

Research Article

Application of Two-Phase Fuzzy Optimization Approach to Multiproduct Multistage Integrated Production Planning with Linguistic Preference under Uncertainty

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This paper tackles the challenges for a production planning problem with linguistic preference on the objectives in an uncertain multiproduct multistage manufacturing environment. The uncertain sources are modelled by fuzzy sets and involve those induced by both the epistemic factors of process and external factors from customers and suppliers. A fuzzy multiobjective mixed integer programming model with different objective priorities is proposed to address the problem which attempts to simultaneously minimize the relevant operations cost and maximize the average safety stock holding level and the average service level. The epistemic and external uncertainty is simultaneously considered and formulated as flexible constraints. By defining the priority levels, a two-phase fuzzy optimization approach is used to manage the preference extent and convert the original model into an auxiliary crisp one. Then a novel interactive solution approach is proposed to solve this problem. An industrial case originating from a steel rolling plant is applied to implement the proposed approach. The numerical results demonstrate the efficiency and feasibility to handle the linguistic preference and provide a compromised solution in an uncertain environment.

1. Introduction

Production planning in a multistage multiproduct production system concerns key production activities of each interconnected element. Material flows involved start from suppliers to the manufacturing network and finally to the customers. Coordination of lot sizes schedule for each element contributes to enhancing the utilization of activities that produce more values and thus attracts substantial interests. Typically, production planning can be categorized into three classes in terms of time horizon for decision making: long-term, medium-term, and short-term. Long-term planning focuses on identifying strategic decisions over a relatively long time, such as facility location and extent of additional investments in processing networks. Medium-term planning involves tactical decisions for the optimal use of various resources and anticipated lot sizing for production stages.

Models for medium-term planning comparatively account for the presence of production system topology and concern controlling the material flow, inventory level, transports, suppliers, and so forth. Short-term planning is related to detailed scheduling operations of a job, such as hour-to-hour job sequencing. The production planning problem in this work focuses on a medium-term level which integrates activities going across the interconnection from suppliers to customers.

Normally, for traditional manufacturers, the material flows within the production system present in two common forms: parallel and spread. Parallel flows usually apply to single-product structure, and switching product families are rather expensive. In contrast, spread flows are related to diversification in product mix and can provide services for customization. However, the dynamics in market and customers challenge the manufacturers to response fast and

accurately. It is usually difficult for the above two production forms to handle overachievement or underachievement for the forecasted demand of specific products, since the production stages within the system are either isolate or less correlate. When the demand for a specific product exceeds the production capacity of its dedicated production stages, it may result in backorders or lost sales, and on the contrary, it may also lead to a high level on inventory or excess capacities. Therefore, many manufacturers nowadays prefer to the production system in which products could be manufactured in a flexible way.

In this regard, we consider a multistage production system in which material and information flows may be interconnected or interwoven with each other. Other than the conventional parallel or spread topology structure, the addressed production system allows each production stage to schedule and allocate the items to be produced independently upon the information received from the customers. In other words, each product is produced through a designated route consisting of a series of production stages, and each stage may process items coming from one or more upstream stages.

To deal with demand fluctuation from the customers, several operations strategies have been developed and incorporated in planning decisions, such as backlogging, lost sales, or inventory holding [1–3]. These techniques show several advantages on obtaining feasible solutions in a more flexible way. However, in practical, the dynamic nature of production environment and markets induces a high degree of uncertainty, thus increasing the risk for decision-makers. Peidro et al. [4] identified several metrics for distinct uncertain sources in supply chain planning and typically addressed the uncertainty for supply, process, and demand. In this paper, we consider three types of practical problems encountered for decision-making: (1) epistemic uncertainty that originates from internal factors due to a lack of knowledge in production parameters or coefficients, (2) external uncertainty concerned with customers and suppliers, and (3) trade-off among the conflicting objectives with decision-makers' preference which is derived from the complicated environment.

Several techniques and researches attempted to model the uncertainty in the production planning, among which four modelling approaches are identified: conceptual, analytical, artificial intelligence, and simulation models [5]. Among the researches, stochastic programming and probability distribution show the applicability in major production categories [6–8]. Stochastic programming formulates the uncertain sources based on the random characteristics and applies probability distributions that can be expressed in different forms such as normal, Beta, and Poisson [9]. Kira et al. [10] proposed a stochastic linear programming model to formulate the hierarchical production planning under random demand. Fleten and Kristoffersen [11] presented a multistage mixed-integer linear stochastic programming model to address a short-term production plan for a hydropower plant. Zanjani et al. [12] developed a hybrid scenario tree with stationary probability distributions to integrate the uncertainty in the quality of raw materials and demand. However, as pointed out by Inuiguchi and Ramík [13], a stochastic programming

problem may not be solved easily, since the computational efforts would be enhanced dramatically with the problem scale. Besides, when there is a lack of evidence on historical or statistical data, the probabilistic reasoning modelling methods are not always reliable and appropriate [14].

Alternatively, fuzzy set theory and possibility theory provide an attractive tool to account for the uncertainty [15]. There are several practical advantages of the fuzzy set theory with respect to handling the uncertainty for production decisions. (1) It is able to deal with the uncertainty that is lack of data or evidence and is thus applicable to ill-defined scenarios. (2) It allows the decision-makers to incorporate judgements for improving the solution interactively. (3) The fuzzy model allows for development of more flexible and reliable decision tools that could be expressed linguistically based on human perception. (4) The distinct fuzzy membership functions provide broader alternatives that are of high computational efficiency. Bellman and Zadeh [16] originally introduced the concept of fuzzy aggregation operators, upon which Zimmermann [17] presented a fuzzy linear programming approach to aggregating the fuzzy goals and constraints.

Following the development of fuzzy optimization approaches, the fuzzy set theory shows its effectiveness and superiority to handle the uncertainty in a production planning problem. Fung et al. [18] discussed a multiproduct aggregate production planning with fuzzy demand and capacity. They formulated the dynamic balance constraints as fuzzy/soft equations and solved the model using parametric programming. Wang and Liang [19] developed a fuzzy multiobjective linear programming model for a multiproduct aggregate production planning problem in a fuzzy environment. The fuzzy objectives were formulated using piecewise linear membership functions and were aggregated by a max operator. Wang and Liang [20] then addressed this problem by incorporating the imprecise forecasted demand, operating cost, and capacity. They adopted the triangular fuzzy number to represent the imprecise data and transform the fuzzy objectives by minimizing the three prominent points. Vasant [21] proposed a fuzzy linear programming model with a modified s-curve membership function to solve a production planning problem with vague parameters and objective coefficients. Mula et al. [22] studied a material resource planning with flexible constraints in a multiproduct multilevel manufacturing environment. They applied a fuzzy linear programming approach with three kinds of aggregation schemes to decompose the original model into three fuzzy models. Torabi and Hassini [23] developed a novel multiobjective possibilistic mixed integer linear programming model for integrating procurement, production, and distribution planning. The model aimed to simultaneously minimize the total cost of logistics and maximize the total value of purchasing. Torabi et al. [24] further studied a fuzzy hierarchical production planning problem. The imprecise parameters along with the soft constraints are introduced to provide required consistency between decisions of the adjacent levels. Peidro et al. [4] proposed a fuzzy model for a tactical supply chain planning which contemplates the different uncertain sources from supply, demand, and process. They adopted a linear ranking function and triangular fuzzy numbers to transform the fuzzy

model into a crisp equivalent problem. Taghizadeh et al. [25] presented a fuzzy multiobjective linear programming model for a multiperiod multiproduct production planning problem that simultaneously minimizes production cost and maximizes machine utilization. The model is solved using piecewise linear membership functions through a nonsymmetric decision for the fuzzy objectives and constraints. Aliev et al. [26] introduced a fuzzy-generic approach for solving an aggregate production-distribution planning problem in a fuzzy environment. Generally, the above researches take advantage of various tools based on the concepts of fuzzy set theory that are applied in goal programming and soft/flexible mathematical programming. Other research studies relevant to fuzzy programming models include Hsu and Wang [27], Lan et al. [28], Figueroa-García et al. [29], Yaghin et al. [30], and Su and Lin [31].

In real-world applications, the production plan usually concerns conflicting objectives regarding the performance evaluated by various factors. Preference on the objectives imposed by the decision-makers which is a critical issue is rather difficult to be represented in the conflicting objectives properly. Fuzzy set theory provides an alternative to express the preference; that is, the more important the objective, the higher the satisfying degree [32]. With respect to the production planning in an uncertain environment, however, few attentions are paid on the decision-makers' preference in the existing contributions in the literature. Conventionally, the weighted additive methods are used by assigning different weights for the objectives [33, 34]. However, the decision-makers still encounter the problems of weighing the relative importance of each objective since the weights are quite dependent on the preference and are not easy to be specified upon the subjective judgment. With regard to the objective priorities, Tiwari et al. [35] constructed priority levels by specifying the membership grades and solved the subproblem for each level in sequence. Chen and Tsai [36] used the concept of desirable achievement degree to reflect the relative importance of goals explicitly; that is, the more important the goal, the higher the desirable achievement degree. However, these models are rigid in nature and may encounter the difficulties in obtaining a feasible solution when acquiring high overall satisfying degrees or desirable preference extent between the objectives.

In this paper, a novel fuzzy multiobjective mixed integer linear programming model (FMO-MILP) is proposed for the multiproduct multistage production planning problem with linguistic preference in a fuzzy environment. The epistemic and external uncertainty is simultaneously formulated as flexible constraints and is handled by using a weighted average method and a fuzzy ranking method. The model defines two key performance indicators as the objectives, along with which the economic objectives are assessed in terms of relative importance. The model attempts to minimize total cost of production, overtime, raw materials, and inventory and in the meanwhile maximize the average safety stock holding level and the average service level. The production constraints include supply and production capacity, warehouse space, production route and product family, inventory balance, and backorders. Linguistic

preference on the objectives imposed by the decision-makers is considered and constructed by priority levels. To acquire desirable solutions that fully reflect the preference, a two-phase interactive satisfying optimization method is applied through relaxing the overall satisfying degree. The FMO-MILP model is transformed into an equivalent crisp model by treating the objectives and constraints separately. Then we developed a novel interactive solution approach to solving the FMO-MILP model until a compromised solution is obtained. Finally, the proposed approach is implemented on real-world production planning in a steel rolling plant.

In summary, the main contributions of this work mainly lie in presenting (1) a novel multiproduct multistage production planning model with linguistic preference and various practical constraints, simultaneously taking into account the epistemic and external uncertainty; (2) definition and formulation of average safety stock holding level and average service level as fuzzy objectives for key indicators, together with fuzzy objectives for the operations cost, where priority levels are constructed; (3) a two-stage interactive solution approach to solving the fuzzy model which is able to treat not only the fuzzy objectives with linguistic preference but also the epistemic and external uncertainty in a flexible way.

