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Research Article

Circuit Tolerance Design Using Belief Rule Base

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A belief rule-based (BRB) system provides a generic nonlinear modeling and inference mechanism. It is capable of modeling complex causal relationships by utilizing both quantitative information and qualitative knowledge. In this paper, a BRB system is firstly developed to model the highly nonlinear relationship between circuit component parameters and the performance of the circuit by utilizing available knowledge from circuit simulations and circuit designers. By using rule inference in the BRB system and clustering analysis, the acceptability regions of the component parameters can be separated from the value domains of the component parameters. Using the established nonlinear relationship represented by the BRB system, an optimization method is then proposed to seek the optimal feasibility region in the acceptability regions so that the volume of the tolerance region of the component parameters can be maximized. The effectiveness of the proposed methodology is demonstrated through two typical numerical examples of the nonlinear performance functions with nonconvex and disconnected acceptability regions and high-dimensional input parameters and a real-world application in the parameter design of a track circuit for Chinese high-speed railway.

1. Introduction

Tolerance has become a crucial design consideration in integrated and discrete circuit designs due to the demand of improved product quality, longer product lifetimes, and shorter design cycle. Designers have to unceasingly seek a central point with the maximum tolerances in the space of circuit component parameters so as to maximize parametric yield and minimize costs while maintaining compliance with design specifications [1–3]. On the other hand, circuit reliability is closely linked to its yield, namely, only those products with high yield would have high reliability. So, tolerance design and yield optimization are also the effective ways to improve circuit reliability [4].

In essence, there are mainly two kinds of methods for tolerance design and yield estimation, that is, the Monte Carlo based statistical methods and the deterministic methods [5–11]. Because the former requires numerous circuit simulations and computationally expensive analysis runs [5, 6], researchers have proposed alternative deterministic methods based on response surface modeling to approximate the performance function and the corresponding acceptability region (R_A). Thus, the optimal center and tolerances (i.e., feasibility region R_F) of component parameters can be found in the approximated region R_A. The deterministic methods mainly include simplicial approximation [7], polyhedral approximation [8], quadratic approximation [9], ellipsoidal method [10], and neural network [11]. However, such approximation methods and low-order polynomial models may not be applicable to some complex cases in which ranges of parameter variables are wide, performance functions are highly nonlinear, and the feasibility regions are nonconvex and even disconnected [1–3]. Hence, there is a need to develop new methods that can be used to model and optimize the design in such a highly complex setting.

This paper develops a novel method of the acceptability region approximation and tolerance optimization to obtain available feasibility region using belief rule-based (BRB) model. In the belief rule base, each possible consequent of a rule is associated with a belief degree. Such a rule base is capable of capturing highly nonlinear and continuous causal relationships between different factors [12, 13]. When applying a belief rule base, the input of an antecedent is transformed into a belief distribution over the referential

values of an antecedent. The distribution is then used to calculate the activation weights of the rules in the rule base. Subsequently, inference in the belief rule base is through the combination of all the activated rules using the evidential reasoning (ER) approach [14, 15]. Compared with polynomial and neural network models, the model parameters in the BRB can be extracted not only from objective data, but also from experts' subjective knowledge [16]. Moreover, the physical meanings of these parameters are easy to understand for experts and engineers, so they can intuitively participate in the whole course of system modeling [17]. The BRB modeling technique has been widely applied in nonlinear system modelling and decision support systems [16–21].

In this paper, a BRB system is designed to model the complex nonlinear relationship between circuit component parameters (i.e., input variables to the BRB system) and a performance index of the circuit (i.e., output) by utilizing the limited knowledge from circuit simulations and its designers. Through rule inference in the BRB and clustering analysis, the acceptability regions can be separated from the value domain of component parameters. Then, an optimization method is presented to seek the optimal feasibility region in the acceptability regions to maximize the volume of tolerance region of the circuit parameters. The remainder of this paper is organized as follows. The research issue is expounded in Section 2. Section 3 describes the use of the BRB modelling technique to approximate the acceptability regions. The tolerance optimization method is presented in Section 4. Section 5 shows some encouraging results obtained from two typical numerical examples of nonlinear performance functions with nonconvex and disconnected acceptability regions and high-dimensional input parameters and a real-world application in the parameter design of track circuit of Chinese high-speed railway.

2. Problem Formulation

Given a product performance or response specification [2–6]

$$\mathrm{Lb} \leqslant g(u) \leqslant \mathrm{Ub}, \tag{1}$$

here $u = (u_1, u_2, \ldots, u_N)$ is a vector of design parameters. g is the performance index or function of an electrical circuit. Ub and Lb are constants, respectively, representing the upper and lower allowable limits of variation of the resonance performance. For discrete component circuits, these parameters may include, but are not limited to, resistances, capacitances, and inductances, whereas for integrated circuits these may be resistivities, linewidths, specific capacitances, and so forth. Commonly, the element u_i of u is characterized by a nominal value u_{i_0} and a tolerance t_i, $i = 1, 2, \ldots, N$.

The acceptability region R_A is defined as [3, 22]

$$R_A = \{u \mid \mathrm{Lb} \leqslant g(u) \leqslant \mathrm{Ub}\}. \tag{2}$$

If $u \in R_A$, then the product is acceptable; otherwise it is unacceptable. The tolerance region R_T is defined as [3, 22]

$$R_T = \{u \mid |u_i - u_{0\_i}| \leq t_i,\ i = 1, 2, \ldots, N\}. \tag{3}$$

When given a nominal value $u_0 = (u_{0\_1}, u_{0\_2}, \ldots, u_{0\_N})$ and a tolerance $t = (t_1, t_2, \ldots, t_N)$, the corresponding parametric yield is defined as [3, 22]

$$Y_u = \frac{V(R_T \cap R_A)}{V(R_T)} \times 100\% = \frac{M}{N} \times 100\%. \tag{4}$$

Here, N is the number of total products, M is the number of acceptable products ($u \in R_A$), and $V(\cdot)$ is the volume of a region. $R_F = R_T \cap R_A$ is defined as the feasibility region which is a subset of R_T at the intersection between R_T and R_A.

When given the design constraints R_A, the actual goal of tolerance design is to maximize R_F (i.e., to seek for the maximum t and corresponding u_0) so that a 100% yield is achievable. In this case, the maximum R_F is equal to the maximum R_T [3, 22]. In tolerance design, the key step is to calculate performance $g(u)$ and estimate the parametric yield. In most cases, the structures of the integrated circuit and analogous circuit are too complex to obtain analytical expressions of circuit performance functions. Hence, designers have to build the circuit simulator using some design software tools (e.g., HSPICE, Simulink) to evaluate the performance function $g(u)$ and estimate yield by simulation runs [22, 23]. However, this kind of simulation-based tolerance design (e.g., Monte Carlo methods) requires numerous circuit simulations and computationally expensive analysis runs [2–6]. In the following section, instead of using a circuit simulator, we will build a BRB system to model the performance function by running as few simulations as possible and using experts' knowledge. The proposed BRB system can be used to approximate the acceptability region and obtain the corresponding feasibility region.

