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Research Article

Impulsive Controllability/Observability for Interconnected Descriptor Systems with Two Subsystems

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The problem of decentralized impulse controllability/observability for large-scale interconnected descriptor systems with two subsystems by derivative feedback is studied. Necessary conditions for the existence of a derivative feedback controller for the first subsystem of the large-scale interconnected descriptor systems ensuring the second subsystem to be impulse controllable and impulse observable are derived, respectively. Based on the results, a derivative feedback controller for the first subsystem of the large-scale interconnected descriptor systems is constructed easily such that the second subsystem is impulse controllable or impulse observable. Finally, examples are given to illustrate the effectiveness of the results obtained in this paper.

1. Introduction

Descriptor systems are also referred to as singular systems, implicit systems, differential algebraic systems, semistate systems, or generalized state space systems. Descriptor system models are more convenient and natural than standard state space system models in order to describe practical systems, such as interconnected large-scale systems, economic systems, networks, power systems, and chemical processes [1, 2]. Due to this reason, the control of descriptor systems has been extensively studied in past years and a great number of results based on the theory of standard state space systems have been generalized to descriptor systems [1–5].

It has been known that the response of a descriptor system may contain impulsive terms. In a practical system, the impulse may cause bad performance and even destroys the system. Therefore, it is important to study the problem of eliminating impulsive behavior of a descriptor systems via certain feedback controllers. For regular descriptor systems, it is pointed out in the work [6] that the necessary and sufficient condition for impulse elimination via proportional state feedback is that the considered system is impulse controllable. The work in [7] shows that homogeneous indices are a complete set of invariants for the action of a natural group of feedback transformations on descriptor linear systems, and the homogeneous indices determine exactly which closed-loop invariant polynomials can be assigned by feedback, thereby generalizing the control structure theorem of Rosenbrock. In [8], the pole placement and connected problems by the use of matrix pencil formulation have also been studied. In [9], the limits in altering the eigenstructure of linear reachable descriptor systems by proportional plus derivative (P-D) state feedback have also been studied, and an explicit necessary and sufficient condition is established for a set of invariant polynomials and positive integers to represent the finite and the infinite eigenstructure of a system obtainable from the given descriptor system by proportional plus derivative (P-D) state feedback. The problem of pole structure assignment (PSA) by proportional state feedback in implicit, linear, and uncontrollable systems has also been studied in [10]. In [11], the problem of the action of proportional plus derivative (P-D) and pure proportional feedback groups on the set of all singular systems has also been studied. In [12], the concepts of impulsive modes and impulsive mode controllability are proposed by the analysis of admissibility of initial conditions. In [13], the problem of impulsive mode elimination via proportional state feedback is considered. It is pointed out that the impulsive mode controllability is necessary and sufficient for the existence of impulsive mode eliminating proportional state feedback controller. The concept of impulsive controllability is proposed in [14]. It is shown [15, 16] that the condition of impulse elimination via proportional plus derivative (P-D) state feedback is that the original system is impulsive controllable one. New criteria are proposed in [17] for impulsive mode controllability of descriptor linear systems by adopting the null space approach. In [18], the problem of impulse elimination via proportional plus derivative (P-D) output feedback has also been studied. In [19], a decentralized (P-D) feedback is explored for the impulsive elimination problem where the centralized (P-D) feedback is regarded as a special case. In [20], a concept of the structured proportional plus derivative (P-D) feedback is first introduced and an explicit necessary and sufficient condition is constructively derived for the closed-loop system to be regular and impulse-free by the structured proportional plus derivative (P-D) feedback. A rank condition of impulse observability is presented for regular descriptor systems [21]. In [12], the presence of impulsive responses in descriptor systems and how they relate to impulse controllability and impulse observability are considered.

The large-scale interconnected descriptor systems are a kind of dynamic systems which are more general and have extensive applied background, such as power systems, economic systems, and network systems [22]. The class of systems can be characterized by a large number of state variables, parametric uncertainties, a complex structure, and a strong interaction between subsystems [23]. Decentralized/interconnected descriptor systems have attracted more and more attention because of their practical backgrounds; for example, [24, 25] investigated dynamic networks governed by decentralized/descriptor systems, respectively. The related research has been widely used in the research of multiagent robot and four-rotor aircraft [26].

Decentralized/interconnected descriptor systems also contain impulse terms. However, to the best of our knowledge, the research on decentralized impulse controllability/observability of interconnected descriptor systems still receives little attention. So in this paper, we study the problem of decentralized impulsive controllability/observability for interconnected descriptor systems. The derivative coefficient matrix $(E + BK)$ is not required to be revertible, so our results include previous related ones as special cases. Decentralized impulsive controllability/observability is investigated by using matrix rank properties, which is a novel way to deal with the impulse. The dynamic order of the closed-loop descriptor systems can be assigned between the minimum rank and maximum rank of $(E + BK)$.

Hypersonic technology is the best stratagem in the realm of aerospace technology in the 21st century. The hypersonic vehicle is characterized by its high speed and high ability of penetration. So it is of great importance in both civilian field and military field. Because of the advanced aerodynamic configuration, fuselage-engine integrated design, large flight envelope, and the tremendous changes of flight environment, the hypersonic aircraft has poor stability, serious couplings among the subsystems, and biggish model uncertainty. Meanwhile, various restrictions for the flight of the vehicles need to be considered. That means that system equations of the vehicles should include the algebraic equations. In most cases, the parameters of hypersonic technology systems change all the time. When derivative coefficient matrix is singular matrix, we can describe the hypersonic technology systems as descriptor systems. So, the impulse problem of vehicle systems is the first problem to be solved. However, to the best of our knowledge, the research on impulse control of hypersonic vehicle still receives little attention. Therefore, the research problem of hypersonic technology impulse controllability is overwhelmingly crucial.

As is known to all, for the descriptor systems, proportional feedback can not alter the impulse controllability of systems. Therefore, this paper focuses on designing a derivative feedback controller for the first subsystem of the large-scale interconnected descriptor system such that the second subsystem is impulse controllable or impulse observable. The impulse controllability/observability problem of descriptor systems involves the rank constraints of matrix pencil with parameter K. Firstly, we use matrix theory to transform the matrix pencil involving parameter K into the form of $A + BKC$. Further by comparing the maximum rank of the left hand of the condition of impulse controllability/observability with the minimum rank of the right hand of the condition of impulse controllability/observability, necessary conditions for the existence of a derivative feedback controller for the first subsystem ensuring the second subsystem to be impulse controllable and impulse observable are derived, respectively. If the maximum rank of the left hand of the condition of impulse controllability is lower than the minimum rank of the right hand of that, the controller to be designed will not exist. If the maximum rank of the left hand of the condition of impulse controllability/observability is higher than the minimum rank of the right hand of that, the intersection of the set of the ranks of the left hand and the set of the ranks of right hand of the condition of impulse controllability/observability is not an empty set, and the controller to be designed will exist and can be selected in the intersection. Finally, illustrative examples are provided to demonstrate the applicability of the proposed method.

This paper is structured as follows. In Section 2, definitions and lemmas are introduced for later use. Section 3 gives conditions of the impulsive controllability and impulsive observability by derivative feedback. In Section 4, illustrative examples are proposed to testify the feasibility of the theorems. Finally, conclusive remarks are made in Section 5.