The remainder of this work is organized as follows. Section 2 analyses the problem, details the assumptions, and formulates the FMO-MILP model for the multiproduct multistage production planning problem. Subsequently, Section 3 presents the approach to handling the uncertainty and the objectives with linguistic preference and develops the two-phase interactive solution approach. Next, an industrial case is used to illustrate the applicability and potential in Section 4. Finally, conclusions and remarks for further researches are given in Section 5.

2. Model Formulation

2.1. Problem Descriptions and Assumptions. The multiproduct multistage production planning problem addresses a small or medium sized plant that manufactures various types of products for the customers over a given planning horizon. Each product is processed stage-by-stage through a designated production route. As the flexible nature of the production topology, the production routes specified for different products may be intersected, thus complicating the modelling. To model the production topology, each production stage is formulated regarding its successor and predecessor stages. Since the operations parameters or coefficients are often incomplete and/or unobtainable, epistemic uncertainty from the production-inventory system involves the available production capacity, planned maintenance time, production efficiency, warehouse space, and safety stock level. These uncertain sources are modelled by fuzzy numbers using possibility theory. External uncertainty from both the suppliers and customers resulting volatile material and information flows is formulated as flexible constraints by fuzzy sets. The objectives evaluated by the decision-makers can be divided into two different categories: operations cost and key performance indicators concerned. The proposed model aims to simultaneously minimize total cost related to production

and inventory operations and maximize the average safety stock holding level and the average service level. In practice, such complex and conflicting objectives along with different constraints imposed by suppliers, production topology, and customers really challenge the efforts on decision-making. The assessment on each objective is vague when considering preference from the decision-makers. The preference decisions are made in linguistic terms to distinguish the relative importance for the objectives, such that “average service level is the most important during this planning horizon” or “average safety stock holding level should be approximately higher than certain value and is relatively more important than inventory cost.” Therefore, this work also defines and incorporates the relative importance of the objectives in a fuzzy environment.

The proposed FMO-MILP model is formulated based on the following assumptions:

- (1) Items produced can be stored to be used in certain periods, but within the planning horizon. The items are shipped to either the successor production stages or customers for delivery.
- (2) Backorders are allowed for multiple periods but should be fulfilled within the planning horizon. This strategy provides more flexibility for the production system to schedule the resources in a feasible way.
- (3) Supply capacity for each raw material is estimated and limited by the suppliers. The total procurement capacity for all the raw materials is limited by purchasing department of the manufacturer.
- (4) Suppliers, different production stages, and warehouses are assumed located in close proximity, so the lead time among them is ignored.
- (5) Raw materials, work-in-process, and end-items are stored in separated warehouses and thus formulated independently.
- (6) Some items can be acted as both work-in-process and end-items according to their usage with regard to the inventory level.
- (7) Dynamic safety stock strategy is considered by providing a buffer to meet fluctuating customer demand.
- (8) Priority levels are constructed for all the objectives to reflect decision-makers’ judgement and preference on the relative importance of each objective. The priority structure may not be constant and is subject to the preference.

Indices, sets, parameters, and decision variables for the proposed FMO-MILP model are defined as follows.

Indices are as follows:

- j : index for production stages,
- j' : index for the immediate successor production stages of j ,
- i : index for items including raw materials, work-in-process, and end items,

- i' : index for the immediate downstream items of i ,
- l : index for item families,
- t : index for time period,
- s : index for raw material suppliers,
- t' : index for the periods after t ,
- t'' : index for the periods before t .

Sets are as follows:

- N : set for all the items,
- J : set for all the production stages,
- T : set for time horizon,
- B : subset of N denoting raw materials,
- W : subset of N denoting work-in-process,
- P : subset of N denoting end items,
- $B(s)$: set for raw materials suppliers s ,
- $C(i)$: set for the production stages that produce i ,
- $O(i)$: set for the first production stage of i ,
- $S(i)$: set for the suppliers that supply raw material i ,
- $T(i)$: set for the terminal production stage of i ,
- $L(j)$: set for item families produced by production stage j ,
- $M(j)$: set for the items produced by production stage j ,
- $F(l)$: set for items involved in family l ,
- $\Omega(j, i)$: set for the immediate predecessor production stages of j for producing i ,
- $\Pi(j, i)$: set for the items produced from i by production stage j .

Deterministic parameters are as follows:

- A_t : hours in period t ,
- cb_i : cost of supplying one unit of raw material i ,
- cba_i : cost of backlogging one unit of end item i for one time period,
- cib_i : cost of holding one unit of raw material i in warehouse for one time period,
- ciw_i : cost of holding one unit of work-in-process i in warehouse for one time period,
- cip_i : cost of holding one unit of end item i in warehouse for one time period,
- co_t : overtime cost per hour in period t ,
- cp_{ij} : cost of producing one unit of i at stage j ,
- cs_{ij} : setup cost of stage j for producing i ,
- R : large positive number,
- TA_{jt} : planned regular production capacity of stage j in period t (in hours),

TM_{jt} : planned maintenance time of stage j in period t (in hours),

TS_{lj} : setup time for family l at stage j ,

β_i : penalty coefficient for backorders that are fulfilled in the next planning horizon,

v_i : warehouse capacity occupied by one unit quantity of the item i .

Fuzzy parameters are as follows:

$\tilde{\eta}_{ij}$: production efficiency of stage j for producing i ,

$M\tilde{D}_{it}$: demand for end item $i \in P$ in period t ,

$U\tilde{M}B_{ist}$: supply capacity of raw material i for supplier s in period t ,

$UR\tilde{M}B_{st}$: raw material supply capacity for supplier s in period t ,

$U\tilde{I}B_t$: raw material warehouse capacity in period t ,

$U\tilde{I}P_t$: end item warehouse capacity in period t ,

$U\tilde{I}W_t$: work-in-process warehouse capacity in period t ,

$S\tilde{I}P_{it}$: end item safety stock in period t .

Continuous variables are as follows:

IB_{it} : inventory level for raw material $i \in B$ in period t ,

IP_{it} : inventory level for end item $i \in P$ in period t ,

IP_{it}^- : end item inventory deviation below the safety stock level in period t ,

IW_{it} : inventory level for work-in-process $i \in W$,

LM_{ljt} : minimum production quantity of family l at stage j in period t ,

$MBA_{i,t,t'}$: backlogging quantity of end item $i \in P$ in period t that will be fulfilled in period t' ,

ML_{it} : quantity of end item $i \in P$ that fails to meet the order demand in period t ,

MB_{ist} : supply quantity of raw material $i \in B$ by supplier s in period t ,

MP_{it} : quantity of end item $i \in P$ shipped to the warehouse in period t ,

MX_{ijt} : production quantity of item $i \in W \cup P$ at stage j in period t ,

MXF_{ljt} : production quantity of family l at stage j in period t ,

TO_{jt} : overtime production capacity at stage j in period t ,

$QB_{i,t'',t'}$: quantity of raw material $i \in B$ shipped to the warehouse in period t'' and which will be stored to be used in period t' ,

$QP_{i,t'',t'}$: quantity of end item $i \in P$ shipped to the warehouse in period t'' and which will be stored to be used in period t' ,

$QW_{i,t'',t'}$: quantity of work-in-process $i \in W$ shipped to the warehouse in period t'' and which will be stored to be used in period t' ,

UM_{ljt} : maximum production quantity of family l at stage j in period t .

Integer variables are as follows:

E_{it} : binary variable that indicates whether the end items shipped in period t will be stored to be used in the future,

G_{it} : binary variable that indicates whether the end item demand in period t will be backlogged,

x_{ljt} : binary variable that indicates whether item family l is produced at stage j in period t .

2.2. Fuzzy Programming Model

2.2.1. Objective Functions. Two conflicting objective categories are considered simultaneously in the FMO-MILP model: the operations cost and the key indicators concerned.

(1) Objectives for the Operations Cost. The operations cost includes those of production, overtime, raw materials, and inventory holding. Accordingly, the following describes these objective functions that are expected to be minimized:

$$\min z_1 \cong \sum_{i \in M(j)} \sum_{j \in J} \sum_{t=1}^T cp_{ij} \cdot MX_{ijt} \quad (1)$$

$$+ \sum_{l \in L(j)} \sum_{j \in J} \sum_{t=1}^T cs_{lj} \cdot X_{ljt},$$

$$\min z_2 \cong \sum_{j \in J} \sum_{t=1}^T co_t \cdot TO_{jt}, \quad (2)$$

$$\min z_3 \cong \sum_{i \in B} \sum_{s \in S(i)} \sum_{t=1}^T cb_{is} \cdot MB_{ist}, \quad (3)$$

$$\min z_4 \cong \sum_{i \in B} \sum_{t=1}^T cib_i \cdot IB_{it} + \sum_{i \in W} \sum_{t=1}^T ciw_i \cdot IW_{it} \quad (4)$$

$$+ \sum_{i \in P} \sum_{t=1}^T cip_i \cdot IP_{it}.$$

The symbol “ \cong ” represents the fuzzified version of “ $=$ ” and reads “essentially equal to.” The production cost in (1) is generated by processing and setup activities. The processing cost is calculated specific to each product and production stage independently. Setup cost is only related to changeovers between products of a family where similar products are grouped. Overtime production is allowed but relatively will

incur high cost in labour, as defined in (2). Raw materials are purchased from diverse suppliers with different prices, as specified in (3). Additionally, inventory holding cost in (4) involves warehouses for raw materials, work-in-process, and end-items.

(2) *Objectives for Key Indicators.* Decisions on operations assessed by cost are those objectives that could be measured by economic performance easily and immediately. However, some objectives having indirect effects on the cost are not suitable to evaluate them by economic performance, such as customer satisfaction and planning nervousness [37]. Conventional researches attempted to estimate the parameters by cost and unify the measuring unit so that the multiobjective optimization problem can be managed through a simple additive method, interactive methods, or fuzzy methods [38–40]. Typical examples include estimation on backloging penalty and lost sale penalty [41]. These estimations are usually empirical and may not satisfy the dynamic and diversified requirements since the production environment and market are rather volatile. With regard to these problems, this work formulates the objectives independently and carries out two key indicators as noneconomic measured objectives that are frequently encountered in the real application: average safety stock holding level and average service level. To incorporate the above indicators into the model, the following definitions are given:

(i) Safety stock holding level (SHL) is as follows:

$$\text{SHL}_i (\%) = \sum_{t=1}^T \left(1 - \frac{IP_{it}^-}{\widetilde{SIP}_{it}} \right) \cdot \frac{100}{T} \quad \forall i \in P. \quad (5)$$

(ii) Average safety stock holding level (ASHL) is as follows:

$$\text{ASHL} (\%) = \sum_{i \in P} \frac{\text{SHL}_i}{\varphi(P)}. \quad (6)$$

SHL evaluates deviation below the safety stock level for each product. Generally, ASHL measures the average level of safety stock holding for all the products, where $\varphi(P)$ indicates the number of elements in set P .

