dx(t) = \left(\frac{1}{2}x(t) + \frac{1}{2x(t)}\right)dt + x\left(t - \frac{1}{20}\right)dB(t),$$
$$t > 0, \quad t \neq \frac{k}{10}, \quad k = 1, 2, \ldots, \tag{36}$$
$$x\left(\frac{k}{10}\right) = \frac{\sqrt{2}}{2}x\left(\left(\frac{k}{10}\right)^-\right),$$

where $B(t)$ is a one-dimension Brownian motion.

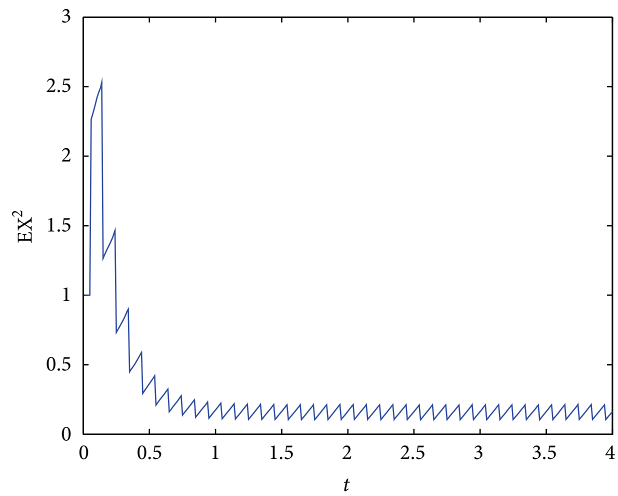

Figure 1: Mean square uniform ultimate boundedness of solution of system (36).

Define $V(t, x) = x^2$; the smoothness requirement is satisfied. Let $a = b = 1$ and $p = 2$; condition (1) of Theorem 2 follows. For any solution $x(t)$ of system (36), we have

$$\begin{aligned}\mathscr{L}V(t, x) &= 2x\left(\frac{1}{2}x(t) + \frac{1}{2x(t)}\right) + x^2\left(t - \frac{1}{20}\right) \\ &= x^2(t) + 1 + x^2\left(t - \frac{1}{20}\right).\end{aligned} \tag{37}$$

Take $\lambda = 1/2$; condition (3) of Theorem 2 is satisfied.

Now let $\rho = 1$; then, when $|x(t)|^2 \geqslant 1$ and $V(t, x) \geqslant \lambda V(t, x(t - \tau))$, that is, $x^2(t) \geqslant (1/2)x^2(t - 1/20)$, we have

$$\mathscr{L}V(t, x) \leqslant x^2(t) + x^2(t) + 2x^2(t) = 4x^2(t) = 4V(t, x). \tag{38}$$

Then let $\gamma = 4$; condition (2) of Theorem 2 is verified.

Condition (4) of Theorem 2 can be verified by taking $\rho_1 = 1$.

Take $\alpha = 1/10$ and then $\alpha\gamma = (1/10) \times 4 = 2/5 < 1/2 = 1 - \lambda$; condition (5) of Theorem 2 is verified.

Therefore, according to Theorem 2, solutions of system (36) are mean square uniformly ultimately bounded. The boundedness can be read from Figure 1, where we take initial condition $x(t) = 1, t \in [-1/20, 0]$.

To see the contribution of impulsive effect on boundedness, we consider the following system:

$$dx(t) = \left(\frac{1}{2}x(t) + \frac{1}{2x(t)}\right)dt + x\left(t - \frac{1}{20}\right)dB(t), \quad t > 0, \tag{39}$$

which is the situation of system (36) without impulses. It is easy to be verified that system (39) is unbounded; see Figure 2, where we also take initial condition $x(t) = 1, t \in [-1/20, 0]$.

Now we give another example to illustrate the correctness of Theorem 4.

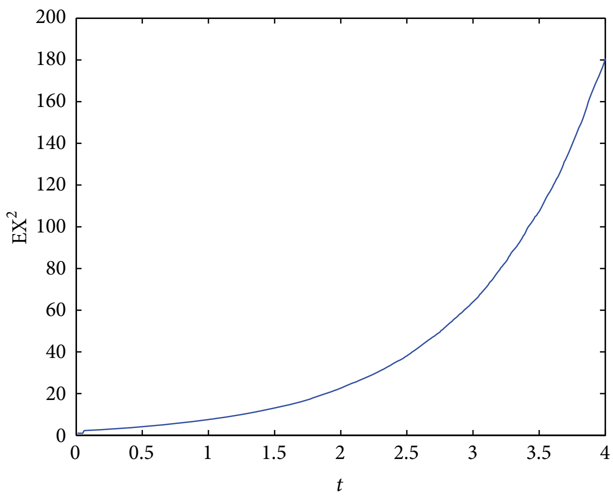

Figure 2: Unboundedness of solution of system (39).