3. Approximating Acceptability Region by Using a BRB System

As an extension of traditional IF-THEN rules, belief rules are the key parts of a BRB system. In a belief rule, each antecedent attribute takes a referential value, and each possible consequent is related to a belief degree [13]. To build a BRB system for circuit performance modelling and acceptability region approximating, we map the relationship between BRB system and circuit performance function in Table 1.

Corresponding to Table 1, the kth ($k = 1, 2, \ldots, K$) referential relationship between input and output of performance function, that is, the belief rule k in the BRB system, can be defined as

Rule k:

$$\begin{cases} \text{IF } (u_1 \text{ is } u_{k,1}) \wedge (u_2 \text{ is } u_{k,2}) \wedge \cdots \wedge (u_N \text{ is } u_{k,N}) \\ \text{THEN } \{(D_1, \beta_{k,1}), (D_2, \beta_{k,2}), \ldots, (D_L, \beta_{k,L})\} \\ \text{with rule weight } \theta_k, \quad k = 1, 2, \ldots, K \\ \text{and attribute weight } \delta_i, \quad i = 1, 2, \ldots, N; \end{cases} \tag{5}$$

here, "$\wedge$" denotes "and."

To use the BRB system to approximate acceptability region R_A involves the following steps: (1) constructing referential values of parameter inputs and performance output

Table 1: Belief rule-based system for circuit performance modelling.

BRB system	Circuit performance function
Antecedent attributes	Parameter inputs $u_1, u_2, \ldots, u_N$
The set of referential values $A_i = \{A_{i,j} \mid j = 1, 2, \ldots, J_i\}$	Referential values of the design parameter u_i
Antecedent in the kth rule, $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots, K$	The kth referential value of the input vector u $u_k = (u_{k,1}, u_{k,2}, \ldots, u_{k,N})$, $u_{k,i} \in A_i$
Consequent in the kth rule $\{(D_1, \beta_{k,1}), (D_2, \beta_{k,2}), \ldots, (D_L, \beta_{k,L})\}$, $\sum_{l=1}^{L} \beta_{k,l} \leq 1$	D_l is the referential value of performance output, $\beta_{k,l}$ is the belief degree of D_l when u is taken as u_k
Rule weights $\theta_k \in [0, 1]$	Relative importance of the kth rule
Attribute weights $\delta_i \in [0, 1]$	Relative importance of u_i in the rule base

about acceptability region R_A; (2) generating new performance output by ER inference of belief rules; (3) training BRB by the selected samples near to R_A.

3.1. Constructing Referential Values of Parameter Inputs and Performance Output about Acceptability Region R_A. The initial belief rules can be established in the following four ways [16]: (1) extracting rules from expert knowledge; (2) extracting rules by examining historical data; (3) using the precious rule bases if available; (4) random rules without any preknowledge. In our context, we use (1) and (2) to determine the parameters in BRB system as listed in Table 1.

In the tolerance design, we first need to specify the upper and lower bounds of each design parameter u_i. Let lb_i and ub_i represent, respectively, the lower and upper bounds on the ith parameter u_i. Thus, the space in which the vector u takes value can be constructed as

$$S_u = \{u = (u_1, u_2, \ldots, u_N) \mid \mathrm{lb}_i \leqslant u_i \leqslant \mathrm{ub}_i,\ i = 1, 2, \ldots, N\}. \tag{6}$$

Generally speaking, we have to set S_u large enough to relatively entirely include the acceptability region. The referential points are uniformly selected from S_u, and then the circuit simulations are implemented to calculate the performance outputs of those points.

As an illustrative example suppose there are two inputs $u = (u_1, u_2)$ with the bounds $1 \leqslant u_1 \leqslant 14$, $1 \leqslant u_2 \leqslant 9$ and its performance function is $g(u)$. We uniformly select 126 initial referential points as $(1, 1), (1, 2), \ldots, (14, 9)$ shown in Figure 1 and calculate their performance outputs by circuit simulations. Assume we have known the acceptability region R_A, which is nonconvex and shown in Figure 1. Then, we can find out 35 points in R_A by constrains in (1). These points are divided into two parts, one is called as "inside point" (12 points), and the other is "inside boundary (IB) point" (23 points). From Figure 1, it can be seen that 126 initial referential points subdivide S_u into 104 gridding cells. The boundary of R_A passes through 26 cells, called boundary gridding (BG) cells and vertexes of each BG cell certainly contain one or two IB points. Hence, in each of BG cells, starting from its IB points, we can seek out three or two vertexes as external boundary (EB) points adjacent to the IB

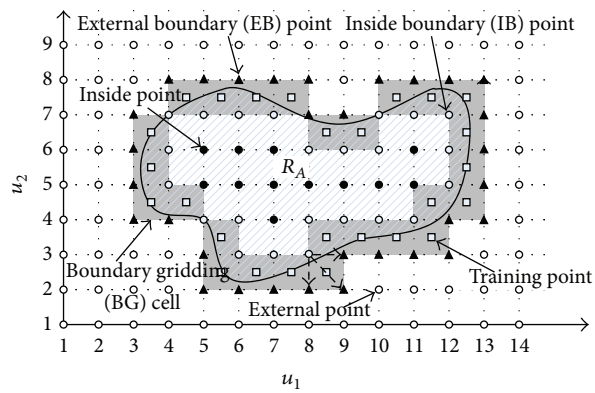

Figure 1: Selection of referential values of parameter inputs and performance output.

points, as demonstrated in the BG cell directly below R_A. In this example, totally, 29 EB points can be found out.

As a result, we totally select 64 points (including 29 EB, 23 IB, and 12 inside points) as the referential values of parameter input. The sets of referential values can be listed as

$$A_1 = \{3, 4, \ldots, 13\}, \qquad A_2 = \{2, 3, \ldots, 8\}. \tag{7}$$

Denote the referential values of parameter input as $u_k = (u_{k,1}, u_{k,2})$, then $u_k \in A_1 \times A_2$, $k = 1, 2, \ldots, K$, $K = 77$; here "×" denotes Cartesian product. Correspondingly, we can construct 77 belief rules.

Next, we need to calculate the bounds of performance output as

$$\begin{gathered} \mathrm{Lb}' \leq g(u_k) \leq \mathrm{Ub}', \\ \mathrm{Lb}' = \min_{u_k \in A_1 \times A_2} (g(u_k)), \qquad \mathrm{Ub}' = \max_{u_k \in A_1 \times A_2} (g(u_k)). \end{gathered} \tag{8}$$

Obviously, $\mathrm{Lb}' \leq \mathrm{Lb}$, $\mathrm{Ub}' \geq \mathrm{Ub}$. According to designers' experiences, the L referential values of performance output can be selected from the interval $[\mathrm{Lb}', \mathrm{Ub}']$ uniformly or not, denoted as $\{D_1, D_2, \ldots, D_L\}$ and $D_1 = \mathrm{Lb}' < D_2 < \cdots < D_L = \mathrm{Ub}'$. For the kth rule, the input $u_k = (u_{k,1}, u_{k,2})$ and

its performance output is $g(u_k)$; then the consequent belief distribution can be given as

$$\{(D_1, \beta_{k,1}), (D_2, \beta_{k,2}), \ldots, (D_L, \beta_{k,L})\}. \tag{9}$$

Here

$$\begin{aligned} \beta_{k,l} &= \frac{(D_{l+1} - g(u_k))}{(D_{l+1} - D_l)}, \quad D_l \le g(u_k) \le D_{l+1}, \\ \beta_{k,l+1} &= \frac{(g(u_k) - D_l)}{(D_{l+1} - D_l)}, \quad D_l \le g(u_k) \le D_{l+1}, \\ \beta_{k,c} &= 0, \quad c = 1, 2, \ldots, l-1, l+2, \ldots, L. \end{aligned} \tag{10}$$

Obviously, $\sum_{l=1}^{L} \beta_{k,l} = 1$, $g(u_k) = \beta_{k,l+1} D_{l+1} + \beta_{k,l} D_l$.