(iii) Service level (SL) is as follows:

$$\text{SL}_i (\%) = \max \left\{ 0, \left[1 - \frac{\sum_{t=1}^{T-1} \sum_{t' > t}^T (t' - t) \cdot \text{MBA}_{i,t,t'} + \sum_{t=1}^T \beta_i \cdot \text{MBA}_{i,t,T+1}}{\sum_{t=1}^T \text{MD}_{it}} \right] \cdot 100 \right\} \quad \forall i \in P. \quad (7)$$

(iv) Average service level (ASL) is as follows:

$$\text{ASL} (\%) = \sum_{i \in P} \frac{\text{SL}_i}{\varphi(P)}. \quad (8)$$

SL concerns customer satisfactory in terms of backloging level for each product. It should be noted that the backorders to be fulfilled in the next planning horizon have a greater impact on those backorders that are fulfilled during the current planning horizon. Thus, a weight penalty coefficient β_i is added for this scenario. Similarly, ASL defines the average level on customer satisfactory for all the products.

Following the concepts of the above indicators, the decision-makers seek to enhance the service and keep the safety stock level, which are presented by the objective functions:

$$\max z_5 \equiv \text{ASHL} \quad (9)$$

$$\max z_6 \equiv \text{ASL}. \quad (10)$$

2.2.2. Constraints

(1) *Material Balance for Raw Material.* The raw materials from various suppliers are stored in the warehouse to be used for further processing. In other words, material flow of raw material is controlled by both suppliers and initial production stages. The following constraints are applied to these two aspects for raw material balance:

$$\sum_{t''=1}^t \text{QB}_{i,t'',t} = \sum_{s \in S(i)} \text{MB}_{i,s,t} - \text{QB}_{i,t,t} \quad (11)$$

$$\forall i \in B, \forall t = 1, \dots, T-1,$$

$$\text{QB}_{i,t'',t} = \sum_{s \in S(i)} \text{MB}_{i,s,t} \quad \forall i \in B, t = T, \quad (12)$$

$$\sum_{t''=1}^t \text{QB}_{i,t'',t} = \sum_{i' \in \Pi(j,i)} \sum_{j \in O(i)} \text{MX}_{i',j,t} \quad i \in B, \forall t. \quad (13)$$

Equations (11) and (12) ensure the material balance at raw material warehouse. Due to the operations strategy, raw materials are allowed to be stored for multiple periods within the planning horizon. Therefore, raw materials supplied during the last period of planning horizon $t = T$ would only be used for processing, as shown in (12). Equation (13) states that the raw materials stored in the warehouse are only used at initial production stages:

$$\sum_{i \in B} \text{MB}_{ist} \leq \text{UR} \widetilde{\text{MB}}_{st} \quad \forall s, t, \quad (14)$$

$$\text{MB}_{ist} \leq \text{U} \widetilde{\text{MB}}_{ist} \quad \forall i, s, t. \quad (15)$$

Equation (14) represents the procurement capacity limit from the perspective of manufactures. This restriction may be related to the shipment capacity or budget. Equation (15) implies that the supply capacity for each supplier is also limited by an upper bound. These two bounds are often lack of fully knowledge and thus are expressed as fuzzy coefficients.

(2) *Material Balance for Work-in-Process.* Similarly, material flow of work-in-process is controlled by its upstream and

downstream flows, especially by its successor and predecessor production stages:

$$\begin{aligned} & \sum_{t' > t}^T QW_{i,t,t'} \\ &= \sum_{t''=1}^{t-1} QW_{i,t'',t} + \sum_{j \in C(i)} MX_{ijt} \\ & \quad - \sum_{i' \in \Pi(j',i)} \sum_{j' \in \Omega(j,i)} MX_{i',j',t} - MP_{it} \\ & \quad \forall i \in W \mid i', j' \notin \emptyset, \forall t = 1, \dots, T-1, \end{aligned} \quad (16)$$

$$\begin{aligned} & \sum_{t''=1}^{t-1} QW_{i,t'',t} + \sum_{j \in C(i)} MX_{ijt} \\ &= \sum_{i' \in \Pi(j',i)} \sum_{j' \in \Omega(j,i)} MX_{i',j',t} + MP_{it} \\ & \quad \forall i \in W \mid i', j' \notin \emptyset, t = T, \end{aligned} \quad (17)$$

$$MP_{it} = 0 \quad \forall i \in W - P, \forall t. \quad (18)$$

Equation (16) presents material balance of the work-in-process warehouse for period $t = 1, \dots, T-1$, in which two adjacent production stages are modelled through constructing sets with respect to their logical relationships. Equation (17) shows the material balance for the last period $t = T$, where the items shipped to the warehouse should be used during the current period. On account of the production topology, items produced by a certain production stage may be either stored for further processing or shipped for delivery in terms of requirements. In this regard, (18) specifies that those items only for further processing are not allowed to be shipped to the end item warehouse.

(3) *Material Balance for End Items.* Consider

$$\begin{aligned} & \sum_{t' > t}^T QP_{i,t,t'} + \tilde{S}IP_{it} - IP_{it}^- - \sum_{t' > t}^T MBA_{i,t,t'} \\ & \cong \sum_{t''=1}^{t-1} QP_{i,t'',t} + \tilde{S}IP_{it-1} - IP_{it-1}^- - \sum_{t''=1}^{t-1} MBA_{i,t'',t} \\ & \quad + MP_{it} - M\bar{D}_{it} \quad \forall i \in P, \forall t = 1, \dots, T-1, \end{aligned} \quad (19)$$

$$\begin{aligned} & \tilde{S}IP_{it} - IP_{it}^- - MBA_{i,t,t+1} \\ & \cong \sum_{t''=1}^{T-1} QP_{i,t'',t} + \tilde{S}IP_{i,t-1} - IP_{i,t-1}^- + MP_{it} - M\bar{D}_{it} \\ & \quad \forall i \in P, t = T, \end{aligned} \quad (20)$$

$$MP_{it} = \sum_{j \in C(i)} MX_{ijt} \quad \forall i \in P - W, \forall t. \quad (21)$$

Equations (19) and (20) formulate the material balance at the end item warehouse for periods $t = 1, \dots, T-1$ and $t = T$, respectively. Safety stock is incorporated in the equations to provide a buffer against uncertainty, where the inventory level is allowed to fluctuate around the safety stock level. Hence, deviation below the safety stock is utilized to relax this restriction on the end item warehouse. Equation (21) indicates the relation of material flow into the end item warehouse with the items produced by terminal production stages. In real-world scenarios, product demand along with the safety stock is usually forecasted and estimated upon the historical statistics and cannot reflect the real market precisely. Therefore, these two variables are treated as fuzziness.

(4) *Inventory Level Constraints.* Due to the fact that raw materials, work-in-process, and end items are stored separately under different inventory management strategies, inventory constraints should be formulated independently upon the dynamic characteristics:

$$IB_{it} \geq \sum_{t''=1}^t \sum_{t' > t}^T QB_{i,t'',t'} \cdot v_i \quad (22)$$

$$\forall i \in B, \forall t = 1, \dots, T-1,$$

$$IB_{it} \geq 0, \quad \forall i \in B, t = T, \quad (23)$$

$$\sum_{i \in B} IB_{it} \lesssim U\bar{I}B_t \quad \forall t. \quad (24)$$

Since the model depicts exact usage of raw materials from one period to another, lower boundary of the raw material inventory level for periods $t = 1, \dots, T-1$ and $t = T$ can be obtained by (22) and (23), respectively. The inventory level should be essentially less than a limited storage capacity as specified by (24):

$$IW_{it} \geq \sum_{t''=1}^t \sum_{t' > t}^T QW_{i,t'',t'} \cdot v_i \quad (25)$$

$$\forall i \in W, \forall t = 1, \dots, T-1,$$

$$IW_{it} \geq 0, \quad \forall i \in W, t = T, \quad (26)$$

$$\sum_{i \in W} IW_{it} \lesssim U\bar{I}W_t \quad \forall t. \quad (27)$$

Similarly, (25)–(27) imply the upper limit and lower limit for the work-in-process warehouse:

$$IP_{it}^- \gtrsim \tilde{S}IP_{it} + \sum_{t''=1}^t \sum_{t' > t}^T QP_{i,t'',t'} \cdot v_i - IP_{it} \quad (28)$$

$$\forall i \in P, \forall t = 1, \dots, T-1,$$

$$IP_{it}^- \gtrsim \tilde{S}IP_{it} - IP_{it} \quad \forall i \in P, t = T, \quad (29)$$

$$IP_{it}^- \lesssim \tilde{S}IP_{it} \quad \forall i \in P, \forall t, \quad (30)$$

$$\sum_{i \in P} IP_{it}^- \lesssim U\bar{I}P_t \quad \forall t. \quad (31)$$

The inventory level for the end items should be appropriately controlled to satisfy the fluctuating demand by utilizing the flexible strategy of safety stock. Equation (28) ensures that the end item inventory level for period $t = 1, \dots, T-1$ should cover both surplus parts of the safety stock and those end items stored for future use, and (29) states the same restriction for period $t = T$. However, the buffer capacity of safety stock is also limited by the setting level itself, thus the lower deviation is restricted by (30). Equation (31) represents the storage capacity for the end item warehouse.

(5) *Production Capacity Constraints.* Consider

$$MXF_{ljt} = \sum_{i \in F(l)} MX_{ijt} \quad \forall l \in L(j), \forall j, \forall t, \quad (32)$$

$$MXF_{ljt} \geq LM_{ljt} \cdot X_{ljt} \quad \forall l \in L(j), \forall j, \forall t, \quad (33)$$

$$MXF_{ljt} \leq UM_{ljt} \cdot X_{ljt} \quad \forall l \in L(j), \forall j, \forall t. \quad (34)$$

Various items are grouped in families by product and processing specification to reduce capacity consumption caused by frequent setup. Equation (32) defines attribution set and quantity relation to the families with the items. Equations (33) and (34) specify the logical relationship between binary setup and production quantity of the families:

$$\sum_{i \in M(j)} \left(\frac{MX_{ijt}}{\tilde{n}_{ij}} \right) + \sum_{l \in L(j)} TS_{lj} \cdot X_{ljt} \leq T\tilde{A}_{jt} + TO_{jt} \quad (35)$$

$\forall j, \forall t,$

$$T\tilde{A}_{jt} + TO_{jt} \leq A_t - T\tilde{M}_{jt} \quad (36)$$

$\forall j, \forall t.$

The production capacity is consumed by both regular processing and setup. Equation (35) ensures that the total capacity occupied is not allowed to exceed the maximum level, which involves planned regular capacity and overtime capacity as well. Since the capacity is measured by time, the upper bound given by (35) should be further restricted by physical time and maintenance plans in a period, as shown by (36). Typically, (35) and (36) are modelled as flexible constraints due to the uncertainty existing in capacity allocation and efficiency.