2. Preliminaries

Consider a class of large-scale interconnected descriptor systems composed of two subsystems of the following form:

$$\begin{aligned} E_1\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + B_1u_1, \\ E_2\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2u_2, \\ y_1 &= C_1x_1, \\ y_2 &= C_2x_2, \end{aligned} \tag{1}$$

where $x_1 \in R^{n_1}$ is the state of the first subsystem, $x_2 \in R^{n_2}$ is the state of the second subsystem, $y_1 \in R^{m_1}$ and $y_2 \in R^{m_2}$ are outputs, and $u_1 \in R^{l_1}$ and $u_2 \in R^{l_2}$ are control inputs. E_1, E_2, A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, C_1, and C_2 are constant matrices with compatible dimensions. In this paper, we suppose that

$$\begin{gathered} \operatorname{rank}(E_1) = r_{e_1}, \\ \operatorname{rank}(B_1) = r_{b_1}, \\ \operatorname{rank}(E_2) = r_{e_2}, \\ \operatorname{rank}(B_2) = r_{b_2}, \\ \operatorname{rank}([E_1 \;\; B_1]) = r_{e_1 b_1}, \\ \operatorname{rank}([E_2 \;\; B_2]) = r_{e_2 b_2}, \\ n_1 + n_2 = n. \end{gathered} \tag{2}$$

Denote

$$\begin{gathered} \overline{E} = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix}, \\ \overline{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ \overline{B} = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \\ \overline{C} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \\ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \\ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \\ y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \end{gathered} \tag{3}$$

Then system (1) can be rewritten into the condensed form

$$\begin{gathered} \overline{E}\dot{x} = \overline{A}x + \overline{B}u, \\ y = \overline{C}x, \end{gathered} \tag{4}$$

and system (1) is assumed to be regular; that is, $\exists s \in \mathbf{C}$, $\det(s\overline{E} - \overline{A}) \neq 0$. In this case, the state response of system (1) exists uniquely for any admissible initial state.

The local derivative feedback can be described as

$$u_1 = -K_1 \dot{x}_1, \tag{5}$$

where $K_1 \in R^{l_1 \times n_1}$ is a gain matrix to be determined. Applying the feedback to the first subsystem of system (1), the resultant closed-loop system is

$$\overline{E}_K \dot{x} = \overline{A}x + \overline{B}_2 u_2, \tag{6}$$

where

$$\begin{gathered} \overline{E}_K = \begin{bmatrix} E_1 + B_1 K_1 & 0 \\ 0 & E_2 \end{bmatrix}, \\ \overline{B}_2 = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}. \end{gathered} \tag{7}$$

Setting

$$\overline{B}_1 = \begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix}, \tag{8}$$

then we have

$$\overline{E}_K = \overline{E} + \overline{B}_1 K_1. \tag{9}$$

The local derivative output feedback can be described as

$$u_1 = -L_1 \dot{y}_1, \tag{10}$$

where $L_1 \in R^{l_1 \times m_1}$ is a gain matrix to be determined. Applying the feedback to the first subsystem of system (1), the resultant closed-loop system is

$$\overline{E}_L \dot{x} = \overline{A}x + \overline{B}_2 u_2, \tag{11}$$

where

$$\begin{gathered} \overline{E}_L = \begin{bmatrix} E_1 + B_1 L_1 C_1 & 0 \\ 0 & E_2 \end{bmatrix}, \\ \overline{C}_2 = [0 \;\; C_2]. \end{gathered} \tag{12}$$

Setting

$$\overline{C}_1 = \begin{bmatrix} C_1 & 0 \\ 0 & 0 \end{bmatrix}, \tag{13}$$

then we have

$$\overline{E}_L = \overline{E} + \overline{B}_1 L_1 \overline{C}_1. \tag{14}$$

For system (1), there exist orthogonal matrices Q_1, Q_2, U_1, U_2, V_1, and V_2 with appropriate dimensions such that

$$\begin{gathered} U_1 E_1 V_1 = \begin{bmatrix} \Sigma_1^1 & 0 & 0 \\ E_{21}^1 & E_{22}^1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ U_1 B_1 Q_1 = \begin{bmatrix} 0 & 0 \\ \Sigma_B^1 & 0 \\ 0 & 0 \end{bmatrix}, \\ U_2 E_2 V_2 = \begin{bmatrix} \Sigma_1^2 & 0 & 0 \\ E_{21}^2 & E_{22}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ U_2 B_2 Q_2 = \begin{bmatrix} 0 & 0 \\ \Sigma_B^2 & 0 \\ 0 & 0 \end{bmatrix}, \end{gathered} \tag{15}$$

where $\Sigma_1^1 \in R^{(r_{e_1b_1}-r_{b_1})\times(r_{e_1b_1}-r_{b_1})}$, $\Sigma_B^1 \in R^{r_{b_1}\times r_{b_1}}$, $\Sigma_1^2 \in R^{(r_{e_2b_2}-r_{b_2})\times(r_{e_2b_2}-r_{b_2})}$, and $\Sigma_B^2 \in R^{r_{b_2}\times r_{b_2}}$ are diagonal positive definite matrices. $E_{21}^1 \in R^{r_{b_1}\times(r_{e_1}+r_{b_1}-r_{e_1b_1})}$, $E_{22}^1 \in R^{r_{b_1}\times(r_{e_1}+r_{b_1}-r_{e_1b_1})}$, $E_{21}^2 \in R^{r_{b_2}\times(r_{e_2}+r_{b_2}-r_{e_2b_2})}$, and $E_{22}^2 \in R^{r_{b_2}\times(r_{e_2}+r_{b_2}-r_{e_2b_2})}$ are full column rank matrices:

$$\begin{aligned} U_1A_{11}V_1 &= \begin{bmatrix} A_{11}^{11} & A_{12}^{11} & A_{13}^{11} \\ A_{21}^{11} & A_{22}^{11} & A_{23}^{11} \\ A_{31}^{11} & A_{32}^{11} & A_{33}^{11} \end{bmatrix}, \\ U_1A_{12}V_2 &= \begin{bmatrix} A_{11}^{12} & A_{12}^{12} & A_{13}^{12} \\ A_{21}^{12} & A_{22}^{12} & A_{23}^{12} \\ A_{31}^{12} & A_{32}^{12} & A_{33}^{12} \end{bmatrix}, \\ U_2A_{21}V_1 &= \begin{bmatrix} A_{11}^{21} & A_{12}^{21} & A_{13}^{21} \\ A_{21}^{21} & A_{22}^{21} & A_{23}^{21} \\ A_{31}^{21} & A_{32}^{21} & A_{33}^{21} \end{bmatrix}, \\ U_2A_{22}V_2 &= \begin{bmatrix} A_{11}^{22} & A_{12}^{22} & A_{13}^{22} \\ A_{21}^{22} & A_{22}^{22} & A_{23}^{22} \\ A_{31}^{22} & A_{32}^{22} & A_{33}^{22} \end{bmatrix}, \\ U_1C_1V_1 &= \begin{bmatrix} C_{11}^1 & C_{12}^1 & C_{13}^1 \end{bmatrix}, \\ U_2C_2V_2 &= \begin{bmatrix} C_{11}^2 & C_{11}^2 & C_{11}^2 \end{bmatrix}. \end{aligned} \tag{16}$$

For linear descriptor systems of the form

$$\begin{aligned} E\dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \tag{17}$$

where $x \in R^n$ is state vector, $u \in R^r$ is control input, and $y \in R^l$ is controlled output, $E, A \in R^{n\times n}$, $B \in R^{n\times r}$, and $C \in R^{l\times n}$ are constant matrices and $\operatorname{rank}(E) = n_0 \le n$.