In Table 1, the remaining rule weights θ_k and the attribute weights δ_i reflect the importance (or unimportance) of the kth rule and ith referential parameter input, respectively. Their values can be determined based on designers' knowledge. When designers' knowledge cannot be collected, θ_k and δ_i will be set equal to 1, respectively, which means equal importance.

Note that, in practice, there may be more than one acceptability region in the parameter space S_u; in this case, some available clustering analysis methods (e.g., k-means clustering [24]) can be used to recognize every disconnected acceptability region. Then, by using the proposed procedure as shown in Figure 1, the belief rules of all disconnected regions can be generated so as to compose a rule base for modelling the whole acceptability regions. In Section 5.1, an example of the disconnected acceptability regions will be given to show the procedure of BRB based modeling.

3.2. Generating New Performance Output by ER Inference of Belief Rules. Section 3.1 gives the BRB system to describe the acceptability region R_A. Actually, the BRB uses the grid-based mechanism to approximate R_A and one vertex of gridding cell corresponds to one belief rule in rule base. Therefore, given a new input $u = (u_1, u_2, \ldots, u_N)$, which certainly falls into a certain N-dimension cell. This input u can activate 2^N rules of this cell. Thus, the activation weight of the kth rule, w_k, is calculated as [16]

$$w_k = \frac{\theta_k \prod_{i=1}^{N} \left(\alpha_i^k\right)^{\overline{\delta}_i}}{\sum_{k=1}^{K} \left[\theta_k \prod_{i=1}^{N} \left(\alpha_i^k\right)^{\overline{\delta}_i}\right]}; \tag{11}$$

here, the relative attribute weight is defined as [16]

$$\overline{\delta}_i = \frac{\delta_i}{\max_{i=1,2,\ldots,N} \{\delta_i\}}. \tag{12}$$

α_i^k is the individual matching degree to which the input u_i matches the ith antecedent referential value $u_{k,i}$ in the kth rule. Here, $u_{k,i} \in A_i$, $A_i = \{A_{i,j} \mid j = 1, 2, \ldots, J_i\}$, J_i is the number of referential values of u_i, and $A_{i,1} < A_{i,2} < \cdots < A_{i,Ji}$. α_i^k is calculated as

$$\alpha_i^k = \begin{cases} \dfrac{(u_i - A_{i,q})}{(A_{i,q+1} - A_{i,q})}, & A_{i,q} \le u_i \le A_{i,q+1},\ u_{k,i} = A_{i,q+1} \\ \dfrac{(A_{i,q+1} - u_i)}{(A_{i,q+1} - A_{i,q})}, & A_{i,q} \le u_i \le A_{i,q+1},\ u_{k,i} = A_{i,q} \\ 1, & u_i \le A_{i,1} = u_{k,i} \text{ or } u_i \ge A_{i,J_i} = u_{k,i} \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Here, $q = 1, 2, \ldots, J_i - 1$.

Having determined the activation weight of each rule in the rule base, the ER approach can be directly applied to combine the rules and generate final conclusions [14]. The output of the new input u by the combination is defined as

$$O(u) = \{(D_1, \beta_1), (D_2, \beta_2), \ldots, (D_L, \beta_L)\}. \tag{14}$$

Since the rule consequent in the BRB system has the form of belief distribution, the result of reference in (14) is also a belief distribution, which expresses that if the input is given as $u = (u_1, u_2, \ldots, u_N)$; then the consequent is D_1 to a degree β_1, D_2 to a degree β, ..., and D_L to a degree β_L. The analytical format of the ER algorithm can be used to calculate the combined belief degree β_l in D_l as [17]

$$\begin{aligned} \beta_l = \Bigg(\mu \times \Bigg[\prod_{k=1}^{K} \Bigg(w_k \beta_{k,l} + 1 - w_k \sum_{l=1}^{L} \beta_{k,l}\Bigg) \\ - \prod_{k=1}^{K} \Bigg(1 - w_k \sum_{l=1}^{L} \beta_{k,l}\Bigg)\Bigg]\Bigg) \\ \cdot \Bigg(1 - \mu \times \Bigg[\prod_{k=1}^{K} (1 - w_k)\Bigg]\Bigg)^{-1}. \end{aligned} \tag{15}$$

Here,

$$\begin{aligned} \mu = \Bigg[\sum_{l=1}^{L} \prod_{k=1}^{K} \Bigg(w_k \beta_{k,l} + 1 - w_k \sum_{l=1}^{L} \beta_{k,l}\Bigg) \\ - (L-1) \prod_{k=1}^{K} \Bigg(1 - w_k \sum_{l=1}^{L} \beta_{k,l}\Bigg)\Bigg]^{-1}. \end{aligned} \tag{16}$$

Next, we can estimate performance output by the weighted average operator

$$g_{\mathrm{BRB}}(u) = \sum_{l=1}^{L} D_l \beta_l. \tag{17}$$

3.3. The Parameter Optimization of the BRB System Using the Selected Training Samples. Although the initial belief rules can be constructed by the limited simulations and designers'

knowledge and the performance outputs can be estimated by ER inference of the initial belief rules, the estimation accuracy can be improved if the parameters in the belief rules are fine-tuned through learning from some selected training samples. The adjustable parameters in a rule base include belief degrees $(\beta_{k,1}, \beta_{k,2}, \ldots, \beta_{k,L})$, rule weights $(\theta_1, \theta_2, \ldots, \theta_K)$, and attribute weight $(\delta_1, \delta_2, \ldots, \delta_N)$.

