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Research Article

On the Propagation of Longitudinal Stress Waves in Solids and Fluids by Unifying the Navier-Lame and Navier-Stokes Equations

Ahmad Barzkar and Hojatollah Adibi

Department of Applied Mathematics, Faculty of Mathematics and Computer Sciences, Amirkabir University of Technology, 424 Hafez Avenue, P.O. Box 15875-4413, Tehran, Iran

Correspondence should be addressed to Hojatollah Adibi; adibih@aut.ac.ir

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Propagation of mechanical waves' phenomenon is the result of infinitely small displacements of integrated individual particles in the materials. These displacements are governed by Navier-Lame and Navier-Stokes equations in solids and fluids, respectively. In the present work, a generalized Kelvin-Voigt model of viscoelasticity has been proposed with the aim of bridging the gap between solids and fluids leading to a new concept of viscoelasticity which unifies the Navier-Lame and the Navier-Stokes equations. On solving this equation in one dimension, propagation of stress disturbance in the so-called "Kelvin-Voigt materials" will be studied. The model of these materials involves all the elastic and viscoelastic solids, as well as fluids and soft materials.

1. Introduction

The subject matter of mechanics is the study of motion, in how a physical object changes position with time and why [1]. In continuum mechanics we are concerned with the mechanical behavior and shape of materials under load. The physical reasons for this behavior can be quite different for different materials. Solid media will deform when forces are applied on them. These materials are called elastic, if the object will return to its initial shape and size when these forces are removed. Hence elasticity is the tendency of solid materials to return to their original shape after being deformed [2]. In continuum mechanics we consider the basic equations describing the physical effects created by external forces acting upon solids and fluids. In addition to the basic equations that are applicable to all continua, there are other equations, called constitutive equations which are constructed to take into account material characteristics. In the study of solids the constitutive equations for a linear elastic material are a set of relations between stress and strain. Hook's law represents the material behavior and relates the Cauchy stress tensor $T = (\sigma_{ij})$ and infinitesimal strain tensor $e = (e_{ij})$. The general form of Hook's law in components is

$$\sigma_{ij} = C_{ijlk} e_{lk}, \tag{1}$$

where C_{ijlk} is the fourth-order stiffness tensor [3]. If the body is isotropic and homogenous, then this law is simply written as [3]

$$T = \lambda\theta I + 2\mu e \tag{2}$$

in which θ is the trace of e, I is the unit tensor, and λ and μ are the Lame constants. Perfect elasticity is an approximation of the real world and few materials remain purely elastic even after very small deformations. When an elastic material is not stressed in tension or compression beyond its elastic limit, its individual particles perform elastic movement. The displacement of the particles mass center, denoted by vector field $u = (u_1, u_2, u_3)$, is related to the strain tensor by the relation [4]

$$e = \frac{1}{2}\left[\operatorname{grad} u + (\operatorname{grad} u)^T\right]. \tag{3}$$

Newton's second law of motion is based on the conservation of momentum and expressed as [5]

$$\operatorname{div} T = \rho \frac{\partial^2 u}{\partial t^2} \tag{4}$$

in which ρ is the mass density of the body. Now, in an isotropic, homogenous elastic solid, by combining (2), (3), and (4) we have

$$u_{tt} = \frac{\lambda + \mu}{\rho} \operatorname{grad} \operatorname{div} u + \frac{\mu}{\rho} \nabla^2 u \tag{5}$$

which is the vector form of the Navier-Lame equations and governs the infinitesimal movements of the body's integrated individual particles [5].

Viscous materials resist shear flow and strain linearly with time, when stress is applied [6]. If a material exhibits a linear response, it is categorized as a Newtonian material [6]. In this case the stress is linearly proportional to the strain rate. If the material exhibits a nonlinear response to the strain rate, it is categorized as non-Newtonian fluid. To get the equation that governs the small movement of integrated particles of fluids, let us first consider the continuity equation, governing the continuity of the integrated individual particles which are derived from the principle of conservation of mass and is given as

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho V) = 0, \tag{6}$$

where the vector field V represents the velocity of the particles and ρ is the mass density of the fluid [7]. In the absence of body forces, conservation of momentum, on which Newton's second law is based, is expressed as [7]

$$\operatorname{div} T = \rho \frac{\partial V}{\partial t}. \tag{7}$$

In the study of fluids, the constitutive equations consist of a set of relations between stress and rate of strain; these equations in tensor form are given as [8]

$$T = -pI + \lambda \operatorname{tr}(D) I + 2\mu D, \tag{8}$$

where the pressure p is induced by tension, that is, the difference between the dynamical and thermodynamical pressures, λ and μ are independent parameters characterizing viscosity, I is the unit tensor, and D is the rate of strain which is a symmetric tensor of order two.

Finally the kinematical relation of fluids describing the relation between tensor D and velocity vector field V is given as

$$D = \frac{1}{2}\left[\operatorname{grad} V + (\operatorname{grad} V)^T\right]. \tag{9}$$

Moreover, the pressure can be written as [8]

$$p = -\left(\lambda + \frac{2}{3}\mu\right) \operatorname{tr}(D). \tag{10}$$

It can be deduced from (7), (8), (9), and (10) that

$$\frac{\partial V}{\partial t} = \frac{\lambda + \mu}{\rho} \operatorname{grad}(\operatorname{div} V) + \frac{\mu}{\rho} \nabla^2 V. \tag{11}$$

However, in terms of displacement vector field u,

$$u_{tt} = \frac{\lambda + \mu}{\rho} \operatorname{grad} \operatorname{div} \dot{u} + \frac{\mu}{\rho} \nabla^2 \dot{u} \tag{12}$$

which is the set of Navier-Stokes equations.

