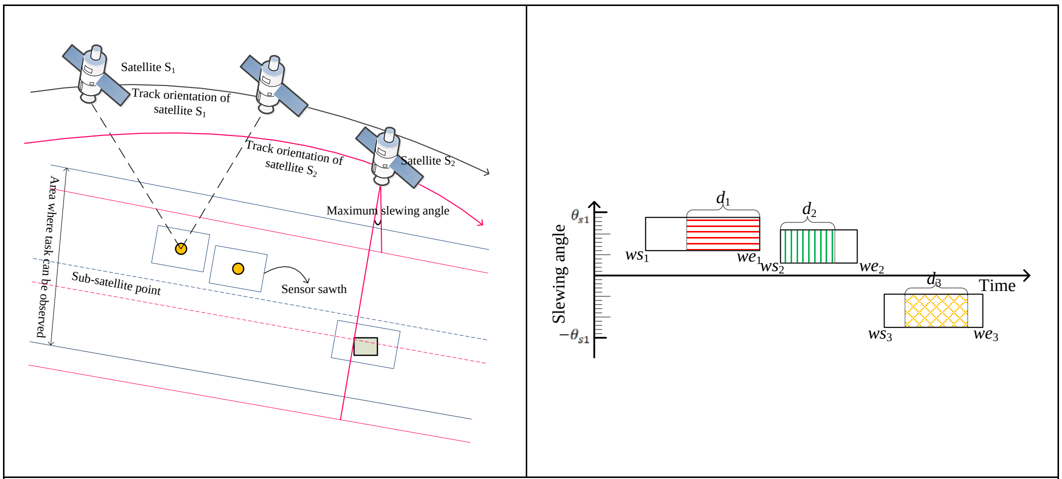
Compact Task Merging Method Considering the Duration of Task Execution

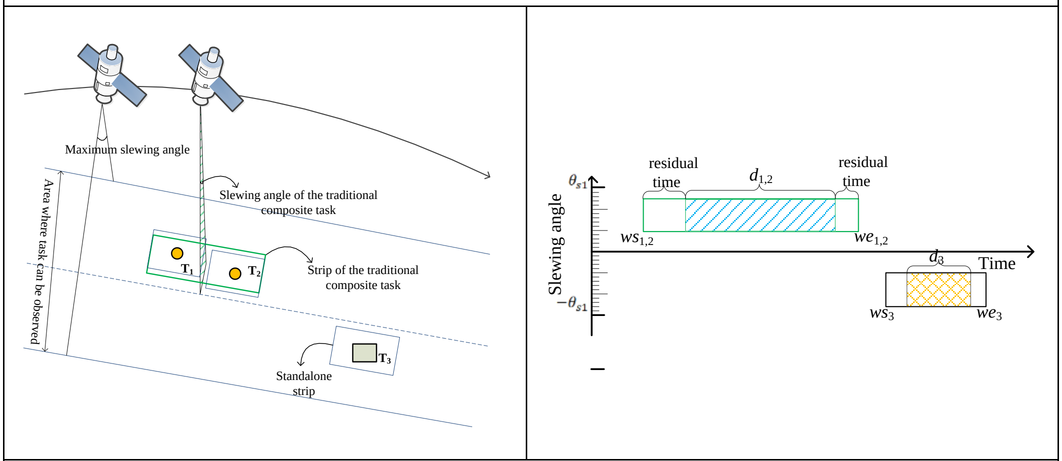
1 Problem statements

Task merging has been demonstrated to be an effectiveness strategy to improve the scheduling efficiency of satellite scheduling problems in previous research studies [1-6].The composite task obtained by the traditional task merging method is characterized as the union of visible time windows and mean of slewing angles of its meta-tasks. In our opinion, the observation duration of the task is an important factor in task merging, which is out of consideration in previous studies. By considering the duration of meta-tasks, we propose a compact task merging method to construct the so-called compact composite tasks in this paper. Specifically, a compact composite task is characterized by the smallest slewing angle, the shortest duration of task execution, and the most compact time window including all of possible chances to executing the merged task.