When we search for the available feasibility region R_F from the approximated acceptability region R_A estimated by the BRB, the vertexes of the maximum R_F certainly reach to the boundary points of R_A, which are all included in the set of critical points $\{u \mid g(u) = \text{Lb}, g(u) = \text{Ub}\}$. To a large extent, the estimation accuracy of the boundary points determines the accuracy of the optimal R_F. Since the boundary points of R_A fall into the boundary gridding (BG) cells as shown in Figure 1, we select the central points of the BG cells as training points and then obtain their performance outputs by circuit simulations as the training samples. Thus, given Q training points u_q $(q = 1, 2, \ldots, Q)$, the error function $d(P)$ between the simulated output $g(u_q)$ and the estimated output $g_{\text{BRB}}(u_q)$ can be defined as [21]

$$d(P) = \frac{1}{Q} \times \sum_{q=1}^{Q} \left(g\left(u_q\right) - g_{\text{BRB}}\left(u_q\right)\right)^2. \tag{18}$$

Here, $d(P)$ is function of the parameter set P

$$P = \{\beta_{k,l}, \theta_k, \delta_i \mid k = 1, 2, \ldots, K, \\ l = 1, 2, \ldots, L,\ i = 1, 2, \ldots, N\}. \tag{19}$$

The objective of the training is to minimize the difference $d(P)$ by adjusting the parameters P. Note that the optimal P can be obtained by using gradient-based search methods or nonlinear optimization software packages, such as the *fmincon* function in the Optimization Toolbox of MATLAB [18].

For example, in the case of two-dimensional inputs as shown in Figure 1, there are 26 BG cells. Correspondingly, 26 training points are selected, only the extra 26 circuit simulations are needed to get training samples. As a result, totally 152 circuit simulations (126 initial referential points and 26 training points) are required to construct the trained BRB with two-dimensional inputs.

4. Tolerance Optimization Methods

In this section, the proposed BRB can be used to identify the optimal feasibility region from the acceptability regions based on the design criterion of maximizing volume of tolerance region.

4.1. Choosing Initial Solution of Optimization. Firstly, we have to select an available point and its tolerance from the acceptability region as the initial solution of optimization, since an arbitrary initial point and its tolerance may cause the optimization process which is time-consuming and even becomes trapped in the local optima [3]. There are two ways of choosing the initial solution.

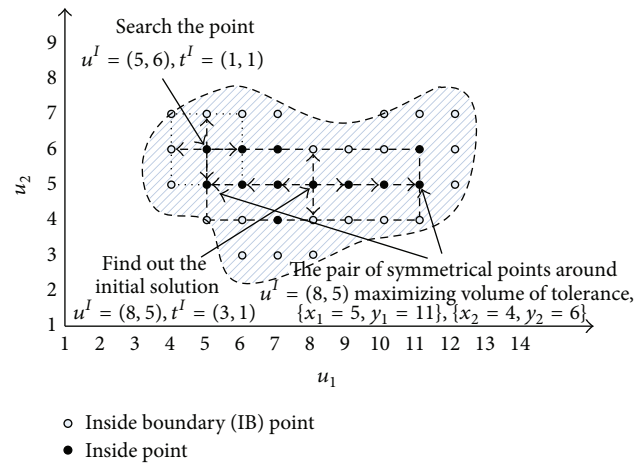

Figure 2: Searching for the initial solution of optimization in the inside points.

The first way is to seek the initial solution directly from the inside points and inside boundary (IB) points that we have obtained by circuit simulations.

For N-dimensional space of design parameters, let $S^I = \{u_z^I \mid z = 1, 2, \ldots, Z\}$ be the set including a total of Z inside points and IB points in which $u_z^I = (u_{z,1}^I, u_{z,2}^I, \ldots, u_{z,N}^I)$. Set u_z^I as the central point and search the other points on its both sides along every coordinate direction of the N-dimensional space. In the ith coordinate direction, we can find a certain pair of symmetrical points around u_z^I; their values in the ith coordination is denoted as $x_{z,i}$ and $y_{z,i}$, respectively. Because of symmetrical characteristic, we have $|u_{z,i}^I - x_{z,i}| = |u_{z,i}^I - y_{z,i}|$ and define $t_{z,i}^I = |u_{z,i}^I - x_{z,i}|$ as the tolerance of u_z^I in the ith coordinate. Thus, we need to search all pairs of the symmetrical points about u_z^I in all coordinate directions and find out such a tolerance vector $t_z^I = (t_{z,1}^I, t_{z,2}^I, \ldots, t_{z,N}^I)$ such that $(2t_{z,1}^I \times 2t_{z,2}^I \times \cdots \times 2t_{z,N}^I)$ is the maximum volume. For $z = 1, 2, \ldots, Z$, the initial solution of optimization can be found in S_I which has the maximum volume of tolerance.

Let us take the 2-dimensional space as an example. Figure 2 shows the inside points and IB points in Figure 1. Searching every point, we can determine that the initial solution is $u^I = (8, 5)$, $t^I = (3, 1)$. Compared with the other points (e.g., $u^I = (5, 6)$), $u^I = (8, 5)$ has the maximum volume of tolerance $(2 \cdot 3 \times 2 \cdot 1 = 12)$.

Instead of circuit simulations, we use the proposed BRB to estimate more inside points so as to increase the density of the cast points in the acceptability region. Hence, the second way is to search for the initial solution from these estimated points using the above procedure given in the first way. Obviously, the initial solution got by the second way is more accurate than that got by the first way, but it needs more computational loads.

Note that, if there are multiple disconnected acceptability regions in the parameter space S_u; then the above two ways can be implemented in every acceptability region to obtain the corresponding initial solution.

4.2. Specifying the Objective Function. According to the criterion of maximizing volume of tolerance region, the objective function can be defined as

$$\begin{aligned} &\max && (V(R_F)) \\ &\text{subject to} && R_F \subseteq R_A. \end{aligned} \tag{20}$$

From (2), (3), and (17), it can be translated into

$$\begin{aligned} \max_{u_0,t} \quad & \prod_i^N (2t_i), \quad u_0 = (u_{0\_1}, u_{0\_2}, \ldots, u_{0\_N}), \\ & t = (t_1, t_2, \ldots, t_N) \\ \text{s.t.} \quad & \text{Lb} \le g_{\text{BRB}}\left(u_{0\_1} \pm \left(\frac{t_1}{j_1}\right), u_{0\_2} \pm \left(\frac{t_2}{j_2}\right), \ldots, \right. \\ & \left. u_{0\_N} \pm \left(\frac{t_N}{j_N}\right)\right) \le \text{Ub}, \\ & \text{Lb} \le g_{\text{BRB}}\left(u_{0\_1} \pm \left(\frac{t_1}{j_1}\right), u_{0\_2}, \ldots, u_{0\_N}\right) \le \text{Ub}, \\ & \text{Lb} \le g_{\text{BRB}}\left(u_{0\_1}, u_{0\_2} \pm \left(\frac{t_2}{j_2}\right), \ldots, u_{0\_N}\right) \le \text{Ub}, \ldots, \\ & \text{Lb} \le g_{\text{BRB}}\left(u_{0\_1}, u_{0\_2}, \ldots, u_{0\_N} \pm \left(\frac{t_N}{j_N}\right)\right) \le \text{Ub}, \\ & 0 \le t_i \le \frac{\text{ub}_i - \text{lb}_i}{2}, \\ & \text{for } i = 1, 2, \ldots, N; \quad j_1 = 1, 2, \ldots, T, \\ & j_2 = 1, 2, \ldots, T, \ldots, \quad j_N = 1, 2, \ldots, T. \end{aligned} \tag{21}$$

R_F is a cube ($N = 3$) or a hypercube ($N > 3$). Hence, if R_A is convex, then $T = 1$. We only need to ensure that 2^N vertexes of R_F fall into the acceptability region R_A. If R_A is nonconvex, then $T \geqslant 2$ because it is necessary that the extra $(2T)^N + 2TN - 2^N$ points on the N sides of R_F must fall into R_A. T can be taken according to experts' experiences about the shape of R_A and the tradeoff between the computational burden and accuracy of optimization. The initial u_0 and t is given by the methods in Section 4.1. Similar with the optimization of the BRB parameters in Section 3.3, the tolerance optimization also can be solved by the *fmincon* function in the MATLAB.