(6) *Backlogging Constraints.* Consider

$$R \cdot E_{it} \geq \sum_{t' > t}^T QP_{i,t,t'} \quad \forall i \in P, \forall t = 1, \dots, T-1, \quad (37)$$

$$R \cdot G_{it} \geq \sum_{t' > t}^T MBA_{i,t,t'} \quad \forall i \in P, \forall t = 1, \dots, T-1, \quad (38)$$

$$E_{it} + G_{it} \leq 1 \quad \forall i \in P, \forall t = 1, \dots, T-1. \quad (39)$$

Although inventory holding and backorders may be sustained for several periods, for each product, they are not allowed to concurrent during a period. Equations (37)–(39)

follow “the best I can do” policy; that is, the decision-makers should make the most of inventory to satisfy the customers before considering backorders [42]. In particular, (37) and (38) convert the inventory holding and backorders into two binary variables, and then (39) can simply restrict the concurrence. It should be noted that, according to the operations strategy, the end items shipped to the warehouse during the last period of the planning horizon can be only used within the same period; however, backorders may be fulfilled during the next planning horizon. In other words, the concurrence would not exist during period $t = T$ and can be neglected.

(7) *Decision Variable Constraints.* Consider

$$X_{ljt} \in \{0, 1\} \quad \forall l \in L(j), \forall j, \forall t, \quad (40)$$

$$E_{it}, G_{it} \in \{0, 1\} \quad \forall i \in P, \forall t, \quad (41)$$

$$IB_{it}, IP_{it}, IP_{it}^-, IW_{it}, LM_{ljt}, MBA_{i,t,t'}, ML_{it}, MB_{ist}, \quad (42)$$

$$MP_{it}, MX_{ijt}, MXF_{ljt}, TO_{jt}, QB_{i,t'',t'}, QP_{i,t'',t'},$$

$$QW_{i,t'',t'}, UM_{ljt} \geq 0, \quad \forall i, j, l, s, t.$$

Equations (40) and (41) indicate the integral constraints. Other decision variables default to nonnegativity as shown in (42).

2.3. *Model Comparisons.* The proposed FMO-MILP model for the multiproduct multistage production planning problem is novel which considers meaningful technical features. The following Table 1 shows the qualitative comparisons of the proposed approach with some representative techniques for the multiproduct multistage production planning, including Torabi and Hassini [23], Peidro et al. [43], and Jamalnia and Soukhakian [44]. Several significant findings and advantages of the proposed approach can be summarized as follows. First, the proposed model involves several practical techniques to formulate the meaningful characteristics in the multiproduct multistage production planning problem, such as constructing the production route using set theory, allowing backorders for multiple periods, and holding the safety stock level. These technical factors provide more flexibility in dealing with the variants on production and market demand. Second, the proposed model defines two significant indicators ASHL and ASL as maximized objectives, together with the operations cost, which are treated to be fuzzy. The construction of ASHL and ASL presents the requirements from both manufactures and customers directly, instead of quantifying them by cost. Third, the multiple objectives have different priorities described by linguistic terms, which can fully express the preference from decision-makers. The two-stage interactive solution approach provides a flexible tool to deal with the objective priority where the preference extent can be quantified. Finally, the proposed model simultaneously considers the epistemic and external uncertainty which affects such a multiproduct multistage production environment, reflecting the uncertainty from production-inventory system, suppliers, and customers. Different strategies by fuzzy

TABLE 1: Comparison of different models for the multiproduct multistage production planning.

Factor	Torabi and Hassini	Peidro et al.	Jamalnia and Soukhakian	The proposed approach
Objective function	Multiple	Single	Multiple	Multiple
Objective for cost	Min	Min	Min	Min
Additional objectives	Max: total value for purchasing	Not considered	Max: customer satisfaction	Max: ASHL and ASL
Objective priority	Not considered	Not considered	Considered	Considered
Preference extent	Not considered	Not considered	Not considered	Considered
Product family	Not considered	Considered	Considered	Considered
Epistemic uncertainty	Considered	Considered	Considered	Considered
External uncertainty	Considered	Considered	Not considered	Considered
Safety stock	Considered	Not considered	Not considered	Considered
Backorder	Not considered	Allowed for single period	Allowed for single period	Allowed for multiperiod
Production route	Not considered	Not considered	Not considered	Considered
Overtime	Not considered	Not considered	Considered	Considered

set theory and possibility theory are jointly developed to deal with the fuzzy constraints.

3. Solution Strategy

In the proposed problem, the uncertainty is expressed as fuzzy numbers to construct the FMO-MILP model. In this section, the original FMO-MILP model can be converted into an equivalent crisp multiobjective mixed integer linear model by treating the constraints in different forms independently. Then we apply a two-phase fuzzy multiobjective optimization approach to handling the objectives taking into account linguistic preference.

3.1. Handling the Uncertain Sources. In the FMO-MILP model, the fuzzy numbers represent the uncertainty on either one side or both sides of the constraints. The constraints with fuzzy numbers on one side hold a strict equality since the converted crisp constraints by defuzzing the uncertainty are still rigid. By contrast, those constraints with fuzzy numbers on both sides may be rather flexible due to the fact that the fuzzy numbers come from different sources.

In the literature, fuzzy numbers can be modelled as various membership functions in terms of application scenarios, such as triangular, trapezoidal, monotonic linear, piecewise linear, and S-shape [45]. Here, we adopt triangular membership function to construct the fuzzy numbers modelled in the constraints. As stated by Liang [46] and Li et al. [47], the triangular fuzzy number is commonly adopted due to its ease in simplistic data acquisition and flexibility of fuzzy arithmetic operations. Decision-makers are easy to estimate the maximum and minimum deviations from the most likely value. Generally, a triangular fuzzy number can be characterized based on the following three prominent values [45]:

- (i) The most likely value that defines the most belonging member to the set of available values (membership degree = 1 if normalized).
- (ii) The most pessimistic value that defines the least belonging member to the set of available values on the minimum accepted value.

- (iii) The most optimistic value that defines the least belonging member to the set of available values on the maximum accepted value.

Considering the above definitions on prominent values, the membership functions for the triangular fuzzy numbers in the k th fuzzy constraint can be represented as follows:

$$\mu_{c_k}(x) = \begin{cases} \frac{c_k(x) - c_k^l}{c_k^m - c_k^l} & \text{if } c_k^l \leq c_k(x) < c_k^m \\ \frac{c_k^u - c_k(x)}{c_k^u - c_k^m} & \text{if } c_k^m \leq c_k(x) < c_k^u \\ 0 & \text{otherwise,} \end{cases} \quad (43)$$

where c_k^m , c_k^l , and c_k^u refer to the most likely, most pessimistic, and most optimistic value for the k th fuzzy constraint, respectively, and μ_{c_k} means the membership function for fuzzy number c_k . If inducing the minimal acceptable degree $\alpha \in [0, 1]$, the prominent values can be reformulated as follows:

$$\begin{aligned} c_{k,\alpha}^l &= c_k^l + \alpha(c_k^m - c_k^l) \quad \forall k, \\ c_{k,\alpha}^m &= c_k^m \quad \forall k, \\ c_{k,\alpha}^u &= c_k^m + (1 - \alpha)(c_k^u - c_k^m) \quad \forall k, \end{aligned} \quad (44)$$

where $c_{k,\alpha}^m$, $c_{k,\alpha}^l$, and $c_{k,\alpha}^u$ denote the most likely, most pessimistic, and most optimistic value with the minimal acceptable degree, respectively. Then the membership functions can be reconstructed with respect to these three values.

Recalling the original model, (14), (15), (24), (27), (28), (29), (30), and (31) are those constraints with fuzzy numbers on one side. Tanaka and Asai [48] handled the triangular fuzzy number by using the weighted average of the most pessimistic value and most optimistic value. Alternatively, the weighted average method is utilized to treat the fuzzy numbers and construct crisp constraints [25, 49]. Then

the above constraints can be converted into the following equations:

$$\begin{aligned}
\sum_{i \in B} MB_{ist} &\leq w_1^l \cdot URM B_{st,\alpha}^l + w_1^m \cdot URM B_{st,\alpha}^m + w_1^u \cdot URM B_{st,\alpha}^u \quad \forall s, t, \\
MB_{ist} &\leq w_2^l \cdot UMB_{ist,\alpha}^l + w_2^m \cdot UMB_{ist,\alpha}^m + w_2^u \cdot UMB_{ist,\alpha}^u \quad \forall i, s, t, \\
\sum_{i \in B} IB_{it} &\leq w_3^l \cdot UIB_{t,\alpha}^l + w_3^m \cdot UIB_{t,\alpha}^m + w_3^u \cdot UIB_{t,\alpha}^u \quad \forall t, \\
\sum_{i \in W} IW_{it} &\leq w_4^l \cdot UIW_{t,\alpha}^l + w_4^m \cdot UIW_{t,\alpha}^m + w_4^u \cdot UIW_{t,\alpha}^u \quad \forall t, \\
IP_{it}^- &\geq w_5^l \cdot SIP_{it,\alpha}^l + w_5^m \cdot SIP_{it,\alpha}^m + w_5^u \cdot SIP_{it,\alpha}^u \\
&\quad + \sum_{t''=1}^t \sum_{t'>t} QP_{i,t'',t'} \cdot v_i - IP_{it} \quad \forall i \in P, \forall t = 1, \dots, T-1, \\
IP_{it}^- &\geq w_5^l \cdot SIP_{it,\alpha}^l + w_5^m \cdot SIP_{it,\alpha}^m + w_5^u \cdot SIP_{it,\alpha}^u \\
&\quad - IP_{it} \quad \forall i \in P, t = T, \\
IP_{it}^- &\leq w_5^l \cdot SIP_{it,\alpha}^l + w_5^m \cdot SIP_{it,\alpha}^m + w_5^u \cdot SIP_{it,\alpha}^u \quad \forall t, \\
\sum_{i \in P} IP_{it} &\leq w_6^l \cdot UIP_{t,\alpha}^l + w_6^m \cdot UIP_{t,\alpha}^m + w_6^u \cdot UIP_{t,\alpha}^u \quad \forall t,
\end{aligned} \tag{45}$$

where w_k^m , w_k^l , and w_k^u represent the corresponding weights of the most likely, most pessimistic, and most optimistic value for the c_k th fuzzy number, respectively. The most likely value with the highest membership should therefore be assigned to more weights than the others. When the weights for the above three prominent values are given, the solution can be further compromised by adjusting the minimal acceptable degree α .

