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Research Article

Stability Criterion of Linear Stochastic Systems Subject to Mixed H_2/*Passivity* Performance

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The H_2 control scheme and passivity theory are applied to investigate the stability criterion of continuous-time linear stochastic system subject to mixed performance. Based on the stochastic differential equation, the stochastic behaviors can be described as multiplicative noise terms. For the considered system, the H_2 control scheme is applied to deal with the problem on minimizing output energy. And the asymptotical stability of the system can be guaranteed under desired initial conditions. Besides, the passivity theory is employed to constrain the effect of external disturbance on the system. Moreover, the Itô formula and Lyapunov function are used to derive the sufficient conditions which are converted into linear matrix inequality (LMI) form for applying convex optimization algorithm. Via solving the sufficient conditions, the state feedback controller can be established such that the asymptotical stability and mixed performance of the system are achieved in the mean square. Finally, the synchronous generator system is used to verify the effectiveness and applicability of the proposed design method.

1. Introduction

In real circumstance, the external disturbance usually causes kinds of the poor control performance and instability of the dynamic systems. For achieving the good control performance of the disturbed systems, many control schemes [1–3] have been proposed to constrain the effect of external disturbance on the systems. In [3–6], the H_∞ control scheme is proposed to achieve the attenuation γ based on the relation between state and disturbance. On the other hand, the passivity theory was applied to deal with the disturbance attenuation performance of the systems in [7, 8]. With setting the power supply function [9], passivity theory can be converted into various constrains including H_∞ control scheme, H_2 control scheme, positive real theory, and several passive constraints. Thus, the passivity theory becomes the powerful and general tool to discuss the disturbance attenuation performance of the systems.

Recently, the mixed control method [10, 11] via combining the H_2 and H_∞ control schemes has been proposed to solve the control problems subject to required performances. Generally, the H_2 control scheme is applied to minimize the output energy of the system and to guarantee the stability of the system under initial conditions. The attenuation performance is discussed via H_∞ control scheme with given attenuation γ for the worst case of external disturbance input. In [12], the mixed H_2/H_∞ performance of uncertain spacecraft systems is achieved via designed PID tracking controller. Based on mixed H_2/H_∞ performance control method, the stability and performance of robotic manipulator have been discussed in [13]. In addition to mixed H_2/H_∞ performance control scheme, the passivity theory was also applied to discuss the mixed performance of the system via combining the H_2 control scheme or H_∞ control scheme. Referring to [14, 15], passivity theory is used to analyze the stability of system and H_2 control scheme is used to constrain the concerned signal resources. However, most of the literatures discussing mixed performance control problem are focused on deterministic systems. Few papers [16] have been proposed to study the mixed performance of stochastic systems. Since the stochastic behavior usually exists in the practical operating conditions, the investigation for mixed performance control problem of stochastic system is worth being discussed.

Stochastic systems have received much attention since stochastic differential equation is proposed with modeling approach [17–21]. Through the modeling approach, the stochastic behaviors can be described via the multiplicative noise term structured by multiplying state and noise. The noise is considered as Brownian motion [18, 19] to imitate natural random variation. For analyzing the stability of stochastic systems, the Itô formula [18] and Lyapunov function are applied to derive the sufficient conditions. During the analyzing process, the mean square calculation is the most useful tool for solving the stability and stabilization problems of stochastic system in the sense of mean square. According to the modeling approach and Itô formula, many stability criteria of the deterministic system have been extended to stochastic cases.

For the above motivation and illustration, the mixed $H_2/passivity$ performance controller design for linear stochastic systems is discussed in this paper. The H_2 control scheme applied in this paper is to minimize the output energy of the system. Moreover, the asymptotical stability of the system can be guaranteed under initial conditions via H_2 control scheme. For constraining the effect of external disturbance on the system, the passivity theory is used with general power supply function which decides the attenuation performance index. Based on the Itô formula and Lyapunov function, the sufficient conditions are derived such that the asymptotical stability and mixed performance of the system are guaranteed in the sense of mean square. For applying convex optimization algorithm, the sufficient conditions are converted into LMI form [22]. The main contribution of this paper is to propose a mixed $H_2/passivity$ performance controller design method for the linear stochastic systems. Through the setting power supply function, the choice of attenuation performance is more flexible and general than previous mixed H_2/H_∞ performance methods. Finally, the synchronous generator system with multiplicative noise is applied to demonstrate the proposed design method.

This paper is organized as follows: In Section 2, the linear stochastic systems and mixed performance control problem are illustrated. Through the Itô formula and Lyapunov function, the sufficient conditions are derived in Section 3. A numerical simulation is proposed in Section 4. Finally, the conclusion is in Section 5.