5. Numerical Studies

In this section, two numerical studies and an industrial case are given to illustrate the procedure of using the BRB system to solve tolerance design problem.

5.1. Two-Dimensional Rosenbrock Function Example. The Rosenbrock function is a well-known benchmark for assessing nonlinear numerical optimization algorithms [22]. In our context, the two-dimensional Rosenbrock function is considered as the performance function to construct the nonconvex and disconnected acceptability regions. The proposed method can be tested using these complex acceptability regions.

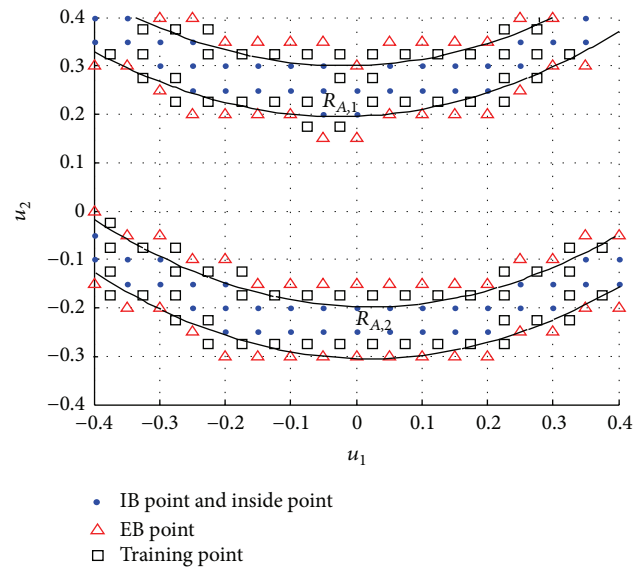

FIGURE 3: The acceptability regions $R_{A,1}$, $R_{A,2}$ and IB, EB inside and training points.

Set the acceptability regions as

$$R_A = \{u \mid 5 \le g(u) \le 10\}. \tag{22}$$

Here, the performance function is

$$g(u) = 100\left(u_2 - u_1^2\right)^2 + (1 - u_1)^2. \tag{23}$$

The design parameters are $-0.4 \leqslant u_1 \leqslant +0.4$, $-0.4 \leqslant u_2 \leqslant +0.4$, $u = (u_1, u_2)$. From Figure 3, it can be seen that the R_A includes two disconnected and nonconvex subregions denoted as $R_{A,1}$ and $R_{A,2}$, respectively.

According to the procedure of building BRB system given in Section 3, suppose 17 referential values are uniformly selected for the two design parameters u_1 and u_2, respectively; we then have 17^2 initial referential points in $S_u = \{u = (u_1, u_2) \mid -0.4 \leqslant u_i \leqslant 0.4_i,\ i = 1, 2\}$. The corresponding 289 circuit simulations are implemented to generate the initial referential performance outputs. In this numerical example, the circuit simulations are replaced by calculations of analytical formula (23). Based on the constraints given by (22), the 68 inside pints and 65 IB points are picked out from the 289 initial referential points as shown in Figure 3. Here, the k-means clustering algorithm is used to divide these 289 points into two parts so as to recognize $R_{A,1}$ and $R_{A,2}$. Thus,

Table 2: The partial parameters of the initial rule base.

k	$u_{k,1}$	$u_{k,2}$	$\beta_{k,1}$	$\beta_{k,2}$	$\beta_{k,3}$	$\beta_{k,4}$	$\beta_{k,5}$	$\beta_{k,6}$	$\beta_{k,7}$
1	−0.4	−0.15	0	0	0	0	0.215	0.785	0
2	−0.4	−0.1	0	0	0	0.64	0.36	0	0
3	−0.4	−0.05	0	0	0.815	0.185	0	0	0
4	−0.4	0	0	0.74	0.26	0	0	0	0
5	−0.4	0.3	0.04	0.96	0	0	0	0	0
6	−0.4	0.35	0	0.215	0.785	0	0	0	0
7	−0.4	0.4	0	0	0.14	0.86	0	0	0
8	−0.35	−0.2	0	0	0	0	0	0.8884	0.1116
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
121	0.35	−0.1	0	0.3134	0.6866	0	0	0	0
122	0.35	−0.05	0.3009	0.6991	0	0	0	0	0
123	0.35	0.3	0.2134	0.7866	0	0	0	0	0
124	0.35	0.35	0	0.2009	0.7991	0	0	0	0
125	0.35	0.4	0	0	0	0.9384	0.0616	0	0
126	0.4	−0.2	0	0	0	0	0	0.34	0.66
127	0.4	−0.15	0	0	0	0.015	0.985	0	0
128	0.4	−0.1	0	0	0.44	0.56	0	0	0

Table 3: The partial parameters in the trained rule base.

k	$u_{k,1}$	$u_{k,2}$	θ_k	$\beta_{k,1}$	$\beta_{k,2}$	$\beta_{k,3}$	$\beta_{k,4}$	$\beta_{k,5}$	$\beta_{k,6}$	$\beta_{k,7}$
1	−0.4	−0.15	1	0.0177	0.0005	0	0	0	0.9818	0
2	−0.4	−0.1	1	0	0	0	1	0	0	0
3	−0.4	−0.05	0.9789	0.0411	0.03	0.9289	0	0	0	0
4	−0.4	0	1	0.0982	0.9018	0	0	0	0	0
5	−0.4	0.3	1	0.0118	0.9882	0	0	0	0	0
6	−0.4	0.35	1	0.0185	0.11	0.8715	0	0	0	0
7	−0.4	0.4	0.9990	0.0185	0	0.9815	0	0	0	0
8	−0.35	−0.2	1	0.0283	0	0	0	0	0.9152	0.0565
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
121	0.35	−0.1	1	0.0308	0.0004	0	0.2358	0.7332	0	0
122	0.35	−0.05	0.9746	0.0625	0	0.9375	0	0	0	0
123	0.35	0.3	0.9990	0.5379	0.4620	0	0	0	0	0
124	0.35	0.35	1	0.0156	0.0587	0.9254	0	0	0	0
125	0.35	0.4	1	0.0158	0	0.1587	0.8255	0	0	0
126	0.4	−0.2	0.9677	0.0391	0.9609	0	0	0	0	0
127	0.4	−0.15	0.9878	0.0429	0.0039	0	0	0	0.2058	0.7474
128	0.4	−0.1	1	0.0308	0.0004	0	0.2355	0.7333	0	0

the 79 EB points can be found out from the 79 BG cells. As a result, we obtain the sets of antecedent referential values as

$$A_1 = \{-0.4, -0.35, -0.3, -0.25, -0.2, -0.15, -0.1, -0.05, 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4\},$$
$$A_2 = \{-0.3, -0.25, -0.2, -0.15, -0.1, -0.05, 0, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4\}. \quad (24)$$

The referential values of parameter input $u_k = (u_{k,1}, u_{k,2})$, $u_k \in A_1 \times A_2$, $k = 1, 2, \ldots, K$, $K = 128$. By (8), we have $\mathrm{Lb}' = 2$, $\mathrm{Ub}' = 14$. Suppose $L = 7$ referential values of performance output are uniformly taken from the interval $[\mathrm{Lb}', \mathrm{Ub}']$; then $D_1 = 2, D_2 = 4, \ldots, D_7 = 14$. Thus, we can construct the initial belief rules by (9) and (10) and list the partial parameters of the initial rule base in Table 2.