On the other hand, (19), (20), (35), and (36) represent the fuzzy numbers on both sides of the constraints. In this context, we treat the constraints based on comparison of their corresponding fuzzy numbers, which is referred to as fuzzy ranking. In the literature, several fuzzy ranking methods for comparing fuzzy numbers are given in terms of various classification schemes [50, 51]. We consider the concept of set-inclusion along with the fuzzy ranking method proposed by Ramík and ímánek [52]. Then (19), (20), (35), and (36) can be converted into crisp constraints by ordering the left-hand

and right-hand side fuzzy numbers. The equivalent auxiliary constraints are expressed as

$$\begin{aligned}
\sum_{t'>t}^T QP_{i,t,t'} + SIP_{it,\alpha}^l - IP_{it}^- - \sum_{t'>t}^T MBA_{i,t,t'} \\
&= \sum_{t''=1}^{t-1} QP_{i,t'',t} + SIP_{it-1,\alpha}^l - IP_{it-1}^- - \sum_{t''=1}^{t-1} MBA_{i,t'',t} \\
&\quad + MP_{i,t} - MD_{it,\alpha}^u \quad \forall i \in P, \forall t = 1, \dots, T-1, \\
\sum_{t'>t}^T QP_{i,t,t'} + SIP_{it,\alpha}^m - IP_{it}^- - \sum_{t'>t}^T MBA_{i,t,t'} \\
&= \sum_{t''=1}^{t-1} QP_{i,t'',t} + SIP_{it-1,\alpha}^m - IP_{it-1}^- - \sum_{t''=1}^{t-1} MBA_{i,t'',t} \\
&\quad + MP_{i,t} - MD_{it,\alpha}^m \quad \forall i \in P, \forall t = 1, \dots, T-1, \\
\sum_{t'>t}^T QP_{i,t,t'} + SIP_{it,\alpha}^u - IP_{it}^- - \sum_{t'>t}^T MBA_{i,t,t'} \\
&= \sum_{t''=1}^{t-1} QP_{i,t'',t} + SIP_{it-1,\alpha}^u - IP_{it-1}^- - \sum_{t''=1}^{t-1} MBA_{i,t'',t} \\
&\quad + MP_{i,t} - MD_{it,\alpha}^l \quad \forall i \in P, \forall t = 1, \dots, T-1, \\
SIP_{it,\alpha}^l - IP_{it}^- - MBA_{i,t,t+1} \\
&= \sum_{t''=1}^{T-1} QP_{i,t'',t} + SIP_{i,t-1}^l - IP_{i,t-1}^- - t + MP_{it} \\
&\quad - MD_{it}^u \quad \forall i \in P, t = T, \\
SIP_{it,\alpha}^m - IP_{it}^- - MBA_{i,t,t+1} \\
&= \sum_{t''=1}^{T-1} QP_{i,t'',t} + SIP_{i,t-1}^m - IP_{i,t-1}^- - t + MP_{it} \\
&\quad - MD_{it}^m \quad \forall i \in P, t = T, \\
SIP_{it,\alpha}^u - IP_{it}^- - MBA_{i,t,t+1} \\
&= \sum_{t''=1}^{T-1} QP_{i,t'',t} + SIP_{i,t-1}^u - IP_{i,t-1}^- - t + MP_{it} \\
&\quad - MD_{it}^l \quad \forall i \in P, t = T, \\
\sum_{i \in M(j)} \left(\frac{MX_{ijt}}{\eta_{ij,\alpha}^u} \right) + \sum_{l \in L(j)} TS_{lj} \cdot X_{ljt} &\leq TA_{jt,\alpha}^l + TO_{jt} \\
&\quad \forall j, \forall t, \\
\sum_{i \in M(j)} \left(\frac{MX_{ijt}}{\eta_{ij,\alpha}^m} \right) + \sum_{l \in L(j)} TS_{lj} \cdot X_{ljt} &\leq TA_{jt,\alpha}^m + TO_{jt}
\end{aligned}$$

$$\begin{aligned}
 & \forall j, \forall t, \\
 & \sum_{i \in M(j)} \left(\frac{MX_{ijt}}{n_{ij,\alpha}^l} \right) + \sum_{l \in L(j)} TS_{lj} \cdot X_{ljt} \leq TA_{jt,\alpha}^u + TO_{jt} \\
 & TA_{jt,\alpha}^l + TO_{jt} \leq A_t - TM_{jt,\alpha}^l \quad \forall j, \forall t, \\
 & TA_{jt,\alpha}^m + TO_{jt} \leq A_t - TM_{jt,\alpha}^m \quad \forall j, \forall t, \\
 & TA_{jt,\alpha}^u + TO_{jt} \leq A_t - TM_{jt,\alpha}^u \quad \forall j, \forall t.
 \end{aligned} \tag{46}$$

3.2. Treatment of the Objectives with Linguistic Preference. The objective functions in the FMO-MILP model concern both the operations cost and the key indicators which are conflicting and noncommensurable. In such a fuzzy environment, decision-makers are even more difficult to obtain feasible solutions as desired among the multiple objectives. Membership approaches and fuzzy set theory are efficient tools to reformulate the objective functions as commensurable units. Conventionally, the relative importance of the objectives is either specified with a weight vector or/and given

satisfying degrees [23, 53] or compared by the corresponding satisfying degrees [44]. The above methods may lead to low computational efficiency, and additional constraints may be too strict to obtain satisfying even feasible solution. Thus, we apply the interactive satisfying method by Li and Hu [32] to overcome these shortcomings, where the satisfying degree is regarded as optimization variables and relaxed using varying-domain optimization method [54]. The trade-off among the conflicting objectives with linguistic preference can then be achieved by relaxing the overall satisfying degree through an interactive procedure. To do so, we implement a two-phase fuzzy optimization approach to solving the proposed problem with linguistic preference.

3.2.1. Constructing the Priority Levels and Objective Membership Functions. Decision-makers usually use linguistic terms to express the preference, such that objective A is more important than B. Considering the fact that the objective with higher priority has higher desirable achievement, the objectives involved can be grouped into different priority levels according to the preference. To quantify as priority levels, the FMO-MILP problem can be constructed as

$$\begin{aligned}
 & \min \text{ or } \max \quad \{P_1(z_1^l(x), \dots, z_{m_1}^l(x)), \dots, P_L(z_1^L(x), \dots, z_{m_L}^L(x))\} \\
 & \text{s.t. } \quad x \in G,
 \end{aligned} \tag{47}$$

where $\{P_n \mid 1 \leq n \leq L, n \in Z\}$ is a set for priority levels in sequence of relative importance; $\{z_n^n(x) \mid 1 \leq n \leq L, 1 \leq h_n \leq m_n, n \in Z, h_n \in Z\}$ represents a set of objectives in the n th priority level; and G refers to system constraints.

By means of fuzzy set theory, the objective functions concerned can be generally expressed in the form of $z_g(x) \lesssim z_g$ or $z_g(x) \gtrsim z_g$, which means the objective $z_g(x)$ is approximately less than or greater than the aspiration level z_g , respectively. To describe the conflict in the objectives, a monotonically decreasing linear membership function is used to normalize the fuzzy objectives in (1)–(4):

$$\mu_{z_g}(x) = \begin{cases} 1 & \text{if } z_g(x) < z_g^l \\ \frac{z_g^u - z_g(x)}{z_g^u - z_g^l} & \text{if } z_g^l \leq z_g(x) \leq z_g^u \\ 0 & \text{if } z_g(x) > z_g^u \end{cases} \tag{48}$$

$\forall g = 1, \dots, 4,$

where z_g^l for $\forall g = 1, \dots, 4$ denotes the highest possible value with the membership degree of 1 and z_g^u for $\forall g = 1, \dots, 4$ is the smallest possible value with the membership degree of 0. Accordingly, the fuzzy objectives modelled by

(9) are identified by fuzzy sets having an increasing linear membership function:

$$\mu_{z_g}(x) = \begin{cases} 0 & \text{if } z_g(x) < z_g^l \\ \frac{z_g(x) - z_g^l}{z_g^u - z_g^l} & \text{if } z_g^l \leq z_g(x) \leq z_g^u \\ 1 & \text{if } z_g(x) > z_g^u \end{cases} \tag{49}$$

$\forall g = 5, 6,$

where z_g^l for $\forall g = 5, 6$ denotes the highest possible value with the membership degree of 0 and z_g^u for $\forall g = 5, 6$ is the smallest possible value with the membership degree of 1.

3.2.2. Phase I Model. Phase I is to obtain the maximum overall satisfying degree for all the objectives without linguistic preference. Introduced by Bellman and Zadeh [16], the min/max operator can aggregate the objective membership functions using a new variable $\lambda \in [0, 1]$. According to the treatment on the fuzzy constraints and the objective membership functions aforementioned, the auxiliary formulation for the FMO-MILP model regardless of linguistic preference can be given by

$$\begin{aligned}
& \max \quad \lambda \\
& \text{s.t.} \quad \text{eqs. (10)–(12), (15)–(17), (20)–(22), (24), (25), (31)–(33), (36)–(41), (44)–(45)} \\
& \quad z_g \leq z_g^u - \lambda (z_g^u - z_g^l) \quad \forall g = 1, \dots, 4 \\
& \quad z_g \geq z_g^l + \lambda (z_g^u - z_g^l) \quad \forall g = 5, 6 \\
& \quad 0 \leq \lambda \leq 1.
\end{aligned} \tag{50}$$

However, the optimal solution solved by the logical min operator in model (50) provides no preference judged by the decision-makers. When considering the relative importance between the objectives, the subjective constraints induced may lead to a decline in the overall satisfying degree.