2. System Description and Definition Statement

In this paper, the linear stochastic system with external disturbance is described as follows:

$$\begin{aligned} dx(t) = {} & (\mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{E}v(t))\,dt \\ & + \left(\overline{\mathbf{A}}x(t) + \overline{\mathbf{B}}u(t)\right) d\beta(t), \end{aligned} \tag{1a}$$

$$y(t) = \mathbf{C}_1 x(t) + \mathbf{D}_1 v(t), \tag{1b}$$

$$z(t) = \mathbf{C}_2 x(t) + \mathbf{D}_2 u(t), \tag{1c}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the measured output vector, $z(t) \in R^p$ is the controlled output vector, $u(t) \in R^q$ is the control input vector, $v(t) \in R^m$ is the disturbance input vector, and $\beta(t)$ is a scalar of Brownian motion which satisfies the independent increment properties [18], such as $E\{d\beta(t)\} = 0$ and $E\{d\beta(t)d\beta(t)\} = 1$. $E\{\cdot\}$ denotes the expected value of $\cdot$. The $\mathbf{A}$, $\overline{\mathbf{A}}$, $\mathbf{B}$, $\overline{\mathbf{B}}$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{D}_1$, $\mathbf{D}_2$, and $\mathbf{E}$ are constant matrices with compatible dimensions.

For the stabilization problem of (1a), (1b), and (1c), the following controller is proposed in this paper:

$$u(t) = \mathbf{F}x(t), \tag{2}$$

where $\mathbf{F}$ is feedback gain with compatible dimensions. Substituting (2) into (1a), (1b), and (1c), the following closed-loop system is formulated:

$$dx(t) = \left(\mathbf{A}_f x(t) + \mathbf{E}v(t)\right) dt + \left(\overline{\mathbf{A}}_f x(t)\right) d\beta(t), \tag{3a}$$

$$y(t) = \mathbf{C}_1 x(t) + \mathbf{D}_1 v(t), \tag{3b}$$

$$z(t) = \mathbf{C}_{2f} x(t), \tag{3c}$$

where $\mathbf{A}_f = \mathbf{A} + \mathbf{B}\mathbf{F}$, $\overline{\mathbf{A}}_f = \overline{\mathbf{A}} + \overline{\mathbf{B}}\mathbf{F}$, and $\mathbf{C}_{2f} = \mathbf{C}_2 + \mathbf{D}_2\mathbf{F}$.

For considering mixed performances, the H_2 control scheme and passivity theory are applied. By setting the power supply function, the passivity theory includes several performance constraints [9]. In this paper, the general power supply function is considered for providing flexible choice of performance index to constrain the external disturbance input. Thus, the general passive constraint is introduced as in the following definition.

Definition 1. The closed-loop system (3a), (3b), and (3c) with external disturbance input $v(t)$ and measured output $y(t)$ is called passive if there exist known matrices $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$ such that

$$\begin{aligned} & E\left\{2\int_0^{t_p} y^T(t)\,\mathbf{S}_1 v(t)\,dt\right\} \\ & \quad > E\left\{\int_0^{t_p} y^T(t)\,\mathbf{S}_2 y(t)\,dt + \int_0^{t_p} v^T(t)\,\mathbf{S}_3 v(t)\,dt\right\}, \end{aligned} \tag{4}$$

where $t_p > 0$.

Remark 2. According to the matrices $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$ defined by [9], the passivity theory is more general and flexible than H_∞ control scheme for constraining the effect of external disturbance on the system. Besides, both of state-dependent noise and input-dependent noise are considered for the stochastic system in this paper. Thus, the proposed mixed $H_2/passivity$ performance control design method is more general and flexible than method in [16] for disturbance attenuation performance of stochastic systems.

In addition, the H_2 control scheme is employed to guarantee the stability of the closed-loop system (3a), (3b), and(3c) under given initial condition. And, the output energy can also be minimized via achieving the H_2 performance. Thus, the H_2 performance is defined in the following definition.

Definition 3. The controller (2) is an H_2 performance measurement if the following inequality is satisfied:

$$E\left\{\int_0^{\infty} z^T(t)\,z(t)\,dt\right\} < \alpha. \tag{5}$$

This definition can also be called as minimizing problem for output with α.

Through the above definitions, the mixed performance of (3a), (3b), and (3c) is discussed to minimize output energy with constraining effect of external disturbance. Based on the above illustration, the sufficient conditions are derived for stability criterion of closed-loop system (3a), (3b), and (3c) in the next section.

3. Mixed Controller Design for Considered System

Applying the Lyapunov function and Itô formula, the sufficient conditions are derived to analyze the stability of closed-loop system (3a), (3b), and (3c). Furthermore, the sufficient conditions are applied to find the feedback gain $\mathbf{F}$ to achieve the mixed $H_2/passivity$ performance of (3a), (3b), and (3c). For using convex optimization algorithm, the conditions are converted into LMI problems for finding the feasible solutions.