Definition 1 (see [6]). System (17) is said to be regular if there exists a constant scalar $s \in \mathbf{C}$ such that

$$\det(sE - A) \neq 0. \tag{18}$$

Lemma 2 (see [6]). *(a) System (17) is impulse-free if and only if the following relation holds:*

$$\operatorname{rank}\left(\begin{bmatrix} E & 0 \\ A & E \end{bmatrix}\right) = n + \operatorname{rank}(E). \tag{19}$$

(b) System (17) is impulse controllable if and only if the following relation holds:

$$\operatorname{rank}\left(\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix}\right) = n + \operatorname{rank}(E). \tag{20}$$

(c) System (17) is impulse observable if and only if the following relation holds:

$$\operatorname{rank}\left(\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix}\right) = n + \operatorname{rank}(E). \tag{21}$$

Lemma 3 (see [2, 21]). *For arbitrary matrices $\mathscr{A} \in R^{n\times r}$, $\mathscr{B} \in R^{n\times m}$, and $\mathscr{C} \in R^{l\times r}$ the following relation holds:*

$$\begin{aligned} &\text{(a)} \min_{\mathscr{K}} \{\operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K})\} \\ &\quad = \operatorname{rank}([\mathscr{A} \;\; \mathscr{B}]) - \operatorname{rank}(\mathscr{B}), \end{aligned} \tag{22}$$

where $\min_{\mathscr{K}}\{*\}$ is the minimum rank of matrix pencil $\{*\}$ with parameter matrix $\mathscr{K} \in R^{m\times r}$.

Consider the following:

$$\begin{aligned} &\text{(b)} \min_{\mathscr{K}} \{\operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})\} \\ &\quad = \operatorname{rank}([\mathscr{A} \;\; \mathscr{B}]) + \operatorname{rank}\left(\begin{bmatrix} \mathscr{A} \\ \mathscr{C} \end{bmatrix}\right) \\ &\quad - \operatorname{rank}\left(\begin{bmatrix} \mathscr{A} & \mathscr{B} \\ \mathscr{C} & 0 \end{bmatrix}\right), \end{aligned} \tag{23}$$

where $\min_{\mathscr{K}}\{*\}$ is the minimum rank of matrix pencil $\{*\}$ with parameter matrix $\mathscr{K} \in R^{m\times l}$.

Consider the following:

$$\begin{aligned} &\text{(c)} \max_{\mathscr{K}} \{\operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})\} \\ &\quad = \min\left\{\operatorname{rank}([\mathscr{A} \;\; \mathscr{B}]), \operatorname{rank}\left(\begin{bmatrix} \mathscr{A} \\ \mathscr{C} \end{bmatrix}\right)\right\}, \end{aligned} \tag{24}$$

where $\max_{\mathscr{K}}\{*\}$ is the maximum rank of matrix pencil $\{*\}$ with parameter matrix $\mathscr{K} \in R^{m\times l}$.

Consider the following:

$$\begin{aligned} M = \Big\{ &\mathscr{K} \in R^{m\times l} \mid \operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C}) \\ &= \max_{\mathscr{K}} \{\operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})\} \Big\}. \end{aligned} \tag{25}$$

It is a Zariski open set, or equivalently almost every matrix $\mathscr{K} \in R^{m\times l}$ can make $\operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})$ *to be maximal.*

By Lemma 2, we have the following results.

Lemma 4. *(a) System (6) is impulse controllable if and only if the following relation holds:*

$$\operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right) = n + \operatorname{rank}(\overline{E}_K). \tag{26}$$

(b) System (11) is impulse observable if and only if the following relation holds:

$$\operatorname{rank}\left(\begin{bmatrix} \overline{E}_L & \overline{A} \\ 0 & \overline{E}_L \\ 0 & \overline{C}_2 \end{bmatrix}\right) = n + \operatorname{rank}(\overline{E}_L). \tag{27}$$

Lemma 5 (see [29]). *If $r_{\min} = \min_{\mathscr{K}} \operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})$ and $r_{\max} = \max_{\mathscr{K}} \operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})$, then there always exists $\mathscr{K} \in R^{m\times l}$ such that $r_0 = \operatorname{rank}(\mathscr{A} + \mathscr{B}\mathscr{K}\mathscr{C})$ for any positive integer r_0 satisfying $r_{\min} \le r_0 \le r_{\max}$.*

3. Main Results

In this section, conditions for the existence of a derivative feedback controller for the first subsystem of the large-scale interconnected descriptor systems ensuring the second subsystem to be impulse controllable and impulse observable are derived, respectively.

Theorem 6. *Consider system (1). If*

$$\operatorname{rank}\left(\begin{bmatrix} A_{32}^{11} & A_{33}^{11} & A_{33}^{12} \\ A_{32}^{21} & A_{33}^{21} & A_{33}^{22} \end{bmatrix}\right) < n - r_{e_1 b_1} - r_{e_2 b_2} - r_{b_1}, \quad (28)$$

there does not exist K_1 such that closed-loop system (6) is impulse controllable.

Proof. We first note that

$$\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix} = \begin{bmatrix} \overline{E} & 0 & 0 \\ \overline{A} & \overline{E} & \overline{B}_2 \end{bmatrix} + \begin{bmatrix} \overline{B}_1 & 0 & 0 \\ 0 & \overline{B}_1 & 0 \end{bmatrix} \overline{K}_1 \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad (29)$$

where

$$\overline{K}_1 = \begin{bmatrix} \widetilde{K}_1 & 0 & 0 \\ 0 & \widetilde{K}_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \widetilde{K}_1 = \begin{bmatrix} K_1 & 0 \\ 0 & K_1 \end{bmatrix}. \quad (30)$$

Denote

$$\mathcal{S}_1 = \operatorname{Diag}\{U_1, U_2, U_1, U_2\}, \qquad \mathcal{S}_2 = \operatorname{Diag}\{V_1, V_2, V_1, V_2, Q_2, Q_1, Q_1\}. \quad (31)$$

Diag{∗} denotes the block diagonal matrix. By Lemma 3, we have

$$\begin{aligned} R_1 &= \max_{K_1}\left\{\operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right)\right\} \\ &= \max_{K_1}\left\{\operatorname{rank}\left(\begin{bmatrix} \overline{E} & 0 & 0 & \overline{B}_1 & 0 \\ \overline{A} & \overline{E} & \overline{B}_2 & 0 & \overline{B}_1 \end{bmatrix}\right)\right\} \\ &= \operatorname{rank}\left(\mathcal{S}_1 \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & B_1 & 0 \\ 0 & E_2 & 0 & 0 & 0 & 0 & 0 \\ A_{11} & A_{12} & E_1 & 0 & 0 & 0 & B_1 \\ A_{21} & A_{22} & 0 & E_2 & B_2 & 0 & 0 \end{bmatrix} \mathcal{S}_2\right) \\ &= 2\operatorname{rank}\left(\Sigma_1^1\right) + 2\operatorname{rank}\left(\Sigma_B^1\right) + 2\operatorname{rank}\left(\Sigma_1^2\right) \\ &\quad + \operatorname{rank}\left(E_{22}^2\right) + \operatorname{rank}\left(\Sigma_B^2\right) \\ &\quad + \operatorname{rank}\left(\begin{bmatrix} A_{32}^{11} & A_{33}^{11} & A_{33}^{12} \\ A_{32}^{21} & A_{33}^{21} & A_{33}^{11} \end{bmatrix}\right) \\ &= 2r_{e_1 b_1} + r_{e_2} + r_{e_2 b_2} + \operatorname{rank}\left(\begin{bmatrix} A_{32}^{11} & A_{33}^{11} & A_{33}^{12} \\ A_{32}^{21} & A_{33}^{21} & A_{33}^{22} \end{bmatrix}\right). \end{aligned} \quad (32)$$