3.2.3. Phase II Model. In phase II, the maximum overall satisfying degree λ^* yielded in phase I will be relaxed to certain extent by the varying-domain method, so that the linguistic preference could be involved. By inducing a relaxation parameter $\Delta\delta \in [0, 1]$, the overall satisfying degree can be relaxed as shown by the following for the objectives $\{z_1, \dots, z_6\}$:

$$\lambda^* - \Delta\delta \leq \mu_{z_g} \quad \forall g = 1, \dots, 6. \tag{51}$$

The extent of relaxation corresponds to the priority imposed by μ_{z_g} . To compare the relative importance between

the objectives in terms of preference, the varying-domain method is applied by using a relaxation parameter $\gamma \in [-1, 0]$ to extend differences among the objective priorities. Concerning the priority levels defined in (47), the satisfying degree comparison can be stated by

$$\mu_{z_{h_{n-1}}} - \mu_{z_{h_n}} \leq \gamma \quad \forall n \geq 2, h_n \in \{1, \dots, m_n\}, \tag{52}$$

where $\mu_{z_{h_n}^n}$ means the satisfying degrees for the objectives in the n th priority level. Equation (52) ensures that the objectives located in the higher priority level will hold higher satisfying degrees. In this regard, phase II also uses min operator to quantify the preference extent. Then the FMO-MILP model can be further formulated by incorporating the priority levels and represented as follows:

$$\begin{aligned}
& \min \quad \gamma \\
& \text{s.t.} \quad \text{eqs. (10)–(12), (15)–(17), (20)–(22), (24), (25), (31)–(33), (36)–(41), (44)–(45)} \\
& \quad \text{membership functions for } z_g, \quad \forall g = 1, \dots, 6 \\
& \quad \lambda^* - \Delta\delta \leq \mu_{z_g} \quad \forall g = 1, \dots, 6 \\
& \quad \mu_{z_{h_n}^n} - \mu_{z_{h_{n+1}}^{n+1}} \leq \gamma \quad \forall 1 \leq n \leq L, h_n \in \{1, \dots, m_n\}, z_{h_n}^n \in \{z_1, \dots, z_6\} \\
& \quad 0 \leq \Delta\delta \leq 1 \\
& \quad -1 \leq \gamma \leq 0.
\end{aligned} \tag{53}$$

In the above model (53), γ is considered to quantify the extent of the relative importance defined by the priority levels, which is relatively dependent on relaxation parameter $\Delta\delta$. When the relaxation on λ^* is not enough, the preference expressed by the objective priorities may not be achieved, and in this case $\gamma = 0$. Therefore, relaxation $\Delta\delta$ should be altered incrementally until the preference extent is achieved. This criterion relies on decision-maker's judgement, since the more the preference extent, the lower the overall satisfying degree.

3.3. Interactive Solution Procedure. The proposed approach concerns treating both the fuzzy constraints in different forms and incorporating the linguistic preference in the objectives. Decision-makers can take fully advantage of the flexible parameters to achieve a preferred trade-off between the overall satisfying degree and objective priorities. In summary, the original fuzzy model with linguistic preference is solved through a novel interactive method which incorporates decision-makers' judgement.

Step 1. Model the fuzzy numbers in the constraints using triangular possibility distributions by specifying the three prominent values.

Step 2. Given the value of minimal acceptable degree α , convert the fuzzy constraints (14), (15), (24), (27), (28), (29), (30), and (31) into crisp ones using the weighted average method as shown in (45) and simultaneously, convert the fuzzy constraints (19), (20), (35), and (36) into crisp ones using the fuzzy ranking method.

Step 3. Model the fuzzy objectives $\{z_1, z_2, z_3, z_4\}$ and $\{z_5, z_6\}$ using the monotonically decreasing linear membership function in (48) and the monotonically increasing linear membership function in (49), respectively.

Step 4. Regardless of linguistic preference on the objectives, aggregate the fuzzy model using a min operator $\lambda \in [0, 1]$ into a crisp phase I model. Then the corresponding overall satisfying degree λ^* can be obtained by solving the model as shown in model (50).

Step 5. Construct the priority levels for the objective functions $\{z_1, \dots, z_6\}$ in terms of linguistic preference.

Step 6. Relax the overall satisfying degree by (51) and formulate the priority comparison by (52).

Step 7. Assume that the relaxation parameter $\Delta\delta = 0$ and aggregate the FMO-MILP model and objective priorities into a crisp phase II model.

Step 8. Solve the crisp model (53), and the minimal value of preference extent γ can be obtained.

Step 9. Check whether the preference extent obtained in Step 7 satisfies the relative importance by the decision-makers. If $\lambda = 0$ which means all the objectives are with the same importance and dissatisfy the linguistic preference, then go to Step 10. If $\lambda < 0$ but the preference extent is not satisfied by the decision-makers, then go to Step 10 as well. Otherwise, stop the interactive procedure and the result obtained provides a compromise solution.

Step 10. Relax the overall satisfying degree λ^* obtained in Step 4 by augmenting the value of parameter $\Delta\delta$, and then go back to Step 8.

4. Case Study

4.1. Case Description. This study investigates a steel rolling plant as a real-world case to illustrate the applicability of the proposed fuzzy programming model and evaluate the performance of the solution approach. The plant manufactures small and midsize steel plates classified in 10 types. These products are generally processed from 3 types of steel slabs as the raw materials, going through different production routes. The production system consists of 6 major production stages, and the production route for each product is constructed by a collection of these production stages according to the process.

TABLE 2: Raw material and product matching matrix.

Raw material $i \in B$	Product $i \in P$									
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
B1	1	1	1	1	0	0	0	0	1	0
B2	0	0	0	0	1	1	0	0	0	1
B3	0	0	0	0	0	0	1	1	0	0

Table 2 presents the raw material and product matching matrix in which pair $(i \in B, i \in P) = 1$ indicates that the product $i \in P$ is manufactured by the raw material $i \in B$, and otherwise $(i \in B, i \in P) = 0$. Table 3 depicts the production stage and product matching matrix in which pair $(j, i \in P) = 1$ implies that the production of product $i \in P$ goes through stage j , and otherwise $(j, i \in P) = 0$. It should be noted that the product is processed in sequence of $j = 1, 2, \dots, 6$, constituting a production route. Further, we can describe the work-in-process for each product upon its specific production route. In addition, the output items processed by production stage j are indicated in terms of each product. Then in Table 3, for example, we can observe that product P10 is manufactured through a production route J1-J2-J4-J5-J6 with corresponding output work-in-process $\{J1D, J2D, J4A, J5F, J6B\}$ by each production stage. Concerning the production families modelled in (32)-(33), Table 4 gives the family structure with regard to each production stage and its output items.

The case addresses a planning horizon of 7 periods. The uncertainty in the constraints modelled by the triangular fuzzy numbers is defined according to the most likely values with their deviations. Table 5 gives the relevant fuzzy parameters within a certain range of intervals in which the three prominent values by (43) are generated. The operational strategy implies that the production planning is made in a make-to-order and make-to-stock environment. It means that the products are manufactured based on the orders received and in the meantime to keep the safety stock. The dynamic demand is forecasted for each period prior to the planning horizon and is given in Table 6.

4.2. Model Implementation. Recalling the interactive solution procedures aforementioned, the triangular fuzzy numbers in the constraints can be easily converted into equivalent crisp ones using the given prominent values by the weighted average method and fuzzy ranking method. In this regard, the model concerns two classes of important parameters: minimal acceptable degree α and weights for the three prominent values (w_k^l, w_k^m, w_k^u) . Considering that the decision-makers usually assign more weights to the most likely values, this case specifies $(w_k^l, w_k^m, w_k^u) = (0.167, 0.666, 0.167)$ and $\alpha = 0.6$.

Membership functions for all the fuzzy objectives involve two prominent values that are subjectively evaluated. Since the objectives $\{z_1, z_2, z_3, z_4\}$ related to cost that is difficult to be quantified directly, this work simply uses the result of additive method for treating $\{z_1, z_2, z_3, z_4\}$ without considering $\{z_5, z_6\}$ as a benchmark to construct the membership functions. On the other hand, the objectives $\{z_5, z_6\}$ concerning

TABLE 3: Production stage and product matching matrix.

Production stage j	Product $i \in P$									
	$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$	$P9$	$P10$
$J1$	1, $J1A$	1, $J1A$	1, $J1B$	1, $J1B$	1, $J1C$	1, $J1D$	1, $J1E$	1, $J1E$	1, $J1A$	1, $J1D$
$J2$	1, $J2A$	1, $J2A$	1, $J2B$	1, $J2B$	1, $J2C$	1, $J2D$	1, $J2E$	1, $J2E$	1, $J2A$	1, $J2D$
$J3$	1, $J3A$	1, $J3B$	1, $J3C$	1, $J3D$	1, $J3E$	0	0	0	1, $J3A$	0
$J4$	0	0	0	0	0	1, $J4A$	1, $J4B$	1, $J4C$	0	1, $J4A$
$J5$	1, $J5A$	1, $J5B$	1, $J5C$	1, $J5D$	1, $J5E$	1, $J5F$	1, $J5G$	1, $J5H$	1, $J5A$	1, $J5F$
$J6$	0	0	0	0	0	0	0	0	1, $J6A$	1, $J6B$

TABLE 4: Item family structure.

Family l	Items grouped $F(l)$
1	$J1A, J1B$
2	$J1C, J1D$
3	$J1E$
4	$J2A, J2B$
5	$J2C, J2D$
6	$J2E$
7	$J3A, J3B, J3C, J3D$
8	$J3E$
9	$J4A$
10	$J4B, J4C$
11	$J5A, J5B, J5C, J5D$
12	$J5E$
13	$J5F$
14	$J5G, J5H$
15	$J6A$
16	$J6B$

TABLE 5: Range of fuzzy parameter values.