In fluid continuum, the motion of substances is described by the Navier-Stokes equations. These equations are strictly the statement of conservation of momentum and are based on the assumption that the fluid, at the scale of interest, is a continuum. In other words, it is not made up of discrete particles but rather a continuous substance. Another necessary assumption is that all the fields of interest like pressure, velocity, density, and temperature are differentiable weakly at least [9].

There are materials for which a suddenly applied and maintained state of uniform stress induces an instantaneous deformation followed by a flow process which may or may not be limited in magnitude as time grows [10]. These materials exhibit both solid and fluid characteristics. Behavior of these materials clearly cannot be described by either elasticity or viscosity theories alone, as it combines features of each and is called viscoelastic. Viscoelasticity is a generalization of elasticity and viscosity [11].

All materials exhibit some viscoelastic response. In common metals such as steel or aluminum, as well as in quartz, at room temperature, and at small strain, the behavior does not deviate much from linear elasticity, which is the simplest response of a viscoelastic material. Synthetic polymers, wood, and human tissue as well as metals at high temperature display significant viscoelastic effects. Some phenomena in viscoelastic materials are as follows.

(i) If the stress is held constant, the strain increases with time (creep).

(ii) If the strain is held constant, the stress decreases with time (relaxation).

(iii) Acoustic waves experience attenuation.

The material creeps, that gives the prefix visco-, and the material fully recovers, which gives the suffix-elasticity [12]. The viscoelastic materials play an important role in many engineering structures. Those materials such as polymers are being used, for example, to dissipate and to insulate vibration caused by rotating or reciprocal movements [13]. They also have potential application in a new Hopkinson pressure bar testing apparatus [14–18]. Therefore, having the knowledge of the behavior of these materials, particularly related to their mechanical parameters, is essential. The theory which illustrates this behavior is the viscoelasticity theory. This theory is used in many fields, such as solid mechanics, seismology, exploration geophysics, acoustics, and engineering [19].

Modeling and model parameter estimation are of great importance for a correct prediction of the foundation behavior [19]. In many cases, elastic constitutive models work well

when time dependent effects can be neglected. However in those cases when time dependent effects cannot be neglected, we will need to utilize different constitutive models. Basically, time dependent effects indicate that the stress-strain behavior of material will change with time. The elastic material model for time dependent effects is viscoelasticity. Many researchers as Alfrey [20], Barberán and Herrera [21], Achenbach and Reddy [22], Bhattacharya and Sengupta [23], and Acharya et al. [24] formulated and developed this theory [19]. Further, Bert and Egle [25], Abd-Alla and Ahmed [26], and Batra [27] successfully applied this theory to wave propagation in homogenous, elastic media [19]. Murayama and Shibata [28] and Schiffman et al. [29] have proposed higher order viscoelastic models of five and seven parameters to represent the soil behavior. Jankowski et al. [30] discussed the linear viscoelastic model and the nonlinear viscoelastic model [19].

One of the most important classic models which is focused on, in this paper is Kelvin-Voigt model. The constitutive relation of this model is expressed as a linear first-order differential equation, which can be derived as below.

The most simple one-dimensional stress and strain tensors are of the form

$$T = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad e = \begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix} \tag{13}$$

which, according to the generalized Hook's law, are related by

$$\sigma = Ee_1, \tag{14}$$

where E is Young's modulus of elasticity [3]. In the Kelvin-Voigt model for viscoelastic materials, (14) is augmented to include viscosity, leading to the generalized equation

$$\sigma = Ee_1 + \eta \frac{\partial e_1}{\partial t}, \tag{15}$$

where η is the constant of viscosity [31, 32].

This model represents a solid undergoing reversible, viscoelastic strain and is extremely good with modeling creep in materials, but with regard to relaxation the model is much less accurate [33].

All material substances are comprised of particles. When a material is not stressed in tension or compression beyond its elastic limit, its individual particles may be forced into vibrational motion about their equilibrium positions. Thus, these particles perform elastic oscillations. Elastic waves are focused on particles that move in unison to produce a mechanical wave. In solids, elastic waves can propagate in four principle modes that are based on the way the particles oscillate. These waves can propagate as longitudinal, shear, and surface waves and in the thin materials as plate waves.

In longitudinal waves, the oscillations occur in the longitudinal direction or the direction of wave propagation. Since compressional and dilatational forces are active in these waves, they are also called pressure or compressional waves. They are also sometimes called density waves because their particle density fluctuates as they move. Compression waves can be generated in liquids, as well as solids, because the energy travels through the atomic structure by a series of compressions and expansion movements [34]. In the transverse or shear wave, the particles oscillate at a right angle or perpendicular to the direction of propagation. Shear waves require a solid material for effective propagation and therefore are not effectively propagated in materials such as liquids or gasses [34]. Waves in an isotropic elastic solid are governed by the vector Navier-Lame equations.

2. New Mathematical Model

2.1. Generalization of Kelvin-Voigt Model. In Kelvin-Voigt model for viscoelastic materials, the viscosity term is augmented to (14), leading to the generalized equation (15). But this model is a generalization of one-dimensional constitutive equation (14). In order to obtain a model for viscoelastic materials in most general form, the constitutive equation (2) should be extended. In this paper this equation is rewritten as

$$T = \lambda\theta I + 2\mu e + \lambda_f \dot{\theta} I \tag{16}$$

in which λ_f is called the first constant of viscosity. Also by symmetric of (16) this equation can be written as

$$T = \lambda\theta I + 2\mu e + \lambda_f \dot{\theta} I + 2\mu_f \dot{e} \tag{17}$$

in which μ_f is the second constant of viscosity. In the simple case (13), (16) is reduced as

$$\sigma = Ee_1 + \frac{\mu\lambda_f}{\lambda+\mu}\dot{e}_1 - \frac{\mu\alpha}{\lambda+\mu}\exp\left(-\frac{\lambda+\mu}{\lambda_f}t\right) \tag{18}$$

in which α is a positive constant. By comparing (18) and (15), it can be easily inferred that

$$\eta = \frac{\mu\lambda_f}{\lambda+\mu}. \tag{19}$$

In the next sections, it has been shown that (16) and (17) can justify the physics.