In Table 2, the other parameters θ_k and δ_i are set equal to 1, respectively.

Next, set the 79 central points of the BG cells as training points and then obtain their performance outputs by circuit simulations as the training samples as shown in Figure 3. By (18), we can optimize the parameter set of the initial rule base and get the trained rule base. Table 3 lists the partial

Table 4: Comparisons of the optimization results for $R_{A,1}$.

R_F^o		$R_{F,\mathrm{BRB}}^o$			$R_{F,\mathrm{BRB}}^I$		
u^o	t^o	u_{BRB}^o	t_{BRB}^o	$\mathrm{re}_{\mathrm{BRB}}^o$	u_{BRB}^I	t_{BRB}^I	$\mathrm{re}_{\mathrm{BRB}}^I$
(−0.026, 0.266)	(0.186, 0.033)	(−0.0255, 0.265)	(0.191, 0.035)	4.4%	(−0.02, 0.26)	(0.15, 0.04)	20.3%

Table 5: Comparisons of the optimization results for $R_{A,2}$.

R_F^o		$R_{F,\mathrm{BRB}}^o$			$R_{F,\mathrm{BRB}}^I$		
u^o	t^o	u_{BRB}^o	t_{BRB}^o	$\mathrm{re}_{\mathrm{BRB}}^o$	u_{BRB}^I	t_{BRB}^I	$\mathrm{re}_{\mathrm{BRB}}^I$
(0.0164, −0.234)	(0.182, 0.033)	(0.0195, 0.234)	(0.185, 0.034)	2.4%	(0.02, −0.24)	(0.14, 0.04)	22.1%

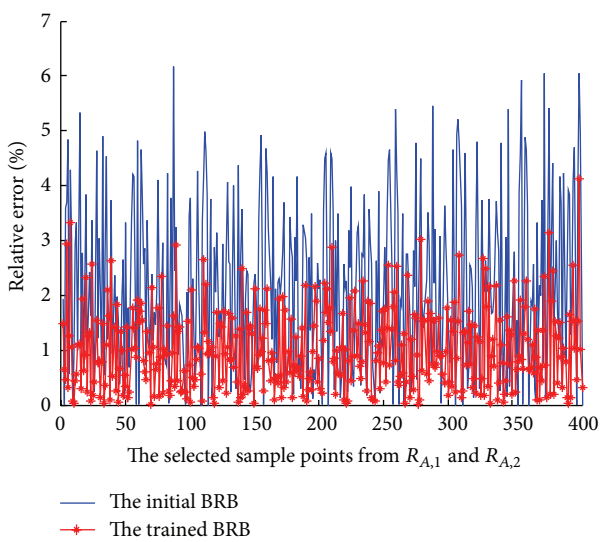

Figure 4: The relative errors of the initial BRB and the trained BRB.

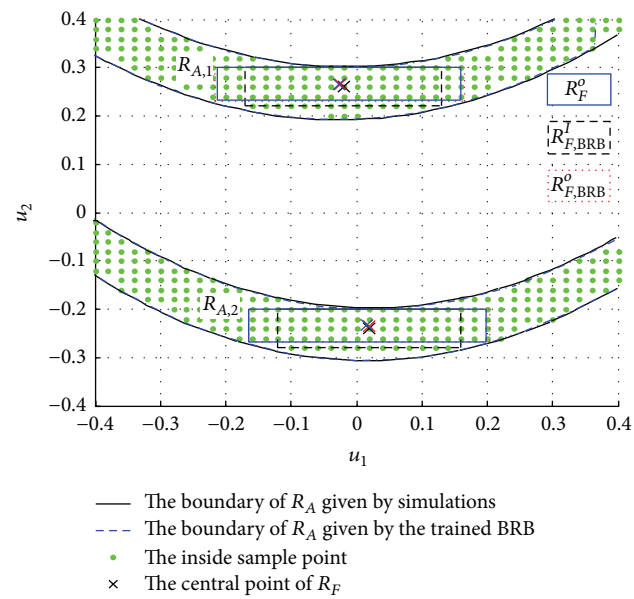

Figure 5: The approximation of R_A and optimization of R_F by using the trained BRB.

parameters obtained by training; here, after training, δ_i is still equal to 1 for $i = 1, 2$.

We uniformly select 404 sample points from $R_{A,1}$ and $R_{A,2}$ and calculate their performance outputs by the initial BRB and trained BRB. Figure 4 shows the relative errors of the trained BRB and the initial BRB, respectively. Obviously, the training process improves the estimation accuracy of the BRB system.

After obtaining the trained BRB, the next step is to optimize tolerances by the proposed method in Section 4. Firstly, according to the second way introduced in Section 4.1, we reuse the inside points in Figure 4 generated by the trained BRB to find out the initial solution $R_{F,\mathrm{BRB}}^I$ of optimization. Secondly, by using the objective function ($T = 10$) in (21), we can get the optimal result $R_{F,\mathrm{BRB}}^o$ together with the optimal R_F^o by circuit simulations shown in Figure 5. Tables 4 and 5 list $R_{F,\mathrm{BRB}}^I$, $R_{F,\mathrm{BRB}}^o$, R_F^o, and the relative errors $\mathrm{re}_{\mathrm{BRB}}^o$ and $\mathrm{re}_{\mathrm{BRB}}^I$ between the estimated $R_{F,\mathrm{BRB}}^I$, $R_{F,\mathrm{BRB}}^o$, and R_F^o for $R_{A,1}$ and $R_{A,2}$, respectively. Here, $\mathrm{re}_{\mathrm{BRB}}^o$ is defined as

$$\mathrm{re}_{\mathrm{BRB}}^o = 100\% \times \frac{1}{N}\sum_{i=1}^{N}\Big(\Big(\Big|\big(u_i^o - t_i^o\big) - \big(u_{\mathrm{BRB},i}^o - t_{\mathrm{BRB}i}^o\big)\Big| + \Big|\big(u_i^o + t_i^o\big) - \big(u_{\mathrm{BRB},i}^o + t_{\mathrm{BRB},i}^o\big)\Big|\Big) \cdot \big(2t_i^o\big)^{-1}\Big). \tag{25}$$

$\mathrm{re}_{\mathrm{BRB}}^I$ can be gotten using the same way, in this example, $N = 2$. Obviously, $R_{F,\mathrm{BRB}}^o$ is very close to the optimal R_F^o.