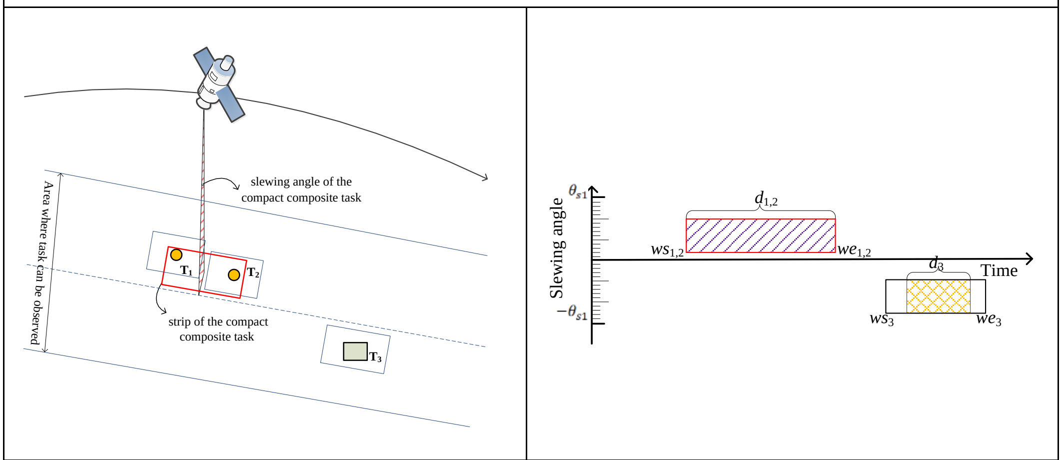
As illustrated in Fig. 1, the compact task merging and traditional task merging are different. Fig. 1 (a) shows that targets 1, 2 and 3 could be imaged by executing three distinct meta-tasks using the two satellites, i.e., T_1, T_2 and T_3. These targets can also be imaged via a composite task $T_{1,2}$ and a meta-task T_3 as shown in Fig. 1 (b) or Fig. 1 (c). As shown in Fig.1 (b), the time window of the composite task obtained by traditional task merging method has residual time. In contrast, no residual time exists in the compact composite task, as shown in Fig.1 (c).



(a)Multiple tasks and time windows



(b) Traditional composite task and time window



(c) Compact composite task and time window

Figure 1 illustration of satellite task merging. T_1, T_2 and T_3 represent three meta-tasks. θ_{s1} denotes the maximum slewing angle of satellite s_1. d_i denotes the indispensable observation duration of the *i*-th task. ws_i and we_i denote the start time and the end time of time window of the *i*-th task, respectively. In Figure 1(a), the three rectangles with the black outlines represent the time windows. In Figure 1(b), the rectangle with the black outline represents the composite time window of the traditional composite task of T_1 and T_2. $d_{1,2}$ denotes the indispensable duration of traditional composite task execution. In Figure 1(c), the rectangle with the black outline represents the compact composite time window of the compact composite task of T_1 and T_2. $d_{1,2}$ denotes the indispensable duration of the compact composite task execution.

To apply a task merging mechanism in satellite scheduling, we investigate the constraints of task merging, including slewing angle related constraint and time window related constraint. Moreover, the method of calculations for the time window and slewing angle of compact composite task is presented.

1.1 Traditional composite tasks

As shown in Fig. 1 (b), meta-tasks T_1 and T_2 can be combined into a composite task $T_{1,2}$ if and only if the following conditions must hold [1-6]

$$\begin{cases} \max\{we_1, we_2\} - \min\{ws_1, ws_2\} \le \Delta d_s \\ |\theta_1 - \theta_2| \le \Delta\theta_s \end{cases} \tag{1}$$

where Δd_s and $\Delta\theta_s$ are the longest open time and field of view of satellite *s*, respectively. The time window and slewing angle of the composite task $T_{1,2}$ are given by

$$\begin{cases} W_{1,2} = \left[\min\{ws_1, ws_2\}, \max\{we_1, we_2\} \right] \\ \theta_{1,2} = \dfrac{\theta_1 + \theta_2}{2} \end{cases}. \tag{2}$$

In the traditional task merging method, the observation duration $d_{1,2}$ of the composite task $T_{1,2}$ is out of consideration. To overcome this shortage, we propose a compact task merging method by considering the duration of meta-tasks.