Theorem 4. *Given matrices $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$, the closed-loop system (3a), (3b), and (3c) is asymptotically stable and satisfies the mixed $H_2/passivity$ performance in the sense of mean square, if there exist positive scalar α, positive definite matrix $\mathbf{P}$, and feedback gain matrix $\mathbf{F}$ such that one has the following:*

minimize α subject to

$$\begin{bmatrix} \mathbf{C}_1^T\mathbf{S}_2\mathbf{C}_1 + \mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f & * \\ -\mathbf{S}_1^T\mathbf{C}_1 + \mathbf{D}_1^T\mathbf{S}_2\mathbf{C}_1 + \mathbf{E}^T\mathbf{P} & \mathbf{S}_3 - \mathbf{D}_1^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_1 + \mathbf{D}_1^T\mathbf{S}_2\mathbf{D}_1 \end{bmatrix} < 0, \tag{6}$$

$$\mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f + \mathbf{C}_{2f}^T\mathbf{C}_{2f} < 0, \tag{7}$$

$$x^T(0)\,\mathbf{P}x(0) < \alpha. \tag{8}$$

Proof. Choose the following Lyapunov function:

$$V(x(t)) = x^T(t)\,\mathbf{P}x(t). \tag{9}$$

And the derivation of the $V(x(t))$ along with trajectories of (3a), (3b), and (3c) can be obtained by Itô formula [18], such as

$$dV(x(t)) = LV(x(t))\,dt + 2x^T(t)\,\overline{\mathbf{A}}_f x(t)\,d\beta(t), \tag{10}$$

where

$$\begin{aligned} LV(x(t)) &= x^T(t)\left(\mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f\right)x(t) \\ &\quad + v^T(t)\left(\mathbf{E}^T\mathbf{P}\right)x(t) + x^T(t)\,(\mathbf{P}\mathbf{E})\,v(t). \end{aligned} \tag{11}$$

Arranging (11), one has

$$LV(x(t)) = \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^T \begin{bmatrix} \mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f & * \\ \mathbf{E}^T\mathbf{P} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}. \tag{12}$$

Let us take expectation of (10); one has the following equation with the independent increment property as $E\{x(t)d\beta(t)\} = 0$ and $E\{v(t)d\beta(t)\} = 0$:

$$E\{dV(x(t))\} = E\{LV(x(t))\,dt\}. \tag{13}$$

In case as nonzero external disturbance $v(t) \neq 0$, the following cost function can be defined with zero initial conditions:

$$\begin{aligned} \Gamma(x, v, t) &= E\left\{\int_0^{t_p}\left(y^T(t)\,\mathbf{S}_2 y(t) + v^T(t)\,\mathbf{S}_3 v(t) \right.\right. \\ &\qquad \left.\left. - 2y^T(t)\,\mathbf{S}_1 v(t)\right)dt\right\} \\ &= E\left\{\int_0^{t_p}\left(y^T(t)\,\mathbf{S}_2 y(t) + v^T(t)\,\mathbf{S}_3 v(t) - 2y^T(t)\right.\right. \\ &\qquad \left.\left. \cdot\,\mathbf{S}_1 v(t) + LV(x(t))\right)dt - V\left(x\left(t_p\right)\right)\right\} \\ &\le E\left\{\int_0^{t_p}\Psi(x, v, t)\,dt\right\}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \Psi(x, v, t) &= y^T(t)\,\mathbf{S}_2 y(t) + v^T(t)\,\mathbf{S}_3 v(t) \\ &\quad - 2y^T(t)\,\mathbf{S}_1 v(t) + LV(x(t)). \end{aligned} \tag{15}$$

Substituting (3b) and (12) into (15), one has the following:

$$\Psi(x, v, t) = \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^T \mathbf{\Lambda} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \tag{16}$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{C}_1^T\mathbf{S}_2\mathbf{C}_1 + \mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f & * \\ -\mathbf{S}_1^T\mathbf{C}_1 + \mathbf{D}_1^T\mathbf{S}_2\mathbf{C}_1 + \mathbf{E}^T\mathbf{P} & \mathbf{S}_3 - \mathbf{D}_1^T\mathbf{S}_1 - \mathbf{S}_1\mathbf{D}_1 + \mathbf{D}_1^T\mathbf{S}_2\mathbf{D}_1 \end{bmatrix}. \quad (17)$$

Obviously, (17) is equal to the left-hand side of inequality (6). Thus, if the condition of this theorem is held then one can find the $\mathbf{\Lambda} < 0$ from (17). Since the $\mathbf{\Lambda} < 0$, the $\Psi(x, v, t) < 0$ is found from (16). Due to $\Psi(x, v, t) < 0$, $\Gamma(x, v, k) < 0$ can be inferred from (14). And then, the following inequalities can be found since $\Gamma(x, v, k) < 0$:

$$E\left\{\int_0^{t_p} y^T(t)\mathbf{S}_2 y(t) + v^T(t)\mathbf{S}_3 v(t) - 2y^T(t)\mathbf{S}_1 v(t)\,dt\right\} < 0 \quad (18a)$$

or

$$E\left\{2\int_0^{t_p} y^T(t)\mathbf{S}_1 v(t)\,dt\right\} > E\left\{\int_0^{t_p} y^T(t)\mathbf{S}_2 y(t)\,dt + \int_0^{t_p} v^T(t)\mathbf{S}_3 v(t)\,dt\right\}. \quad (18b)$$

