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Research Article

High-Order Feedback Iterative Learning Control Algorithm with Forgetting Factor

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A novel iterative learning control (ILC) algorithm is proposed to produce output curves that pass close to the desired trajectory. The key advantage of the proposed algorithm is introducing forgetting factor, which is a function of the number of iterations. Due to the forgetting factor characteristic of ILC, the proposed scheme not only stabilizes the nonlinear system with uncertainties but also weakens interference on the tracking desired trajectory. Simulation examples are included to demonstrate feasibility and effectiveness of the proposed algorithm.

1. Introduction

Control schemes for tracking problems can be divided into two steps: trajectory planning and tracking control. In these schemes, the trajectory planner attempts to generate a desired trajectory, which is set in advance. Then, the controller, which is designed to track the desired trajectory, focuses on the system dynamics to generate a sequence of inputs. To improve the accuracy in trajectory tracking, various control schemes such as feedback control [1], robust control [2], and iterative learning control [3–5] have been developed. Iterative learning control (ILC) is a control methodology for tracking a desired trajectory in repetitive systems; those were widely applied in practical engineering such as robotics [6, 7], semiconductors [8], and chemical processes [9, 10]. The prime strategy of ILC algorithms is to refine the input from one trial in order to improve the performance of the system on the next trial.

The algorithm based on ILC is further improved by combining with existing feedback controller, such as PID [11], ALINEA [12], model-free adaptive control [13], and other general feedback controls [14, 15]. These combinations can retain the functionality of existing feedback loop such as robustness and meanwhile enjoy the extra performance improvement from ILC. However, some ILC algorithm's convergence process is slow. To make the system track expectations more quickly and precisely, a high-order feedback iterative learning control [16] is proposed. Although it is applied to the linear system, the algorithm itself is of great value. However, when the system exist uncertainty and nonrepetitive disturbance, the control methods often achieve poor effect in practical applications. In order to reduce the effects of disturbance throughout the iterative process, the paper introduces forgetting factor, which can filter the signal towards the direction of iteration.

The contribution of this paper is a combination of the high-order feedback iterative learning control and forgetting factor. Through a new ILC algorithm, the perfect tracking control performance of the robot manipulators can be achieved in the repetitive nonlinear time-varying systems [17, 18] with uncertainty and disturbance [19, 20]. A rigorous proof based on the Lipschitz-like approach is given to guarantee the stability and convergence of nonlinear system.

The remainder of this paper is organized as follows. The iterative learning control problem is described in a general setting in Section 2. A designed controller is presented briefly in Section 3. A robust convergence analysis of tracking properties under the high-order feedback ILC controller with forgetting factor is performed in Section 4. Simulation results are given in Section 5. Finally, in Section 6, conclusions are drawn.

2. Problem Description

Now, let us consider the repetitive nonlinear time-varying systems with uncertainty and disturbance:

$$\begin{aligned} \dot{x}_i(t) &= \mathbf{f}(x_i(t),t) + \mathbf{B}(x_i(t),t)u_i(t) + \mathbf{w}_i(x_i(t),t), \\ y_i(t) &= \mathbf{C}x_i(t) + v_i(t), \end{aligned} \tag{1}$$

where i and t are the iteration index and continuous time, respectively. $\mathbf{u}_i(t) \in R^m$ is the control variable. $x_i(t) \in R^m$ is the state variable; $y_i(t) \in R^m$ is the output of the system at the ith trial. $w_i(x_i(t),t)$ and $v_i(t)$ are the uncertain item and disturbance term, respectively. $\mathbf{C} \in R^{r\times n}$ is constant matrix. $\mathbf{f}(\cdot)$, $\mathbf{B}(\cdot)$, and $\mathbf{w}(\cdot)$ are nonlinear functions.

Assumption 1. Functions $\mathbf{f}$, $\mathbf{B}$, and $\mathbf{w}$ are uniformly globally Lipschitz with respect to x on a compact set $\Omega \in R^m \times R^m \times [1,T]$:

$$\begin{aligned} \|\mathbf{f}(x_{i+1}(t),t) - \mathbf{f}(x_i(t),t)\| &\le k_f \|x_{i+1}(t) - x_i(t)\|, \\ \|\mathbf{B}(x_{i+1}(t),t) - \mathbf{B}(x_i(t),t)\| &\le k_B \|x_{i+1}(t) - x_i(t)\|, \\ \|\mathbf{w}(x_{i+1}(t),t) - \mathbf{w}(x_i(t),t)\| &\le k_w \|x_{i+1}(t) - x_i(t)\|, \end{aligned} \tag{2}$$

where k_f, k_B, and k_w are the Lipschitz constants.