1.2 Compact composite tasks

Definition 1: The residual time in a composite task's time window is referred to as the wasted time, which is equal to the difference between the span of the task's time window and its indispensable time duration.

Definition 2: A composite task is called a compact composite task, only when there is no residual time in the combined time window and the slewing angle is the smallest one among all feasible angles to simultaneously execute the meta-tasks.

Without loss of generality, between two meta-tasks T_i and T_j, the window start time ws_i of meta-task T_i is assumed to be earlier than that of meta-task T_j in the following.

Theorem 1: Two feasible meta-tasks T_i and T_j can be combined into a compact composite task $T_{i,j}$ if and only if they satisfy

$$\left(ws_j + d_j\right) - \left(we_i - d_i\right) \le \Delta d_s \tag{3}$$

and

$$\left|\theta_i - \theta_j\right| \le \Delta\theta_s \tag{4}$$

where $\left|\theta_i - \theta_j\right|$ is the absolute difference of the slewing angles between the two meta-tasks.

Theorem 2: If two feasible meta-tasks T_i and T_j can be merged into a compact composite task $T_{i,j}$, then its time window $W_{i,j} = \left[ws_{i,j}, we_{i,j} \right]$ should range from

$$ws_{i,j} = \begin{cases} we_i - d_i, & if \ |W_i \cap W_j| \le \min\left(d_i, d_j\right) \\ \min\left\{ws_j, \max\left(ws_i, ws_j + d_j - d_i\right)\right\}, & else \end{cases} \tag{5}$$

to

$$we_{i,j} = \begin{cases} ws_j + d_j, & if \ |W_i \cap W_j| \le \min\left(d_i, d_j\right) \\ \min\left(we_i - d_i, we_j - d_j\right) + \max\left\{d_i, d_j\right\}, & else \end{cases} \tag{6}$$

its indispensable duration of task execution should be

$$d_{i,j} = \begin{cases} ws_j + d_j - (we_i - d_i), & if \ |W_i \cap W_j| \le \min(d_i, d_j) \\ \max\left\{d_i, d_j\right\}, & else \end{cases} \tag{7}$$

and the slewing angle is given by

$$\theta_{i,j} = \begin{cases} \max\left\{\theta_i - \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\, \theta_i \ge 0, |\theta_i| \ge |\theta_j| \\ \min\left\{\theta_i + \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\, \theta_i < 0, |\theta_i| \ge |\theta_j| \\ \max\left\{\theta_j - \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\, \theta_j \ge 0, |\theta_j| > |\theta_i| \\ \min\left\{\theta_j + \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\, \theta_j < 0, |\theta_j| > |\theta_i| \end{cases} . \tag{8}$$

In section 2, the sufficient and necessary condition for a compact composite task is proved by enumerating the relationship between time windows and durations of task execution. In section 3, the slewing angle of a compact composite task is proved to be smallest one among all of the feasible angles.

2 The time window and duration of task execution

For a meta-task, the indispensable duration for its execution is given in advance, and it could be actually executed in any continuous time range within the time window. The indispensable duration for a composite task should be the shortest time range for satellite to execute them simultaneously. Therefore, the indispensable duration depends on both the intersection between meta-tasks' time windows and indispensable durations.