On the other hand, by Lemma 3, we have

$$\begin{aligned} r_1 &= n + \operatorname{rank}\left(\overline{E}_K\right) \\ &= \min_{K_1}\left\{n + \operatorname{rank}\left(\begin{bmatrix} E_1 + B_1 K_1 & 0 \\ 0 & E_2 \end{bmatrix}\right)\right\} \\ &= n + \operatorname{rank}\left(\begin{bmatrix} E_1 & 0 & B_1 & 0 \\ 0 & E_2 & 0 & 0 \end{bmatrix}\right) - \operatorname{rank}\left(\begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix}\right) \\ &= n + r_{e_2} + r_{e_1 b_1} - r_{b_1}. \end{aligned} \quad (33)$$

When $R_1 < r_1$, namely, (28) holds, by Lemma 4, we can know that there does not exist K_1 such that closed-loop system (6) is impulse controllable. This completes the proof. □

Theorem 7. *Consider system (1). If*

$$\begin{aligned} &\operatorname{rank}\left(\begin{bmatrix} A_{32}^{11} & A_{33}^{11} & A_{33}^{12} \\ A_{32}^{21} & A_{33}^{21} & A_{33}^{22} \end{bmatrix}\right) \geq n - r_{e_1 b_1} - r_{e_2 b_2} - r_{b_1}, \\ &\Omega_K = \bigcup_{r_1 \leq i \leq R_1} \left\{K_{1i} \cap K_{2i}\right\} \neq \phi, \end{aligned} \quad (34)$$

where

$$\begin{aligned} K_{1i} &= \left\{K_1 \mid \operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right) = i\right\}, \\ K_{2i} &= \left\{K_1 \mid n + \operatorname{rank}\left(\overline{E}_K\right) = i\right\}, \end{aligned} \quad (35)$$

then one can find $K_1 \in \Omega_K$ such that the closed-loop system is impulse controllable.

Proof. The proof of Theorem 7 is similar to that of Theorem 6; the details are omitted. □

Remark 8. In the case of $R_1 < r_1$, there does not exist gain matrix K_1 such that the closed-loop system is impulse controllable. When $R_1 \geq r_1$, the existence of gain matrix K_1 can be considered in terms of probability. Namely, assume there exists a topological space, in which we can define a nonnull set $\widetilde{\Omega}$ that expresses the intersection of the dense set and any subset of that topological space. Then based on the perspective of probability, we can easily obtain this kind of $\widetilde{\Omega}$.

Based on this idea, we know that K_{1R_1} is a Zariski open set according to Lemma 3. We also know that almost every K_1 can make $\operatorname{rank}(\left[\begin{smallmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{smallmatrix}\right]) = R_1$. Namely, K_{1R_1} is a dense set. According to Lemma 5 we know that there always exists K_1 such that $n + \operatorname{rank}(\overline{E}_K) = R_1$, and K_{2R_1} is a subset of the topological space $R^{m\times n_1}$. Then based on the perspective of probability, we can easily get this kind of $\widetilde{\Omega}_K$. If $\widetilde{\Omega}_K \neq \emptyset$, we can find $K_1 \in \widetilde{\Omega}_K$ such that the closed-loop system is impulse controllable, where

$$\widetilde{\Omega}_K = K_{1R_1} \cap K_{2R_1}. \tag{36}$$

Remark 9. When $\widetilde{\Omega}_K \neq \emptyset$ and $n + \operatorname{rank}(\overline{E}_K) = R_1$, we can obtain K_1 as the following form:

$$K_1 = Q\begin{bmatrix} K_{d11} & \left[-\left(\Sigma^1_{B_1}\right)^{-1} E^1_{22} \;\; 0\right] + \Delta \\ K_{d21} & \left[K_{d22} \;\; K_{d23}\right] \end{bmatrix} V^T, \tag{37}$$

where $K_{d11} \in R^{r_b\times(r_{e_1b_1}-r_{b_1})}$, $K_{d21} \in R^{(l_1-r_{b_1})\times(r_{e_1b_1}-r_{b_1})}$, $K_{d22} \in R^{(l_1-r_{b_1})\times(r_{e_1}+r_{b_1}-r_{e_1b_1})}$, and $K_{d23} \in R^{(l_1-r_{b_1})\times(n-r_{e_1})}$ are four arbitrary parameters matrices, $\Delta \in R^{r_{b_1}\times(n-r_{e_1b_1}+r_{b_1})}$ is full column rank parameter matrix, and $\operatorname{rank}(\Delta) = R_1 - n - r_{e_1b_1} + r_{b_1} - r_{e_2}$. At the same time, if K_1 which is obtained by the above process satisfies formula (27), then K_1 is the gain matrix. Let

$$Q_1{}^T K_1 V_1 = \begin{bmatrix} K_{d11} & K_{d12} & K_{d13} \\ K_{d21} & K_{d22} & K_{d23} \end{bmatrix}, \tag{38}$$

in which matrix-blocks are compatible. Then we have

$$U\overline{E}_K V = \begin{bmatrix} \Sigma^1_1 & 0 & 0 & 0 & 0 & 0 \\ E^1_{21} + \Sigma^1_1 K_{d11} & E^1_{22} + \Sigma^1_{B_1} K_{d12} & \Sigma^1_{B_1} K_{d13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma^2_1 & 0 & 0 \\ 0 & 0 & 0 & E^2_{21} & E^2_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{39}$$

where $U = \left[\begin{smallmatrix} U_1 & \\ & U_2 \end{smallmatrix}\right]$, $V = \left[\begin{smallmatrix} V_1 & \\ & V_2 \end{smallmatrix}\right]$.

By $\operatorname{rank}(\overline{E}_K) = R_1 - n$ and (39) we can get the following formula:

$$\begin{aligned} &\operatorname{rank}\left(\left[E^1_{22} + \Sigma^1_{B_1} K_{d12} \;\; \Sigma^1_{B_1} K_{d13}\right]\right) \\ &\quad = R_1 - n - r_{e_1b_1} + r_{b_1} - r_{e_2}. \end{aligned} \tag{40}$$

Obviously, the matrix $[K_{d12} \;\; K_{d13}]$ can be expressed as the following form:

$$\begin{gathered} \left[K_{d12} \;\; K_{d13}\right] = \left[-\left(\Sigma^1_{B_1}\right)^{-1} E^1_{22} \;\; 0\right] + \Delta, \\ \operatorname{rank}(\Delta) = R_1 - n - r_{e_1b_1} + r_{b_1} - r_{e_2}. \end{gathered} \tag{41}$$

Next, we discuss the problem of impulse observability.