Assumption 2. Initial error of the system at the ith trial satisfies this criterion:

$$\|x_{i+1}(0) - x_i(0)\| \le b_{x_0}, \quad \forall i. \tag{3}$$

Let us make the definitions of norm to simplify formula:

$$\begin{aligned} b_v &\triangleq \sup_{t\in[0,T]} \|v_{l+1}(t)\|_\lambda, \\ b_C &\triangleq \sup \|\mathbf{C}\|_\lambda, \\ b_\varphi &\triangleq \sup_{t\in[0,T]} \|\boldsymbol{\varphi}_k(t)\|_\lambda, \\ b_B &\triangleq \sup_{\forall x\in R^n} \left(\sup_{t\in[0,T]} \|\mathbf{B}(x_i(t),t)\|_\lambda \right), \\ \partial\tilde{x}_l &= \max\{\sup \|\partial x_{l+1}\|_\lambda\}. \end{aligned} \tag{4}$$

Let

$$\begin{aligned} e_i &= y_d - y_i = \mathbf{C}\partial x_i - v_i, \\ \dot{e}_i &= \dot{y}_d - \dot{y}_i = \mathbf{C}\partial\dot{x}_i - \dot{v}_i, \\ \mathbf{B}_i &\triangleq \mathbf{B}(x_i(t),t), \\ \partial u_i(t) &\triangleq \mathbf{u}_d(t) - \mathbf{u}_i(t), \\ \dot{x}_d(t) &= \mathbf{f}(x_d(t),t) + \mathbf{B}(x_d(t),t)u_d(t) \\ &\quad + \mathbf{w}(x_d(t),t) \triangleq f_d + B_d u_d + w_d, \\ \partial x_i(t) &\triangleq x_d(t) - x_i(t), \\ \partial f_i(t) &\triangleq f_d - \mathbf{f}(x_i(t),t), \\ \partial B_i(t) &\triangleq B_d - \mathbf{B}(x_i(t),t), \\ \partial w_i(t) &\triangleq w_d - \mathbf{w}_i(x_i(t),t); \end{aligned} \tag{5}$$

we have

$$\begin{aligned} \partial\dot{x}_i &= \dot{x}_d - \dot{x}_i \\ &= f_d + B_d u_d + w_d \\ &\quad - (\mathbf{f}(x_i(t),t) + \mathbf{B}(x_i(t),t)u_i(t) + \mathbf{w}(x_i(t),t)) \\ &= \partial f_i(t) + B_d u_d - \mathbf{B}(x_i(t),t)[u_d - \partial u_i(t)] \\ &\quad + \partial w_i(t) \\ &= \partial f_i(t) + \partial B_i u_d - \mathbf{B}(x_i(t),t)\partial u_i(t) + \partial w_i(t), \end{aligned} \tag{6}$$

where x_d, u_d, and y_d are the desired state, desired control input, and desired output of system and $\triangleq$ indicate a formula is defined as another formula.

The high-order feedback iterative learning control algorithm with forgetting factor is applied to nonlinear time-varying systems. As $i \to \infty$, the bound of tracking error converges to a small neighborhood of the origin. If $\|\partial x_i(0)\|_\lambda$ and b_v tend to zero, the bound of tracking error asymptotically reaches zero, by ILC.

When $\varepsilon = 0$, $b_v = 0$, and $\|\partial x_i(0)\|_\lambda = 0$, we obtain $\lim_{i\to\infty}\|\partial u_i\|_\lambda = 0$. If $\lim_{i\to\infty}\|\partial x_i\|_\lambda = 0$, we have $\lim_{i\to\infty}\|e_i(t)\|_\lambda = \lim_{i\to\infty}\|\partial y_i\|_\lambda = 0$. At any given bound of tracking error ε^*, we can find a group parameters of $\boldsymbol{\varphi}_k$ and $\mathbf{G}_k$ to satisfy $\sup_{t\in[0,T]}\|e_i(t)\|_\lambda \le \varepsilon^*$, with $\forall i \ge M$.

3. Designed Controller

To make further improvement on the tracking precision of system, we will find an ideal input u_d by the method of iterative learning. And it can make the output of system close to the desired trajectory as far as possible. In this section, we will design an ILC to realize perfect tracking.

Denote the output tracking error $e_i = y_d - y_i$, where y_d is the given desired output trajectory, which is a solution to system (1); that is, there exists a unique desired input u_d, such that when $u = u_d$, the system has a unique desired state x_d satisfying

$$\begin{aligned} \dot{x}_d(t) &= \mathbf{f}(x_d(t),t) + \mathbf{B}(x_d(t),t)u_d(t) \\ &\quad + \mathbf{w}(x_d(t),t) \triangleq f_d + B_d u_d + w_d, \\ y_d(t) &= \mathbf{C}x_d(t) \triangleq \mathbf{C}x_d. \end{aligned} \tag{7}$$

Starting from an arbitrary continuous initial control input u_0, obtain the next control input u_1 and the subsequent series $\{u_i \mid i = 2,3,\ldots\}$ for system (1) by using a proper learning control updating law in such a way that when $i \to \infty$, $y_i \to y_d \pm \varepsilon^*$ in the presence of bounded uncertainty and disturbance.