Fuzzy parameters	Range of values	Fuzzy parameters	Range of values
$\tilde{\eta}_{ij}$	(27, 63)	$U\tilde{I}P_t$	(136, 170)
$U\tilde{M}B_{ist}$	(120, 230)	$U\tilde{I}W_t$	(85, 105)
$UR\tilde{M}B_{st}$	(300, 500)	$S\tilde{I}P_{it}$	(8, 11)
$U\tilde{I}B_t$	(85, 105)	$U\tilde{I}P_t$	(136, 170)

general operations indicators are easily formulated based on the historical statistics. Then the membership functions for the case study are constructed as follows:

$$(z_1^l, z_1^u) = (3430000, 4000000),$$

$$\mu_{z_1} = \begin{cases} 1 & \text{if } z_1 < 3430000 \\ \frac{4000000 - z_1}{570000} & \text{if } 3430000 \leq z_1 \leq 4000000 \\ 0 & \text{if } z_1 > 4000000, \end{cases}$$

$$(z_2^l, z_2^u) = (40000, 65000),$$

$$\mu_{z_2} = \begin{cases} 1 & \text{if } z_2 < 40000 \\ \frac{65000 - z_2}{25000} & \text{if } 40000 \leq z_2 \leq 65000 \\ 0 & \text{if } z_2 > 65000, \end{cases}$$

$$(z_3^l, z_3^u) = (4400000, 5000000),$$

$$\mu_{z_3} = \begin{cases} 1 & \text{if } z_3 < 4400000 \\ \frac{5000000 - z_3}{600000} & \text{if } 4400000 \leq z_3 \leq 5000000 \\ 0 & \text{if } z_3 > 5000000, \end{cases}$$

$$(z_4^l, z_4^u) = (150000, 200000),$$

$$\mu_{z_4} = \begin{cases} 1 & \text{if } z_4 < 150000 \\ \frac{200000 - z_4}{50000} & \text{if } 150000 \leq z_4 \leq 200000 \\ 0 & \text{if } z_4 > 200000, \end{cases}$$

$$(z_5^l, z_5^u) = (50, 80),$$

$$\mu_{z_5} = \begin{cases} 0 & \text{if } z_5 < 50 \\ \frac{z_5 - 50}{30} & \text{if } 50 \leq z_5 \leq 80 \\ 1 & \text{if } z_5 > 80, \end{cases}$$

$$(z_6^l, z_6^u) = (80, 95),$$

$$\mu_{z_6} = \begin{cases} 0 & \text{if } z_6 < 80 \\ \frac{z_6 - 80}{15} & \text{if } 80 \leq z_6 \leq 95 \\ 1 & \text{if } z_6 > 95. \end{cases}$$

(54)

Followed by the solution procedures, the crisp phase I model is implemented with the above membership functions. The model is solved using LINGO 11.0 optimization software on an Inter 2.5 GHz CPU. The maximum overall satisfying degree λ^* can be obtained and is equal to 0.916. Then, in phase II, preference on the objectives is incorporated, and the decision-makers in the case use linguistic terms to categorize the objectives into four levels: very important, somewhat

TABLE 6: Demand forecast for each product.

Product $i \in P$	Demand forecast ($MD_{it}^l, MD_{it}^m, MD_{it}^u$)						
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$
$P1$	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(120, 130, 135)	(120, 130, 135)	(120, 130, 135)	(120, 130, 135)
$P2$	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(0, 0, 0)	(0, 0, 0)
$P3$	(80, 90, 95)	(80, 90, 95)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(80, 90, 95)	(80, 90, 95)
$P4$	(110, 120, 125)	(110, 120, 125)	(110, 120, 125)	(110, 120, 125)	(110, 120, 125)	(110, 120, 125)	(110, 120, 125)
$P5$	(90, 100, 105)	(90, 100, 105)	(90, 100, 105)	(90, 100, 105)	(0, 0, 0)	(0, 0, 0)	(90, 100, 105)
$P6$	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
$P7$	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)	(70, 80, 85)
$P8$	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)
$P9$	(80, 90, 95)	(80, 90, 95)	(80, 90, 95)	(80, 90, 95)	(80, 90, 95)	(0, 0, 0)	(0, 0, 0)
$P10$	(0, 0, 0)	(0, 0, 0)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)	(100, 110, 115)

important, important, and general. And the priority levels are constructed as follows:

Level 1: z_6 ,

Level 2: z_5 ,

Level 3: z_1, z_3, z_4 ,

Level 4: z_2 .

In the context of the above priority structure and membership functions, phase II model is then implemented as follows:

$$\begin{aligned}
 & \min \quad \gamma \\
 & \text{s.t.} \quad \text{eqs. (10)–(12), (15)–(17), (20)–(22), (24), (25), (31)–(33), (36)–(41), (44)–(45)} \\
 & \quad \text{membership functions for } \mu_{z_g} \quad \forall g = 1, \dots, 6 \\
 & \quad \mu_{z_g} \geq 0.916 - \Delta\delta \quad \forall g = 1, \dots, 6 \\
 & \quad \mu_{z_5} - \mu_{z_6} \leq \gamma \\
 & \quad \mu_{z_1} - \mu_{z_5} \leq \gamma \\
 & \quad \mu_{z_3} - \mu_{z_5} \leq \gamma \\
 & \quad \mu_{z_4} - \mu_{z_5} \leq \gamma \\
 & \quad \mu_{z_2} - \mu_{z_1} \leq \gamma \\
 & \quad \mu_{z_2} - \mu_{z_3} \leq \gamma \\
 & \quad \mu_{z_2} - \mu_{z_4} \leq \gamma \\
 & \quad 0 \leq \Delta\delta \leq 1 \\
 & \quad -1 \leq \gamma \leq 0.
 \end{aligned} \tag{55}$$

4.3. Results and Performance Analysis. It is obvious that phase II model is MILP problem and is subject to the value of relaxation parameter $\Delta\delta$. Initially we solve phase II model assuming $\Delta\delta = 0$ by LINGO 11.0, and get $\gamma = 0$. The solution is equal to that obtained by phase I model, which conforms to the consistency with respect to phase I and II models. In this case, all the objectives are considered coequally without any priorities.

To seek a compromised solution and investigate the effects of relaxation, phase II model is conducted interactively by increasing $\Delta\delta$ linearly. Since $\Delta\delta$ is used to relax the overall satisfying degree so that the objective priorities could be incorporated, a relative large value is not desirable. The numerical experiment tests phase II model with a maximum value of $\Delta\delta = 0.30$. Then, a list of feasible solutions regarding the priority structure can be identified, as shown in Table 7.

TABLE 7: Optimal results of phase II model under different relaxation levels.

$\Delta\delta$	γ	Individual satisfying degree	Individual objectives
0.03	-0.0168	(0.9028, 0.8860, 0.9028, 0.9028, 0.9197, 0.9365)	(3485388, 42850, 4458303, 154859, 77.59, 94.05)
0.06	-0.0336	(0.8896, 0.8560, 0.8896, 0.8896, 0.9231, 0.9567)	(3492949, 43600, 4466262, 155522, 77.69, 94.35)
0.09	-0.0502	(0.8762, 0.8260, 0.8762, 0.8762, 0.9264, 0.9767)	(3500552, 44350, 4474265, 156189, 77.79, 94.65)
0.12	-0.0668	(0.8628, 0.7960, 0.8628, 0.8628, 0.9295, 0.9963)	(3508227, 45100, 4482344, 156862, 77.89, 94.94)
0.15	-0.0780	(0.8440, 0.7660, 0.8440, 0.8440, 0.9220, 1.0000)	(3518918, 45850, 4493598, 157800, 77.66, 95.00)
0.18	-0.0880	(0.8240, 0.7360, 0.8240, 0.8240, 0.9120, 1.0000)	(3525426, 46600, 4505598, 158800, 77.36, 95.00)
0.21	-0.0980	(0.8040, 0.7060, 0.8040, 0.8040, 0.9020, 1.0000)	(3541718, 47350, 4517598, 159800, 77.06, 95.00)
0.24	-0.1080	(0.7840, 0.6760, 0.7840, 0.7840, 0.8920, 1.0000)	(3553118, 48100, 4529598, 160800, 76.76, 95.00)
0.27	-0.1180	(0.7640, 0.6460, 0.7640, 0.7640, 0.8820, 1.0000)	(3564518, 48850, 4541598, 161800, 76.46, 95.00)
0.30	-0.1280	(0.7440, 0.6160, 0.7440, 0.7440, 0.8720, 1.0000)	(3575918, 49600, 4553598, 162800, 76.16, 95.00)

It is clear that, in each test, the satisfying degrees obtained accord with priorities among the objectives. As the parameter $\Delta\delta$ increases, several significant implications with regard to the proposed approach can be explored. First, the extent variable γ decreases monotonously, but with a diminished rate. A relaxation on the overall satisfying degree would enlarge the preference extent between the objective membership degrees; however the effect decreases when intensifying the relaxation. Second, the satisfying degrees for $\{z_1, z_2, z_3, z_4\}$ decrease and those for $\{z_6\}$ increase. The satisfying degrees for $\{z_5\}$ show a trend of ascending followed by leveling off from the value of $\Delta\delta = 0.12$ approximately, on which the satisfying degree for $\{z_6\}$ is maximized. Third, the minimum satisfying degree among the objectives is equal to the overall satisfying degree λ^{**} in phase II. For example, when $\Delta\delta = 0.09$, $\lambda^{**} = \lambda^* - \Delta\delta = 0.826$ and $\min_g(\mu_{z_g}) = \mu_{z_2} = 0.826$. It indicates the consistency between the overall satisfying degree and the relaxation degree. Fourth, the objectives follow the same trend as the satisfying degrees and are between the two prominent values with the membership degree of 0 and 1. Taking a comprehensive view of the above results, the decision-makers can select a preferred solution considering the preference extent and individual satisfying degree.

In order to analyze the effectiveness of proposed approach in terms of handling linguistic preference for each objective, we define three performance measure indicators: (1) the range of satisfaction degree (RSD), (2) maximum preference extent between the objectives, and (3) minimum PE between the objectives. RSD is used to measure the maximum difference between the satisfying degrees for all the objectives and is defined as follows [23]:

$$\text{RSD} = \max_g(\mu_{z_g}) - \min_g(\mu_{z_g}) \quad \forall g \in G. \quad (56)$$

Here, preference extent refers to the difference between the satisfying degrees for the objectives of adjacent priority levels. With reference to the definition of the priority levels in (47), we define the maximum and minimum preference extents as follows:

$$\text{PE}^{\max} = \max_{g, g'} |\mu_{z_g} - \mu_{z_{g'}}| \quad (57)$$

$$\forall z_g \in P_n, z_{g'} \in P_{n+1}, g, g' \in G,$$

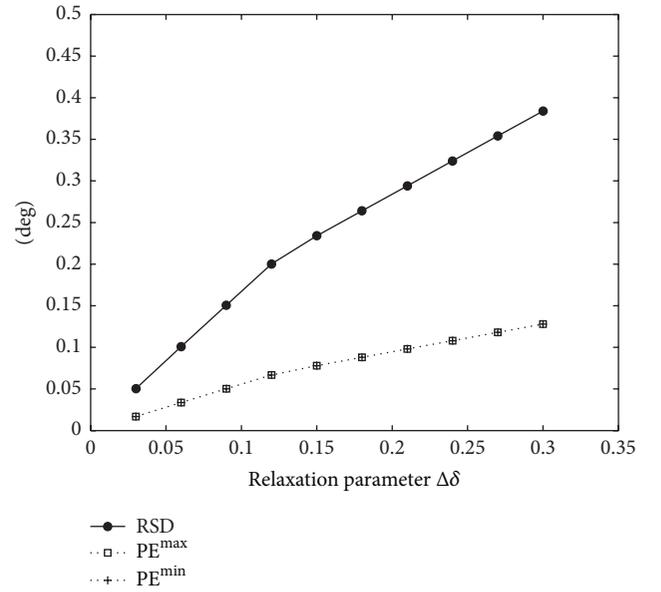


FIGURE 1: RSD versus preference extent.