2.2. Generalization of Navier-Lame Equation. In the first section, the Navier-Lame equation was deduced by combining (2), (3), and (4). Now (2) is replaced by (16); therefore, by combining (16), (3), and (4), a generalized Navier-Lame equation is obtained as follows:

$$u_{tt} = \frac{\lambda+\mu}{\rho}\operatorname{grad}\operatorname{div} u + \frac{\mu}{\rho}\nabla^2 u + \frac{\lambda_f}{\rho}\operatorname{grad}\operatorname{div}\dot{u}, \tag{20}$$

where $\dot{u} = \partial u/\partial t$. Also if (2) is replaced by (17), then, by combining (17), (3), and (4), another generalization of Navier-Lame equation can be deduced as follows:

$$\begin{aligned} u_{tt} = {} & \left(\frac{\lambda+\mu}{\rho}\operatorname{grad}\operatorname{div} u + \frac{\mu}{\rho}\nabla^2 u\right) \\ & + \left(\frac{\lambda_f+\mu_f}{\rho}\operatorname{grad}\operatorname{div}\dot{u} + \frac{\mu_f}{\rho}\nabla^2\dot{u}\right), \end{aligned} \tag{21}$$

where λ_f and μ_f resemble the form of equations for λ and μ, respectively.

2.3. Unifying Navier-Lame and Navier-Stokes Equations. At this point, by comparing (21) and (12), it is apparent that (21) can be written in two parts. The first and second parts are concerned with the solid and fluid properties of the body, respectively. In fact, (21) governs the displacements of the particles in both solids and fluids and in bodies with both solid and fluid properties, but not in materials involving two or more different phases. Equation (21) is a unification of Navier-Lame and Navier-Stokes equations; therefore, (16) and (17) can be used as constitutive equation to all viscoelastic materials.

3. Longitudinal Waves in Materials

Equation (21) governs displacement of integrated individual particles of any continuum body. To study the propagation of longitudinal stress waves in a body, we note that (21) can be written in one dimension as follows:

$$u_{tt} = c^2 u_{xx} + d^2 u_{txx}, \quad \infty < x < +\infty, \ t \geq 0 \tag{22}$$

in which

$$c^2 = \frac{\lambda_s + 2\mu_s}{\rho}, \qquad d^2 = \frac{\lambda_f + 2\mu_f}{\rho}, \tag{23}$$

where the only initial condition is $u(0,0) = 0$. This equation can be solved by separation of variables method, so $u(x,t) = X(x)T(t)$. Using this in (22) leads to

$$\frac{X}{X''} = c^2 \frac{T}{T''} + d^2 \frac{T'}{T''} = \gamma \tag{24}$$

in which γ is constant. Now, the following five cases are considered.

Case 1 (γ is positive). In this case one can assume $\gamma = 1/k^2$, $k \neq 0$; hence, (24) becomes

$$\begin{cases} X'' - k^2 X = 0 \\ T'' - k^2 d^2 T' - k^2 c^2 T = 0 \end{cases} \xrightarrow{\text{yields}} \begin{cases} X(x) = c_1 e^{kx} + c_2 e^{-kx} \\ T(t) = d_1 e^{r_1 t} + d_2 e^{r_2 t} \end{cases} \tag{25}$$

in which

$$r_1 = \frac{1}{2}\left(k^2 d^2 + k\sqrt{k^2 d^4 + 4c^2}\right), \qquad r_2 = \frac{1}{2}\left(k^2 d^2 - k\sqrt{k^2 d^4 + 4c^2}\right). \tag{26}$$

The solution, therefore, can be written as

$$u(x,t) = \left(c_1 e^{kx} + c_2 e^{-kx}\right)\left(d_1 e^{r_1 t} + d_2 e^{r_2 t}\right). \tag{27}$$

By applying the initial condition, one arrives at

$$u(0,0) = (c_1 + c_2)(d_1 + d_2) = 0 \longrightarrow c_2 = -c_1 \quad \text{or} \quad d_2 = -d_1. \tag{28}$$

If $c_2 = -c_1$, then the velocity of the particle, placed at the origin, is zero, since

$$u_t(0,t) = (c_1 + c_2)\left(d_1 r_1 e^{r_1 t} + d_2 r_2 e^{r_2 t}\right). \tag{29}$$

Therefore, in this case, there is no motion in the medium.

On the other hand, since the medium is assumed to be elastic, the movement of the particle, located at the origin, stops after a while. Hence, if $d_2 = -d_1$, at time T, then (29) can be written as

$$u_t(0,T) = d_1(c_1 + c_2)\left(r_1 e^{r_1 T} - r_2 e^{r_2 T}\right) = 0. \tag{30}$$

Since r_1 is positive and r_2 is negative, the second part of (30) is not zero and so $c_2 = -c_1$. This means that there is no motion in the medium. Therefore, in this case, the equation does not have a nontrivial solution.

Case 2. If $\gamma = 0$, then the solution is trivial.