We propose high-order feedback iterative learning controller as follows. At the ith ILC iteration, the control input u_i to system (1) satisfies

$$\mathbf{u}_{i+1}(t) = \mathbf{u}_{i+1}^{s}(t), \tag{8}$$

where $\mathbf{u}_{i+1}^{s}(t)$ is from the feedback iterative learning controller. The high-order feedback iterative learning controller is assumed to be in the following general form. The ILC law which includes tracking error of N previous iterations is used; that is,

$$\mathbf{u}_{i+1}^{s}(t) = \mathbf{u}_{i}(t) + \sum_{k=1}^{N} \boldsymbol{\varphi}_k(t)\, e_{l+1}(t), \tag{9}$$

where i indicates the iteration number, $l = i - k + 1$, N is the order of ILC law with $N \geq 1$, $\boldsymbol{\varphi}_k(t)$ is the feedback gain matrix, and $e_{l+1} = y_d - y_{l+1}$.

In the work, the forgetting factor β is introduced to the designed controller. The factor β is used to make a tradeoff between perfect learning and robustness, which can increase the robustness of ILC against uncertainty, disturbance, initialization error, and fluctuation of system dynamics:

$$\beta(i) = H^{-i}, \tag{10}$$

where i indicates the iteration number; we use β instead of $\beta(i)$ to simplify formula. $0 \leq \beta < 1$ is time-variant forgetting factor, which is based on its iterations. H is constant. If $i \to \infty$, then $\beta \to 0$.

The high-order feedback ILC controller with forgetting factor is constructed as follows:

$$\mathbf{u}_{i+1}(t) = \beta u_0(t) + (1-\beta)\,\mathbf{u}_i(t) + \sum_{k=1}^{N} \boldsymbol{\varphi}_k(t)\, e_{l+1}(t), \tag{11}$$

where i indicates the iteration number, $l = i - k + 1$, N is the order of ILC law with $N \geq 1$, and β is time-variant forgetting factor. $u_0(t)$ is the initial value of input, $\boldsymbol{\varphi}_k(t)$ is the feedback gain matrix, and $e_{l+1} = y_d - y_{l+1}$.

4. Convergence Analysis

From (1) and (11), we get

$$\begin{aligned}
\partial u_{i+1} &= u_d - \mathbf{u}_{i+1} \\
&= u_d - \beta u_0 - (1-\beta)\,\mathbf{u}_i - \sum_{k=1}^{N} \boldsymbol{\varphi}_k(t)\, e_{l+1}(t) \\
&= \beta \partial u_0 - (1-\beta)\,\partial u_i - \sum_{k=1}^{N} \boldsymbol{\varphi}_k(t)\left[C\partial x_{l+1} - v_{l+1}\right] \\
&= (1-\beta)\,\partial u_i - \sum_{k=1}^{N} \boldsymbol{\varphi}_k(t)\, C\partial x_{l+1} + \sum_{k=1}^{N} \boldsymbol{\varphi}_k(t)\, v_{l+1} \\
&\quad + \beta \partial u_0.
\end{aligned} \tag{12}$$

Let us take norm on both sides of (12) yields and define $b_{u_d} \triangleq \sup_{t\in[0,T]} \|u_d(t)\|_\lambda$. Using Assumptions 1 and 2 gives

$$\begin{aligned}
\|\partial u_{i+1}\|_\lambda &\leq \sum_{k=1}^{N} p_k (1-\beta)\,\|\partial u_l\|_\lambda + \sum_{k=1}^{N} b_c b_\varphi\,\|\partial \tilde{x}_l\|_\lambda \\
&\quad + \sum_{k=1}^{N} b_v b_\varphi + \beta\,\|\partial u_0\|_\lambda \\
&= \sum_{k=1}^{N} p_k (1-\beta)\,\|\partial u_l\|_\lambda + \xi_0 \sum_{k=1}^{N} \|\partial \tilde{x}_l\|_\lambda + \mu_0,
\end{aligned} \tag{13}$$

where $\mu_0 = N b_v b_\varphi + \beta\|\partial u_0\|_\lambda$, $\xi_0 = b_c b_\varphi$, and $\partial u_l = u_d - u_l$; we have

$$\begin{aligned}
\|\partial \tilde{x}_i\|_\lambda &= \left\|\partial \tilde{x}_i(0) + \int_0^t \partial \dot{\tilde{x}}\, d\tau\right\|_\lambda = \left\|\partial \tilde{x}_i(0)\right. \\
&\quad + \int_0^t \left[\partial f_i(t) + \partial B_i(t)\, u_d - \mathbf{B}(\tilde{x}_i(t), t)\,\partial u_i(t)\right. \\
&\quad \left.\left. + \partial w_i(t)\right] d\tau\right\|_\lambda \leq b_{x_0} + \int_0^t \left(\hat{e}\,\|\partial \tilde{x}_i\|_\lambda\right. \\
&\quad \left. + b_B\,\|\partial u_i\|_\lambda\right) d\tau
\end{aligned} \tag{14}$$

