# $\mathscr{H}_{\infty}$ Output-Feedback Tracking Control for Networked Control Systems 

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#### Abstract

This paper investigates the observer-based $\mathscr{H}_{\infty}$ tracking problem of networked output-feedback control systems with consideration of data transmission delays, data-packet dropouts, and sampling effects. Different from other approaches, this paper offers a singlestep procedure to handle nonconvex terms that appear in the process of designing observer-based output-feedback control, and then establishes a set of linear matrix inequality conditions for the solvability of the tracking problem. Finally, two numerical examples are given to illustrate the effectiveness of our result.


## 1. Introduction

Recently, the research on networked control systems (NCSs) has been rapidly growing due to both the fast development of technology of communication networks and the benefits of NCSs that include (1) overcoming the spatial limits of the traditional control system, (2) expanding system setups, (3) increasing flexibility, (4) multitasking, and (5) improving system diagnosis and maintenance (see [1-4]). In particular, more recently, the development of the embedded system that has various communication modules and digital signal processing (DPS) core has confirmed the necessity of further investigations on NCSs. However, it is worth noticing here that the signal transmission over communication channels inevitably gives rise to data transmission delay problem, data-packet dropout problem, and sampling problem (see [3, 5-8]), which may cause instability or serious deterioration in the performance of the resultant control systems. Thus, exploring such problems has been recognized as one of the most important issues in the application of control theory.

Over the past several years, numerous researchers have made considerable efforts to propose methods for solving the aforementioned problems, especially based on LyapunovKrasovskii functional approach (see [9-11] for stabilization of NCSs (S-NCSs); [12, 13] for stabilization of NOCSs (S-NOCSs); and $[5,14-16]$ for tracking control of NCSs
(T-NCSs), where NOCSs is the abbreviation of networked output-feedback control systems). In addition, [17] investigated the problem of output tracking for NCSs on the basis of the Lyapunov function approach. However, it is worth pointing out here that, regardless of such abundant literature, little progress has been made toward solving the tracking problem of NOCSs (T-NOCSs) in light of the LyapunovKrasovskii functional approach. In fact, all states of the controlled plant are not fully measurable in many engineering applications, and thus the tracking problem has emerged as a topic of significant interest in parallel to the stabilization problem. Thus, it is quite meaningful to study the method of designing T-NOCSs, especially by establishing a set of linear matrix inequality (LMI) conditions for the solvability of the tracking problem.

Motivated by the above concern, we investigate the problem of designing an observer-based T-NOCS with consideration of data transmission delays, data-packet dropouts, and sampling effects. Specifically, the attention is focused on designing an observer-based NOCS in such a way that the plant state tracks the reference signal in the $\mathscr{H}_{\infty}$ sense. The contributions of this paper are mainly threefold.
(1) The problem of designing T-NOCSs is systematically covered with the help of the Lyapunov-Krasovskii functional approach, which helps our results to have more wide applications.
(2) A single-step procedure is proposed to handle nonconvex terms that inherently appear in the process of designing observer-based output-feedback control, which allows the derived sufficient conditions for the solvability of the tracking problem to be established in terms of LMIs.
(3) Through the control synthesis process, this paper shows that the stability criteria derived from the reciprocally convex approach [18] can be clearly applied to the problem of designing T-NOCSs, which offers the possibilities for the extension of the results $[19,20]$ on the stability analysis toward the design of T-NOCSs.

Finally, two numerical examples are given to illustrate the effectiveness of our result.

Notation. The Lebesgue space $\mathscr{L}_{2+}=\mathscr{L}_{2}[0, \infty)$ consists of square-integrable functions on $[0, \infty)$. Throughout this paper, standard notions will be adopted. The notations $X \geq Y$ and $X>Y$ mean that $X-Y$ is positive semidefinite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. For a square matrix $\mathbb{Q}$, the notation $\mathrm{He}(\mathbb{Q})$ denotes $\mathbb{Q}+\mathbb{Q}^{T}$, where $\mathbb{Q}^{T}$ is the transpose of $\mathbb{Q} \cdot \operatorname{col}\left(q_{1}, q_{2}\right)$ is a column vector with entries $q_{1}$ and $q_{2}$ and $\operatorname{diag}\left(\mathbb{Q}_{1}, Q_{2}\right)$ is a diagonal matrix with diagonal entries $\mathbb{Q}_{1}$ and $Q_{2}$. All matrices, if their dimensions are not explicitly stated, are assumed to be compatible with algebraic operation.

## 2. System Description and Preliminaries

Consider a continuous-time plant of the following form:

$$
\begin{gather*}
\dot{x}(t)=A x(t)+B u(t)+D w(t), \\
y(t)=C x(t), \tag{1}
\end{gather*}
$$

where $x(t) \in \mathbb{R}^{n_{x}}, u(t) \in \mathbb{R}^{n_{u}}$, and $y(t) \in \mathbb{R}^{n_{y}}$ denote the state to be estimated, the control input, and the output, respectively, and $w(t) \in \mathbb{R}^{n_{w}}$ denotes the disturbance input such that $w(t) \in \mathscr{L}_{2+}$. Here, as a way to estimate the immeasurable state variables of (1), we employ the following usual state observer:

$$
\begin{gather*}
\dot{\hat{x}}(t)=A \widehat{x}(t)+B u(t)-L(y(t)-\hat{y}(t)),  \tag{2}\\
\hat{y}(t)=C \widehat{x}(t),
\end{gather*}
$$

where $\widehat{x}(t) \in \mathbb{R}^{n_{x}}$ denotes the estimated state and $L \in \mathbb{R}^{n_{x} \times n_{y}}$ is the observer gain to be designed. Further, in parallel to (1) and (2), we incorporate the following dynamic system that generates the reference signal $x_{r}(t) \in \mathbb{R}^{n_{x}}$ :

$$
\begin{equation*}
\dot{x}_{r}(t)=A_{r} x_{r}(t)+r(t), \tag{3}
\end{equation*}
$$

where $r(t) \in \mathbb{R}^{n_{x}}$ denotes the reference input such that $r(t) \in$ $\mathscr{L}_{2+}$ and $A_{r}$ is constructed to be an asymptotically stable matrix. In this paper, our interest is to design an observer-


Figure 1: Networked output-feedback control systems (NOCSs) with observer-based controller.
based networked output-feedback control system (NOCS), based on (1)-(3), such that
(1) the estimated state $\widehat{x}(t)$ can approach the real state $x(t)$ asymptotically;
(2) the estimated state $\widehat{x}(t)$ can track a reference signal $x_{r}(t)$ over a communication network; that is, the state $x(t)$ can track $x_{r}(t)$ by (1);
(3) a guaranteed $\mathscr{H}_{\infty}$ tracking performance can be achieved.

To this end, we first employ the networked control system (NCS) architecture proposed in [3], which contains an observer with time-driven sampler, an event-driven controller, and a packet analyzer with event-driven holder (see Figure 1). For brevity, this paper omits the sophisticated description for the NCS under consideration since it is analogue to that of [3]. However, different from [3], we assume that the initial condition of (2) is given as $\widehat{x}(t)=\phi(t)$, for $t \in\left[t_{0}-d_{M}, t_{0}\right]$, and the initial condition of (3) is given as $x_{r}(t)=\varphi(t)$, for $t \in\left[t_{0}-d_{M}, t_{0}\right]$, where $t_{0}$ denotes the initial time.

Remark 1. Here, it should be noted that, by the NCS architecture of [3], the communication constraints, such as data transmission delays and packet dropouts, can be represented in terms of piecewise continuous-time-varying delays with the lower and upper bounds.