Because (18b) is equivalent to (4), the closed-loop system (3a), (3b), and (3c) is passive with given $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$. Next, the asymptotical stability of (3a), (3b), and (3c) is proven. Assuming the disturbance input is zero ($v(t) = 0$), (12) can be rewritten as follows:

$$LV(x(t)) = x^T(t)\left(\mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f\right)x(t). \quad (19)$$

Furthermore, the following relation can be derived with (3c):

$$\begin{aligned} LV(x(t)) &= x^T(t)\left(\mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f\right)x(t) \\ &\quad + z^T(t)z(t) - z^T(t)z(t) \\ &= x^T(t)\left(\mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f + \mathbf{C}_{2f}^T\mathbf{C}_{2f}\right)x(t) \\ &\quad - z^T(t)z(t) < \Xi, \end{aligned} \quad (20)$$

where $\Xi = x^T(t)(\mathbf{A}_f^T\mathbf{P} + \mathbf{P}\mathbf{A}_f + \overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f + \mathbf{C}_{2f}^T\mathbf{C}_{2f})x(t)$.

One can find that if condition (7) is held then $\Xi < 0$ can easily be found. Based on the $\Xi < 0$, the $LV(x(t)) < 0$ can also be obtained from (20). Since $LV(x(t)) < 0$ and $v(t) = 0$, the following inequality can be obtained from (13):

$$E\{dV(x(t))\} = E\{LV(x(t))\,dt\} < 0. \quad (21)$$

According to $E\{dV(x(t))\} < 0$ and referring to [17], the closed-loop system (3a), (3b), and (3c) is asymptotically stable in the sense of mean square with zero external disturbance.

Integrating both side of (13) from 0 to t_g, one has the following equation with nonzero initial condition:

$$\begin{aligned} &E\left\{x^T(t_g)\mathbf{P}x(t_g)\right\} - E\left\{x^T(0)\mathbf{P}x(0)\right\} \\ &\quad = E\left\{\int_0^{t_g} LV(x(t))\,dt\right\} \\ &\quad = E\left\{\int_0^{t_g} \left(\Xi - z^T(t)z(t)\right)dt\right\}. \end{aligned} \quad (22)$$

Also, (22) can be rewritten as follows:

$$\begin{aligned} &E\left\{x^T(t_g)\mathbf{P}x(t_g)\right\} - E\left\{x^T(0)\mathbf{P}x(0)\right\} \\ &\quad + E\left\{\int_0^{t_g}\left(z^T(t)z(t)\right)dt\right\} = E\left\{\int_0^{t_g}\Xi\,dt\right\}. \end{aligned} \quad (23)$$

Since condition (7) is held, the $\Xi < 0$ can easily be found. And then, the following inequalities can also be obtained:

$$\begin{aligned} &E\left\{x^T(t_g)\mathbf{P}x(t_g)\right\} - E\left\{x^T(0)\mathbf{P}x(0)\right\} \\ &\quad + E\left\{\int_0^{t_g} z^T(t)z(t)\,dt\right\} < 0, \end{aligned} \quad (24a)$$

$$\begin{aligned} &E\left\{x^T(t_g)\mathbf{P}x(t_g)\right\} - E\left\{x^T(0)\mathbf{P}x(0)\right\} \\ &\quad < -E\left\{\int_0^{t_g} z^T(t)z(t)\,dt\right\}. \end{aligned} \quad (24b)$$

Due to the fact that the closed-loop system (3a), (3b), and (3c) is asymptotically stable in the mean square, one can find that the $x(t_g) \rightarrow 0$ as $t_g \rightarrow +\infty$. Thus, the $x^T(t_g)\mathbf{P}x(t_g) \rightarrow 0$ can also be found with $t_g \rightarrow +\infty$. And (24b) can be rewritten as in the following inequality:

$$E\left\{\int_0^{+\infty} z^T(t)z(t)\,dt\right\} < E\left\{x^T(0)\mathbf{P}x(0)\right\}. \quad (25)$$

Obviously, the $x^T(0)\mathbf{P}x(0)$ is the upper bound of output energy from (25). If condition (8) is satisfied, one has

$$x^T(0)\mathbf{P}x(0) < \alpha. \quad (26)$$

From (25) and (26), the following relation can directly be found:

$$E\left\{\int_0^{+\infty} z^T(t)z(t)\,dt\right\} < \alpha. \quad (27)$$

Since (27) is equivalent to (5), one can find that if conditions (7) and (8) are held, then the H_2 performance of closed-loop system is achieved. The proof of this theorem is completed. □

In Theorem 4, the stability problem of closed-loop system(3a), (3b), and (3c) is discussed via finding the feasible solutions. However, the conditions of Theorem 4 belong to BMI (bilinear matrix inequality) problems which cannot directly be calculated by convex optimization algorithm. For this reason, the conditions of Theorem 4 are converted into LMI problems in the next theorem.