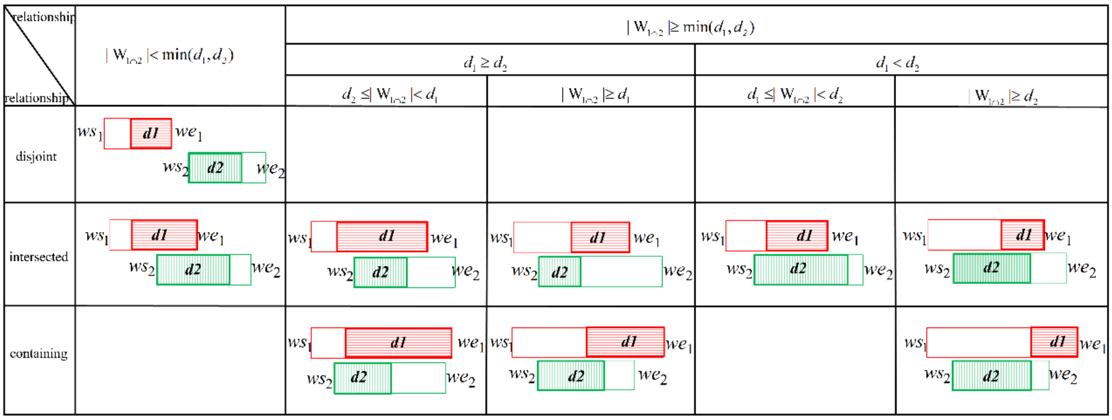


Figure 2 relationship between two time windows：disjoint, intersected, containing.

$|W_{1\cap 2}|$ denotes the length of the intersection between W_1 and W_2. d_1 and d_2 denote the indispensable time duration of task T_1 execution and task T_2 execution, respectively. ws_1 and we_1 denote the start time and the end time of the time window of task T_1. ws_2 and we_2 denote the start time and the end time of time window of task T_2, respectively.

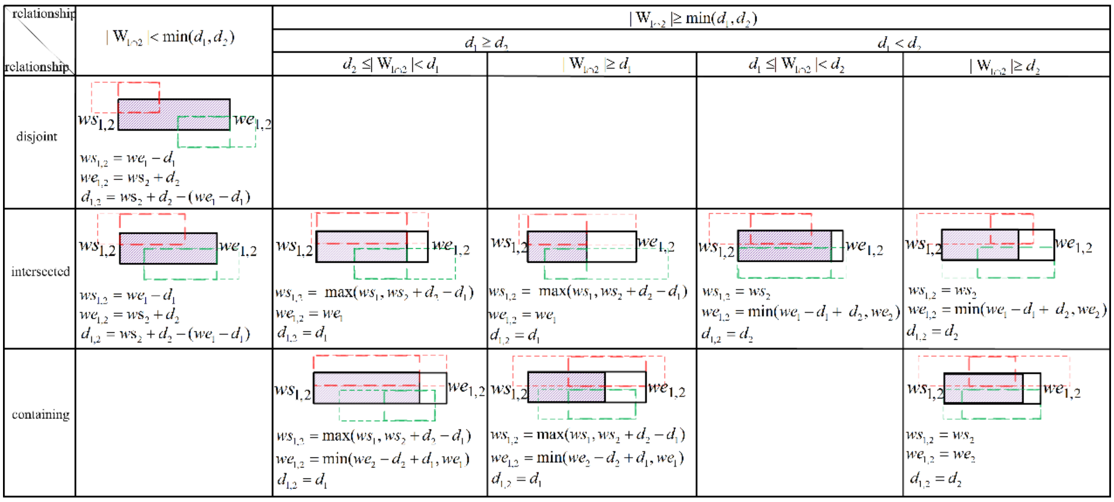


Figure3 time window of a compact composite task. $|W_{1\cap 2}|$ denotes the length of the intersection between W_1 and W_2. d_1 and d_2 denote the indispensable duration of task T_1 execution and task T_2 execution, respectively. $ws_{1,2}$ and $we_{1,2}$ represent the start time and the end time of the compact composite time window, respectively. $d_{1,2}$ denotes the indispensable time duration of the compact composite task.