Case 3. If γ is negative and $k^2 d^4 - 4c^2$ is positive, then one can assume $\gamma = -1/k^2$, $k \neq 0$, so

$$\begin{cases} X'' + k^2 X = 0 \\ T'' + k^2 d^2 T' + k^2 c^2 T = 0 \end{cases} \xrightarrow{\text{yields}} \begin{cases} X(x) = c_1 \cos kx + c_2 \sin kx \\ T(t) = d_1 e^{r_1 t} + d_2 e^{r_2 t} \end{cases} \tag{31}$$

in which

$$r_1 = \frac{1}{2}\left(-k^2 d^2 + k\sqrt{k^2 d^4 - 4c^2}\right), \qquad r_2 = \frac{1}{2}\left(-k^2 d^2 - k\sqrt{k^2 d^4 - 4c^2}\right). \tag{32}$$

Thus the solution is as follows:

$$u(x,t) = (c_1 \cos kx + c_2 \sin kx)\left(d_1 e^{r_1 t} + d_2 e^{r_2 t}\right). \tag{33}$$

Also, the velocity is

$$u_t(x,t) = (c_1 \cos kx + c_2 \sin kx)\left(r_1 d_1 e^{r_1 t} + r_2 d_2 e^{r_2 t}\right). \tag{34}$$

Now, from the initial condition, one obtains

$$u(0,0) = c_1(d_1 + d_2) = 0 \xrightarrow{\text{yields}} c_1 = 0, \quad \text{or} \quad d_2 = -d_1. \tag{35}$$

If $c_1 = 0$, then $u_t(0,t) = 0$, which means that there is no motion and the solution is trivial. Otherwise, $d_2 = -d_1$, which yields

$$u_t(0,t) = c_1 d_1\left(r_1 e^{r_1 t} - r_2 e^{r_2 t}\right). \tag{36}$$

Since after a time T the particle that is at the origin stops, then

$$r_1 e^{r_1 T} - r_2 e^{r_2 T} = 0 \xrightarrow{\text{yields}} T = \frac{1}{r_1 - r_2} \ln\left(\frac{r_2}{r_1}\right). \tag{37}$$

Consequently, the solution can be written as

$$u(x,t) = d_1(c_1 \cos kx + c_2 \sin kx)\left(e^{r_1 t} - e^{r_2 t}\right). \tag{38}$$

On the other hand, it can be written as follows:

$$u(x,t) = (\alpha_1 \cos kx + \alpha_2 \sin kx)\left(e^{r_1 t} - e^{r_2 t}\right). \tag{39}$$

Case 4 (γ is negative and $k^2d^4 - 4c^2 = 0$). In this case the solution is trivial.

Case 5 (γ is negative and $k^2d^4 - 4c^2 < 0$). In this case one can assume $\gamma = -1/k^2$, $k \neq 0$, so that

$$\begin{gathered}\begin{cases} X'' + k^2X = 0 \\ T'' + k^2d^2T' + k^2c^2T = 0 \end{cases} \\ \xrightarrow{\text{yields}} \begin{cases} X(x) = c_1 \cos kx + c_2 \sin kx \\ T(t) = e^{-(k^2d^2t)/2}(d_1 \cos rt + d_2 \sin rt) \end{cases}\end{gathered} \tag{40}$$

in which

$$r = \frac{1}{2}k\sqrt{4c^2 - k^2d^4}. \tag{41}$$

Hence, the solution is

$$\begin{aligned} u(x,t) &= e^{-(k^2d^2t)/2}(c_1 \cos kx + c_2 \sin kx) \\ &\quad \times (d_1 \cos rt + d_2 \sin rt). \end{aligned} \tag{42}$$

Also, the velocity is given as

$$\begin{aligned} u_t(x,t) &= e^{-(k^2d^2t)/2}(c_1 \cos kx + c_2 \sin kx) \\ &\quad \times \left[\left(rd_2 - \frac{1}{2}k^2d^2d_1\right)\cos rt \right. \\ &\quad \left. - \left(rd_1 + \frac{1}{2}k^2d^2d_2\right)\sin rt\right]. \end{aligned} \tag{43}$$

From the initial condition one obtains

$$u(0,0) = c_1d_1 = 0 \xrightarrow{\text{yields}} c_1 = 0, \quad \text{or} \quad d_1 = 0. \tag{44}$$

If $c_1 = 0$, then $u_t(0,t) = 0$, and hence there is no motion in the medium and the solution is trivial. Therefore, $c_1 \neq 0$, and d_1 must be zero, so that

$$u_t(0,t) = e^{-(k^2d^2t)/2}c_1d_2\left[r\cos rt - \frac{1}{2}k^2d^2 \sin rt\right]. \tag{45}$$

Since after a time T the particle which is at the origin stops, then

$$\begin{aligned} & r\cos rT - \frac{1}{2}k^2d^2 \sin rT = 0 \\ & \longrightarrow T_n = \frac{1}{r}\left(\tan^{-1}\left(\frac{2r}{k^2d^2}\right) + n\pi\right). \end{aligned} \tag{46}$$

Consequently the solution can be written as

$$\begin{aligned} u(x,t) &= 2c_1d_2\sqrt{{c_1}^2 + {c_2}^2} \\ &\quad \times e^{-(k^2d^2t)/2}\cos(kx - \varphi)\sin(rt) \end{aligned} \tag{47}$$

in which

$$\varphi = \cos^{-1}\left(\frac{c_1}{\sqrt{{c_1}^2 + {c_2}^2}}\right). \tag{48}$$

Therefore, the solution can be written as

$$u(x,t) = \beta e^{-(k^2d^2t)/2}\cos(kx - \varphi)\sin(rt). \tag{49}$$

4. Results

4.1. Longitudinal Waves in a Perfectly Elastic Solid. In a metal like aluminum, whose characteristics are listed in Table 1, the wave that propagates in the medium is of the form (49).