with $\hat{e} \triangleq k_f + b_{u_d} k_B + k_w$. Taking norm, we can obtain

$$\begin{aligned}
\left\|\int_0^t \|\tilde{x}(\tau)\|_\lambda\, d\tau\right\|_\lambda &= \sup_{t\in[0,T]} e^{-\lambda t} \int_0^t \|\tilde{x}(\tau)\|\,\lambda d\tau \\
&= \sup_{t\in[0,T]} e^{-\lambda t} \int_0^t \|\tilde{x}(\tau)\|_\lambda\, e^{-\lambda\tau} e^{-\lambda\tau} d\tau \\
&\leq \|\tilde{x}(\tau)\|_\lambda \sup_{t\in[0,T]} e^{-\lambda t} \int_v^t e^{\lambda\tau} d\tau \\
&= \|\tilde{x}(\tau)\|_\lambda \sup_{t\in[0,T]} \left[\frac{\left(1 - e^{-\lambda t}\right)}{\lambda}\right] \\
&\leq \|\tilde{x}(\tau)\|_\lambda\, \bigcirc\left(\lambda^{-1}\right)
\end{aligned} \tag{15}$$

with $\bigcirc(\lambda^{-1}) \triangleq (1 - e^{-\lambda T})/\lambda$; we get $\|\partial \tilde{x}_i\|_\lambda \leq b_{x_0} + \hat{e}\bigcirc(\lambda^{-1})\|\partial \tilde{x}_i\|_\lambda + b_B \bigcirc(\lambda^{-1})\|\partial u_i\|_\lambda$. This equation means

$$\|\partial \tilde{x}_i\|_\lambda \leq \frac{\left(b_{x_0} + b_B \bigcirc\left(\lambda^{-1}\right)\|\partial u_i\|_\lambda\right)}{1 - \hat{e}\bigcirc\left(\lambda^{-1}\right)}. \tag{16}$$

Inserting (16) into (13) gives

$$\begin{aligned}
\|\partial u_{i+1}\|_\lambda &\leq \sum_{k=1}^{N} p_k (1-\beta)\,\|\partial u_l\|_\lambda + \xi_0 \sum_{k=1}^{N} \|\partial \tilde{x}_l\|_\lambda + \mu_0 \\
&\leq \sum_{k=1}^{N} p_k (1-\beta)\,\|\partial u_l\|_\lambda
\end{aligned}$$

$$+\xi_0\sum_{k=1}^{N}\frac{\left(b_{x_0}+b_B\bigcirc\left(\lambda^{-1}\right)\left\|\partial u_l\right\|_\lambda\right)}{1-\hat{e}\bigcirc\left(\lambda^{-1}\right)}+\mu_0$$
$$=\sum_{k=1}^{N}\rho_k\left\|\partial u_l\right\|_\lambda+\varepsilon, \tag{17}$$

where

$$\rho_k=p_k\left(1-\beta\right)+\xi_0\frac{b_B\bigcirc\left(\lambda^{-1}\right)}{1-\hat{e}\bigcirc\left(\lambda^{-1}\right)},$$
$$\varepsilon=\xi_0\sum_{k=1}^{N}\frac{b_{x_0}}{1-\hat{e}\bigcirc\left(\lambda^{-1}\right)}+\mu_0. \tag{18}$$

Lemma 3. *Assume that there is a positive real sequence $\{a_n\}_1^\infty$ and the condition $a_n\le\tilde{\rho}_1a_{n-1}+\tilde{\rho}_2a_{n-2}+\cdots+\tilde{\rho}_Na_{n-N}+\tilde{\varepsilon}$, $(n=N+1,N+2,\ldots)$; $\tilde{\varepsilon}\ge 0$ is satisfied with $\tilde{\rho}_i\ge 0$ $(i=1,2,\ldots,N)$.*

If $\tilde{\rho}\le\sum_{i=1}^{N}\tilde{\rho}_i<1$, then $\lim_{n\to\infty}a_n\le\tilde{\varepsilon}/(1-\tilde{\rho})$. By Lemma 3, we choose a sufficiently large constant λ such that the following inequality holds when conditions $\tilde{\rho}_k<1$ and $\tilde{\rho}=\sum_{k=1}^{N}\rho_k<1$ are satisfied:

$$\lim_{i\to\infty}\left\|\partial u_i\right\|_\lambda\le\frac{\varepsilon}{\left(1-\tilde{\rho}\right)}. \tag{19}$$

Inserting $\partial y_i=y_d-y_i=\mathbf{C}x_d-\mathbf{C}x_i-v_i=\mathbf{C}\partial x_i-v$, we get $\|\partial y_i\|_\lambda\le b_c\|\partial x_i\|_\lambda+b_v$. Adding (16), (17), and (19), we get that the tracking error bound converges to a small neighborhood of the origin, as i goes to infinity. Meanwhile, the tracking error $\|\partial y_i\|_\lambda$, initial state error $\|\partial x_i(0)\|_\lambda$, and the bound of output disturbance item b_v have a linear relationship. If $\|\partial x_i(0)\|_\lambda$ and b_v tend to zero, the tracking error bound asymptotically reaches zero, by ILC.