As shown in Fig. 2, three types of temporal relationships exist between the meta-tasks' time windows, i.e., disjoint, intersected, and containing. Two cases exist in terms of the relationship between the length of the intersection and the maximal duration of the meta-tasks. In total, we enumerated 9 relationships between two meta-tasks. Corresponding with the cases in Fig. 2, the compact composite tasks of those meta-tasks are shown in Fig. 3, coupled with its time window and duration of task execution.

2.1 Proof of the sufficient condition defined in (3)

According to Eq.(1), a time window $W = [ws, we]$ is a feasible composite time window of W_i and W_j if they satisfy

$$\begin{cases} |W \cap W_i| \geq d_i \\ |W \cap W_j| \geq d_j \\ we - ws \leq \Delta d_s \end{cases} \tag{9}$$

where $|W \cap W_i|$ is the length of the intersection between W and W_i. In other words, meta-tasks T_i and T_j can be merged into a composite task in terms of the time windows.

Given $(ws_j + d_j) - (we_i - d_i) \leq \Delta d_s$, we present an example of the imagined time window W as follows:

1) $|W_i \cap W_j| \leq \min(d_i, d_j)$

Let $ws = we_i - d_i$ and $we = ws_j + d_j$; then $|W \cap W_i| = d_i$, $|W \cap W_j| = d_j$ and $we - ws = (ws_j + d_j) - (we_i - d_i) \leq \Delta d_s$. Therefore, Eq. (9) is satisfied.

2) $|W_i \cap W_j| \geq \min(d_i, d_j)$

Without lows of generality, assume $d_i \geq d_j$. Let $ws = ws_i$ and $we = we_i$; then $W = W_i$. Because W_i is the time window of a feasible meta-task, $d_i \leq |W_i| \leq \Delta d_s$. In addition, since $|W_i \cap W_j| \geq \min(d_i, d_j) = d_j$, so $|W \cap W_j| \geq d_j$. Therefore, Eq.(9) is satisfied.

Because at least one instance can be found, the condition defined in Eq.(3) is sufficient for the combination of meta-tasks T_i and T_j in terms of time windows.

2.2 Proof of the necessary condition defined in (3)

If meta-tasks' time windows can be combined into a composite task, then its time window $W_{i,j} = [ws_{i,j}, we_{i,j}]$ should satisfy

$$\begin{cases} \left|W_{i,j} \cap W_i\right| \geq d_i \\ \left|W_{i,j} \cap W_j\right| \geq d_j \\ we_{i,j} - ws_{i,j} \leq \Delta d_s \\ d_{i,j} \geq max\left(d_i, d_j\right) \end{cases} \tag{10}$$

since the two meta-tasks should can be executed in the time window.

Because $\left|W_{i,j} \cap W_i\right| \geq d_i$, we have $ws_{i,j} \leq we_i - d_i$. Since $\left|W_{i,j} \cap W_j\right| \geq d_j$, we have $we_{i,j} \geq ws_j + d_j$ .Thus, $\left(ws_j + d_j\right) - \left(we_i - d_i\right) \leq we_{i,j} - ws_{i,j} \leq \Delta d_s$. Therefore, $\left(ws_j + d_j\right) - \left(we_i - d_i\right) \leq \Delta d_s$. In other words, the necessary condition defined in Eq. (3) holds.

2.3 Characteristics of a compact composite task defined in Eq. (5-7)

According to definition 2, the indispensable duration of a compact composite task is the shortest one among all feasible composite tasks. In the following, we discuss the characteristics of a compact composite task according to two situations:

1) $|W_i \cap W_j| \leq \min\left(d_i, d_j\right)$

When $|W_i \cap W_j| \leq \min\left(d_i, d_j\right)$, both the latest start-time of T_i to the earliest end time of T_j should be within a composite task, so that the two meta-tasks can be executed simultaneously. Therefore, the shortest execution time $d_{i,j}$ of a composite task is equal to $\left(ws_j + d_j\right) - \left(we_i - d_i\right)$ as given in Eq.(7). Accordingly, the time window of a compact composite task should be $\left[\left(we_i - d_i\right), \left(ws_j + d_j\right)\right]$. Otherwise, the length of combined task must be larger than $d_{i,j}$, so that the two meta-tasks can be executed simultaneously. Therefore, Eqs. (5-6) hold for a compact composite task.