Theorem 5. *Given matrices $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$, the closed-loop system (3a), (3b), and (3c) is asymptotically stable and satisfies mixed H_2/passivity performances in the sense of mean square if there exist positive scalar α, positive definite matrix $\mathbf{P}$, and feedback gain matrix $\mathbf{F}$ such that one has the following:*

minimize α subject to

$$\begin{bmatrix} \mathbf{X}^T\mathbf{A}^T + \mathbf{K}^T\mathbf{B}^T + \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{K} & * & * & * \\ -\mathbf{S}_1^T\mathbf{C}_1\mathbf{X} + \mathbf{E}^T & \mathbf{S}_3 - \mathbf{D}_1^T\mathbf{S}_1 - \mathbf{S}_1^T\mathbf{D}_1 & * & * \\ \overline{\mathbf{A}}\mathbf{X} + \overline{\mathbf{B}}\mathbf{K} & 0 & -\mathbf{X} & * \\ \sqrt{\mathbf{S}_2}\mathbf{C}_1\mathbf{X} & \sqrt{\mathbf{S}_2}\mathbf{D}_1 & 0 & -\mathbf{I} \end{bmatrix} < 0, \quad (28)$$

$$\begin{bmatrix} \mathbf{X}^T\mathbf{A}^T + \mathbf{K}^T\mathbf{B}^T + \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{K} & * & * \\ \overline{\mathbf{A}}\mathbf{X} + \overline{\mathbf{B}}\mathbf{K} & -\mathbf{X} & * \\ \mathbf{C}_2\mathbf{X} + \mathbf{D}_2\mathbf{K} & 0 & -\mathbf{I} \end{bmatrix} < 0, \quad (29)$$

$$\begin{bmatrix} -\alpha & x^T(0) \\ x(0) & -\mathbf{X} \end{bmatrix} < 0, \quad (30)$$

where $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{K} = \mathbf{F}\mathbf{X}$. $\mathbf{I}$ denotes the identity matrix with approximate dimension.

Proof. Multiplying the both sides of (6) by $\left[\begin{smallmatrix} \mathbf{P}^{-1} & * \\ 0 & \mathbf{I} \end{smallmatrix}\right]$, the following inequality can be obtained:

$$\begin{bmatrix} \mathbf{P}^{-1}\mathbf{C}_1^T\mathbf{S}_2\mathbf{C}_1\mathbf{P}^{-1} + \mathbf{P}^{-1}\mathbf{A}_f^T + \mathbf{A}_f\mathbf{P}^{-1} + \mathbf{P}^{-1}\overline{\mathbf{A}}_f^T\mathbf{P}\overline{\mathbf{A}}_f\mathbf{P}^{-1} & * \\ -\mathbf{S}_1^T\mathbf{C}_1\mathbf{P}^{-1} + \mathbf{D}_1^T\mathbf{S}_2\mathbf{C}_1\mathbf{P}^{-1} + \mathbf{E}^T & \mathbf{S}_3 - \mathbf{D}_1^T\mathbf{S}_1 - \mathbf{S}_1^T\mathbf{D}_1 + \mathbf{D}_1^T\mathbf{S}_2\mathbf{D}_1 \end{bmatrix} < 0. \quad (31)$$

Applying Schur complement to (31), one has

$$\begin{bmatrix} \mathbf{P}^{-1}\mathbf{A}_f^T + \mathbf{P}^{-1}\mathbf{A}_f^T & * & * & * \\ -\mathbf{S}_1^T\mathbf{C}_1\mathbf{P}^{-1} + \mathbf{E}^T & \mathbf{S}_3 - \mathbf{D}_1^T\mathbf{S}_1 - \mathbf{S}_1^T\mathbf{D}_1 & * & * \\ \overline{\mathbf{A}}_f\mathbf{P}^{-1} & 0 & -\mathbf{P}^{-1} & * \\ \sqrt{\mathbf{S}_2}\mathbf{C}_1\mathbf{P}^{-1} & \sqrt{\mathbf{S}_2}\mathbf{D}_1 & 0 & -\mathbf{I} \end{bmatrix} < 0. \quad (32)$$

According to $\mathbf{A}_f = \mathbf{A} + \mathbf{B}\mathbf{F}$ and $\overline{\mathbf{A}}_f = \overline{\mathbf{A}} + \overline{\mathbf{B}}\mathbf{F}$, one has the following inequality from (32) with setting $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{K} = \mathbf{F}\mathbf{X}$:

$$\begin{bmatrix} \mathbf{X}^T\mathbf{A}^T + \mathbf{K}^T\mathbf{B}^T + \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{K} & * & * & * \\ -\mathbf{S}_1^T\mathbf{C}_1\mathbf{X} + \mathbf{E}^T & \mathbf{S}_3 - \mathbf{D}_1^T\mathbf{S}_1 - \mathbf{S}_1^T\mathbf{D}_1 & * & * \\ \overline{\mathbf{A}}\mathbf{X} + \overline{\mathbf{B}}\mathbf{K} & 0 & -\mathbf{X} & * \\ \sqrt{\mathbf{S}_2}\mathbf{C}_1\mathbf{X} & \sqrt{\mathbf{S}_2}\mathbf{D}_1 & 0 & -\mathbf{I} \end{bmatrix} < 0. \quad (33)$$

Thus, (28) is obtained from (33). And, one can find that if (28) is satisfied, condition (6) can be held. Based on the above procedure, condition (34) can easily be obtained. And, if (29) is held, condition (7) can be satisfied. Thus, the proof of (29) is omitted here.