When conditions $b_v=0$ and $\|\partial x_i(0)\|_\lambda=0$ are satisfied, from (17), we get $\varepsilon=0$ and from (19), we obtain $\lim_{i\to\infty}\|\partial u_i\|_\lambda=0$. Then, (16) gives $\lim_{i\to\infty}\|\partial x_i\|_\lambda=0$; we have $\lim_{i\to\infty}\|e_i(t)\|_\lambda=\lim_{i\to\infty}\|\partial y_i\|_\lambda=0$. According to limit definition, at any bound of given tracking error ε^*, we can choose a group parameter of $\boldsymbol{\varphi}_k$ and $\mathbf{G}_k$ in order to reach the conclusion of this theorem $\sup_{t\in[0,T]}\|e_i(t)\|_\lambda\le\varepsilon^*$, with $\forall i\ge M$.

5. Simulation

To demonstrate the effectiveness of the proposed ILC algorithm, we consider a two-degree-of-freedom planar manipulator with revolute joints and use the most basic nonlinear strong coupling example. This paper deals with nonlinear dynamical system (20) by using simulation method:

$$\mathbf{M}\left(\theta_i\right)\ddot{\theta}_i+\mathbf{C}\left(\theta_i,\dot{\theta}_i\right)\dot{\theta}_i+\mathbf{G}\left(\theta_i\right)+\mathbf{f}\left(\theta_i,t\right)=\mathbf{u}_i \quad (i=1,2,3,\ldots), \tag{20}$$

where $\theta_i\in R^n$ is the vector of state variables, $\mathbf{M}(\theta_i)\in R^{n\times n}$ is the inertia matrix, $\mathbf{C}(\theta_i,\dot{\theta}_i)\dot{\theta}_i\in R^n$ is the vector of the Coriolis and centripetal torques, $\mathbf{G}(\theta_i)\in R^n$ is the gravitational term, $\mathbf{f}(\theta_i,t)\in R^n$ is interference terms, and $\mathbf{u}_i\in R^n$ is the vector of control input.

Equation (20) can be rewritten. Let $x_{1i}=\theta_i$, $x_{2i}=\dot{\theta}_i=\dot{x}_{1i}$; we get

$$\begin{aligned}\dot{x}_{1i}&=x_{2i},\\ \dot{x}_{2i}&=\ddot{\theta}_i=\mathbf{M}^{-1}\left(\theta_i\right)\\ &\quad\cdot\left[\mathbf{u}_i-\mathbf{C}\left(\theta_i,\dot{\theta}_i\right)\dot{\theta}_i-\mathbf{G}\left(\theta_i\right)-\mathbf{f}\left(\theta_i,\dot{\theta}_i\right)\right]\\ &=\mathbf{M}^{-1}\left(x_{1i}\right)\\ &\quad\cdot\left[\mathbf{u}_i-\mathbf{C}\left(x_{1i},x_{2i}\right)x_{2i}-\mathbf{G}\left(x_{1i}\right)-\mathbf{f}\left(x_{1i},x_{2i}\right)\right].\end{aligned} \tag{21}$$

Let $\mathbf{X}_i=[x_{1i},x_{2i}]^T$, $y_i=\mathbf{X}_i$; (21) can be rewritten as

$$\begin{aligned}\dot{\mathbf{X}}_i&=\begin{bmatrix}x_{2i}\\ -\mathbf{M}^{-1}\left(x_{1i}\right)\left[\mathbf{C}\left(x_{1i},x_{2i}\right)+\mathbf{G}\left(x_{1i}\right)\right]\end{bmatrix}\\ &\quad+\begin{bmatrix}0\\ \mathbf{M}^{-1}\left(x_{1i}\right)\end{bmatrix}u_i\\ &\quad+\begin{bmatrix}0\\ -\mathbf{M}^{-1}\left(x_{1i}\right)\mathbf{f}\left(x_{1i},x_{2i}\right)\end{bmatrix},\\ y_i&=\mathbf{X}_i.\end{aligned} \tag{22}$$

Simulations are carried out on a planar directly driving two-joint robot. And the matrices for the robot arm in the state space are

$$\mathbf{M}\left(\theta_i\right)=\begin{bmatrix}a & b\cos\left(\theta_{2i}-\theta_{1i}\right)\\ b\cos\left(\theta_{2i}-\theta_{1i}\right) & c\end{bmatrix},$$
$$\mathbf{C}\left(\theta_i,\dot{\theta}_i\right)=\begin{bmatrix}-b\dot{\theta}_{2i}^2\sin\left(\theta_{2i}-\theta_{1i}\right)\\ -b\dot{\theta}_{2i}^2\sin\left(\theta_{2i}-\theta_{1i}\right)\end{bmatrix}, \tag{23}$$

where a, b, and c are three constants. $a=5.76794\,\mathrm{kg{\cdot}m^2}$, $b=1.473\,\mathrm{kg{\cdot}m^2}$, and $c=1.76794\,\mathrm{kg{\cdot}m^2}$.