2) $|W_i \cap W_j| \geq \min\left(d_i, d_j\right)$

As shown in Eq.(9), $d_{ij} \geq \max\left(d_i, d_j\right)$. When $|W_i \cap W_j| \geq \min\left(d_i, d_j\right)$, it is possible for a meta-task with smaller indispensable execution duration to actually be completed within the intersection between time windows of two meta-tasks. Therefore, the shortest execution time $d_{i,j}$ of the composite task is equal to $\max\left(d_i, d_j\right)$. However, as shown in Fig. 3, the earliest start-time and the latest end-time depend on the relationship between d_i, d_j and $|W_i \cap W_j|$.

Without loss of generality, assume $d_i \geq d_j$ and the intersected time window is $W_{i\cap j} = \left[ws_{i\cap j}, we_{i\cap j}\right]$. The earliest start-time of the compact composite time window is $\max\left(ws_i, ws_j + d_j - d_i\right)$, and the latest end-time is $\min\left(we_i, we_j, we_{i\cap j} - d_j + d_i\right)$.

3 The slewing angle

If two meta-tasks could be constructed as a composite task, the slewing angles of these meta-tasks must be in one field of view of the sensor simultaneously. In other word, the discrepancy of their look angle cannot surpass the sensor's field of view, which can be expressed by Eq. (4). As a result, the composite task's slewing angle depends on the field of view of the sensor and the slewing angle of each meta-task. The smallest slewing angle of the composite task is given by Eq.(8).

3.1 Proof of sufficient condition defined in Eq. (4)

Given an imagined slewing angle $\theta_{i,j}$, if it satisfies

$$\theta_{i,j} \in \left(\left[\theta_i - \frac{\Delta\theta}{2}, \theta_i + \frac{\Delta\theta}{2}\right] \cap \left[\theta_j - \frac{\Delta\theta}{2}, \theta_j - \frac{\Delta\theta}{2}\right]\right) \tag{11}$$

then the slewing angle $\theta_{i,j}$ is feasible. In other words, meta-tasks T_i and T_j can be merged in terms of the slewing angles.

We consider the example of the slewing angle $\theta_{i,j}=\frac{\theta_i+\theta_j}{2}$. since $\left|\theta_i-\theta_j\right|\le\Delta\theta$,

we have $-\Delta\theta\le\theta_i-\theta_j\le\Delta\theta$, $-\Delta\theta\le\theta_j-\theta_i\le\Delta\theta$, and

$$\begin{cases}\theta_{i,j}-(\theta_i-\frac{\Delta\theta}{2})=\frac{\theta_i+\theta_j}{2}-\theta_i+\frac{\Delta\theta}{2}=\frac{\theta_j-\theta_i}{2}+\frac{\Delta\theta}{2}\ge-\frac{\Delta\theta}{2}+\frac{\Delta\theta}{2}=0\\ \theta_{i,j}-(\theta_i+\frac{\Delta\theta}{2})=\frac{\theta_i+\theta_j}{2}-\theta_i-\frac{\Delta\theta}{2}=\frac{\theta_j-\theta_i}{2}-\frac{\Delta\theta}{2}\le\frac{\Delta\theta}{2}-\frac{\Delta\theta}{2}=0\end{cases}. \tag{12}$$

So $\theta_{i,j}\in\left[\theta_i-\frac{\Delta\theta}{2},\theta_i+\frac{\Delta\theta}{2}\right]$.Similarly, $\theta_{i,j}\in\left[\theta_j-\frac{\Delta\theta}{2},\theta_j+\frac{\Delta\theta}{2}\right]$.

Therefore, $\theta_{i,j}\in\left(\left[\theta_i-\frac{\Delta\theta}{2},\theta_i+\frac{\Delta\theta}{2}\right]\cap\left[\theta_j-\frac{\Delta\theta}{2},\theta_j+\frac{\Delta\theta}{2}\right]\right)$.