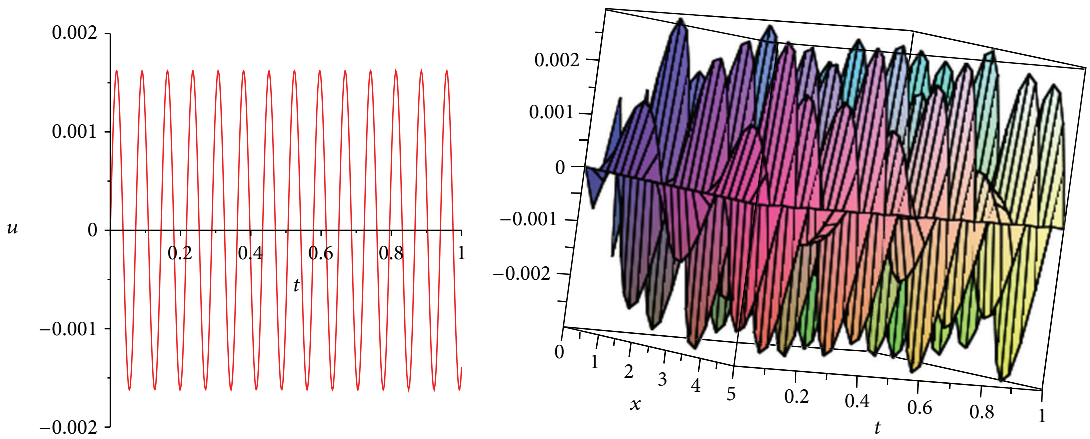
If one considers $k = 2$, $\beta = 0.003$, and $\varphi = 1$, the depiction of the longitudinal wave that propagates from the origin is shown in Figure 1. It is clear that it is a wave with no attenuation.

4.2. Longitudinal Waves in Viscoelastic Solids. If the viscoelastic properties are characterized as in Table 2, the depiction of the longitudinal stress wave is shown in Figure 2. It is also clear that this wave is attenuated.

4.3. Longitudinal Waves in a Viscoelastic Newtonian Fluid. In a liquid with parameters shown in Table 3, the wave propagating in the medium is of the form (39). So then the wave characteristics are discussed in the following for material with properties as mentioned in Table 3.

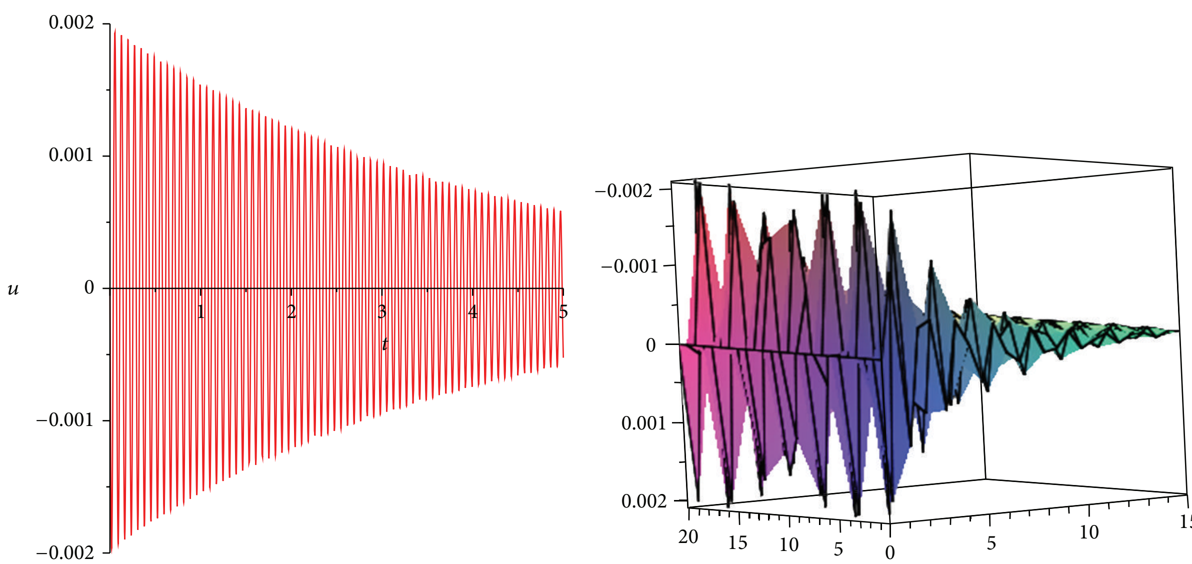
If one assumes $k = 2$, $\alpha = 0.01$, and $\beta = 0.003$, then the displacement graph of particles in this medium is as shown in Figure 3. In Figure 3(a) the displacement of particles at the origin without returning to their initial position and in Figure 3(b) the displacement of a particle in each position x without returning to its initial position are seen.

4.4. Longitudinal Waves in an Ideal Newtonian Fluid. If a liquid is viscoelastic with viscosity coefficients shown in Table 4, the graph of longitudinal stress wave in this medium is as shown in Figure 4. In Figure 4(a), the distance between the particle originally located in the origin and the same particle moved away from the origin in time T is seen. This distance starts to decrease with the particles returning to the origin. In Figure 4(b), the distance between the particle at the time T and the same particle approaching its equilibrium position at the origin is shown; in Figure 4(c) the distance between the particles located in their respective equilibrium position and the same particles moved away from the equilibrium position in time T is shown. This distance then starts to decrease with the particles returning to their respective equilibrium position.



(a) Particle at the origin (b) Particle at each position x

Figure 1: The vibration of particles.



(a) Particle at the origin (b) Particle at each position x

Figure 2: Vibration of a particle with attenuation.

Table 1: Parameters of aluminum that is considered to be perfectly elastic.