$$\text{PE}^{\min} = \min_{g, g'} |\mu_{z_g} - \mu_{z_{g'}}| \quad (58)$$

$$\forall z_g \in P_n, z_{g'} \in P_{n+1}, g, g' \in G.$$

Since the minimum satisfying degree among the objectives equals the overall satisfying degree, the indicator RSD also implies the level of consistency between the preference extent and the satisfying degrees. For example, if the decision-makers consider a larger preference extent, the higher RSD will be desirable. As seen in Figure 1, the RSD and preference extents are compared with different $\Delta\delta$. Figure 1 presents that the maximum preference extent holds the same level as the minimum preference extent and is equal to the obtained value of the extent variable γ . It means the degree of difference between the objectives of the adjacent priority levels keeps the same. The RSD shows a trend of ascending with an increase in $\Delta\delta$. The larger the value of RSD is, the larger the difference of the satisfying degrees between the objectives of the adjacent priority levels is. Noticeably, the ascending rate for RSD slows down when $\Delta\delta$ goes around 0.15, on which the objective

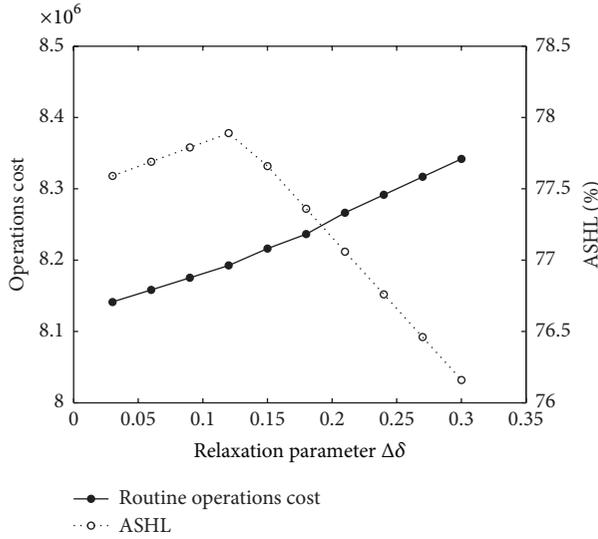


FIGURE 2: Operations cost versus ASHL.

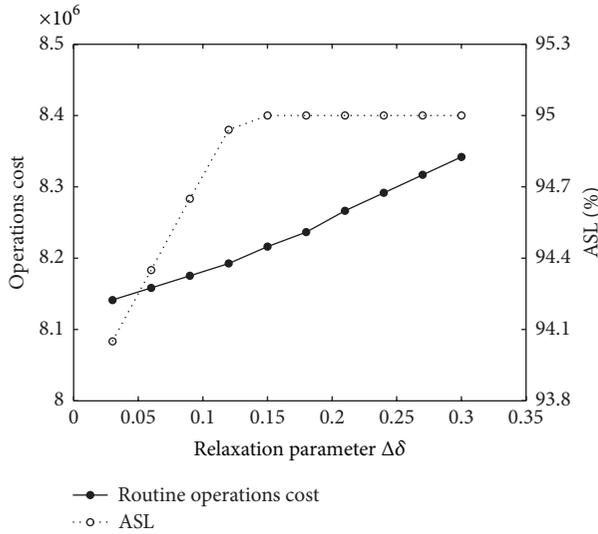


FIGURE 3: Operations cost versus ASL.

$\{z_6\}$ in the highest priority level has reached its maximum satisfying degree. So the increase of RSD when $\Delta\delta \geq 0.15$ is originated from the decrease of the overall satisfying degree.

Obviously, obtaining a decision vector that complies with the decision-makers is subject to the relaxation degree with a broad range of possible values, though the solutions are all feasible. To gain insights into the performance of the above results, we evaluate the objectives in a more efficient way as an auxiliary tool for determining the relaxation degree. As the objectives $\{z_1, z_2, z_3, z_4\}$ are measured in terms of cost, we define the operations cost which is the sum of all the cost for processing, setup, overtime, raw material, and inventory. Figures 2 and 3, respectively, present the operations cost with regard to the two key indicators ASHL(%) and ASL(%) under different relaxation degrees. A higher relaxation degree of the overall satisfying degree implies an increased operations

cost. Regarding the ASHL in Figure 2, decisions with $\Delta\delta \leq 0.12$ should be selected since the ASHL drops off when $\Delta\delta > 0.12$. As seen in Figure 3, the ASL rises dramatically with relaxation and reaches maximum value at approximate $\Delta\delta = 0.15$. Therefore, decisions with $\Delta\delta \leq 0.15$ are superior regarding the ASL. Taking into account the above two scenarios, an increase of relaxation degree when $\Delta\delta > 0.15$ will enhance the operations cost without improving the ALS but deteriorate the ASHL in the meanwhile. Rationally, the decision-makers should choose a compromised solution with $\Delta\delta \leq 0.15$.

4.4. Comparisons. To further illustrate the effectiveness of the proposed approach, we implement the addressed production planning problem using different approaches, by which four application scenarios are set as follows.

Scenario I. Implement the model by the proposed two-phase fuzzy optimization approach with $\Delta\delta = 0.13$.

Scenario II. Implement the model applying the additive method by Chen and Tsai [36] to deal with the objective priorities.

Scenario III. Remove the objective priorities from the model, and implement it with an aggregating “max” operator, as shown in phase I model.

Scenario IV. Regard the uncertain conditions as their deterministic versions and remove the objective priorities from the model.

As seen in Table 8, the results deriving from the four scenarios are compared, and several implications can be found. First, from the view of the individual satisfying degree, the results obtained in Scenario I can fully present the preference defined in the priority structure. And the Scenario I obtains the largest value of RSD, which implies a high level of preference difference among the objectives. By contrast, the objective priorities obtained by Scenario II are actually categorized into two levels in terms of the satisfying degrees, which cannot properly express the preference imposed by the decision-makers. The results of PE^{\max} and PE^{\min} also reflect the above implication. Since Scenarios III and IV ignore the preference on the objectives, the values of RSD are rather small. Second, the higher the level of the preference extent is, the smaller the overall satisfying degree is. In Scenario I, the results show the highest level of the preference extent but obtain the smallest overall satisfying degree. Therefore, Scenarios III and IV regardless of preference gain larger overall satisfying degrees. Third, there are trade-off relations between the objectives for the operations cost and the two indicators. Scenario I gains higher values in ASHL and ASL where greater preference is attached by the decision-makers and increases the operations cost as a compromise. In comparison, Scenarios II, III, and IV obtain lower values of the two indicators and the operations cost as well. It implies that the preference would affect the values of the objectives according to the individual priority structure imposed. Consequently, the proposed approach as applied in Scenario I

TABLE 8: Result comparison under different scenarios.

	Scenario I	Scenario II	Scenario III	Scenario IV
Operations cost	8199399	8132932	8124575	8119202
Processing	3511318	3496019	3477878	3479953
Overtime	45350	42896	42100	42190
Raw material	4485598	4440631	4450397	4442677
Inventory	157133	153386	154200	154382
ASHL	77.86	77.97	77.48	77.37
ASL	95.00	93.98	93.74	93.69
Individual satisfying degree	(0.8576, 0.7860, 0.8573, 0.8573, 0.9287, 1.0000)	(0.8842, 0.8842, 0.9323, 0.9323, 0.9323, 0.9323)	(0.9160, 0.9160, 0.9160, 0.9160, 0.9160, 0.9160)	(0.9124, 0.9124, 0.9289, 0.9124, 0.9124, 0.9124)
Overall satisfying degree	0.7860	0.8842	0.9169	0.9124
RSD	0.2140	0.0481	0	0.0165
PE ^{max}	0.0716	0.0481	—	—
PE ^{min}	0.0713	0	—	—

shows its efficiency in expressing the linguistic preference by the decision-makers and in the meantime provides a flexible tool to deal with trade-off among the conflicting objectives.

5. Conclusions

In industrial practice, production planning in an integrated framework for a multistage multiproduct manufacturing system should concern coordinated decisions in an uncertain environment. This work develops a novel fuzzy multiobjective mixed integer linear programming model to consider the uncertainty in various forms using fuzzy set theory and simultaneously handles multiple imprecise conflicting objectives with linguistic preference imposed by the decision-makers. The proposed fuzzy model aims to minimize the total cost of processing, setup, overtime, raw material, and inventory and in the meanwhile maximize the average safety stock holding level and the average service level. Further, a two-phase fuzzy optimization approach is used to obtain a compromised solution with respect to the linguistic preference for the objectives. The fuzzy model is then implemented on an industrial case and solved through a novel interactive solution procedure. The numerical experiment shows the effectiveness of making decisions on production planning in such an uncertain environment along with decision-makers' preference. The results also indicate that the proposed approach can quantify the preference extent between the objective priorities in a flexible way so that a compromised solution is more obtainable.

Finally, some suggestions or directions for further researches can be considered. One of the main concerns is to qualify or quantify the fuzzy objectives using linguistic terms, since it is not easy for the decision-makers to specify the prominent values for the fuzzy membership functions. In addition, this work assumes trapezoidal and triangular membership functions to, respectively, formulate the fuzzy numbers in the constraints and the fuzzy objectives. Future researches can express uncertain sources in different forms applying specific fuzzy membership functions, such as piecewise linear or S-shape membership functions. Moreover,

the lead time which existed is usually uncertain in nature. How to coordinate the production regarding the uncertain lead time between the adjacent production stages is practically important.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

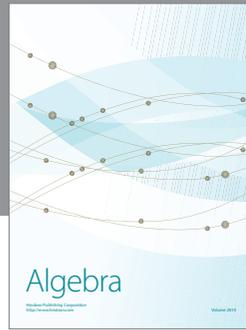
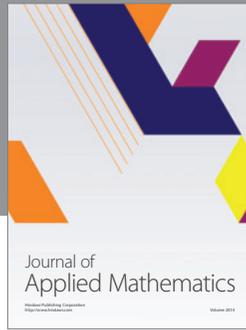
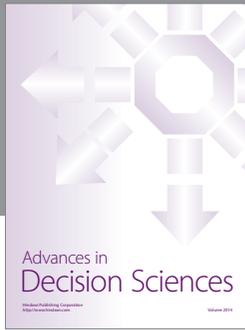
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