Since at least one example can be found, the condition defined in Eq. (4) is sufficient for the combination of meta-tasks T_i and T_j in terms of slewing angles.

3.2 Proof of the necessary condition defined in Eq. (4)

If meta-tasks T_i and T_j can be merged, then they can be observed within one strip. Thus, the difference between the slewing angles of the meta-tasks must less than the field of view, i.e., $\left|\theta_i-\theta_j\right|\le\Delta\theta$. Therefore, the necessary condition defined in Eq.(4) holds.

3.3 Characteristics of a compact composite task defined in Eq. (8)

According to definition 2, the slewing angle of a compact composite task is the smallest one among all feasible angles. Without loss of generality, we assume that $\left|\theta_i\right|$ is larger or equal to $|\theta_j|$, i.e., $\left|\theta_i\right|\ge\left|\theta_j\right|$. We discuss the characteristics of a compact composite task according to two situations as follows:

(1) The case of $\theta_i\ge0$.

If $\theta_i - \frac{\Delta\theta}{2} \geq 0$, then $\theta_i - \frac{\Delta\theta}{2} - \frac{\Delta\theta}{2} \leq \theta_i \leq \theta_i - \frac{\Delta\theta}{2} + \frac{\Delta\theta}{2}$. Because $\left|\theta_i - \theta_j\right| \leq \Delta\theta$, we have $\theta_i - \Delta\theta \leq \theta_j \leq \theta_i + \Delta\theta$ .As a result, $\theta_i - \frac{\Delta\theta}{2} - \frac{\Delta\theta}{2} \leq \theta_j \leq \theta_i - \frac{\Delta\theta}{2} + \frac{\Delta\theta}{2}$, and $\theta_i - \frac{\Delta\theta}{2}$ is an appropriate slewing angle. Suppose there exists a smaller angle $\theta' < \theta_i - \frac{\Delta\theta}{2}$, then $\theta' + \frac{\Delta\theta}{2} < \theta_i - \frac{\Delta\theta}{2} + \frac{\Delta\theta}{2} = \theta_i$, so task T_i is beyond the field of view of the sensor. Therefore, $\theta_i - \frac{\Delta\theta}{2}$ is the smallest angle of composite task.

If $\theta_i - \frac{\Delta\theta}{2} < 0$, then $0 - \frac{\Delta\theta}{2} \leq \theta_i \leq 0 + \frac{\Delta\theta}{2}$. Since $\left|\theta_i\right| \geq \left|\theta_j\right|$ is given, then $\theta_i > \theta_j > -\theta_i$. As a result, $0 - \frac{\Delta\theta}{2} \leq \theta_j \leq 0 + \frac{\Delta\theta}{2}$.Therefore, 0 is the smallest angle of composite task.

Thus， $\theta_{i,j} = \max\left\{\theta_i - \frac{\Delta\theta_s}{2}, 0\right\}$.

(2) The case of $\theta_i < 0$.

If $\theta_i + \frac{\Delta\theta}{2} \leq 0$, then $\theta_i + \frac{\Delta\theta}{2} - \frac{\Delta\theta}{2} \leq \theta_i \leq \theta_i + \frac{\Delta\theta}{2} + \frac{\Delta\theta}{2}$. Given $\left|\theta_i - \theta_j\right| \leq \Delta\theta$ and $\left|\theta_i\right| \geq \left|\theta_j\right|$, we have $\theta_j - \theta_i \leq \Delta\theta$, and $\theta_i + \frac{\Delta\theta}{2} - \frac{\Delta\theta}{2} \leq \theta_j \leq \theta_i + \frac{\Delta\theta}{2} + \frac{\Delta\theta}{2}$. Thus, $\theta_i + \frac{\Delta\theta}{2}$ is an appropriate slewing angle. Suppose there exists an angle $\theta' > \theta_i + \frac{\Delta\theta}{2}$, we have $\theta' - \frac{\Delta\theta}{2} > \theta_i + \frac{\Delta\theta}{2} - \frac{\Delta\theta}{2} = \theta_i$. In this case, the task T_i is out of range of satellite. Therefore, $\theta_i - \frac{\Delta\theta}{2}$ is the smallest angle of composite task.