The expected output trail is $\boldsymbol{\theta}_d=\left[2.63t\quad 2.8t\right]^T$. The interference term is $\mathbf{f}(t)=\left[\cos(5t)\quad\sin(5t)\right]^T$ and the initial state is considered as

$$x(0)=\left[0\quad 2.81\quad 0.18\quad 3.12\right]^T. \tag{24}$$

We choose a high-order feedback iterative learning control algorithm with forgetting factor:

$$\mathbf{u}_{i+1}(t)=\beta u_0(t)+\left(1-\beta\right)\mathbf{u}_i(t)+\sum_{k=1}^{N}\boldsymbol{\varphi}_k(t)\,e_{l+1}(t) \tag{25}$$

with $l=i-k+1$.

We choose a high-order feedback iterative learning control algorithm:

$$\mathbf{u}_{i+1}(t)=\mathbf{u}_i(t)+\sum_{k=1}^{N}\boldsymbol{\varphi}_k(t)\,e_{l+1}(t) \tag{26}$$

with $l=i-k+1$.

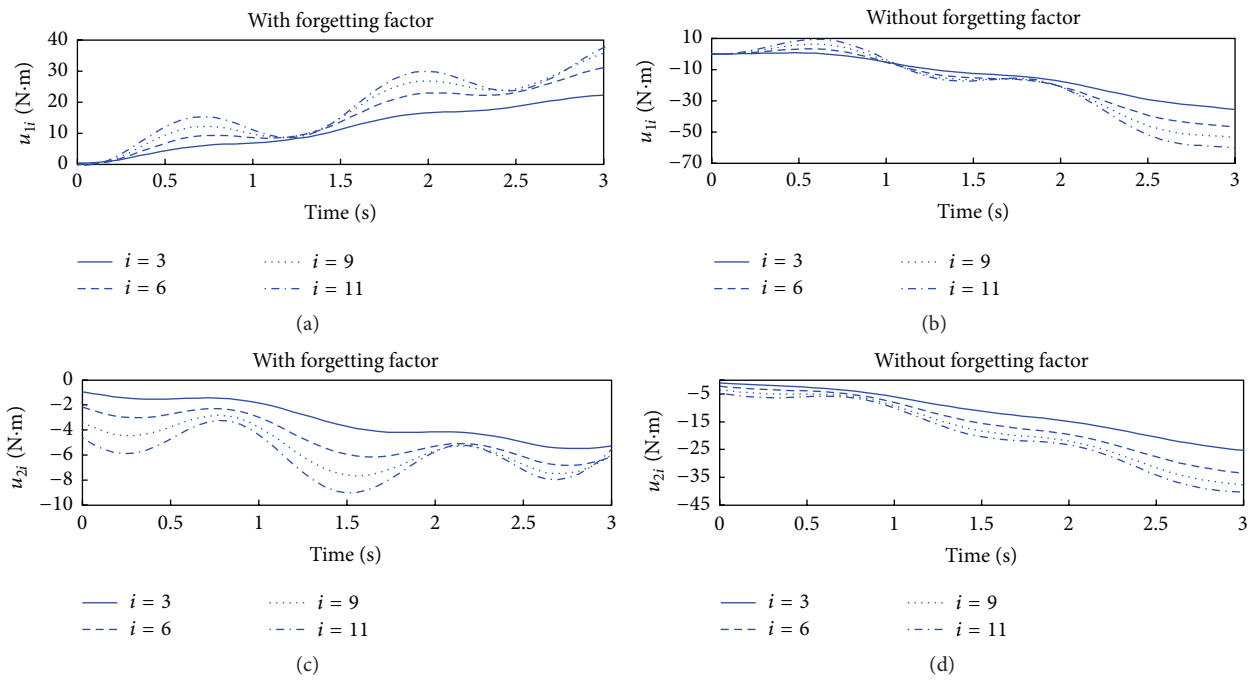

Figure 1: Control input trajectories.