λ_s	λ_f	μ_s	μ_f	ρ
50.6×10^6 KN/m^2	0	27.6×10^6 KN/m^2	0	56000 Kg/m^3

Table 2: Parameters of aluminum with fluid properties pertaining to viscoelasticity.

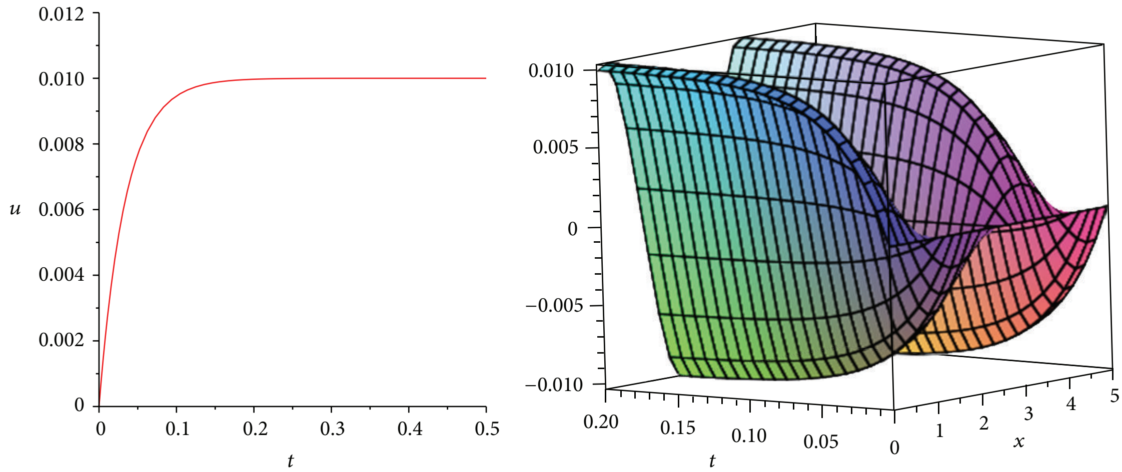
λ_s	λ_f	μ_s	μ_f	ρ
50.6×10^6 KN/m^2	3016	27.6×10^6 KN/m^2	2130	56000 Kg/m^3

Table 3: Parameters of a Newtonian liquid like water which is perfectly fluid with no solid parameters.

λ_s	λ_f	μ_s	μ_f	ρ
0	30.12×10^2	0	21.3×10^2	996.3 Kg/m^3

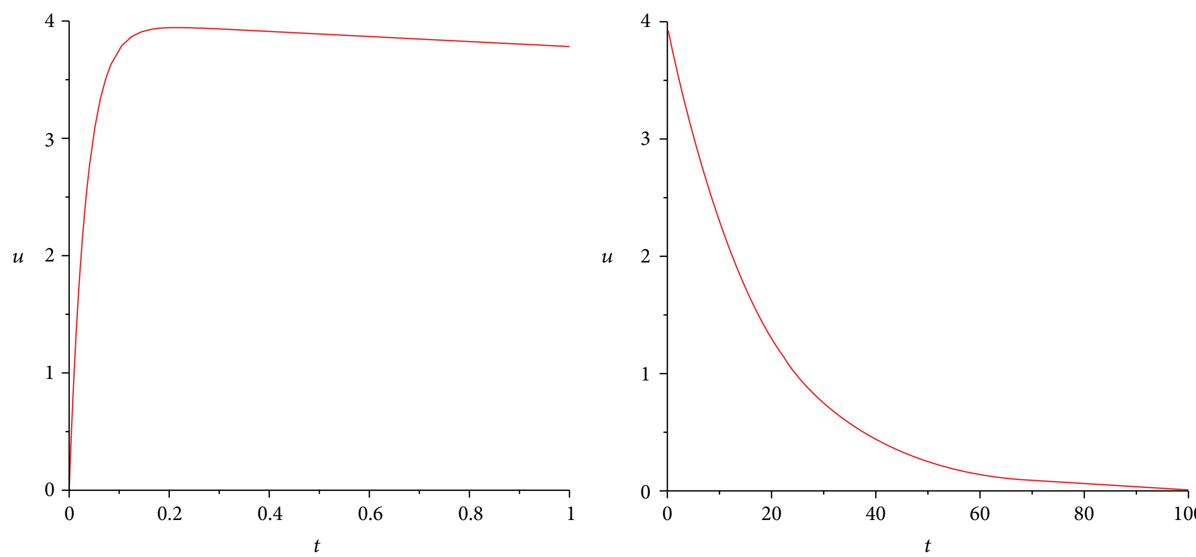
Table 4: Parameters of a liquid like water which is not perfectly fluid and has solid parameters.

λ_s	λ_f	μ_s	μ_f	ρ
50.6	30.12×10^2	27.6	21.3×10^2	996.3 Kg/m^3



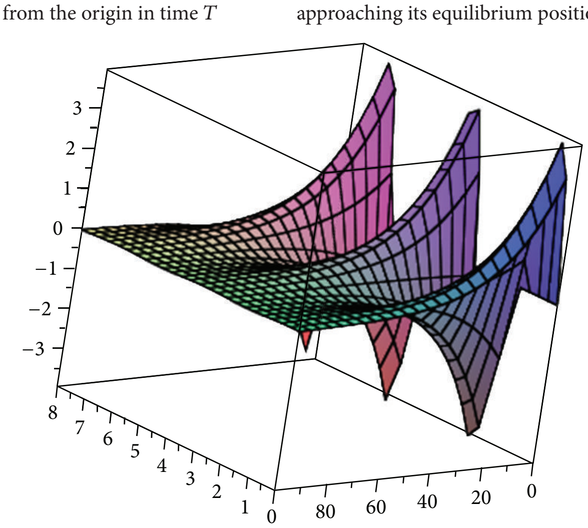
(a) Particle at the origin (b) Particle at each position x

FIGURE 3: The displacement of particles.