Based on (8), one has the following inequality:

$$x^T(0)\mathbf{P}x(0) - \alpha < 0. \quad (34)$$

Using Schur complement for (34), the following inequality can be found with $\mathbf{X} = \mathbf{P}^{-1}$:

$$\begin{bmatrix} -\alpha & x^T(0) \\ x(0) & -\mathbf{X} \end{bmatrix} < 0. \quad (35)$$

Since (35) is equivalent to (30), one knows that if condition (8) is held then (30) can also be satisfied. On the other hand, if the conditions in this theorem are held then the conditions in Theorem 4 are satisfied. And the asymptotical stability and mixed H_2/*passivity* performance of the closed-loop system (3a), (3b), and (3c) are achieved in the sense of mean square. Thus, the proof of the theorem is complete. □

Applying the convex optimization algorithm, the feasible solutions of Theorem 5 can directly be found to establish controller (2). Based on the proposed stability criterion, the closed-loop system (3a), (3b), and (3c) driven by designed controller is asymptotically stable and satisfies mixed H_2/*passivi*ty performance in the sense of mean square. In the next section, the synchronous generator system is utilized to verify the effectiveness and application of the proposed design method.

4. Simulation

Referring to [8], the dynamic equations of the synchronous generator system are given as follows:

$$\begin{aligned}
\dot{x}_1(t) &= 2\pi f_0 x_2(t) - 0.1v(t),\\
\dot{x}_2(t) &= -\frac{D}{H}x_2(t)\\
&\quad + \frac{\omega_0}{H}\left(P_m - \frac{V_s}{x'_{d\Sigma}}\left(x_3(t)+E'_{q0}\right)\sin\left(x_1(t)+\delta_0\right)\right.\\
&\qquad \left. + \frac{\left(x_d - x'_d\right)V_s^2}{x_{d\Sigma}x'_{d\Sigma}}\right)\\
&\quad \times \sin\left(x_1(t)+\delta_0\right)\cos\left(x_1(t)+\delta_0\right),\\
\dot{x}_3(t) &= -\frac{x_{d\Sigma}}{T_{D0}x'_{d\Sigma}}\left(x_3(t)+E'_{q0}\right)\\
&\quad + \frac{x_d - x'_d}{T_{D0}x'_{d\Sigma}}V_s\cos\left(x_1(t)+\delta_0\right) + \frac{k_A x_{ad}}{T_{D0}R_f}\left(u_0 + u(t)\right),
\end{aligned} \tag{36}$$

where $x_1(t)$ is the angular position of the rotor of the generator with respect to synchronously rotating reference, which is selected here to be the infinite bus; $x_2(t)$ is the angular velocity of the rotor, $x_3(t)$ is the electromotive force the q-axis of the generator, $u(t)$ is control input, and $v(t)$ is a zero mean white noise with unit variance. And, the parameters of the synchronous generator system (36) are given as $f_0 = 50\,\text{Hz}$, $D = 0.8$, $H = 8\,\text{s}$, $\omega_0 = 1\,\text{p.u.}$, $P_m = 0.79\,\text{p.u.}$, $V_s = 1\,\text{p.u.}$, $x'_{d\Sigma} = 1.1808\,\text{p.u.}$, $x_{d\Sigma} = 2.3108\,\text{p.u.}$, $E'_{q0} = 1.2723\,\text{p.u.}$, $\delta_0 = 60^\circ$, $x_d = 1.5\,\text{p.u.}$, $x'_d = 0.3\,\text{p.u.}$, $T_{D0} = 3\,\text{s}$, $k_A = 10$, $R_f = 0.0045\,\text{p.u.}$, $u_0 = 7.2942\times10^{-4}\,\text{p.u.}$, and $x_{ad} = 1.3\,\text{p.u.}$ Base on the linearization technique applied in [8], the linear equation of (36) can be obtained with equilibrium point as $x^{\text{ep}}(t) = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$. Considering the stochastic behavior, the constant matrices $\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$ of the multiplicative noise terms are assumed as follows:

$$\overline{\mathbf{A}} = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \qquad \overline{\mathbf{B}} = 96.2963. \tag{37}$$