Given to Lemma 3, the data used in the simulation is chosen as

$$\beta = 8^{-i},$$
$$u_0 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$N = 3,$$
$$\boldsymbol{\varphi}_1 = \begin{bmatrix} 1.7 & 0 \\ 0 & 1.6 \end{bmatrix}, \tag{27}$$
$$\boldsymbol{\varphi}_2 = \begin{bmatrix} 0.85 & 0 \\ 0 & 0.65 \end{bmatrix},$$
$$\boldsymbol{\varphi}_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix}.$$

After checking the convergence condition in Lemma 3, we can obtain the following:

The first algorithm is as follows:

$$\tilde{\rho} = \sum_{k=1}^{N} \rho_k = \sum_{k=1}^{N} p_k (1-\beta) + b_c b_\varphi \frac{b_B \bigcirc (\lambda^{-1})}{1 - \hat{e} \bigcirc (\lambda^{-1})} = 0.54 < 1. \tag{28}$$

The second algorithm is as follows:

$$\tilde{\rho} = \sum_{k=1}^{N} \rho_k = \sum_{k=1}^{N} p_k + b_c b_\varphi \frac{b_B \bigcirc (\lambda^{-1})}{1 - \hat{e} \bigcirc (\lambda^{-1})} = 0.68 < 1. \tag{29}$$

So, $\lim_{i\to\infty} \|\partial u_i\|_\lambda \le \varepsilon/(1-\tilde{\rho})$. Finally, at any bound of given tracking error ε^*, we can reach the conclusion of this theorem $\sup_{t\in[0,T]} \|e_i(t)\|_\lambda \le \varepsilon^*$, with $\forall i \ge M$.

Simulation results are shown in Figures 1, 2, and 3.

In Figure 1, there is a small amplitude of control input in the ILC algorithm with forgetting factor. From Figure 2, we can see that a good tracking performance can be achieved using the high-order feedback ILC algorithm with forgetting factor, even in the conditions of uncertainty and nonrepetitive disturbance from iteration to iteration. That is attributed to the forgetting factor, which can weaken the disturbance. But for the traditional ILC algorithm without forgetting factor, there are large stable tracking errors.

Figure 3 shows the absolute maximum tracking error from iteration to iteration, although the error curve is not perfectly smooth due to the presence of uncertainty and disturbance. From this figure, we can see that the ILC algorithm with forgetting factor can obtain a very fast convergent rate and very small and monotonic decreased tracking errors. But for the classic ILC algorithm without forgetting factor, although high-order feedback controller is used, the tracking errors were unsatisfactory and more iterations were needed to obtain a relatively acceptable tracking performance. We can see that the tracking errors are still relatively large compared with approach 1. It demonstrated that the forgetting factor is more useful in terms of reducing tracking error and speeding up the convergence.

$|x_i(0) - x_d(0)| \ne 0$ indicates that the system exist initialization errors. So the error of the last iteration cannot be close to zero; it only converges to a compact domain around zero.

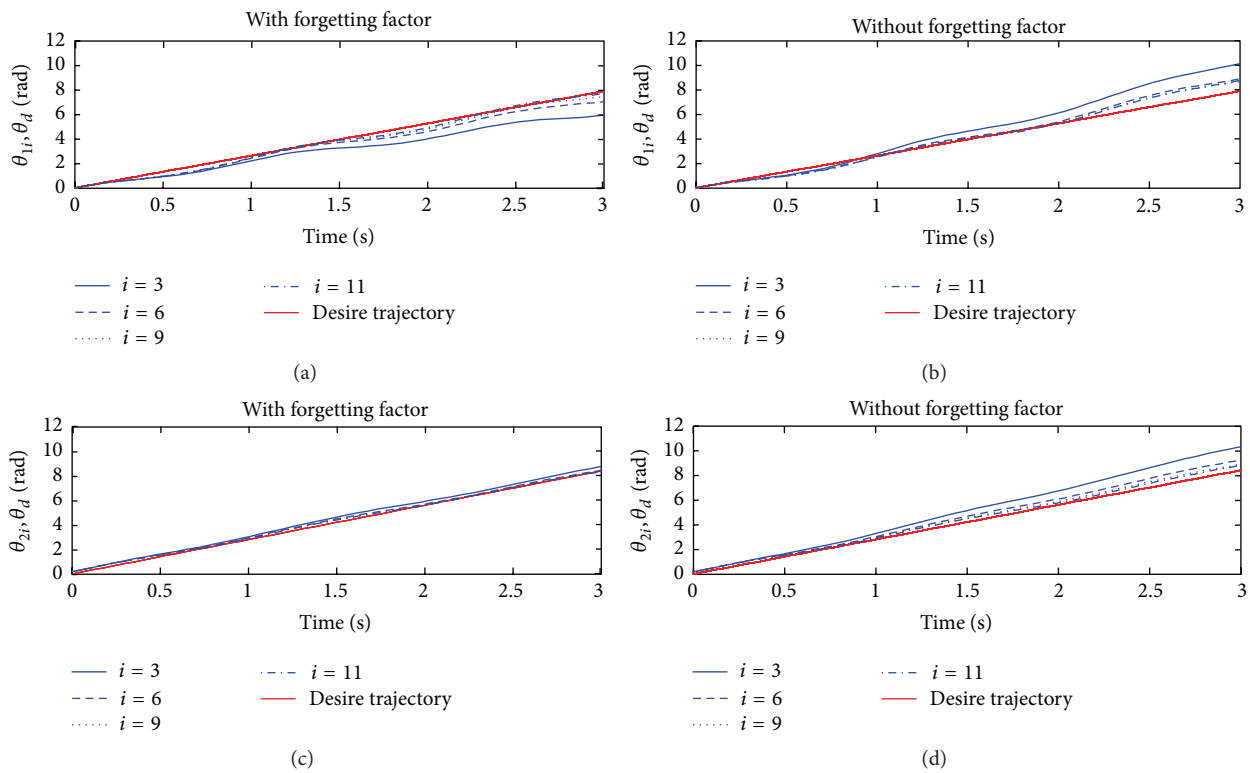

Figure 2: Desired and actual position trajectories.