Next, let us consider the following control law, inferred by [3]:

$$
\begin{equation*}
u(t)=F\left(\widehat{x}(t-d(t))-x_{r}(t-d(t))\right), \tag{4}
\end{equation*}
$$

where $d(t) \in\left[d_{m}, d_{M}\right]$ corresponds to the piecewise continuous-time-varying delay that occurs from data transmission delays and packet dropouts. Then, by letting $e(t)=$ $x(t)-\widehat{x}(t)$ and $e_{r}(t)=x(t)-x_{r}(t)$, the control law (4) can be rewritten as

$$
\begin{equation*}
u(t)=F\left(e_{r}(t-d(t))-e(t-d(t))\right) . \tag{5}
\end{equation*}
$$

Further, by setting $\widetilde{x}(t)=\operatorname{col}\left(x_{r}(t), e_{r}(t), e(t)\right) \in \mathbb{R}^{3 n_{x}}$ and $\widetilde{w}(t)=\operatorname{col}(w(t), r(t)) \in \mathbb{R}^{n_{w}+n_{x}}$ and by combining (1), (2), (3), and (5), the closed-loop system is described as

$$
\begin{gather*}
\dot{\tilde{x}}(t)=\widetilde{A} \widetilde{x}(t)+\widetilde{A}_{d} \widetilde{x}(t-d(t))+\widetilde{D} \widetilde{w}(t),  \tag{6}\\
\widetilde{z}(t)=\widetilde{C} \widetilde{x}(t)\left(=C e_{r}(t)\right),
\end{gather*}
$$

where $\widetilde{z}(t) \in \mathbb{R}^{n_{x}}$ denotes the desired output,

$$
\begin{gather*}
\widetilde{A}=\left[\begin{array}{ccc}
A_{r} & 0 & 0 \\
A-A_{r} & A & 0 \\
0 & 0 & A+L C
\end{array}\right], \quad \widetilde{A}_{d}=\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & B F & -B F \\
0 & 0 & 0
\end{array}\right], \\
\widetilde{C}^{T}=\left[\begin{array}{c}
0 \\
C^{T} \\
0
\end{array}\right], \quad \widetilde{D}=\left[\begin{array}{cc}
0 & I \\
D & -I \\
D & 0
\end{array}\right] . \tag{7}
\end{gather*}
$$

Before ending this section, we present the following lemma that will be used in the proof of our main results.

Lemma 2 (see [21]). For real matrices $X, Y$, and $S>0$ with appropriate dimensions, it is satisfied that $0 \leq(X-$ $S Y)^{T} S^{-1}(X-S Y)$ and thus the following inequality holds: $Y^{T} S Y \geq \mathbf{H e}\left(X^{T} Y\right)-X^{T} S^{-1} X$. Further if $X=\mu I$, then

$$
\begin{equation*}
Y^{T} S Y \geq \mathbf{H e}(\mu Y)-\mu^{2} S^{-1} \tag{8}
\end{equation*}
$$

where $\mu$ is a scalar. On the other hand, if $S<0$, then

$$
\begin{equation*}
Y^{T} S Y \leq-\mathbf{H e}(\mu Y)-\mu^{2} S^{-1} \tag{9}
\end{equation*}
$$

## 3. Main Results

Choose a Lyapunov-Krasovskii functional of the following form:

$$
\begin{gather*}
V(t)=V_{1}(t)+V_{2}(t)+V_{3}(t) \\
V_{1}(t)=\tilde{x}^{T}(t) P \widetilde{x}(t) \\
V_{2}(t)=\int_{t-d_{m}}^{t} \tilde{x}^{T}(\alpha) Q_{1} \tilde{x}(\alpha) d \alpha+\int_{t-d_{M}}^{t} \tilde{x}^{T}(\alpha) Q_{2} \tilde{x}(\alpha) d \alpha \\
V_{3}(t)=d_{m} \int_{-d_{m}}^{0} \int_{t+\alpha}^{t} \dot{\tilde{x}}^{T}(\beta) R_{1} \dot{\tilde{x}}(\beta) d \beta d \alpha \\
+d_{I} \int_{-d_{M}}^{-d_{m}} \int_{t+\alpha}^{t} \dot{\tilde{x}}^{T}(\beta) R_{2} \dot{\tilde{x}}(\beta) d \beta d \alpha \tag{10}
\end{gather*}
$$

where $P, Q_{1}, Q_{2}, R_{1}$, and $R_{2} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}}$ are positive definite matrices and $d_{I}=d_{M}-d_{m}$. For later convenience, we define an augmented state $\zeta(t)=\operatorname{col}\left(\widetilde{x}(t), \widetilde{x}\left(t-d_{m}\right), \widetilde{x}(t-d(t)), \widetilde{x}(t-\right.$ $\left.\left.d_{M}\right), \widetilde{w}(t)\right) \in \mathbb{R}^{n_{\eta}}, n_{\eta}=13 n_{x}+n_{w}$, and then establish some block entry matrices $\mathbf{e}_{i}$ such that $\widetilde{x}(t)=\mathbf{e}_{1} \zeta(t), \widetilde{x}\left(t-d_{m}\right)=$ $\mathbf{e}_{2} \zeta(t), \widetilde{x}(t-d(t))=\mathbf{e}_{3} \zeta(t), \widetilde{x}\left(t-d_{M}\right)=\mathbf{e}_{4} \zeta(t)$, and $\widetilde{w}(t)=$ $\mathbf{e}_{5} \zeta(t)$. Then the closed-loop system (6) can be rewritten as $\dot{\tilde{x}}(t)=\Phi_{t} \zeta(t)$, where $\Phi_{t}=\widetilde{A} \mathbf{e}_{1}+\widetilde{A}_{d} \mathbf{e}_{3}+\widetilde{D} \mathbf{e}_{5}$. As a result, the time derivative of $V_{i}(t)$ along the trajectories of (6) is given by

$$
\begin{gather*}
\dot{V}_{1}(t)=\zeta^{T}(t) \operatorname{He}\left(\mathbf{e}_{1}^{T} P \Phi_{t}\right) \zeta(t), \\
\dot{V}_{2}(t)=\zeta^{T}(t)\left(\mathbf{e}_{1}^{T}\left(Q_{1}+Q_{2}\right) \mathbf{e}_{1}-\mathbf{e}_{2}^{T} Q_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} Q_{2} \mathbf{e}_{4}\right) \zeta(t), \\
\dot{V}_{3}(t)=\zeta^{T}(t) \Phi_{t}^{T}\left(d_{m}^{2} R_{1}+d_{I}^{2} R_{2}\right) \Phi_{t} \zeta(t)+\mathcal{O}, \tag{11}
\end{gather*}
$$

where

$$
\begin{align*}
\mathcal{O}= & -d_{m} \int_{t-d_{m}}^{t} \dot{\tilde{x}}^{T}(\alpha) R_{1} \dot{\tilde{x}}(\alpha) d \alpha \\
& -d_{I} \int_{t-d(t)}^{t-d_{m}} \dot{\tilde{x}}^{T}(\alpha) R_{2} \dot{\tilde{x}}(\alpha) d \alpha  \tag{12}\\
& -d_{I} \int_{t-d_{M}}^{t-d(t)} \dot{\tilde{x}}^{T}(\alpha) R_{2} \dot{\tilde{x}}(\alpha) d \alpha .
\end{align*}
$$

By (11), the time derivative of $V(t)$ becomes

$$
\begin{equation*}
\dot{V}(t)=\zeta^{T}(t) \Pi_{0} \zeta(t)+\mathcal{O}, \tag{13}
\end{equation*}
$$

where $\Pi_{0}=\operatorname{He}\left(\mathbf{e}_{1}^{T} P \Phi_{t}\right)+\mathbf{e}_{1}^{T}\left(Q_{1}+Q_{2}\right) \mathbf{e}_{1}-\mathbf{e}_{2}^{T} Q_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} Q_{2} \mathbf{e}_{4}+$ $\Phi_{t}^{T}\left(d_{m}^{2} R_{1}+d_{I}^{2} R_{2}\right) \Phi_{t}$. To deal with $\mathcal{O}$, we apply the Jensen inequality [22] to $\mathcal{O}$, which results in

$$
\begin{align*}
\mathcal{O} \leq & -\left(\int_{t-d_{m}}^{t} \dot{\tilde{x}}(\alpha) d \alpha\right)^{T} R_{1}\left(\int_{t-d_{m}}^{t} \dot{\tilde{x}}(\alpha) d \alpha\right) \\
& -\frac{1}{\theta_{1}(t)}\left(\int_{t-d(t)}^{t-d_{m}} \dot{\tilde{x}}(\alpha) d \alpha\right)^{T} R_{2}\left(\int_{t-d(t)}^{t-d_{m}} \dot{\tilde{x}}(\alpha) d \alpha\right) \\
& -\frac{1}{\theta_{2}(t)}\left(\int_{t-d_{M}}^{t-d(t)} \dot{\tilde{x}}(\alpha) d \alpha\right)^{T} R_{2}\left(\int_{t-d_{M}}^{t-d(t)} \dot{\tilde{x}}(\alpha) d \alpha\right)  \tag{14}\\
= & -\zeta^{T}(t)\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} R_{1}\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right) \zeta(t) \\
& -\frac{1}{\theta_{1}(t)} \zeta^{T}(t)\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} R_{2}\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right) \zeta(t) \\
& -\frac{1}{\theta_{2}(t)} \zeta^{T}(t)\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} R_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right) \zeta(t),
\end{align*}
$$

where $\theta_{1}(t)=\left(d(t)-d_{m}\right) / d_{I} \geq 0, \theta_{2}(t)=\left(d_{M}-d(t)\right) / d_{I} \geq 0$, and $\theta_{1}(t)+\theta_{2}(t)=1$; that is, the set of $\theta_{i}(t)$ is convex. Furthermore, by taking the convexity of $\theta_{i}(t)$ into account, we can get the following equality:

RHS of (14)

$$
\begin{aligned}
& =\zeta^{T}(t)\left(\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} R_{1}\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)+\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} R_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)\right. \\
& \left.\quad+\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} R_{2}\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right)\right) \zeta(t) \\
& -\frac{\theta_{2}(t)}{\theta_{1}(t)} \zeta^{T}(t)\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} R_{2}\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right) \zeta(t) \\
& -\frac{\theta_{1}(t)}{\theta_{2}(t)} \zeta^{T}(t)\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} R_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right) \zeta(t)
\end{aligned}
$$

$$
\begin{align*}
& =\zeta^{T}(t) \Pi_{1} \zeta(t)-\left[\begin{array}{l}
\sqrt{\frac{\theta_{2}}{\theta_{1}}}\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right) \zeta(t) \\
\sqrt{\frac{\theta_{1}}{\theta_{2}}}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right) \zeta(t)
\end{array}\right]^{T} \\
& \quad \times \Pi_{2}\left[\sqrt{\sqrt{\frac{\theta_{2}}{\theta_{1}}}\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right) \zeta(t)} \sqrt{\frac{\theta_{1}}{\frac{\theta_{2}}{\theta_{2}}}}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right) \zeta(t)\right] \tag{15}
\end{align*}
$$

where $\Pi_{1}=\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} R_{1}\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)+\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} R_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)+$ $\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} R_{2}\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right)+\operatorname{He}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} S\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)$,

$$
\Pi_{2}=\left[\begin{array}{cc}
R_{2} & S  \tag{16}\\
(*) & R_{2}
\end{array}\right] .
$$

Hence, we can see that the time derivative of $V(t)$ satisfies that $\dot{V}(t) \leq \zeta^{T}(t)\left(\Pi_{0}+\Pi_{1}\right) \zeta(t)-(*)^{T} \Pi_{2}(*)$, where $(*)=$ $\operatorname{col}\left(\sqrt{\theta_{2} / \theta_{1}}\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right) \zeta(t), \sqrt{\theta_{1} / \theta_{2}}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right) \zeta(t)\right)$. As a result, based on this derivation, the following stability criteria can be established.

Lemma 3 (stability criterion). For $\widetilde{w}(t)=0$, the stability criterion is given by

$$
\begin{equation*}
0>\Pi_{0}+\Pi_{1}, \quad 0 \leq \Pi_{2}, \tag{17}
\end{equation*}
$$

where $\Phi_{t}=\widetilde{A} \mathbf{e}_{1}+\widetilde{A}_{d} \mathbf{e}_{3}$.
Proof. If $\Pi_{2} \geq 0$ holds, then $\dot{V}(t) \leq \zeta^{T}(t)\left(\Pi_{0}+\Pi_{1}\right) \zeta(t)$.
Lemma 4 (stability criterion in the $\mathscr{H}_{\infty}$ sense). The stability criterion in the $\mathscr{H}_{\infty}$ sense is given by

$$
\begin{equation*}
0>\Pi_{0}+\Pi_{1}+\Pi_{3}, \quad 0 \leq \Pi_{2} \tag{18}
\end{equation*}
$$

where $\Phi_{t}=\widetilde{A} \mathbf{e}_{1}+\widetilde{A}_{d} \mathbf{e}_{3}+\widetilde{D} \mathbf{e}_{5}, \Pi_{3}=\mathbf{e}_{1}^{T} \widetilde{C}^{T} \widetilde{C} \mathbf{e}_{1}-\gamma^{2} \mathbf{e}_{5}^{T} \mathbf{e}_{5}$.
Proof. Let us consider the $\mathscr{H}_{\infty}$ tracking performance such that $\sup _{\widetilde{w}}\left(\|\widetilde{z}\|_{2} /\|\widetilde{w}\|_{2}\right)<\gamma$. Then, as reported in [19], the $\mathscr{H}_{\infty}$ stability criterion can be readily derived by $\dot{V}(t)+\widetilde{z}^{T}(t) \widetilde{z}(t)-$ $\gamma^{2} \widetilde{w}^{T}(t) \widetilde{w}(t)<0$, which is assured by (18).

Based on Lemma 3, the stabilization problem of (6) with $\widetilde{w}(t)=0$ will be addressed in Section 3.1, and further, based on Lemma 4, the $\mathscr{H}_{\infty}$ stabilization problem of (6) with $\widetilde{w}(t) \neq 0$ will be investigated in Section 3.2. Here, to derive a set of linear matrix inequalities (LMIs), we first set $P=\operatorname{diag}\left(P_{1}, P_{2}, P_{3}\right)$ and $\bar{P}=P^{-1}=\operatorname{diag}\left(\bar{P}_{1}, \bar{P}_{2}, \bar{P}_{3}\right)$, where $\bar{P}_{1}=P_{1}^{-1}, \bar{P}_{2}=P_{2}^{-1}$, and $\bar{P}_{3}=P_{3}^{-1}$. Then, from (7), it follows that

$$
\begin{gather*}
P \widetilde{A}=\left[\begin{array}{ccc}
P_{1} A_{r} & 0 & 0 \\
P_{2} A-P_{2} A_{r} & P_{2} A & 0 \\
0 & 0 & P_{3} A+\bar{L} C
\end{array}\right], \\
P \widetilde{A}_{d}=\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & P_{2} B \bar{F} P_{2} & -P_{2} B \bar{F} P_{2} \\
0 & 0 & 0
\end{array}\right], \quad P \widetilde{D}=\left[\begin{array}{cc}
0 & P_{1} \\
P_{2} D & -P_{2} \\
P_{3} D & 0
\end{array}\right], \tag{19}
\end{gather*}
$$

where $\bar{L}=P_{3} L$ and $\bar{F}=F \bar{P}_{2}$. Accordingly, the term $P \Phi_{t}$ becomes

$$
\begin{equation*}
P \Phi_{t}=X \breve{A} X \mathbf{e}_{1}+X \breve{A}_{d} X \mathbf{e}_{3}+X \breve{D} \mathbf{e}_{5}, \tag{20}
\end{equation*}
$$

where $X=\boldsymbol{\operatorname { d i a g }}\left(I, P_{2}, P_{2}\right)$,

$$
\begin{gather*}
\breve{A}=\left[\begin{array}{ccc}
P_{1} A_{r} & 0 & 0 \\
A-A_{r} & A \bar{P}_{2} & 0 \\
0 & 0 & \bar{P}_{2}\left(P_{3} A+\bar{L} C\right) \bar{P}_{2}
\end{array}\right],  \tag{21}\\
\breve{A}_{d}=\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & B \bar{F} & -B \bar{F} \\
0 & 0 & 0
\end{array}\right], \quad \breve{D}=\left[\begin{array}{cc}
0 & P_{1} \\
D & -I \\
\bar{P}_{2} P_{3} D & 0
\end{array}\right] .
\end{gather*}
$$

Remark 5. Inspired by the work of [18], this paper also applied the reciprocally convex approach to reduce the computational complexity and the conservatism of the delay-dependent stability criteria that will be used to derive our main results.

### 3.1. Control Design for $\widetilde{w}(t)=0$

Lemma 6. Let $\mu_{1}>0, \mu_{2}>0$, and $\epsilon>0$ be prescribed. Suppose that there exist matrices $\bar{F} \in \mathbb{R}^{n_{u} \times n_{x}}, \bar{L} \in \mathbb{R}^{n_{x} \times n_{y}}$, and $\widetilde{S} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}}$ and symmetric matrices $0<P_{1}, \bar{P}_{2}, P_{3} \in \mathbb{R}^{n_{x} \times n_{x}}$, $0<\widetilde{Q}_{1}, \widetilde{Q}_{2} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}}, 0<\widetilde{R}_{1}, \widetilde{R}_{2} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}}$ such that

$$
0>\left[\begin{array}{cc|cccc|c}
(1,1) & 0 & d_{m} \bar{A} & 0 & d_{m} \breve{A}_{d} & 0 & \epsilon E_{1}  \tag{22}\\
0 & (2,2) & d_{I} \bar{A} & 0 & d_{I} \breve{A}_{d} & 0 & \epsilon E_{2} \\
\hline(*) & (*) & (3,3) & \widetilde{R}_{1} & \breve{A}_{d} \widetilde{0} & 0 & \epsilon E_{3} \\
0 & 0 & (*) & (4,4) & \widetilde{R}_{2}+\widetilde{S} & -\widetilde{S} & 0 \\
(*) & (*) & (*) & (*) & (5,5) & \widetilde{R}_{2}+\widetilde{S} & 0 \\
0 & 0 & 0 & (*) & (*) & (6,6) & 0 \\
\hline(*) & (*) & (*) & 0 & 0 & 0 & (7,7)
\end{array}\right],
$$