Theorem 10. *Consider system (1). If*

$$\begin{gathered} \operatorname{rank}\left(\begin{bmatrix} \overline{E} & \overline{A} & \overline{B}_1 & 0 \\ 0 & \overline{E} & 0 & \overline{B}_1 \\ 0 & \overline{C}_2 & 0 & 0 \end{bmatrix}\right) \leq \operatorname{rank}\left(\begin{bmatrix} \overline{E} & \overline{A} \\ 0 & \overline{E} \\ 0 & \overline{C}_2 \\ \overline{C}_1 & 0 \\ 0 & \overline{C}_1 \end{bmatrix}\right), \\ \operatorname{rank}\left(\begin{bmatrix} A^1_{32} & A^1_{33} & A^{12}_{33} \\ A^{21}_{22} & A^{21}_{23} & A^2_{23} \\ A^{21}_{32} & A^{21}_{33} & A^2_{33} \\ 0 & 0 & C^2_{13} \end{bmatrix}\right) \geq n - r_{e_1b_1} - r_{b_1} - r_{e_2}, \\ \Omega_L = \bigcup_{r_2\leq i\leq R_2} \{L_{1i} \cap L_{2i}\} \neq \phi, \end{gathered} \tag{42}$$

where

$$\begin{gathered} L_{1i} = \left\{ L \mid \operatorname{rank}\left(\begin{bmatrix} \overline{E}_L & \overline{A} \\ 0 & \overline{E}_L \\ 0 & \overline{C}_2 \end{bmatrix}\right) = i \right\}, \\ L_{2i} = \left\{ L \mid n + \operatorname{rank}\left(\overline{E}_L\right) = i \right\}, \end{gathered} \tag{43}$$

then one can find $L \in \Omega_L$ such that closed-loop system (11) is impulse observable.

Proof. Note that

$$\begin{bmatrix} \overline{E}_L & \overline{A} \\ 0 & \overline{E}_L \\ 0 & \overline{C}_2 \end{bmatrix} = \begin{bmatrix} \overline{E} & \overline{A} \\ 0 & \overline{E} \\ 0 & \overline{C}_2 \end{bmatrix} + \begin{bmatrix} \overline{B}_1 & 0 \\ 0 & \overline{B}_1 \\ 0 & 0 \end{bmatrix} \overline{L}_1 \begin{bmatrix} \overline{C}_1 & 0 \\ 0 & \overline{C}_1 \end{bmatrix}, \tag{44}$$

where

$$\begin{gathered} \overline{L}_1 = \begin{bmatrix} \widetilde{L}_1 & 0 \\ 0 & \widetilde{L}_1 \end{bmatrix}, \\ \widetilde{L}_1 = \begin{bmatrix} L_1 & 0 \\ 0 & L_1 \end{bmatrix}. \end{gathered} \tag{45}$$

Denote

$$\begin{gathered} \mathcal{S}_3 = \operatorname{Diag}\{U_1, U_2, U_1, U_2, U_1, U_2\}, \\ \mathcal{S}_4 = \operatorname{Diag}\{V_1, V_2, V_1, V_2, Q_1, Q_1\}, \end{gathered} \tag{46}$$

when

$$\operatorname{rank}\left(\begin{bmatrix}\overline{E} & \overline{A} & \overline{B}_1 & 0\\ 0 & \overline{E} & 0 & \overline{B}_1\\ 0 & \overline{C}_2 & 0 & 0\end{bmatrix}\right) \le \operatorname{rank}\left(\begin{bmatrix}\overline{E} & \overline{A}\\ 0 & \overline{E}\\ 0 & \overline{C}_2\\ \overline{C}_1 & 0\\ 0 & \overline{C}_1\end{bmatrix}\right). \tag{47}$$

By Lemma 3, (15), and (16), we have

$$\begin{aligned} R_2 &= \max_{L_1}\left\{\operatorname{rank}\left(\begin{bmatrix}\overline{E}_L & \overline{A}\\ 0 & \overline{E}_L\\ 0 & \overline{C}_2\end{bmatrix}\right)\right\}\\ &= \operatorname{rank}\left(\begin{bmatrix}\overline{E} & \overline{A} & \overline{B}_1 & 0\\ 0 & \overline{E} & 0 & \overline{B}_1\\ 0 & \overline{C}_2 & 0 & 0\end{bmatrix}\right)\\ &= \operatorname{rank}\left(\begin{bmatrix}E_1 & 0 & A_1 & A_{12} & B_1 & 0\\ 0 & E_2 & A_{21} & A_2 & 0 & 0\\ 0 & 0 & E_1 & 0 & 0 & B_1\\ 0 & 0 & 0 & E_2 & 0 & 0\\ 0 & 0 & 0 & C_2 & 0 & 0\end{bmatrix}\right)\\ &= \operatorname{rank}\left(\mathcal{S}_3\begin{bmatrix}E_1 & 0 & A_{11} & A_{12} & B_1 & 0\\ 0 & E_2 & A_{21} & A_{22} & 0 & 0\\ 0 & 0 & E_1 & 0 & 0 & B_1\\ 0 & 0 & 0 & E_2 & 0 & 0\\ 0 & 0 & 0 & C_2 & 0 & 0\end{bmatrix}\mathcal{S}_4\right)\\ &= 2r_{e_1b_1} + 2r_{e_2} + \operatorname{rank}\left(\begin{bmatrix}A_{32}^1 & A_{33}^1 & A_{33}^{12}\\ A_{22}^{21} & A_{23}^{21} & A_{23}^2\\ A_{32}^{21} & A_{33}^{21} & A_{33}^2\\ 0 & 0 & C_{13}^2\end{bmatrix}\right). \end{aligned} \tag{48}$$

On the other hand, by Lemma 3, we have

$$\begin{aligned} r_2 &= \min_{L_1}\left\{n + \operatorname{rank}\left(\overline{E}_L\right)\right\}\\ &= n + \operatorname{rank}\left(\begin{bmatrix}\overline{E} & \overline{B}_1\end{bmatrix}\right) - \operatorname{rank}\left(\overline{B}_1\right)\\ &= n + \operatorname{rank}\left(\begin{bmatrix}E_1 & 0 & B_1 & 0\\ 0 & E_2 & 0 & 0\end{bmatrix}\right) - \operatorname{rank}\left(\begin{bmatrix}B_1 & 0\\ 0 & 0\end{bmatrix}\right)\\ &= n + r_{e_2} + r_{e_1b_1} - r_{b_1}. \end{aligned} \tag{49}$$

Since $R_2 > r_2$ and $\Omega_L \neq \phi$, by Lemma 4 there exists L_1 such that closed-loop system (11) is impulse observable. This completes the proof. □

Theorem 11. *Consider system (1). If*

$$\begin{gathered} \operatorname{rank}\left(\begin{bmatrix}\overline{E} & \overline{A} & \overline{B}_1 & 0\\ 0 & \overline{E} & 0 & \overline{B}_1\\ 0 & \overline{C}_2 & 0 & 0\end{bmatrix}\right) > \operatorname{rank}\left(\begin{bmatrix}\overline{E} & \overline{A}\\ 0 & \overline{E}\\ 0 & \overline{C}\\ \overline{C}_1 & 0\\ 0 & \overline{C}_1\end{bmatrix}\right),\\ \operatorname{rank}\left(\begin{bmatrix}0 & A_{23}^1 & A_{23}^{12}\\ 0 & A_{33}^1 & A_{33}^{12}\\ 0 & A_{23}^{21} & A_{23}^2\\ 0 & A_{33}^{21} & A_{33}^2\\ 0 & 0 & C_{13}^2\\ C_{13}^1 & 0 & 0\\ 0 & C_{13}^1 & 0\end{bmatrix}\right) \ge n + r_{e_1b_1} - 2r_{e_1} - r_{e_2} - r_{b_1},\\ \Omega_L = \bigcup_{r_2\le i\le R_2}\left\{L_{1i}\cap L_{2i}\right\} \neq \phi, \end{gathered} \tag{50}$$

then one can find $L \in \Omega_L$ such that closed-loop system (11) is impulse observable.