(a) The distance between the particle originally located in the origin and the same particle moved away from the origin in time T

(b) The distance between the particle at the time T and the same particle approaching its equilibrium position at the origin



(c) The distance between the particles located in their respective equilibrium position and the same particles moved away from the equilibrium position in time T

FIGURE 4: The distance between the particles at the time T.

5. Conclusions

When a material is subjected to an impulsive force, there occur infinitesimal displacements of the integrated particles of that body. In solids and in fluids the Navier-Lame and the Navier-Stokes equations govern these displacements, respectively. In the present work it was shown that through the use of a generalized form of Kelvin-Voigt model of viscoelasticity the unification of these two equations is possible and one obtains an equation that can represent solid materials, fluids, and soft materials. A new concept of viscoelasticity was defined in which every material has some degree of both solid and fluid properties depending on the particular parameters of viscosity. This is of particular importance in relation to the soft materials. Using this generalized equation, propagation of stress disturbance, pulse, and waves in solids as well as fluids can be studied in one single frame.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] K. Hackl and M. Goodarzi, *An Introduction to Linear Continuum Mechanics*, Ruhr-University Bochum, 2010.

[2] L. Treloar, *The Physics of Rubber Elasticity*, Clarendon Press, Oxford, UK, 1975.

[3] N. Muskhilishvili, *Some Basic Problem of the Mathematical Theory of Elasticity*, Noordhoff, Leiden, The Netherlands, 1975.

[4] C. Truesdell, *Mechanics of Solids*, vol. 2, Springer, New York, NY, USA, 1972.

[5] J. Allard, *Propagation of Sound in Porous Media*, Elsevier Science, Essex, UK, 1993.

[6] M. A. Meyers and K. K. Chawla, *Mechanical Behavior of Materials*, Cambridge University Press, London, UK, 2008.

[7] C.-e. Foias, *Navier-Stokes Equation and Turbulence*, Cambridge University, Cambridge, UK, 2001.

[8] "Overtheads," 2014, http://www.ucd.ie/mecheng/ccm/Overheads_6,7.pdf.

[9] "Derivation of Naveir Stokes equations," 2014, http://en.wikipedia.org/wiki/Derivation_of_the_Navier%E2%80%93Stokes_equations.

[10] R. M. Christensen, *Theory of Viscoelasticity*, Academic Press, New York, NY, USA, 1971.

[11] A. Musa, "Predicting wave velocity and stress of impact of elastic slug and viscoelastic rod using viscoelastic discotinuity analysis (standard linear solid model)," in *Proceedings of the World Congress on Engineering*, London, UK, 2012.

[12] N. McCrum, C. Buckley, and C. Bucknall, *Principles of Polymer Engineering*, Oxford University Press, Oxford, UK, 1997.

[13] A. D. Drozdov and A. Dorfmann, "The nonlinear viscoelastic response of carbon black-filled natural rubbers," *International Journal of Solids and Structures*, vol. 39, no. 23, pp. 5699–5717, 2002.

[14] H. Zhao and G. Gary, "A three dimensional analytical solution of the longitudinal wave propagation in an infinite linear viscoelastic cylindrical bar. Application to experimental techniques," *Journal of the Mechanics and Physics of Solids*, vol. 43, no. 8, pp. 1335–1348, 1995.

[15] H. Zhao, G. Gary, and J. R. Klepaczko, "On the use of a viscoelastic split Hopkinson pressure bar," *International Journal of Impact Engineering*, vol. 19, no. 4, pp. 319–330, 1997.

[16] C. Salisburg, *Wave propagation through a polymeric Hopkinson bar [M.S. thesis]*, Department of Mechanical Engineering, Univesity of Waterloo, Waterloo, Canada, 2001.

[17] A. Benatar, D. Rittel, and A. L. Yarin, "Theoretical and experimental analysis of longitudinal wave propagation in cylindrical viscoelastic rods," *Journal of the Mechanics and Physics of Solids*, vol. 51, no. 8, pp. 1413–1431, 2003.

[18] D. T. Casem, W. Fourney, and P. Chang, "Wave separation in viscoelastic pressure bars using single-point measurements of strain and velocity," *Polymer Testing*, vol. 22, no. 2, pp. 155–164, 2003.

[19] K. Rajneesh and K. Kanwaljeet, "Mathematical analysis of five parameter model on the propagation of cylindrical shear waves in non- homogeneous viscoelastic media," *International Journal of Physical and Mathematical Science*, vol. 4, no. 1, pp. 45–52, 2013.

[20] T. Alfrey, "Non-homogeneous stresses in visco-elastic media," *Quarterly of Applied Mathematics*, vol. 2, pp. 113–119, 1944.

[21] J. Barberán and I. Herrera, "Uniqueness theorems and the speed of propagation of signals in viscoelastic materials," *Archive for Rational Mechanics and Analysis*, vol. 23, pp. 173–190, 1966.

[22] J. D. Achenbach and D. P. Reddy, "Note on wave propagation in linearly viscoelastic media," *Zeitschrift für angewandte Mathematik und Physik ZAMP*, vol. 18, no. 1, pp. 141–144, 1967.

[23] S. Bhattacharya and P. Sengupta, "Disturbances in a general viscoelastic medium due to impulsive forces on a spherical cavity," *Gerlands Beiträge zur Geophysik*, vol. 87, no. 8, pp. 57–62, 1978.