$$
0 \leq\left[\begin{array}{cc}
\widetilde{R}_{2} & \widetilde{S}^{\prime}  \tag{23}\\
(*) & \widetilde{R}_{2}
\end{array}\right]
$$

where

$$
\begin{gathered}
(1,1)=\mu_{1}^{2} \widetilde{R}_{1}+\operatorname{diag}\left(-2 \mu_{1} P_{1},-2 \mu_{1} \bar{P}_{2},-2 \epsilon \bar{P}_{2}\right), \\
(2,2)=\mu_{2}^{2} \widetilde{R}_{2}+\operatorname{diag}\left(-2 \mu_{2} P_{1},-2 \mu_{2} \bar{P}_{2},-2 \epsilon \bar{P}_{2}\right), \\
(3,3)=\overline{\bar{A}}+\widetilde{Q}_{1}+\widetilde{Q}_{2}-\widetilde{R}_{1}, \\
(4,4)=-\widetilde{Q}_{1}-\widetilde{R}_{1}-\widetilde{R}_{2}, \\
(5,5)=-2 \widetilde{R}_{2}-\operatorname{He}(\widetilde{S}), \quad(6,6)=-\widetilde{Q}_{2}-\widetilde{R}_{2}, \\
(7,7)=\left[\begin{array}{ccc}
-2 \mu_{1} P_{3} & 0 & d_{m}\left(P_{3} A+\bar{L} C\right) \\
0 & -2 \mu_{2} P_{3} & d_{I}\left(P_{3} A+\bar{L} C\right) \\
(*) & (*) & \operatorname{He}\left(P_{3} A+\bar{L} C\right)
\end{array}\right], \\
\breve{A}_{d}=\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & B \bar{F} & -B \bar{F} \\
0 & 0 & 0
\end{array}\right],
\end{gathered}
$$

$$
\begin{gather*}
\bar{A}=\left[\begin{array}{ccc}
P_{1} A_{r} & 0 & 0 \\
A-A_{r} & A \bar{P}_{2} & 0 \\
0 & 0 & 0
\end{array}\right], \\
\overline{\bar{A}}=\left[\begin{array}{ccc}
\operatorname{He}\left(P_{1} A_{r}\right) & \left(A-A_{r}\right)^{T} & 0 \\
(*) & \operatorname{He}\left(A \bar{P}_{2}\right) & 0 \\
0 & 0 & -2 \epsilon \bar{P}_{2}
\end{array}\right], \\
E_{1}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
I & 0 & 0
\end{array}\right], \quad E_{2}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & I & 0
\end{array}\right], \\
E_{3}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & I
\end{array}\right] . \tag{24}
\end{gather*}
$$

Then the closed-loop system (6) is asymptotically stable in the absence of $\widetilde{w}(t)$ for any time-varying delay $d(t)$ satisfying $d_{m} \leq$ $d(t) \leq d_{M}$. Moreover, the control and observer gain matrices can be reconstructed as follows:

$$
\begin{equation*}
F=\bar{F} \bar{P}_{2}^{-1}, \quad L=P_{3}^{-1} \bar{L} \tag{25}
\end{equation*}
$$

Proof. From Lemma 3, the stabilization condition is given as follows: (i) $0 \leq \Pi_{2}$ and (ii) $0>\Pi_{0}+\Pi_{1}=\operatorname{He}\left(\mathbf{e}_{1}^{T} P \Phi_{t}\right)+$ $\mathbf{e}_{1}^{T}\left(Q_{1}+Q_{2}\right) \mathbf{e}_{1}-\mathbf{e}_{2}^{T} Q_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} Q_{2} \mathbf{e}_{4}+\Phi_{t}^{T}\left(d_{m}^{2} R_{1}+d_{I}^{2} R_{2}\right) \Phi_{t}+$ $\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} R_{1}\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)+\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} R_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)+\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} R_{2}\left(\mathbf{e}_{4}-\right.$ $\left.\mathbf{e}_{3}\right)+\operatorname{He}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} S\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)$, where $\Phi_{t}=\widetilde{A} \mathbf{e}_{1}+\widetilde{A}_{d} \mathbf{e}_{3}$ and thus $P \Phi_{t}=X \breve{A} X \mathbf{e}_{1}+X \breve{A}_{d} X \mathbf{e}_{3}$. Let us consider the condition given in (ii). Then, by letting $\bar{R}_{1}=X \bar{P} R_{1} \bar{P} X$ and $\bar{R}_{2}=X \bar{P} R_{2} \bar{P} X$, we can rewrite the condition, $0>\Pi_{0}+\Pi_{1}$, as follows:

$$
\begin{align*}
0> & H e\left(\mathbf{e}_{1}^{T} X \breve{A} X \mathbf{e}_{1}+\mathbf{e}_{1}^{T} X \breve{A}_{d} X \mathbf{e}_{3}\right)+\mathbf{e}_{1}^{T}\left(Q_{1}+Q_{2}\right) \mathbf{e}_{1} \\
& -\mathbf{e}_{2}^{T} Q_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} Q_{2} \mathbf{e}_{4}+\left(X \breve{A} X \mathbf{e}_{1}+X \breve{A}_{d} X \mathbf{e}_{3}\right)^{T} \\
& \times \bar{X}\left(d_{m}^{2} \bar{R}_{1}+d_{I}^{2} \bar{R}_{2}\right) \bar{X}\left(X \breve{A} X \mathbf{e}_{1}+X \breve{A}_{d} X \mathbf{e}_{3}\right) \\
& +\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} P \overline{X R_{1}} \bar{X} P\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)  \tag{26}\\
& +\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} P \overline{X R}_{2} \bar{X} P\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right) \\
& +\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} P \overline{X R}_{2} \bar{X} P\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right) \\
& +\operatorname{He}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} S\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)
\end{align*}
$$

where $\bar{X}=X^{-1}$. Further, since $\operatorname{diag}(\bar{X}, \bar{X}, \bar{X}, \bar{X}) \mathbf{e}_{i}^{T}=\mathbf{e}_{i}^{T} \bar{X}$ ( $i=1,2,3,4$ ), pre- and postmultiplying both sides of (26) by $\operatorname{diag}(\bar{X}, \bar{X}, \bar{X}, \bar{X})$ and its transpose yield

$$
\begin{equation*}
0>Q+\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}\right)^{T}\left(d_{m}^{2} \bar{R}_{1}+d_{I}^{2} \bar{R}_{2}\right)\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}\right), \tag{27}
\end{equation*}
$$

where $\mathbb{Q}=\mathbf{H e}\left(\mathbf{e}_{1}^{T} \breve{A} \mathbf{e}_{1}+\mathbf{e}_{1}^{T} \breve{A}_{d} \mathbf{e}_{3}\right)+\mathbf{e}_{1}^{T}\left(\widetilde{Q}_{1}+\widetilde{Q}_{2}\right) \mathbf{e}_{1}-\mathbf{e}_{2}^{T} \widetilde{Q}_{1} \mathbf{e}_{2}-$ $\mathbf{e}_{4}^{T} \widetilde{Q}_{2} \mathbf{e}_{4}+\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} \widetilde{R}_{1}\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)+\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} \widetilde{R}_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)+\left(\mathbf{e}_{3}-\right.$ $\left.\mathbf{e}_{4}\right)^{T} \widetilde{R}_{2}\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right)+\mathbf{H e}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} \widetilde{S}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)$ in which $\widetilde{Q}_{1}=\bar{X} Q_{1} \bar{X}$,
$\widetilde{Q}_{2}=\bar{X} Q_{2} \bar{X}, \widetilde{R}_{1}=\widetilde{X} \bar{R}_{1} \widetilde{X}, \widetilde{R}_{2}=\widetilde{X} \bar{R}_{2} \widetilde{X}, \widetilde{S}=\bar{X} S \bar{X}$, and $\widetilde{X}=\bar{X} P \bar{X}=\operatorname{diag}\left(P_{1}, \bar{P}_{2}, \bar{P}_{2} P_{3} \bar{P}_{2}\right)>0$. That is, by applying the Schur complement to (27), we can get

$$
0>\left[\begin{array}{cc|c}
-\bar{R}_{1}^{-1} & 0 & d_{m}\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}\right)  \tag{28}\\
0 & -\bar{R}_{2}^{1} & d_{I}\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}\right) \\
\hline(*) & (*) & \overparen{Q}
\end{array}\right]
$$