Proof. The proof of Theorem 11 is similar to that of Theorem 10; the details are omitted. □

4. Illustrative Examples

In this section, illustrative examples are considered to show the effectiveness of the proposed approach.

Example 1. Consider system (1) with the following parameters:

$$E_1 = \begin{bmatrix}1 & 3\\ 2 & 6\end{bmatrix},$$

$$E_2 = \begin{bmatrix}-2.3 & 4.5\\ 1.15 & -2.25\end{bmatrix},$$

$$A_{11} = \begin{bmatrix}-1 & 0.5\\ -2 & 1\end{bmatrix},$$

$$A_{22} = \begin{bmatrix}1 & 6\\ -0.5 & -3\end{bmatrix},$$

$$A_{12}=\begin{bmatrix}4 & 0.7\\ 8 & 1.4\end{bmatrix},\quad A_{21}=\begin{bmatrix}-3 & 1.5\\ 1.5 & -0.75\end{bmatrix},\quad B_1=\begin{bmatrix}0.5\\ 1\end{bmatrix},\quad B_2=\begin{bmatrix}2\\ -1\end{bmatrix}. \tag{51}$$

Note that

$$\begin{aligned}R_1 &= \max_{K_1}\left\{\operatorname{rank}\left(\begin{bmatrix}\overline{E}_K & 0 & 0\\ \overline{A} & \overline{E}_K & \overline{B}_2\end{bmatrix}\right)\right\}\\ &= \operatorname{rank}\left(\begin{bmatrix}E_1 & 0 & 0 & 0 & 0 & B_1 & 0\\ 0 & E_2 & 0 & 0 & 0 & 0 & 0\\ A_{11} & A_{12} & E_1 & 0 & 0 & 0 & B_1\\ A_{21} & A_{22} & 0 & E_2 & B_2 & 0 & 0\end{bmatrix}\right)\\ &= 1+\operatorname{rank}\left(\begin{bmatrix}2 & 6 & 0 & 0\\ 0 & 0 & -2.3 & 4.5\\ -1 & 0.5 & 4 & 0.7\end{bmatrix}\right)=4,\end{aligned}$$

$$\begin{aligned}r_1 &= \min_{K_1}\left\{n+\operatorname{rank}\left(\overline{E}_K\right)\right\}\\ &= \min_{K_1}\left\{n+\operatorname{rank}\left(\begin{bmatrix}E_1+B_1K_1 & 0\\ 0 & E_2\end{bmatrix}\right)\right\}\\ &= n+\operatorname{rank}\left(\begin{bmatrix}E_1 & 0 & B_1 & 0\\ 0 & E_2 & 0 & 0\end{bmatrix}\right)\\ &\quad -\operatorname{rank}\left(\begin{bmatrix}B_1 & 0\\ 0 & 0\end{bmatrix}\right)\\ &= 4+\operatorname{rank}\left(\begin{bmatrix}1 & 3 & 0 & 0 & 0.5\\ 2 & 6 & 0 & 0 & 1\\ 0 & 0 & -2.3 & 4.5 & 0\\ 0 & 0 & 1.15 & -2.25 & 0\end{bmatrix}\right)\\ &\quad -\operatorname{rank}\left(\begin{bmatrix}0.5\\ 1\end{bmatrix}\right)=5.\end{aligned} \tag{52}$$

$R_1 < r_1$ does not meet the necessary condition of impulse controllability.

By Theorem 6, there does not exist K_1 such that closed-loop system (6) is impulse controllable.

We can design a derivative feedback controller for the first subsystem as follows:

$$u_1=-K_1\dot{x}_1,\quad K_1=\begin{bmatrix}a & b\end{bmatrix}, \tag{53}$$

where a, b are arbitrary parameters. We have

$$\begin{aligned}\operatorname{rank}\left(\begin{bmatrix}\overline{E}_K & 0 & 0\\ \overline{A} & \overline{E}_K & \overline{B}_2\end{bmatrix}\right) &= \operatorname{rank}\left(\begin{bmatrix}E_1+B_1K_1 & 0 & 0 & 0 & 0\\ 0 & E_2 & 0 & 0 & 0\\ A_{11} & A_{12} & E_1+B_1K_1 & 0 & 0\\ A_{21} & A_{22} & 0 & E_2 & B_2\end{bmatrix}\right)\\ &= \operatorname{rank}\left(\begin{bmatrix}\frac{a}{2}+1 & \frac{b}{2}+3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ a+2 & b+6 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -2.3 & 4.5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1.15 & -2.25 & 0 & 0 & 0 & 0 & 0\\ -1 & 0.5 & 4 & 0.7 & \frac{a}{2}+1 & \frac{b}{2}+3 & 0 & 0 & 0\\ -2 & 1 & 8 & 1.4 & a+2 & b+6 & 0 & 0 & 0\\ -3 & 1.5 & 1 & 6 & 0 & 0 & -2.3 & 4.5 & 2\\ -1.5 & 0.75 & -0.5 & -3 & 0 & 0 & 1.15 & -2.25 & -1\end{bmatrix}\right)\le 5,\end{aligned} \tag{54}$$

$$n+\operatorname{rank}\left(\overline{E}_K\right)=n+\operatorname{rank}\left(\begin{bmatrix}E_1+B_1K_1 & 0\\ 0 & E_2\end{bmatrix}\right)=4+\operatorname{rank}\left(\begin{bmatrix}\frac{a}{2}+1 & \frac{b}{2}+3 & 0 & 0\\ a+2 & b+6 & 0 & 0\\ 0 & 0 & -2.3 & 4.5\\ 0 & 0 & 1.15 & -2.25\end{bmatrix}\right)\ge 6.$$

For any derivative feedback gain K_1, equality (26) in Lemma 4 can not hold. That means there does not exist K_1 such that closed-loop system (6) is impulse controllable.

Example 2. Consider system (1) with the following parameters:

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\quad E_2 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix},\quad A_{11} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix},\quad A_{22} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\quad A_{12} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix},\quad A_{21} = \begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix},\quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\quad B_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}. \tag{55}$$

Note that

$$\begin{aligned} R_1 &= \max_{K_1} \left\{ \operatorname{rank} \left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix} \right) \right\} \\ &= \operatorname{rank} \left(\begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & B_1 & 0 \\ 0 & E_2 & 0 & 0 & 0 & 0 & 0 \\ A_{11} & A_{12} & E_1 & 0 & 0 & 0 & B_1 \\ A_{21} & A_{22} & 0 & E_2 & B_2 & 0 & 0 \end{bmatrix} \right) \\ &= 6, \\ r_1 &= \min_{K_1} \left\{ n + \operatorname{rank} \left(\overline{E}_K \right) \right\} \\ &= \min_{K_1} \left\{ n + \operatorname{rank} \left(\begin{bmatrix} E_1 + B_1 K_1 & 0 \\ 0 & E_2 \end{bmatrix} \right) \right\} \\ &= n + \operatorname{rank} \left(\begin{bmatrix} E_1 & 0 & B_1 & 0 \\ 0 & E_2 & 0 & 0 \end{bmatrix} \right) \\ &\quad - \operatorname{rank} \left(\begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix} \right) \\ &= 4 + \operatorname{rank} \left(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right) - \operatorname{rank} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = 5. \end{aligned} \tag{56}$$