[24] D. P. Acharya, I. Roy, and P. K. Biswas, "Vibration of an infinite inhomogeneous transversely isotropic viscoelastic medium with cylindrical hole," *Applied Mathematics and Mechanics (English Edition)*, vol. 29, no. 3, pp. 367–378, 2008.

[25] C. Bert and D. Egle, "Wave propagation in a finite length bar with variable area of cross-section," *Journal of Applied Mechanics*, vol. 36, pp. 908–909, 1969.

[26] A. M. Abd-Alla and S. M. Ahmed, "Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress," *Earth, Moon, and Planets*, vol. 75, no. 3, pp. 185–197, 1996.

[27] R. C. Batra, "Linear constitutive relations in isotropic finite elasticity," *Journal of Elasticity*, vol. 51, pp. 269–273, 1998.

[28] S. Murayama and T. Shibata, "Rheological properties of clays," in *Proceedings of the 5th International Conference of Soil Mechanics and Foundation Engineering*, Paris, France, 1961.

[29] R. Schiffman, C. Ladd, and A. Chen, "The secondary consolidation of clay, rheology and soil mechanics," in *Proceedings of the International Union of Theoritical and Applied Mechanivs Symposium*, Berlin, Germany, 1964.

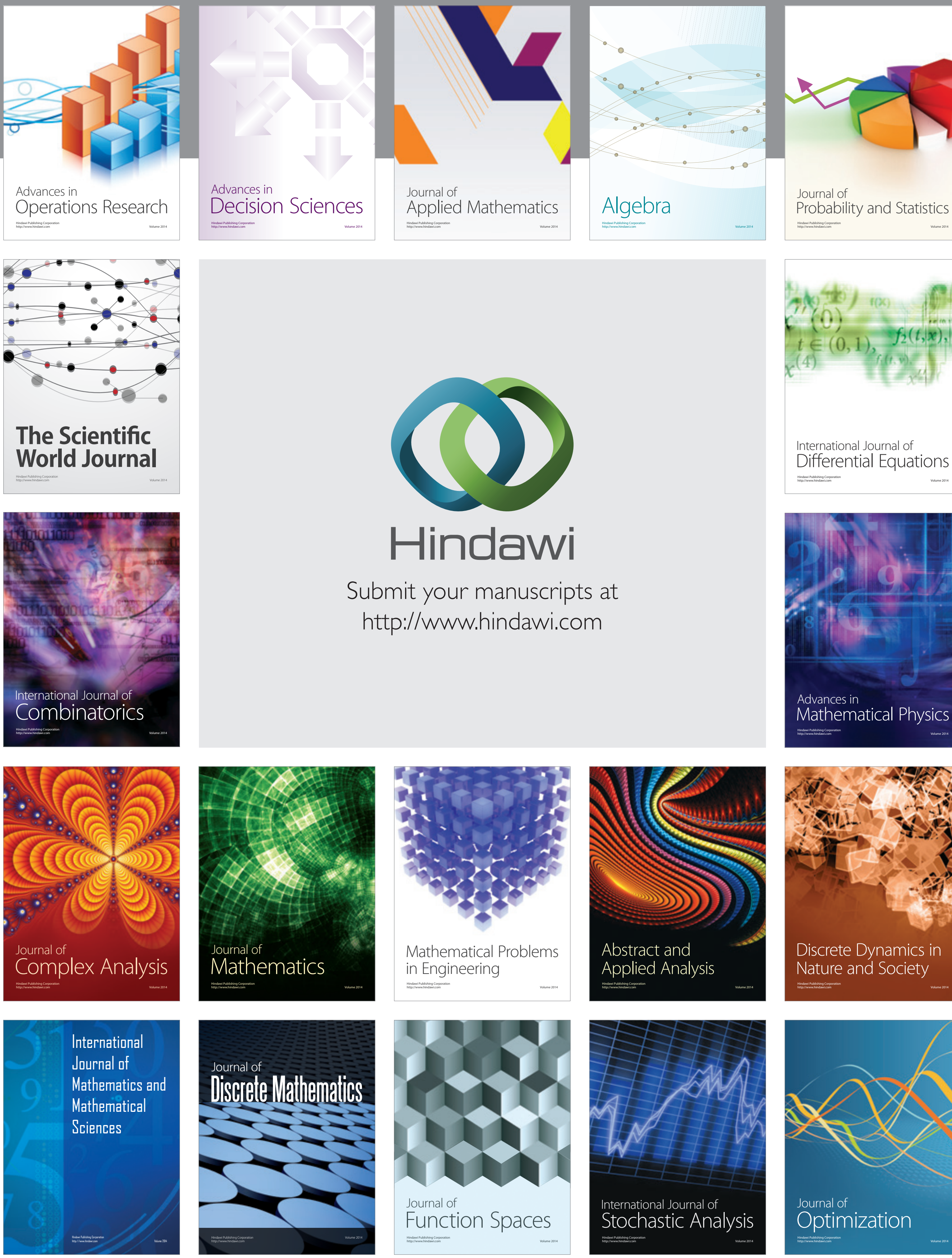
[30] R. Jankowski, K. Wilde, and Y. Fujino, "Pounding of superstructure segments in isolated elevated bridge during earthquakes," *Earthquake Engineering and Structural Dynamics*, vol. 27, no. 5, pp. 487–502, 1998.

[31] V. N. Nikolaevskij, *Mechanics of Porous and Fractured Media*, World Scientific, 1990.

[32] N. Thin, *Understanding Viscoelasticity*, Springer, Heidelberg, Germany, 2002.

[33] "viscoelasticity," 2014, http://en.wikipedia.org/wiki/Viscoelasticity.

[34] "Wavepropagation," 2014, https://www.nde-ed.org/EducationResources/CommunityCollege/Ultrasonics/Physics/wavepropagation.htm.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