Here, since $\bar{R}_{1}^{-1}=\widetilde{X} \widetilde{R}_{1}^{-1} \widetilde{X}$ and $\bar{R}_{2}^{-1}=\widetilde{X} \widetilde{R}_{2}^{-1} \widetilde{X}$, it follows from Lemma 2 that $\bar{R}_{1}^{-1} \geq 2 \mu_{1} \widetilde{X}-\mu_{1}^{2} \widetilde{R}_{1}$ and ${\overline{R_{2}}}^{-1} \geq 2 \mu_{2} \widetilde{X}-\mu_{2}^{2} \widetilde{R}_{2}$. In this sense, it is clear that (28) holds if

$$
0>\left[\begin{array}{cc|cccc}
(1,1)^{\prime} & 0 & (1,3)^{\prime} & 0 & d_{m} \breve{A}_{d} & 0  \tag{29}\\
0 & (2,2)^{\prime} & (2,3)^{\prime} & 0 & d_{I} \breve{A}_{d} & 0 \\
\hline(*) & (*) & (3,3)^{\prime} & \widetilde{R}_{1} & \breve{A}_{d} & 0 \\
0 & 0 & (*) & (4,4) & \widetilde{R}_{2}+\widetilde{S} & -\widetilde{S} \\
(*) & (*) & (*) & (*) & (5,5) & \widetilde{R}_{2}+\widetilde{S} \\
0 & 0 & 0 & (*) & (*) & (6,6)
\end{array}\right],
$$

where

$$
\begin{gather*}
(1,1)^{\prime}=\mu_{1}^{2} \widetilde{R}_{1}+\operatorname{diag}\left(-2 \mu_{1} P_{1},-2 \mu_{1} \bar{P}_{2}, \bar{P}_{2}\left(-2 \mu_{1} P_{3}\right) \bar{P}_{2}\right), \\
(2,2)^{\prime}=\mu_{2}^{2} \widetilde{R}_{2}+\operatorname{diag}\left(-2 \mu_{2} P_{1},-2 \mu_{2} \bar{P}_{2}, \bar{P}_{2}\left(-2 \mu_{2} P_{3}\right) \bar{P}_{2}\right), \\
(3,3)^{\prime}=\operatorname{He}(\breve{A})+\widetilde{Q}_{1}+\widetilde{Q}_{2}-\widetilde{R}_{1}, \\
(1,3)^{\prime}=d_{m} \breve{A}, \quad(2,3)^{\prime}=d_{I} \breve{A} . \tag{30}
\end{gather*}
$$

However, as shown in (29), there exist some nonconvex terms in $(1,1)^{\prime},(1,3)^{\prime},(2,2)^{\prime},(2,3)^{\prime}$, and $(3,3)^{\prime}$ as follows:

$$
\begin{gather*}
(1,1)^{\prime}=\left[\begin{array}{ccc}
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star)+\bar{P}_{2}\left(-2 \mu_{1} P_{3}\right) \bar{P}_{2}
\end{array}\right], \\
(2,2)^{\prime}=\left[\begin{array}{lll}
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star)+\bar{P}_{2}\left(-2 \mu_{2} P_{3}\right) \bar{P}_{2}
\end{array}\right], \\
(1,3)^{\prime}=\left[\begin{array}{ccc}
(\star) & 0 & 0 \\
(\star) & (\star) & 0 \\
0 & 0 & \bar{P}_{2}\left(d_{m}\left(P_{3} A+\bar{L} C\right)\right) \bar{P}_{2}
\end{array}\right],  \tag{31}\\
(2,3)^{\prime}=\left[\begin{array}{ccc}
(\star) & 0 & 0 \\
(\star) & (\star) & 0 \\
0 & 0 & \bar{P}_{2}\left(d_{I}\left(P_{3} A+\bar{L} C\right)\right) \bar{P}_{2}
\end{array}\right], \\
(3,3)^{\prime}=\left[\begin{array}{ccc}
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star)+\bar{P}_{2}\left(\mathbf{H e}\left(P_{3} A+\bar{L} C\right)\right) \bar{P}_{2}
\end{array}\right] .
\end{gather*}
$$

Here, note that all terms associated with $\bar{P}_{2}(*) \bar{P}_{2}$ in

$$
\left[\begin{array}{ccc}
(1,1)^{\prime} & 0 & (1,3)^{\prime}  \tag{32}\\
0 & (2,2)^{\prime} & (2,3)^{\prime} \\
(*) & (*) & (3,3)^{\prime}
\end{array}\right]
$$

can be separated as follows:

$$
\left[\begin{array}{c}
E_{1}  \tag{33}\\
E_{2} \\
E_{3}
\end{array}\right] \underbrace{\left[\begin{array}{ccc}
\bar{P}_{2} & 0 & 0 \\
0 & \bar{P}_{2} & 0 \\
0 & 0 & \bar{P}_{2}
\end{array}\right]}_{\Psi} \underbrace{\left[\begin{array}{c}
E_{1} \\
E_{2} \\
E_{3}
\end{array}\right]^{T} . . . . . . . ~}_{\left[\begin{array}{ccc}
-2 \mu_{1} P_{3} & 0 & d_{m}\left(P_{3} A+\bar{L} C\right) \\
0 & -2 \mu_{2} P_{3} & d_{I}\left(P_{3} A+\bar{L} C\right) \\
(*) & (*) & \operatorname{He}\left(P_{3} A+\bar{L} C\right)
\end{array}\right]\left[\begin{array}{ccc}
\bar{P}_{2} & 0 & 0 \\
0 & \bar{P}_{2} & 0 \\
0 & 0 & \bar{P}_{2}
\end{array}\right]}
$$

Furthermore, from Lemma 2, it follows that

$$
\begin{align*}
\Psi \leq & -2 \epsilon\left[\begin{array}{ccc}
\bar{P}_{2} & 0 & 0 \\
0 & \bar{P}_{2} & 0 \\
0 & 0 & \bar{P}_{2}
\end{array}\right] \\
& -\epsilon^{2}\left[\begin{array}{ccc}
-2 \mu_{1} P_{3} & 0 & d_{m}\left(P_{3} A+\bar{L} C\right) \\
0 & -2 \mu_{2} P_{3} & d_{I}\left(P_{3} A+\bar{L} C\right) \\
(*) & (*) & \operatorname{He}\left(P_{3} A+\bar{L} C\right)
\end{array}\right]^{-1}, \tag{34}
\end{align*}
$$

which allows that (22) implies (29), based on the Schur complement. Next, we need to convert the given condition in (i), that is, $0 \leq \Pi_{2}$, into an LMI. To this end, let us pre-
and postmultiply both sides of $0 \leq \Pi_{2}$ by $\operatorname{diag}(\bar{X}, \bar{X})$ and its transpose. Then we can get

$$
0 \leq\left[\begin{array}{cc}
\bar{X} R_{2} \bar{X} & \tilde{S}  \tag{35}\\
(*) & \bar{X} R_{2} \bar{X}
\end{array}\right]
$$

which becomes (23) due to $\widetilde{R}_{2}=\widetilde{X} \bar{R}_{2} \widetilde{X}=$ $(\bar{X} P \bar{X})\left(X \bar{P} R_{2} \bar{R} X\right)(\bar{X} P \bar{X})=\bar{X} R_{2} \bar{X}$.

### 3.2. Control Design for $\widetilde{w}(t) \neq 0$

Theorem 7. Let $\mu_{1}>0, \mu_{2}>0, \epsilon_{p}>0$ be prescribed. Suppose that there exist scalars $\epsilon_{q}>0, \gamma>0$; matrices $\bar{F} \in \mathbb{R}^{n_{u} \times n_{x}}$, $\bar{L} \in \mathbb{R}^{n_{x} \times n_{y}}, \widetilde{S} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}} ;$ and symmetric matrices $0<P_{1}$, $\bar{P}_{2}, P_{3} \in \mathbb{R}^{n_{x} \times n_{x}}, 0<\widetilde{Q}_{1}, \widetilde{Q}_{2} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}}, 0<\widetilde{R}_{1}, \widetilde{R}_{2} \in \mathbb{R}^{3 n_{x} \times 3 n_{x}}$ such that