$R_1 > r_1$ satisfies the necessary condition of impulse controllability. By Theorem 7, we can design proportional feedback controller for the first subsystem as follows:

$$u_1 = K_d x, \quad K_d = \begin{bmatrix} a & b \end{bmatrix}. \tag{57}$$

We have

$$\begin{aligned} &\operatorname{rank} \left(\begin{bmatrix} \overline{E} & 0 & 0 \\ \overline{A}_K & \overline{E} & \overline{B}_2 \end{bmatrix} \right) \\ &= \operatorname{rank} \left(\begin{bmatrix} E_1 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 \\ A_{11} + B_1 K_d & A_{12} & E_1 & 0 & 0 \\ A_{21} & A_{22} & 0 & E_2 & B_2 \end{bmatrix} \right) \\ &= \operatorname{rank} \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1+a & 2+b & 7 & 3 & 1 & 0 & 0 & 0 & 0 \\ 5 & 6 & 2 & 4 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 3 & 0 & 0 & 2 & 0 & 3 \\ 4 & 6 & 2 & 4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right) \\ &= 5, \\ &n + \operatorname{rank} \left(\overline{E} \right) = 4 + \operatorname{rank} \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right) = 6, \\ &\operatorname{rank} \left(\begin{bmatrix} \overline{E} & 0 & 0 \\ \overline{A}_K & \overline{E} & \overline{B}_2 \end{bmatrix} \right) \neq n + \operatorname{rank} \left(\overline{E} \right), \end{aligned} \tag{58}$$

where $\overline{A}_K = \begin{bmatrix} A_{11} + B_1 K_d & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$.

We can conclude that there does not exist proportional feedback K_d such that closed-loop system (6) is impulse controllable.

Example 3. Consider system (1) with the parameters as Example 2.

$R_1 > r_1$ satisfies the necessary condition of impulse controllability. Further, by Theorem 7 we can design a derivative feedback controller for the first subsystem as follows:

$$u_1 = -K_1 \dot{x}_1, \quad K_1 = \begin{bmatrix} a & b \end{bmatrix}. \tag{59}$$

Then we have

$$\begin{aligned}
&\operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right) \\
&= \operatorname{rank}\left(\begin{bmatrix} E_1 + B_1K_1 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 \\ A_{11} & A_{12} & E_1 + B_1K_1 & 0 & 0 \\ A_{21} & A_{22} & 0 & E_2 & B_2 \end{bmatrix}\right) \\
&= \operatorname{rank}\left(\begin{bmatrix} 1+a & b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 7 & 3 & 1+a & b & 0 & 0 & 0 \\ 5 & 6 & 4 & 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 3 & 0 & 0 & 2 & 0 & 3 \\ 4 & 6 & 2 & 4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right) \\
&= 3 + \operatorname{rank}\left(\begin{bmatrix} 1+a & b & 0 \\ 5 & 6 & 2 \\ 4 & 6 & 4 \end{bmatrix}\right), \\
&n + \operatorname{rank}\left(\overline{E}_K\right) = 5 + \operatorname{rank}\left(\begin{bmatrix} 1+a & b \end{bmatrix}\right).
\end{aligned} \tag{60}$$

In fact, by the above derivation, we can give arbitrary derivative feedback controller such that closed-loop system (6) is impulse controllable. When $K_1 = [1 \ \ -1]$, we have

$$\begin{aligned}
&\operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right) \\
&= \operatorname{rank}\left(\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 7 & 3 & 2 & -1 & 0 & 0 \\ 5 & 6 & 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 3 & 0 & 0 & 2 & 3 \\ 4 & 6 & 2 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}\right) = 6, \\
&n + \operatorname{rank}\left(\overline{E}_K\right) = 4 + \operatorname{rank}\left(\begin{bmatrix} 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right) = 6,
\end{aligned} \tag{61}$$

and we can conclude that closed-loop system (6) is impulse controllable under the above derivative feedback controller.

Illustrative Examples 2 and 3 are provided for demonstrating the different effects of proportional feedback and derivative feedback. We can find that our method of derivative feedback can change impulsive controllability when the method of proportional feedback is unable to do it.

Example 4. The hypersonic vehicle is modeled as in [27], and the mathematical model (62) of hypersonic vehicle is established in [28]. Consider

$$\begin{aligned}
\dot{\gamma} &= \omega_x - \tan\vartheta\left(\omega_y \cos\gamma - \omega_z \sin\gamma\right), \\
\dot{\alpha} &= b_3\omega_z + b_4\alpha + b_4\dot{\alpha} + b_8\delta z - \frac{\omega_x\beta}{57.3}, \\
\dot{\beta} &= a_1\omega_x + a_2\omega_y + a_5\beta + a_5\dot{\beta} + b_6\delta x + b_6\delta x + a_7\delta y \\
&\quad + \frac{\omega_x\alpha}{57.3}, \\
\dot{\omega}_x &= c_1\omega_x + c_2\omega_y + c_4\alpha + c_5\beta + c_6\delta x + c_7\delta y \\
&\quad + \left(I_y - I_z\right)\frac{\omega_z\omega_x}{57.3I_x}, \\
\dot{\omega}_y &= b_1\omega_x + b_2\omega_y + b_5\beta + b_5\dot{\beta} + b_6\delta x + b_7\delta y \\
&\quad + \left(I_z - I_x\right)\frac{\omega_z\omega_x}{57.3I_y}, \\
\dot{\omega}_z &= a_3\omega_z + a_4\alpha + a_4\dot{\alpha} + a_8\delta z + \left(I_x - I_y\right)\frac{\omega_x\omega_y}{57.3I_z},
\end{aligned} \tag{62}$$

where γ denotes roll angle, α denotes angle of attack, β denotes yaw angle, and ω_x, ω_y, and ω_z denote rotating angular velocity components in the coordinates x, y, and z.

Various restrictions for the flight of the hypersonic technology need to be considered. In most cases, the parameters of hypersonic technology systems change all the time, with derivative coefficient matrix into singular matrix; by [29] we describe the hypersonic technology systems as descriptor systems. By Assumption 2.1 of [30] if the equilibrium of nonlinear singular system $E\dot{x} = f(x)$ is $x_e = 0$ then the locally nonimpulsiveness of this system is equivalent to (deg(det($sE - A$)) = rank(E) $\forall s \in \mathbf{C}$, where $A = (\partial f/\partial x)(0)$). Therefore nonlinear large-scale interconnected descriptor systems (62) can be linearized at equilibrium point $(\gamma, \alpha, \beta, \omega_x, \omega_y, \omega_z) = (0,0,0,0,0,0)$; then we obtain the following system:

$$\begin{aligned}
E_1\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + B_1u_1, \\
E_2\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2u_2,
\end{aligned} \tag{63}$$

where

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - b_4 & 0 \\ 0 & -a_4 & -1 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - a_5 & 1 \\ 0 & -b_5 & 1 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b_4 & a_3 \\ 0 & a_4 & a_4 \end{bmatrix},$$

$$A_{12}=\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix},$$
$$A_{21}=\begin{bmatrix}c_1&c_5&c_2\\a_1&a_5&a_2\\b_1&b_5&b_2\end{bmatrix},$$
$$A_{22}=\begin{bmatrix}0&0&0\\0&0&0\\0&c_4&0\end{bmatrix},$$
$$B_1=\begin{bmatrix}0\\b_8\\a_8\end{bmatrix},$$
$$B_2=\begin{bmatrix}b_6&b_7\\a_6&a_7\\b_6&b_6\end{bmatrix},$$
$$x_1=\begin{bmatrix}\gamma\\\alpha\\\beta\end{bmatrix},$$
$$x_2=\begin{bmatrix}\omega_x\\\omega_y\\\omega_z\end{bmatrix},$$
$$u_1=\delta_z,$$
$$u_2=\begin{bmatrix}\delta_x\\\delta_y\end{bmatrix}. \tag{64}$$