5.2. Eight-Dimensional Quadratic Performance Function. In the yield estimation of integrated circuits, the high-dimensional quadratics or higher order polynomials are usually

Table 6: Comparisons of the optimization results for R_A.

	R_F^o			$R_{F,\mathrm{BRB}}^o$			$R_{F,\mathrm{BRB}}^I$	
i	u_i^o	t_i^o	$u_{\mathrm{BRB},i}^o$	$t_{\mathrm{BRB},i}^o$	$\mathrm{re}_{\mathrm{BRB}}^o$	$u_{\mathrm{BRB},i}^I$	$t_{\mathrm{BRB},i}^I$	$\mathrm{re}_{\mathrm{BRB}}^I$
1	4.9929	0.0738	4.9891	0.0743		4.99	0.065	
2	4.9913	0.0887	4.9896	0.0889		4.99	0.065	
3	4.9994	0.0157	4.9969	0.0164		4.99	0.045	
4	4.9925	0.0774	4.9891	0.0778	5.8%	4.99	0.065	43.2%
5	4.9922	0.0802	4.9893	0.0808		4.99	0.065	
6	4.9928	0.0747	4.9891	0.0753		4.99	0.055	
7	4.9915	0.0868	4.9895	0.0872		4.99	0.055	
8	4.9940	0.0642	4.9888	0.0652		4.99	0.05	

used to model the performances of interest. Here, we give an eight-dimensional quadratic function to test the proposed method. Without loss of generality, assume a circuit performance can be expressed as an 8-dimension quadratic function whose symmetric matrixes are diagonal or block diagonal [6, 25, 26]. The function for this example is taken as

$$g(u) = a_0 + Ju + \frac{1}{2}u^T Hu, \tag{26}$$

where $a_0 = 6$ and matrices J and H are as follows:

$$J = \begin{bmatrix} -1 & 1 & 0 & 2 & 3 & 1 & 2 & 4 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 \end{bmatrix}. \tag{27}$$

Set the acceptability region as

$$R_A = \{u \mid 346 \le g(u) \le 360\}. \tag{28}$$

The design parameter vector $u = (u_1, u_2, \ldots, u_8)$, $S_u = \{u \mid 4.5 \leqslant u_i \leqslant 5.5_i,\ i = 1, 2, \ldots, 8\}$.

Suppose the numbers of the referential values uniformly selected for u_i are $J_i = 3$ for $i = 1, 4, 6, 8$ and $J_i = 3$ for $i = 2, 3, 5, 7$, respectively. The corresponding 1296 circuit simulations are implemented to generate the initial referential performance outputs. Based on the constraints given by (28), 165 IB points are picked out from 1296 initial referential points. Thus, 458 EB points can be found out. As a result, we obtain the initial rule base consisting of 623 belief rules, in which $L = 26$ referential values of performance output are uniformly taken from the interval $[\mathrm{Lb}', \mathrm{Ub}']$; here, $\mathrm{Lb}' = 302$, $\mathrm{Ub}' = 406$, and then $D_1 = 302, D_2 = 306, \ldots, D_{26} = 406$. The rule weights and attribute weights are all set equal to 1, respectively.

By testing the selected 1373 sample points in R_A, the maximum of the relative errors of the initial BRB is only 0.25%; it is accurate enough for tolerance design. Hence, the training process is not implemented any longer so as to reduce the computational burden. According to the second way introduced in Section 4.1, we reuse the 1373 sample points in R_A generated by the initial BRB to find out the initial solution $R_{F,\mathrm{BRB}}^I$ of optimization. Secondly, by using the objective function in (21) ($T = 2$), we can get the optimal result $R_{F,\mathrm{BRB}}^o$ together with the optimal R_F^o by circuit simulations. Table 6 lists $R_{F,\mathrm{BRB}}^I$, $R_{F,\mathrm{BRB}}^o$, R_F^o, and r_{BRB}^o and r_{BRB}^I between the estimated $R_{F,\mathrm{BRB}}^I$, $R_{F,\mathrm{BRB}}^o$, and R_F^o respectively. Obviously, $R_{F,\mathrm{BRB}}^o$ accurately approximates the optimal R_F^o.

5.3. Application in the Tolerance Design of Railway Track Circuit. Railway track circuit is an essential component of information transmission system between track and vehicle and the automatic train control system [6, 27]. It uses a specific carrier frequency to transmit the coded control information to the train. The application considered in this paper concerns the parameters designs of the insulating section of a jointless track circuit named as ZPW 2000A widely used on Chinese high-speed railway lines. This device will first be described and the problem addressed will be exposed.

The railway track is divided into different sections. Each one of them has a specific ZPW 2000A consisting of main track circuit and short track circuit [28] (see Figure 6). In main track circuit, a transmitter in sending end delivers an alternating current with the specific modulation frequency; a receiver in receiving end demodulates the currents signal transmitted along two rails and controls the track relay. The short track circuit is an electric insulating section composed of two tuning units (BA1 and BA2) and a track air-core inductor (SVA), which can achieve good signal insulation between adjacent track circuits through its resonance characteristic.

Figure 7 shows the schematic diagram of the short track circuit of G2. We take the analysis of the resonance performance of the tuning unit 2 (BA2) as an example. $L_2\ C_2\ C_3$ parallel resonance with the track inductance Lv happens at its frequency 2300 Hz. Lv is given as $Lv = 0.5L + 0.5L//Ls$. $L(\mu H)$ and $Ls(\mu H)$ are respective inductance values of rails and SVA in short track circuit of G2. "//" denotes parallel operator. The impedance value of $L_2\ C_2\ C_3\ Lv$ circuit gets to maximum about 2 Ω. It is equivalent to "open circuit" and reduces the attenuation of G2 signal intensity. In our context, parallel resonance frequency $g(L_2, C_2, C_3)$ is considered as the performance function. In practical parameters designs, it

Table 7: Comparisons of the optimization results for R_A.

	R_F^o		$R_{F,\mathrm{BRB}}^o$			$R_{F,\mathrm{BRB}}^I$		
i	u_i^o	t_i^o	$u_{\mathrm{BRB},i}^o$	$t_{\mathrm{BRB},i}^o$	$\mathrm{re}_{\mathrm{BRB}}^o$	$u_{\mathrm{BRB},i}^I$	$t_{\mathrm{BRB},i}^I$	$\mathrm{re}_{\mathrm{BRB}}^I$
1	97.0506	0.5335	97.0153	0.5402		96.9	0.6	
2	91.0297	0.8900	91.1083	0.8917	6.5%	91	0.9	11.1%
3	262.2203	1.1877	262.2363	1.2363		262.2	1.2	