$$
\begin{gather*}
{\left[\begin{array}{cc|cccccc|c}
(1,1) & 0 & d_{m} \bar{A} & 0 & d_{m} \breve{A}_{d} & 0 & d_{m} \bar{D} & \epsilon_{p} E_{1} & 0 \\
0 & (2,2) & d_{I} \bar{A} & 0 & d_{I} \breve{A}_{d} & 0 & d_{I} \bar{D} & \epsilon_{p} E_{2} & 0 \\
\hline(*) & (*) & (3,3) & \widetilde{R}_{1} & \breve{A}_{d} & 0 & \bar{D} & \epsilon_{p} E_{3} & \widetilde{X}^{T} \\
0 & 0 & (*) & (4,4) & \widetilde{R}_{2}+\widetilde{S} & -\widetilde{S} & 0 & 0 & 0 \\
(*) & (*) & (*) & (*) & (5,5) & \widetilde{R}_{2}+\widetilde{S} & 0 & 0 & 0 \\
0 & 0 & 0 & (*) & (*) & (6,6) & 0 & 0 & 0 \\
(*) & (*) & (*) & 0 & 0 & 0 & (7,7) & \epsilon_{q} E_{4} & 0 \\
(*) & (*) & (*) & 0 & 0 & 0 & (*) & (8,8) & 0 \\
\hline 0 & 0 & (*) & 0 & 0 & 0 & 0 & 0 & -I
\end{array}\right],}  \tag{36}\\
0 \leq\left[\begin{array}{cc}
\widetilde{R}_{2} & \widetilde{S} \\
(*) & \widetilde{R}_{2}
\end{array}\right], \tag{37}
\end{gather*}
$$

where

$$
\begin{gathered}
(1,1)=\mu_{1}^{2} \widetilde{R}_{1}+\operatorname{diag}\left(-2 \mu_{1} P_{1},-2 \mu_{1} \bar{P}_{2},-2 \epsilon_{p} \bar{P}_{2}\right) \\
(2,2)=\mu_{2}^{2} \widetilde{R}_{2}+\operatorname{diag}\left(-2 \mu_{2} P_{1},-2 \mu_{2} \bar{P}_{2},-2 \epsilon_{p} \bar{P}_{2}\right) \\
(3,3)=\overline{\bar{A}}+\widetilde{Q}_{1}+\widetilde{Q}_{2}-\widetilde{R}_{1} \\
(4,4)=-\widetilde{Q}_{1}-\widetilde{R}_{1}-\widetilde{R}_{2} \\
(5,5)=-2 \widetilde{R}_{2}-\operatorname{He}\left(\widetilde{S}_{S}\right) \\
(6,6)=-\widetilde{Q}_{2}-\widetilde{R}_{2}, \quad(7,7)=\operatorname{diag}\left(-2 \epsilon_{q} I,-\gamma^{2} I\right)
\end{gathered}
$$

$$
(8,8)=\left[\begin{array}{cccc}
-2 \mu_{1} P_{3} & 0 & d_{m}\left(P_{3} A+\bar{L} C\right) & d_{m} P_{3} D \\
0 & -2 \mu_{2} P_{3} & d_{I}\left(P_{3} A+\bar{L} C\right) & d_{I} P_{3} D \\
(*) & (*) & \operatorname{He}\left(P_{3} A+\bar{L} C\right) & P_{3} D \\
(*) & (*) & (*) & -\gamma^{2} I
\end{array}\right]
$$

$$
\begin{gathered}
\breve{A}_{d}=\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & B \bar{F} & -B \bar{F} \\
0 & 0 & 0
\end{array}\right], \\
\bar{A}=\left[\begin{array}{ccc}
P_{1} A_{r} & 0 & 0 \\
A-A_{r} & A \bar{P}_{2} & 0 \\
0 & 0 & 0
\end{array}\right],
\end{gathered}
$$

$$
\begin{array}{cc}
\overline{\bar{A}}=\left[\begin{array}{ccc}
\operatorname{He}\left(P_{1} A_{r}\right) & \left(A-A_{r}\right)^{T} & 0 \\
(*) & \operatorname{He}\left(A \bar{P}_{2}\right) & 0 \\
0 & 0 & -2 \epsilon_{p} \bar{P}_{2}
\end{array}\right], \\
\bar{D}=\left[\begin{array}{cc}
0 & P_{1} \\
D & -I \\
0 & 0
\end{array}\right], \quad \widetilde{C}^{T}=\left[\begin{array}{c}
0 \\
C^{T} \\
0
\end{array}\right], \\
E_{1}=\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
I & 0 & 0 & 0
\end{array}\right], \quad E_{2}=\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & I & 0 & 0
\end{array}\right], \\
E_{3}=\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & I & 0
\end{array}\right], \quad E_{4}=\left[\begin{array}{llll}
0 & 0 & 0 & I \\
0 & 0 & 0 & 0
\end{array}\right] . \tag{38}
\end{array}
$$

Then the closed-loop system (6) is asymptotically stable and satisfies $\|\widetilde{z}\|_{2}<\gamma\|\widetilde{w}\|_{2}$ for all nonzero $\widetilde{w}(t) \in \mathscr{L}_{2+}$ and for any time-varying delay $d(t)$ satisfying $d_{m} \leq d(t) \leq d_{M}$. Moreover, the control and observer gain matrices can be reconstructed as follows:

$$
\begin{equation*}
F=\bar{F} \bar{P}_{2}^{-1}, \quad L=P_{3}^{-1} \bar{L} \tag{39}
\end{equation*}
$$

Proof. From Lemma 4, the $\mathscr{H}_{\infty}$ stabilization condition is given as follows: (i) $0 \leq \Pi_{2}$ and (ii) $0>\Pi_{0}+\Pi_{1}+\Pi_{3}=$ $\mathrm{He}\left(\mathbf{e}_{1}^{T} P \Phi_{t}\right)+\mathbf{e}_{1}^{T}\left(Q_{1}+Q_{2}\right) \mathbf{e}_{1}-\mathbf{e}_{2}^{T} Q_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} \mathrm{Q}_{2} \mathbf{e}_{4}+\Phi_{t}^{T}\left(d_{m}^{2} R_{1}+\right.$ $\left.d_{I}^{2} R_{2}\right) \Phi_{t}+\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} R_{1}\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)+\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} R_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)+\left(\mathbf{e}_{3}-\right.$ $\left.\mathbf{e}_{4}\right)^{T} R_{2}\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right)+\operatorname{He}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} S\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)+\mathbf{e}_{1}^{T} \widetilde{C}^{T} \widetilde{C} \mathbf{e}_{1}-\gamma^{2} \mathbf{e}_{5}^{T} \mathbf{e}_{5}$, where $\Phi_{t}=\widetilde{A} \mathbf{e}_{1}+\widetilde{A}_{d} \mathbf{e}_{3}+\widetilde{D} \mathbf{e}_{5}$. As in the proof of Lemma 6, we first consider the condition given in (ii) by letting $\bar{R}_{1}=$ $X \bar{P} R_{1} \bar{P} X$ and $\bar{R}_{2}=X \bar{P} R_{2} \bar{P} X$. Then the condition, $0>\Pi_{0}+$ $\Pi_{1}+\Pi_{3}$, can be converted by (20) into

$$
\begin{aligned}
0> & H e\left(\mathbf{e}_{1}^{T} X \breve{A} X \mathbf{e}_{1}+\mathbf{e}_{1}^{T} X \breve{A}_{d} X \mathbf{e}_{3}+\mathbf{e}_{1}^{T} X \breve{D} \mathbf{e}_{5}\right) \\
& +\mathbf{e}_{1}^{T}\left(Q_{1}+Q_{2}\right) \mathbf{e}_{1}-\mathbf{e}_{2}^{T} Q_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} Q_{2} \mathbf{e}_{4}
\end{aligned}
$$

$$
\begin{align*}
& +\left(X \breve{A} X \mathbf{e}_{1}+X \breve{A}_{d} X \mathbf{e}_{3}+X \breve{D} \mathbf{e}_{5}\right)^{T} \bar{X}\left(d_{m}^{2} \bar{R}_{1}+d_{I}^{2} \bar{R}_{2}\right) \\
& \times \bar{X}\left(X \breve{A} X \mathbf{e}_{1}+X \breve{A}_{d} X \mathbf{e}_{3}+X \breve{D} \mathbf{e}_{5}\right) \\
& +\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} P \overline{X R} \bar{X} P\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right) \\
& +\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} P \overline{X R} \bar{X} P\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)+\mathbf{e}_{1}^{T} \widetilde{C}^{T} \widetilde{C} \mathbf{e}_{1} \\
& +\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} P \overline{X R} \bar{X} P\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right) \\
& +\mathbf{H e}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} S\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)-\gamma^{2} \mathbf{e}_{5}^{T} \mathbf{e}_{5} \tag{40}
\end{align*}
$$