System (63) is the large-scale interconnected descriptor systems for the conditions that $1-b_4=0$, $1-a_5=0$, and E_1 and E_2 are singular matrices. Furthermore, we find $\mathrm{rank}(\left[\begin{smallmatrix}\overline{E}&0\\\overline{A}&\overline{E}\end{smallmatrix}\right])\neq n+\mathrm{rank}(\overline{E})$, so impulse exists in the system. For hypersonic vehicle, it may generate impulse response to the vehicle with the vehicle flight with acceleration, deceleration, and sharp turns, which may lead to the instability of the system or may even destroy the vehicle, and hence it is not expected to exist.

Consider system (63) with the following parameters [22, 23]:

$$a_1=0.5,$$
$$a_2=0.8,$$
$$a_3=2,$$
$$a_4=0.5,$$
$$a_5=1,$$
$$a_6=0,$$
$$a_7=0.6,$$
$$a_8=0.5,$$
$$b_1=0.3,$$
$$b_2=0.4,$$
$$b_4=1,$$
$$b_5=1,$$
$$b_6=0,$$
$$b_7=1,$$
$$b_8=0.5,$$
$$c_1=0.1,$$
$$c_2=0.4,$$
$$c_4=0.3,$$
$$c_5=0.4. \tag{65}$$

Note that

$$\begin{aligned}R_1&=\max_{K_1}\left\{\mathrm{rank}\left(\begin{bmatrix}\overline{E}_K&0&0\\\overline{A}&\overline{E}_K&\overline{B}_2\end{bmatrix}\right)\right\}\\&=\mathrm{rank}\left(\begin{bmatrix}E_1&0&0&0&0&B_1&0\\0&E_2&0&0&0&0&0\\A_1&A_{12}&E_1&0&0&0&B_1\\A_{21}&A_{22}&0&E_2&B_2&0&0\end{bmatrix}\right)\\&=11,\\r_1&=\min_{K_1}\left\{n+\mathrm{rank}\left(\overline{E}_K\right)\right\}\\&=\min_{K_1}\left\{n+\mathrm{rank}\left(\begin{bmatrix}E_1+B_1K_1&0\\0&E_2\end{bmatrix}\right)\right\}\end{aligned}$$

$$= n + \operatorname{rank}\left(\begin{bmatrix} E_1 & 0 & B_1 & 0 \\ 0 & E_2 & 0 & 0 \end{bmatrix}\right) - \operatorname{rank}\left(\begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix}\right) = 10. \quad (66)$$

$R_1 > r_1$ satisfies the necessary condition of impulse controllability. By Theorem 7, we can design a proportional feedback controller for the first subsystem as follows:

$$u_1 = K_d x, \quad K_d = \begin{bmatrix} a & b & c \end{bmatrix}. \quad (67)$$

Then we have

$$\operatorname{rank}\left(\begin{bmatrix} \overline{E} & 0 & 0 \\ \overline{A}_K & \overline{E} & \overline{B}_2 \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} E_1 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 \\ A_{11} + B_1 K_d & A_{12} & E_1 & 0 & 0 \\ A_{21} & A_{22} & 0 & E_2 & B_2 \end{bmatrix}\right) = 9 + \operatorname{rank}\left(\begin{bmatrix} -0.5 & -1 \\ 1 & 2 \end{bmatrix}\right) = 10,$$

$$n + \operatorname{rank}\left(\overline{E}\right) = 6 + \operatorname{rank}\left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}\right) = 11. \quad (68)$$

Obviously,

$$\operatorname{rank}\left(\begin{bmatrix} \overline{E} & 0 & 0 \\ \overline{A}_K & \overline{E} & \overline{B}_2 \end{bmatrix}\right) \neq n + \operatorname{rank}\left(\overline{E}\right). \quad (69)$$

By the above derivation, for any proportional feedback gain K_d, equality (26) in Lemma 4 can not hold. That means there does not exist K_d such that closed-loop system (6) is impulse controllable. Further, by Theorem 7 we can design a derivative feedback controller for the first subsystem as follows:

$$u_1 = -K_1 \dot{x}_1, \quad K_1 = \begin{bmatrix} a & b & c \end{bmatrix}. \quad (70)$$

Then we have

$$\operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} E_1 + B_1 K_1 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 \\ A_{11} & A_{12} & E_1 + B_1 K_1 & 0 & 0 \\ A_{21} & A_{22} & 0 & E_2 & B_2 \end{bmatrix}\right)$$
$$= 6 + \operatorname{rank}\left(\begin{bmatrix} 1 + 0.1a & 0.1b & 0.1c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2a & -0.5 + 0.2a & -1 + 0.2c & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 + 0.1a & 0.1b & 0.1c \\ 0 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.2a & -0.5 + 0.2b & -1 + 0.2c \end{bmatrix}\right), \quad (71)$$
$$n + \operatorname{rank}\left(\overline{E}_K\right) = 9 + \operatorname{rank}\left(\begin{bmatrix} 1 + 0.1a & 0.1b & 0.1c \\ 0.2a & -0.5 + 0.2b & -1 + 0.2c \end{bmatrix}\right).$$

When a, b, and c satisfy

$$\operatorname{rank}\left(\begin{bmatrix} 1 + 0.1a & 0.1b & 0.1c \\ 0.2a & -0.5 + 0.2b & 1 + 0.2c \end{bmatrix}\right) = 2, \quad (72)$$

for example, $a = 0$, $b = 1$, $c = 0$, we have

$$\operatorname{rank}\left(\begin{bmatrix} \overline{E}_K & 0 & 0 \\ \overline{A} & \overline{E}_K & \overline{B}_2 \end{bmatrix}\right) = n + \operatorname{rank}\left(\overline{E}_K\right) = 11. \quad (73)$$

Then closed-loop system (6) is impulse controllable.

In the practical example, we can find that our method of derivative feedback can change impulse controllability when the method of proportional feedback is unable to do it. So, derivative feedback method is more effective than proportional feedback in the problem of impulse elimination.

5. Conclusions

In this paper, the problem of impulse controllability and impulse observability of large-scale interconnected descriptor system by derivative feedback has been studied. Necessary conditions for the existence of a derivative feedback controller for the first subsystem of the large-scale interconnected descriptor systems ensuring the second subsystem to be impulse controllable and impulse observable are derived, respectively. The examples have been presented to demonstrate the applicability of the proposed approach. Recently, to ensure the reliability and safety of modern large-scale industrial processes, data-driven methods have been receiving considerably increasing attention. The research on the modeling and the impulsive controllability/observability of interconnected descriptor systems based on large data and fault diagnosis of complex system will be the next research focus [31–33].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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