If $\theta_i + \frac{\Delta\theta}{2} > 0$, then $0 - \frac{\Delta\theta}{2} \leq \theta_i \leq 0 + \frac{\Delta\theta}{2}$. Given $\left|\theta_i\right| \geq \left|\theta_j\right|$, we have $\theta_i \leq \theta_j \leq -\theta_i$. As a result, $0 - \frac{\Delta\theta}{2} \leq \theta_j \leq 0 + \frac{\Delta\theta}{2}$. Therefore, 0 is the smallest angle of composite task.

Thus, $\theta_{i,j} = \min\left\{\theta_i + \frac{\Delta\theta_s}{2}, 0\right\}$.

In conclusion, the slewing angle of the compact composite task is

$$\theta_{i,j} = \begin{cases} \max\left\{\theta_i - \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\,\theta_i \ge 0, |\theta_i| \ge |\theta_j| \\ \min\left\{\theta_i + \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\,\theta_i < 0, |\theta_i| \ge |\theta_j| \\ \max\left\{\theta_j - \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\,\theta_j \ge 0, |\theta_j| > |\theta_i| \\ \min\left\{\theta_j + \dfrac{\Delta\theta_s}{2}, 0\right\}, & if\,\theta_j < 0, |\theta_j| > |\theta_i| \end{cases}.$$

4 Combined method for multiple meta-tasks

Based on the theory of constructing the compact composite task with two meta-tasks, we further develop a strategy regarding the construction of multiple meta-tasks to a compact composite task.

Let

$$\theta_i = \max(\max(\theta_1, 0), \max(\theta_2, 0), \ldots \max(\theta_n, 0)) \tag{13}$$

$$\theta_j = \min(\min(\theta_1, 0), \min(\theta_2, 0), \ldots \min(\theta_n, 0)) \tag{14}$$

$$ws_i = \begin{cases} ws_n & if\ ws_{1.n-1} > ws_n \\ ws_{1.n-1} & else \end{cases} \tag{15}$$

$$we_i = \begin{cases} we_n & if\ ws_{1.n-1} > ws_n \\ we_{1.n-1} & else \end{cases} \tag{16}$$

$$d_i = \begin{cases} d_n & if\ ws_{1.n-1} > ws_n \\ d_{1.n-1} & else \end{cases} \tag{17}$$

$$ws_j = \begin{cases} ws_{1.n-1} & if\ ws_{1.n-1} > ws_n \\ ws_n & else \end{cases} \tag{18}$$

$$we_j = \begin{cases} we_{1.n-1} & if\ ws_{1.n-1} > ws_n \\ we_n & else \end{cases} \tag{19}$$

$$d_j = \begin{cases} d_{1.n-1} & if\ ws_{1.n-1} > ws_n \\ d_n & else \end{cases} \tag{20}$$

where ws_n, we_n, d_n, and θ_n denote the start time , end time, indispensable time duration of task execution and slewing angle of Task *n*, respectively. $ws_{1.n-1}$, $we_{1.n-1}$ and $d_{1.n-1}$ denote the start time , end time, indispensable time duration of task execution of compact composite task consisting of task 1,task 2,⋯, and task *n*.

If $\{ws_i, we_i, d_i, \theta_i\}$ and $\{ws_j, we_j, d_j, \theta_j\}$ obtained by Eqs.(13)-(20) can satisfy Eqs. (3) and (4), then the *n* meta-tasks (task 1, task 2,⋯, and task *n*) can be used to construct a compact composite task $T_{1.n}$. The time window $W_{i,j} = [ws_{i,j}, we_{i,j}]$ should follow Eqs.(5)-(6) and the slewing angle should follow Eq. (7).

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