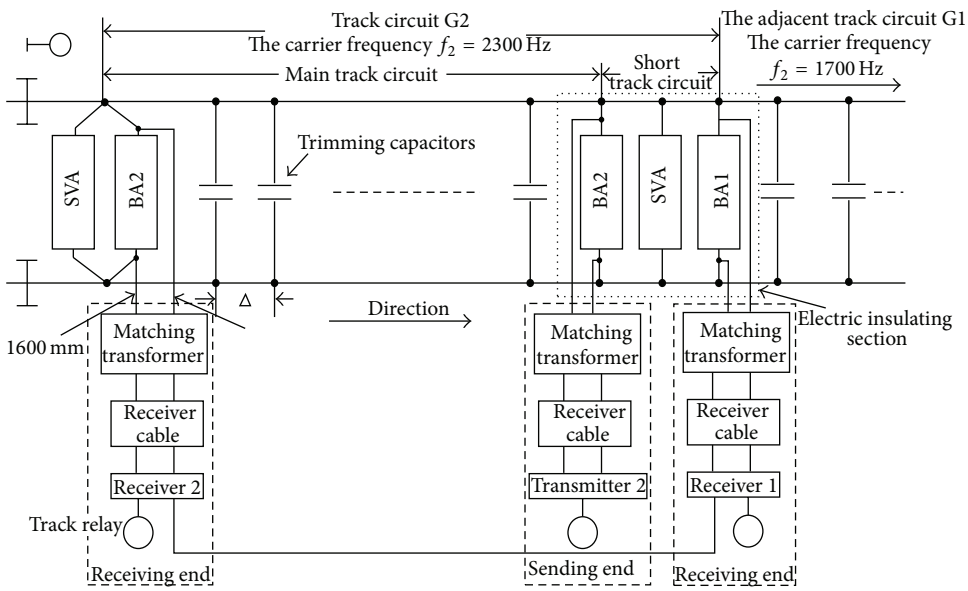

Figure 6: System structure of ZPW 2000A.

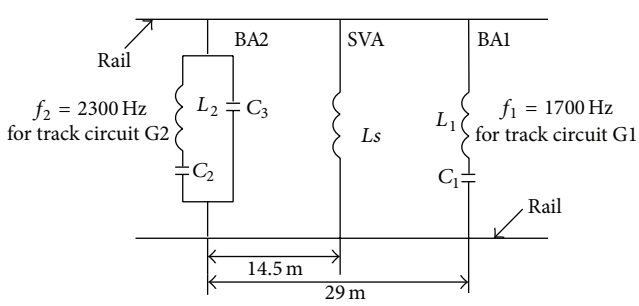

Figure 7: The short track circuit of G2.

is generally required that the nominal values of g is 2300 Hz [28]. The maximum frequency drifts are ±11 Hz. So we set the acceptability region as

$$R_A = \{u \mid 2289\ \mathrm{Hz} \leq g(u) \leq 2311\ \mathrm{Hz}\}. \tag{29}$$

Here, the design parameters $u_1 = L_2$, $u_2 = C_2$, $u_3 = C_3$, $u = (u_1, u_2, u_3)$, and $S_u = \{u \mid 96\mu H \leqslant u_1 \leqslant 98\mu H, 88\mu F \leqslant u_2 \leqslant 92\mu F, 261\mu F \leqslant u_3 \leqslant 264\mu F\}$. The whole circuit model of ZPW 2000A has been built using the software Simulink in [29]. Here, we can use the Simulink-based circuit simulator to generate the simulation data of performance output $g(u)$.

We uniformly select $J_1 = 7$, $J_2 = 9$, and $J_3 = 8$ referential values for u_1, u_2, and u_3, respectively. The corresponding 504 circuit simulations are implemented to generate the initial referential performance outputs. Based on the constraints given by (29), 347 inside points and 41 IB points are picked out from the 504 initial referential points. Thus, the 41 EB points can be found out. As a result, we obtain the initial rule base consisting of 429 belief rules, in which $L = 10$ referential values of performance output are uniformly taken from the interval $[\mathrm{Lb}', \mathrm{Ub}']$; here, $\mathrm{Lb}' = 2287$, $\mathrm{Ub}' = 2314$, and then $D_1 = 2287, D_2 = 2290, \ldots, D_{10} = 2314$. The rule weights and attribute weights are all set equal to 1, respectively.

Similar with example 2, the given initial BRB is accurate enough for tolerance design, so the training process is cancelled. Figure 8 shows the approximated R_A which is a complex polyhedron. The selected 2470 sample points are generated by the initial BRB, which are used to seek for the initial solution $R_{F,\mathrm{BRB}}^I$. Using the objective function in (21) ($T = 2$), we can get the optimal result $R_{F,\mathrm{BRB}}^o$ (see Figure 8) together with the optimal R_F^o by Simulink simulations. Table 7 lists $R_{F,\mathrm{BRB}}^I$, $R_{F,\mathrm{BRB}}^o$, R_F^o, and r_{BRB}^o and r_{BRB}^I between the estimated $R_{F,\mathrm{BRB}}^I$, $R_{F,\mathrm{BRB}}^o$, and R_F^o, respectively. Obviously, $R_{F,\mathrm{BRB}}^o$ accurately approximates the optimal R_F^o.

6. Conclusions

In this paper, a BRB system is designed to model the acceptability region and optimize the feasibility region of circuit

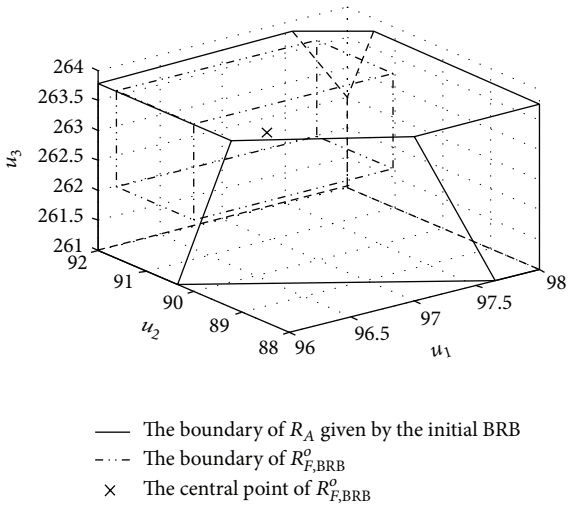

Figure 8: The approximated R_A and the optimal R_F given by the initial BRB.

parameters so that the volume of the tolerance region of the circuit can be maximized. Its virtues can be demonstrated by several examples in this paper.

The main advantages of this new method are as follows.

(i) The physical meanings of the parameters and structures of the BRB system are transparent and intuitively easy to understand by experts and engineers, so they can participate in the main steps of system modeling (e.g., determining the number of rules by considering the inside points and IB points, choosing the training samples from the BG cells, and determining the attribute weights and rule weights).

(ii) The proposed BRB system is applicable to complex cases, such as highly nonlinear performance function and nonconvex and disconnected feasibility regions.

(iii) The proposed optimization algorithm provides alternative ways to obtain the initial solutions of the optimization problem so as to avoid local optima with a higher chance and improve the efficiency of the algorithm.

Like all other deterministic methods, the proposed method also suffers from exponential explosion of computational costs when the dimension of a design parameter space increases. However, when there are strong correlations between the design parameters, the correlation analysis methods can be used to find out less independent variables or principal components so as to reduce the dimension of design parameter space as analyzed in [30].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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