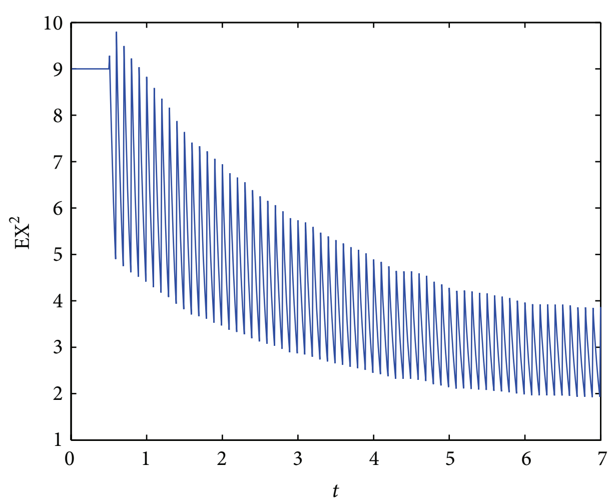

Figure 3: Mean square uniform ultimate boundedness of solution of system (40).

Example 2. Consider

$$\begin{aligned} dx(t) &= \left(-4x(t) + \frac{1}{2x(t)}\right)dt + x\left(t - \frac{1}{2}\right)dB(t), \\ &\qquad t > 0, \quad t \neq 2k, \quad k = 1, 2, \ldots, \\ x(2k) &= \sqrt{2}x\left((2k)^-\right), \end{aligned} \tag{40}$$

where $B(t)$ is a one-dimension Brownian motion.

Define $V(t, x) = x^2$; the smoothness requirement is satisfied. Let $a = b = 1$ and $p = 2$; condition (1) of Theorem 4 follows. For any solution $x(t)$ of system (40), we have

$$\begin{aligned} \mathscr{L}V(t, x) &= 2x\left(-4x(t) + \frac{1}{2x(t)}\right) + x^2\left(t - \frac{1}{2}\right) \\ &= -8x^2(t) + 1 + x^2\left(t - \frac{1}{20}\right). \end{aligned} \tag{41}$$

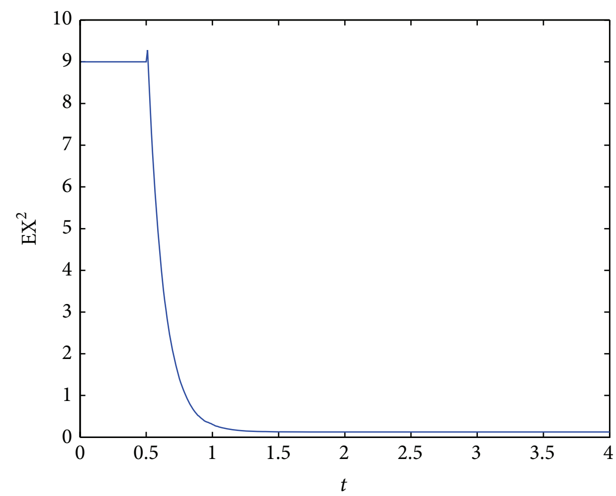

Figure 4: Simulation of system (43).

Take $\lambda_1 = 2$, condition (3) of Theorem 4 is satisfied.

Now let $\rho = 1$ and $\lambda_2 = 3$; then, when $|x(t)|^2 \geqslant 1$ and $V(t, x) \geqslant \lambda_2 V(t, x(t - \tau))$, that is, $3x^2(t) \geqslant x^2(t - 1/2)$, we have

$$\begin{aligned} \mathscr{L}V(t, x) &\leqslant -8x^2(t) + x^2(t) + 3x^2(t) \\ &= -4x^2(t) = -4V(t, x). \end{aligned} \tag{42}$$

Then, let $\gamma = 4$; condition (2) of Theorem 2 is verified.

Condition (4) of Theorem 2 can be verified by taking $\rho_1 = 2$.

Take $\mu = 2$ and then $\mu\gamma = 2 \times 8 = 16 > 3 - 1 = \lambda_2 - 1$ and condition (5) of Theorem 4 is verified.

Therefore, according to Theorem 4, solutions of system (40) are mean square uniformly ultimately bounded. The boundedness can be seen in Figure 3, where we take initial condition $x(t) = 3, t \in [-1/2, 0]$.

We also present the simulation of system (40) without impulsive effects; that is,

$$dx(t) = \left(-4x(t) + \frac{1}{2x(t)}\right)dt + x\left(t - \frac{1}{2}\right)dB(t), \quad t > 0. \tag{43}$$

The property of system (43) can be read from Figure 4, where we take initial condition $x(t) = 3, t \in [-1/2, 0]$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding to the publications of this paper.

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