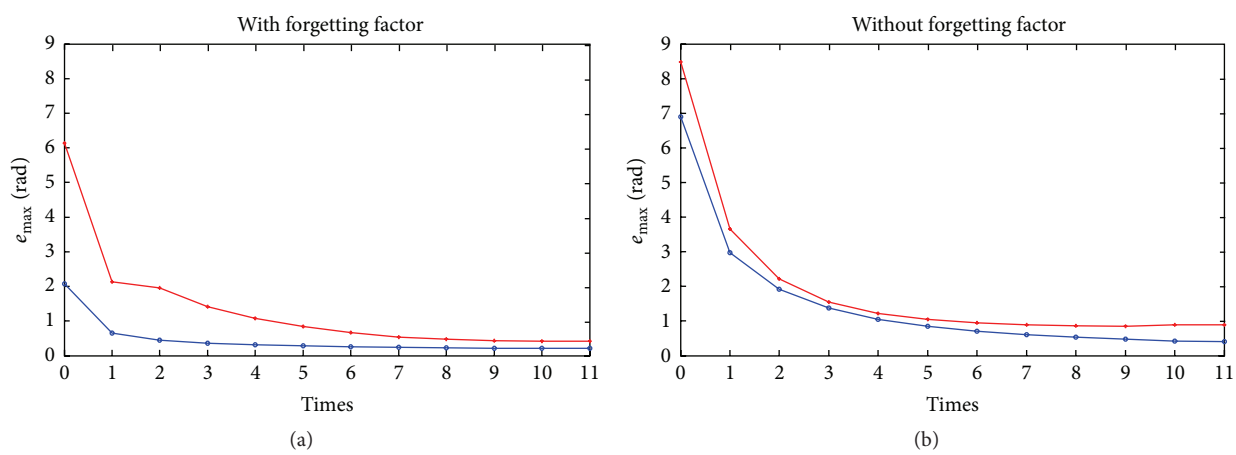

Figure 3: Max absolute values of the tracking error.

6. Conclusions

A high-order feedback ILC algorithm with forgetting factor for a class of nonlinear systems with uncertain and non-repetitive disturbance is introduced in this paper. The main contribution of this paper is adding a forgetting factor to the existing high-order feedback ILC method for nonlinear systems. A rigorous proof is given to show the effectiveness of the proposed algorithm and the asymptotic error convergence along the iteration axis. Simulation results demonstrate that the proposed algorithm has faster convergence speed, greatly reduced tracking error, and better performance in stability and robustness. The future work aims to apply the proposed algorithm to actual robot control and biochemical reactions process.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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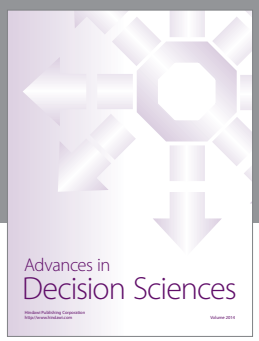

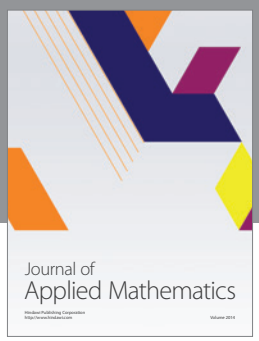

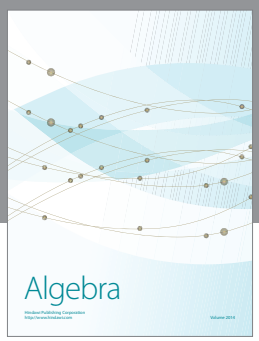

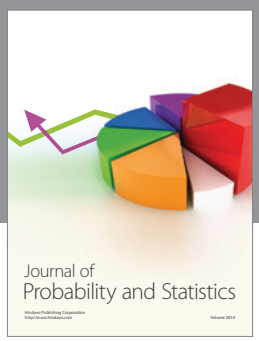

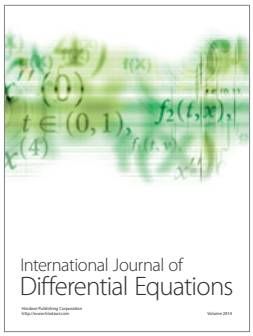

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