Based on the stochastic modeling approach, one has the following dynamic linear stochastic equation to characterize the local trajectories of (36) with added multiplicative noise term around equilibrium point:

$$\begin{aligned}
dx(t) = {} & (\mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{E}v(t))\,dt\\
& + \left(\overline{\mathbf{A}}x(t) + \overline{\mathbf{B}}u(t)\right)d\beta(t),
\end{aligned} \tag{38}$$

where $\mathbf{A} = \begin{bmatrix}0 & 314.1593 & 0\\ -0.1009 & -0.1 & -0.0957\\ -0.3121 & 0 & -0.6937\end{bmatrix}$, $\mathbf{B} = \begin{bmatrix}0\\ 0\\ 962.963\end{bmatrix}$, and $\mathbf{E} = \begin{bmatrix}0.1 & 0 & 0\end{bmatrix}^T$. Moreover, the measured output $y(t)$ and controlled output $z(t)$ for (38) are given as in the following equations:

$$\begin{aligned}
y(t) &= \mathbf{C}_1 x(t) + \mathbf{D}_1 v(t),\\
z(t) &= \mathbf{C}_1 x(t) + \mathbf{D}_2 u(t),
\end{aligned} \tag{39}$$

where $\mathbf{C}_1 = \mathbf{C}_2 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$ and $\mathbf{D}_1 = \mathbf{D}_2 = 1$. With given $\mathbf{S}_1 = 1$, $\mathbf{S}_2 = 1$, and $\mathbf{S}_3 = 0.98$ and initial condition $x(0) = \begin{bmatrix}30^\circ & 0.1 & 0\end{bmatrix}^T$, the following feasible solutions can be obtained via using LMI Toolbox of MATLAB to solve the sufficient conditions in Theorem 5:

$$\begin{aligned}
\mathbf{P} &= \begin{bmatrix}1.9325 & 13.6451 & -0.0139\\ 13.6451 & 160.5379 & -0.2368\\ -0.0139 & -0.2368 & 0.0005\end{bmatrix},\\
\mathbf{F} &= \begin{bmatrix}2.3515 & 37.9586 & -0.0863\end{bmatrix},\\
\alpha &= 3.5641.
\end{aligned} \tag{40}$$

With obtained feedback gain matrix $\mathbf{F}$, the linear controller can be designed as follows:

$$u(t) = \mathbf{F}x(t). \tag{41}$$

In order to emphasize the advantages of the proposed design method, the approaches proposed by [6, 16] are also applied to control the synchronous generator system (36) with added multiplicative noise term (37). Studying [16], the mixed H_2/H_∞ performance controller design method has been proposed for state-dependent noised systems. Applying [16], the following controller can be obtained via setting the $\gamma^2 = 1.01$ performance index for H_∞ control scheme and the same initial condition $x(0) = \begin{bmatrix}30^\circ & 0.1 & 0\end{bmatrix}^T$:

$$u(t) = \mathbf{F}x(t), \tag{42}$$

where $\mathbf{F} = \begin{bmatrix}1.9878 & 27.8338 & -0.0771\end{bmatrix}$.

Besides, the state feedback H_∞ control for stochastic systems has been developed in [6] without considering the H_2 performance. And, the following controller can be established by [6] with $\gamma^2 = 1.01$ performance index for H_∞ control scheme:

$$u(t) = \mathbf{F}x(t), \tag{43}$$

where $\mathbf{F} = \begin{bmatrix}0.0384 & 3.2799 & -0.0151\end{bmatrix}$.

Applying controller (41) to the synchronous generator system (36) with added multiplicative noise term, the simulation results are presented by Figures 1–3 with initial condition as $x(0) = \begin{bmatrix}30^\circ & 0.1 & 0\end{bmatrix}^T$. From the figures, system (36) driven by (41) is asymptotically stable in the mean square. In order to check the achievement of mixed $H_2/\textit{passivity}$ performance, the ratio value of the following functions can be obtained by introducing simulation responses:

$$\frac{E\left\{2\int_0^{t_p} y^T(t)\,\mathbf{S}_1 v(t)\,dt\right\}}{E\left\{\int_0^{t_p} y^T(t)\,\mathbf{S}_2 y(t)\,dt + \int_0^{t_p} v^T(t)\,\mathbf{S}_3 v(t)\,dt\right\}} = 1.002, \tag{44}$$

$$\int_0^{t_p} z^T(t)\,z(t) = 0.2293. \tag{45}$$

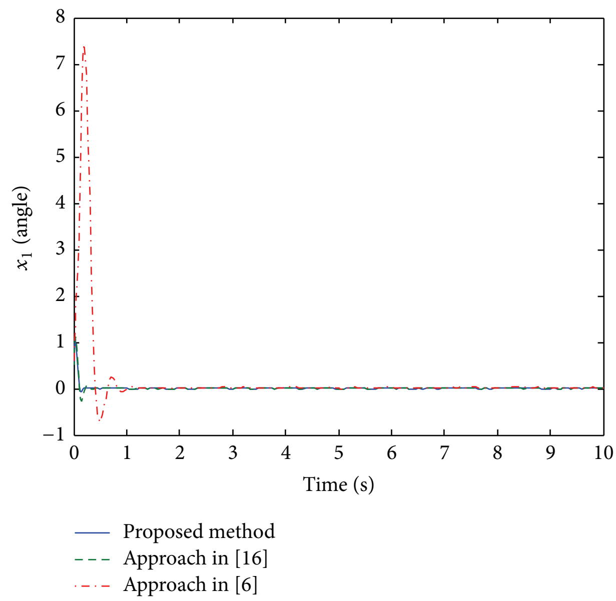

FIGURE 1: Responses of state $x_1(t)$.