where $\bar{X}=X^{-1}$. Further, since $\operatorname{diag}(\bar{X}, \bar{X}, \bar{X}, \bar{X}, I) \mathbf{e}_{i}^{T}=\mathbf{e}_{i}^{T} \bar{X}$, for $i=1,2,3,4$, and $\operatorname{diag}(\bar{X}, \bar{X}, \bar{X}, \bar{X}, I) \mathbf{e}_{5}^{T}=\mathbf{e}_{5}^{T} I$, pre- and postmultiplying both sides of (40) by $\operatorname{diag}(\bar{X}, \bar{X}, \bar{X}, \bar{X}, I)$ and its transpose yield

$$
\begin{align*}
0> & \mathbb{Q}+\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}+\breve{D} \mathbf{e}_{5}\right)^{T}  \tag{41}\\
& \times\left(d_{m}^{2} \bar{R}_{1}+d_{I}^{2} \bar{R}_{2}\right)\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}+\breve{D} \mathbf{e}_{5}\right),
\end{align*}
$$

where $\mathbb{Q}=\mathbf{H e}\left(\mathbf{e}_{1}^{T} \breve{A} \mathbf{e}_{1}+\mathbf{e}_{1}^{T} \breve{A}_{d} \mathbf{e}_{3}+\mathbf{e}_{1}^{T} \breve{D} \mathbf{e}_{5}\right)+\mathbf{e}_{1}^{T}\left(\widetilde{Q}_{1}+\widetilde{Q}_{2}\right) \mathbf{e}_{1}-$ $\mathbf{e}_{2}^{T} \widetilde{Q}_{1} \mathbf{e}_{2}-\mathbf{e}_{4}^{T} \widetilde{Q}_{2} \mathbf{e}_{4}+\left(\mathbf{e}_{1}-\mathbf{e}_{2}\right)^{T} \widetilde{R}_{1}\left(\mathbf{e}_{2}-\mathbf{e}_{1}\right)+\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} \widetilde{R}_{2}\left(\mathbf{e}_{3}-\mathbf{e}_{2}\right)+$ $\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)^{T} \widetilde{R}_{2}\left(\mathbf{e}_{4}-\mathbf{e}_{3}\right)+\operatorname{He}\left(\left(\mathbf{e}_{2}-\mathbf{e}_{3}\right)^{T} \widetilde{S}\left(\mathbf{e}_{3}-\mathbf{e}_{4}\right)\right)+\mathbf{e}_{1}^{T} \bar{X} \widetilde{C}^{T} \widetilde{C X} \mathbf{e}_{1}-$ $\gamma^{2} \mathbf{e}_{5}^{T} \mathbf{e}_{5}$, in which $\widetilde{Q}_{1}=\bar{X} Q_{1} \bar{X}, \widetilde{Q}_{2}=\bar{X} Q_{2} \bar{X}, \widetilde{R}_{1}=\widetilde{X} \bar{R}_{1} \widetilde{X}$, $\widetilde{R}_{2}=\widetilde{X} \bar{R}_{2} \widetilde{X}, \widetilde{S}=\bar{X} S \bar{X}, \widetilde{X}=\bar{X} P \bar{X}=\operatorname{diag}\left(P_{1}, \bar{P}_{2}, \bar{P}_{2} P_{3} \bar{P}_{2}\right)>$ 0 . That is, by applying the Schur complement to (41), we can get

$$
0>\left[\begin{array}{cc|c}
-\bar{R}_{1}^{-1} & 0 & d_{m}\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}+\breve{D} \mathbf{e}_{5}\right)  \tag{42}\\
0 & -\bar{R}_{2}^{-1} & d_{I}\left(\breve{A} \mathbf{e}_{1}+\breve{A}_{d} \mathbf{e}_{3}+\breve{D} \mathbf{e}_{5}\right) \\
\hline(*) & (*) & \mathscr{Q}
\end{array}\right]
$$

Here, since $\bar{R}_{1}^{-1}=\widetilde{X} \widetilde{R}_{1}^{-1} \widetilde{X}$ and $\bar{R}_{2}^{-1}=\widetilde{X} \widetilde{R}_{2}^{-1} \widetilde{X}$, it follows from Lemma 2 that $\bar{R}_{1}^{-1} \geq 2 \mu_{1} \widetilde{X}-\mu_{1}^{2} \widetilde{R}_{1}$ and $\bar{R}_{2}^{-1} \geq 2 \mu_{2} \widetilde{X}-\mu_{2}^{2} \widetilde{R}_{2}$. In this sense, it is clear that (42) holds if

$$
0>\left[\begin{array}{cc|ccccc}
(1,1)^{\prime} & 0 & (1,3)^{\prime} & 0 & d_{m} \breve{A}_{d} & 0 & (1,7)^{\prime}  \tag{43}\\
0 & (2,2)^{\prime} & (2,3)^{\prime} & 0 & d_{I} \breve{A}_{d} & 0 & (2,7)^{\prime} \\
\hline(*) & (*) & (3,3)^{\prime \prime} & \widetilde{R}_{1} & \breve{A}_{d} & 0 & (3,7)^{\prime} \\
0 & 0 & (*) & (4,4) & \widetilde{R}_{2}+\widetilde{S} & -\widetilde{S} & 0 \\
(*) & (*) & (*) & (*) & (5,5) & \widetilde{R}_{2}+\widetilde{S} & 0 \\
0 & 0 & 0 & (*) & (*) & (6,6) & 0 \\
(*) & (*) & (*) & 0 & 0 & 0 & (7,7)^{\prime}
\end{array}\right]
$$

where

$$
\begin{gather*}
(1,1)^{\prime}=\mu_{1}^{2} \widetilde{R}_{1}+\operatorname{diag}\left(-2 \mu_{1} P_{1},-2 \mu_{1} \bar{P}_{2}, \bar{P}_{2}\left(-2 \mu_{1} P_{3}\right) \bar{P}_{2}\right), \\
(2,2)^{\prime}=\mu_{2}^{2} \widetilde{R}_{2}+\operatorname{diag}\left(-2 \mu_{2} P_{1},-2 \mu_{2} \bar{P}_{2}, \bar{P}_{2}\left(-2 \mu_{2} P_{3}\right) \bar{P}_{2}\right), \\
(3,3)^{\prime \prime}=\operatorname{He}(\breve{A})+\widetilde{Q}_{1}+\widetilde{Q}_{2}-\widetilde{R}_{1}+\bar{X}^{T} \widetilde{\mathrm{CX}}, \tag{44}
\end{gather*}
$$

However, as shown in (43), there exist some nonconvex terms in $(1,1)^{\prime},(1,3)^{\prime},(1,7)^{\prime},(2,2)^{\prime},(2,3)^{\prime},(2,7)^{\prime},(3,3)^{\prime \prime},(3,7)^{\prime}$, and $(7,7)^{\prime}$ as follows: $(1,1)^{\prime},(1,3)^{\prime},(2,2)^{\prime}$, and $(2,3)^{\prime}$ are the same as those defined in the proof of Lemma 6, and

$$
\begin{aligned}
& (1,7)^{\prime}=\left[\begin{array}{cc}
0 & (\star) \\
(\star) & (\star) \\
\bar{P}_{2}\left(d_{m} P_{3} D\right) I & 0
\end{array}\right], \\
& (2,7)^{\prime}=\left[\begin{array}{cc}
0 & (\star) \\
(\star) & (\star) \\
\bar{P}_{2}\left(d_{I} P_{3} D\right) I & 0
\end{array}\right], \\
& (3,7)^{\prime}=\left[\begin{array}{cc}
0 & (\star) \\
{ }^{(\star)} & (\star) \\
\bar{P}_{2}\left(P_{3} D\right) I & 0
\end{array}\right],
\end{aligned}
$$

$$
(7,7)^{\prime}=\left[\begin{array}{cc}
-\gamma^{2} I & 0 \\
0 & (\star)
\end{array}\right]
$$

$$
(3,3)^{\prime \prime}=\left[\begin{array}{ccc}
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star) \\
(\star) & (\star) & (\star)+\bar{P}_{2}\left(\operatorname{He}\left(P_{3} A+\bar{L} C\right)\right) \bar{P}_{2}
\end{array}\right]
$$

$$
\begin{equation*}
+\bar{X} \widetilde{C}^{T} \widetilde{C X} \tag{45}
\end{equation*}
$$

where $I$ denotes the identity matrix. As in the proof of Lemma 6, to deal with the nonconvex terms, we apply Lemma 2 to (43), which boils down to