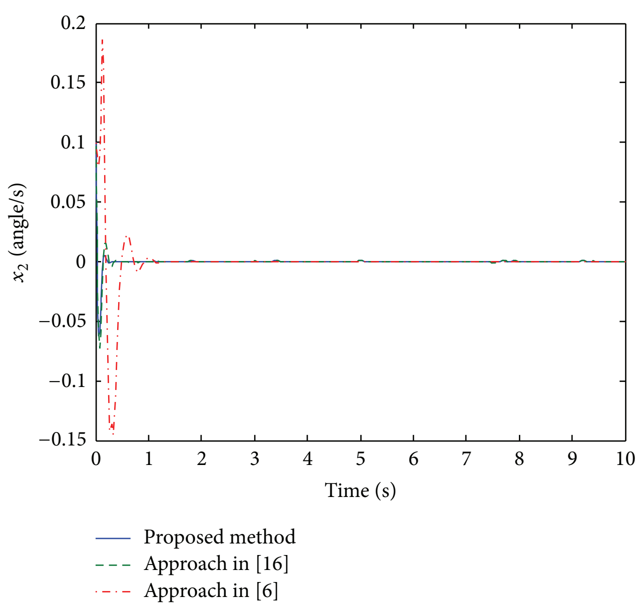

FIGURE 2: Responses of state $x_2(t)$.

Since the ratio value of (44) is bigger than one, one can find that Definition 1 is satisfied with given matrices as $\mathbf{S}_1 = 1$, $\mathbf{S}_2 = 1$, and $\mathbf{S}_3 = 0.98$. Hence, the passivity of considered system is achieved by designed controller (41). Besides, (41) shows that the output energy of system is smaller than $\alpha = 3.5641$. Thus, the inequality of Definition 3 can be satisfied.

On the other hand, the simulated responses of (36) driven by (42) and (43) are also shown in Figures 1–3. From these responses, the overshoot of (43) is the biggest among the others because the H_2 performance is not considered. Thus, one can find that the control performance of (43) is the worst case in this simulation. In addition, the requirements for attenuation performance and H_2 performance of (36) driven by (42) are achieved. However, the overshoot and setting time of (41) are smaller than ones of (42). Therefore, in this simulation, the state responses of the proposed design method are better than that driven by the approaches in [6, 16]. Based on the results, the asymptotical stability and mixed $H_2/passivity$ performance of the stochastic system (36) are achieved by designed controller (41).

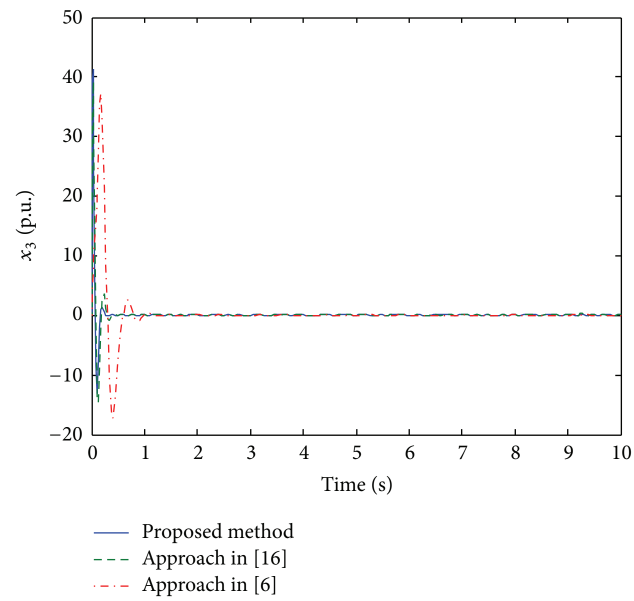

FIGURE 3: Responses of state $x_3(t)$.

5. Conclusions

In this paper, the H_2 control scheme was applied to minimize the output energy and to stabilize the considered system with initial condition. In addition, the disturbance attenuation performance of the system was discussed via the passivity theory with general power supply function. Through the H_2 control scheme and passivity theory, the stability criterion for the linear stochastic system has been proposed subject to mixed $H_2/passivity$ performance. Furthermore, the Lyapunov function and Itô formula were employed to derive the sufficient conditions which have been converted into LMI problems. Thus, the feasible solutions can directly be found via convex optimization algorithm for establishing the controller such that the asymptotical stability and mixed $H_2/passivity$ performance of linear stochastic system are achieved in the sense of mean square. Through the simulation results, the proposed design method is useful and effective for stabilizing linear stochastic systems subject to mixed $H_2/passivity$ performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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