$$
0>\left[\begin{array}{cc|ccccc|c}
(1,1) & 0 & d_{m} \bar{A} & 0 & d_{m} \breve{A}_{d} & 0 & d_{m} \bar{D} & E_{1}  \tag{46}\\
0 & (2,2) & d_{I} \bar{A} & 0 & d_{I} \breve{A}_{d} & 0 & d_{I} \bar{D} & E_{2} \\
\hline(*) & (*) & (3,3)^{\prime} & \widetilde{R}_{1} & \breve{A}_{d} \widetilde{0} & 0 & \bar{D} & E_{3} \\
0 & 0 & (*) & (4,4) & \widetilde{R}_{2}+\widetilde{S} & -\widetilde{S} & 0 & 0 \\
(*) & (*) & (*) & (*) & (5,5) & \widetilde{R}_{2}+\widetilde{S} & 0 & 0 \\
0 & 0 & 0 & (*) & (*) & (6,6) & 0 & 0 \\
(*) & (*) & (*) & 0 & 0 & 0 & (7,7) & E_{4} \\
\hline(*) & (*) & (*) & 0 & 0 & 0 & (*) & (8,8)
\end{array}\right],
$$

where $(3,3)^{\prime}=\overline{\bar{A}}+\widetilde{Q}_{1}+\widetilde{Q}_{2}-\widetilde{R}_{1}+\bar{X} \widetilde{C}^{T} \widetilde{C X}$. As a result, by applying the Schur complement to $\bar{X} \widetilde{C}^{T} \widetilde{C X}$ in $(3,3)^{\prime}$, we can obtain (36). The next step is to convert the given condition in (i), that is, $0 \leq \Pi_{2}$, into an LMI. To this end, let us preand postmultiply both sides of $0 \leq \Pi_{2}$ by $\operatorname{diag}(\bar{X}, \bar{X})$ and its transpose. Then we can get

$$
0 \leq\left[\begin{array}{cc}
\bar{X} R_{2} \bar{X} & \widetilde{S}  \tag{47}\\
(*) & \bar{X} R_{2} \bar{X}
\end{array}\right]
$$

which becomes (23) due to $\widetilde{R}_{2}=\bar{X} R_{2} \bar{X}$.

## 4. Numerical Example

We provide two examples to verify the effectiveness of the proposed methods in Lemma 6 and Theorem 7. For the networked output-feedback control system (NOCS), we assume that the sampling period $h=0.01$ and the data transmission delay bounds are given by $\tau_{m}=0.005$ and $\tau_{M}=$ 0.01 . As a result, from [3], it follows that $d_{m}=\tau_{m}=0.005[s$ ] and $d_{M}=(\bar{o}+1) h+\tau_{M}=0.01 \bar{o}+0.02[s]$, where $\bar{o}$ denotes the maximum number of data-packet dropouts.
4.1. Example 1. Consider a continuous-time system of the following form:

$$
\begin{gather*}
\dot{x}(t)=\left[\begin{array}{cc}
0 & 1 \\
-1 & \alpha
\end{array}\right] x(t)+\left[\begin{array}{l}
0 \\
1
\end{array}\right] u(t)+\left[\begin{array}{l}
0 \\
1
\end{array}\right] w(t),  \tag{48}\\
y(t)=\left[\begin{array}{ll}
1 & 0
\end{array}\right] x(t),
\end{gather*}
$$

where $\alpha>0$ is a variable element. First of all, to show the applicability of the proposed method in Lemma 6, we search the maximum allowable upper bounds (MAUBs) for (48) with $\widetilde{w}(t)=\operatorname{col}(w(t), r(t))=0 \in \mathbb{R}^{n_{x}+n_{w}}$. To this end, let us set $\mu_{1}=\mu_{2}=0.1, \epsilon=10$, and $A_{r}=\boldsymbol{\operatorname { d i a g }}(1,1)$. Then, from Lemma 6 , we can obtain the MAUBs for $\alpha=1,2,3$, which are tabulated in Table 1. Now, let us analyze the behavior of the tracking response for $x(t)$ and $x_{r}(t)$ of the NOCS in the case where $\widetilde{w}(t) \neq 0$ by using the derived condition in Theorem 7. For this purpose, we set $\alpha=1, \bar{o}=10$, $\mu_{1}=\mu_{2}=0.1, \epsilon=10$, and $A_{r}=\operatorname{diag}(1,1)$. Then, from Theorem 7, we can obtain the following control and observer gain matrices: $F=\left[\begin{array}{ll}-5.5501 & -4.9075\end{array}\right], L=\left[\begin{array}{ll}-31.1609 & - \\ \hline\end{array}\right.$ $179.7242]^{T}$. In addition, the disturbance attenuation is given by $\gamma=1.1679$. Here we assume that $w(t)=e^{-0.5 t} \sin (2 \pi t)$, $x(0)=\operatorname{col}( \pm 0.5, \pm 0.5), x_{r}(t)=\widehat{x}(t)=0$, for $t \in\left[-d_{M}, 0\right]$, and $r(t)=\operatorname{col}(0.2 \sin 0.2 \pi t, 0.04 \pi \cos 0.2 \pi t)$, for $t \geq 0$, where the initial time $t_{0}$ is set to zero. Figure 2(a) shows the $x_{1}-x_{2}$ trajectories for four different initial conditions $x(0)$, which form a specific ellipse, made by the given reference input $r(t)$, as the time $t$ increases. Further, the behavior of the estimation error $e_{1}(t)=y(t)-\widehat{y}(t)$ is depicted in Figure 2(d), from which we can see that the estimation error goes to zero as the time $t$ increases. Figures 2(b) and 2(c) show the behavior of the state $x(t)$ of (49) for initial condition $x(0)=(0.5,-0.5)$, where the network-induced delay $d(t)$ is generated as shown in Figure 2(e) such that the data transmission delay $\tau(t) \in$ [ $0.005,0.01$ ] and the data-packet dropouts $\bar{o}=10$. From Figures 2(b) and 2(c), we can see that the state $x(t)$ tracks the reference signal $x_{r}(t)$ well; that is, the tracking response of


FIGURE 2: (a) The $x_{1}-x_{2}$ trajectory for each different initial condition $x(0)$; (b) the tracking response of $x_{1}(t)$ for $x_{r, 1}(t)$; (c) the tracking response of $x_{2}(t)$ for $x_{r, 2}(t)$; (d) the estimation error $e_{1}(t)=y(t)-\widehat{y}(t)$; and (e) the network-induced delay $d(t)$. Here, $x_{i}, \widehat{x}_{i}, e_{i}$, and $x_{r, i}$ denote the $i$ th element of $x, \widehat{x}, e$, and $x_{r}$, respectively.

Table 1: Maximum allowable upper bounds (MAUBs) for $\alpha$ of Example 1.

| $\alpha$ | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: |
| $\tau_{M}$ | $0.32(\bar{o}=30)$ | $0.19(\bar{o}=17)$ | $0.09(\bar{o}=7)$ |

the NOCS with (2), (5), and (48) is in a good shape with respect to our control goal.
4.2. Example 2. Consider the following satellite system, modified from [5]: $A_{r}=\operatorname{diag}(-1,-1,-1,-1)$,

$$
\begin{gather*}
A=\left[\begin{array}{cccc}
0.000 & 0.000 & 1.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 1.000 \\
-0.300 & 0.300 & -0.004 & 0.004 \\
0.300 & -0.300 & 0.004 & -0.004
\end{array}\right] \\
B=\left[\begin{array}{l}
0 \\
0 \\
1 \\
0
\end{array}\right], \quad C^{T}=\left[\begin{array}{l}
0 \\
1 \\
0 \\
0
\end{array}\right], \quad D=\left[\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right] . \tag{49}
\end{gather*}
$$

Through this example, we will achieve the $\mathscr{H}_{\infty}$ performance for (49) based on Theorem 7 to design an observer-based

TABLE 2: $\mathscr{H}_{\infty}$ performance for each upper bound $d_{M}$.

| $d_{M}$ | 0.02 | 0.03 | 0.05 | 0.1 |
| :--- | :---: | :---: | :---: | :---: |
| $\gamma$ | 0.8265 | 0.9072 | 1.1337 | 2.3290 |

NOCS in such a way that the state $x(t)$ of (49) tracks the reference signal $x_{r}(t)$ in the $\mathscr{H}_{\infty}$ sense. The obtained $\mathscr{H}_{\infty}$ performance for each upper bound $d_{M}$ is tabulated in Table 2, where $\mu_{1}=\mu_{2}=0.1$ and $\epsilon_{p}=10$ are assumed. From Table 2, we can see that the $\mathscr{H}_{\infty}$ performance is improved as $d_{M}$ decreases from 0.1 to 0.02 , which is reasonable.

## 5. Concluding Remarks

This paper has addressed the observer-based $\mathscr{H}_{\infty}$ tracking problem of NOCSs with network-induced delays. In the derivation, a single-step procedure is proposed to handle nonconvex terms that appear in the process of designing observer-based output-feedback control, and then a set of linear matrix inequality conditions are established for the solvability of the tracking problem.

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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