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Research Article

Adaptive Synchronization of Chaotic SC-CNN with Uncertain State Template

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The problem of synchronization of chaotic State Controlled Cellular Neural Network (SC-CNN) with uncertain state template is investigated. In detail, the following three cases are solved: firstly, synchronization of two identical chaotic SC-CNNs with uncertain state template, secondly, synchronization of two nonidentical chaotic SC-CNNs with all uncertain state templates, and, thirdly, synchronization between chaotic SC-CNN with uncertain state template and different uncertain parameter chaotic systems. The controllers and update laws proposed in each case are proved closely based on Lyapunov stability theory. In addition, some illustrative corresponding examples are presented to demonstrate the effectiveness and usefulness of the proposed control laws.

1. Introduction

State Controlled Cellular Neural Network (SC-CNN) [1, 2] is a simple model of Cellular Neural Networks (CNNs) [3]. It is able to generate the dynamics of nonlinear chaotic circuits [4–6]. Chaotic systems in general and chaotic CNN in particular are very complex nonlinear dynamical systems. They have several special characteristics such as sensitivity to initial conditions, topological mixture, and dense periodic orbits. Based on these characteristics, chaotic CNN has some useful applications in fields such as encryption, secure communications, and information processing. Therefore, the problems of chaos generator, chaos control, and chaotic CNN synchronization are interesting to solve.

In the previous studies, the authors have solved some cases with special, certain state template chaotic SC-CNN such as synchronization of an uncertain unified chaotic system and a CNN [7, 8], synchronization of Lorenz system and third-order CNN with uncertain parameters [9], synchronization of CNN with delays based on OPNCL control [10], and synchronization of CNN based on Rossler cells [11]. In more general works, some classes of chaotic CNNs with uncertain parameters were reported [12–16]. However, the uncertainty state template of chaotic SC-CNN has not been mentioned.

From the above discussion, the main contribution of this paper is to solve the problem of synchronization of chaotic SC-CNN with uncertain state template. The cases studied include synchronization of two identical chaotic SC-CNNs with uncertain state template, synchronization of two nonidentical chaotic SC-CNNs with all uncertain state templates, and synchronization of uncertain state template of chaotic SC-CNN with different uncertain parameter chaotic system.

The paper is organized as follows. After the introduction, the problem formulation and preliminaries are given in Section 2. In Section 3, the main results of the paper are presented. Some numerical simulations are included in Section 4. Finally, concluding remarks are provided in Section 5.

2. Problem Formulation and Preliminaries

First, system description and problem formulation are shown as follows.

SC-CNN was introduced by Arena et al. in 1996 [2]. The generalized SC-CNN equations describing system dynamics can be written for each of the cells as

$$\dot{x}_i(t) = -x_i + \sum_{C_k \in N(i)} a_{ik} f(x_k) + \sum_{C_k \in N(i)} b_{ik} v_k + \sum_{C_k \in N(i)} s_{ik} x_k + I_i, \quad (1)$$

where $\mathbf{x}(t) = (x_1, x_2, \ldots, x_n)^T$ is the state vector of SC-CNN, v_i and $N(i)$, $i = 1, 2, \ldots, n$, are the input and the neighborhood set of cell C_i, respectively, $f(x_i)$, $i = 1, 2, \ldots, n$, is the output nonlinear function of cell C_i, and $f(x_i)$ is defined as follows:

$$f(x_i) = \frac{1}{2}\left(|x_i + 1| - |x_i - 1|\right). \quad (2)$$

I_i, $i = 1, 2, \ldots, n$, is the threshold value. Matrixes $\mathbf{A} = (a_{ik})$, $\mathbf{B} = (b_{ik})$, and $\mathbf{S} = (s_{ik})$, $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots, m$, are called feedback, control, and state template, respectively. General chaotic system can be expressed as follows [17]:

$$\dot{y}_i = g_i(y_1, y_2, \ldots, y_n) + G_i(y_1, y_2, \ldots, y_n)\theta, \quad (3)$$

where $y(t) = (y_1, y_2, \ldots, y_n)^T$ is the state vector of chaotic system, $g_i(y)$, $i = 1, 2, \ldots, n$, is a continuous nonlinear function, $G_i(y)$, $i = 1, 2, \ldots, n$, is ith row of the $n \times p$ matrix ($\mathbf{G}(y)$) whose elements are continuous nonlinear functions, and θ is a $p \times 1$ parameter vector of the chaotic system.

Definition 1 (see [18]). The problem of drive-response chaotic synchronization is to determine control law for the response system to guarantee the convergence of the trajectories of the response system to the trajectories of the drive system.

In this paper, three problems in terms of Definition 1 are investigated. From here, the superscripts d and r denote drive and response systems, respectively.

Problem 2. Synchronization of two identical chaotic SC-CNNs with uncertain state template is as follows.

Drive System. Consider the following:

$$\dot{x}_i^d(t) = -x_i^d + \sum_{C_k \in N(i)} a_{ik} f\left(x_k^d\right) + \sum_{C_k \in N(i)} b_{ik} v_k + \sum_{C_k \in N(i)} s_{ik} x_k^d + I_i. \quad (4)$$

Response System. Consider the following:

$$\dot{x}_i^r(t) = -x_i^r + \sum_{C_k \in N(i)} a_{ik} f\left(x_k^r\right) + \sum_{C_k \in N(i)} b_{ik} v_k + \sum_{C_k \in N(i)} s_{ik} x_k^r + I_i + u_i(t). \quad (5)$$

Problem 3. Synchronization of two nonidentical chaotic SC-CNNs with all uncertain state templates is as follows.

Drive System. Consider the following:

$$\dot{x}_i^d(t) = -x_i^d + \sum_{C_k \in N^d(i)} a_{ik}^d f\left(x_k^d\right) + \sum_{C_k \in N^d(i)} b_{ik}^d v_k^d + \sum_{C_k \in N^d(i)} s_{ik}^d {x_k}^d + I_i^d. \quad (6)$$

Response System. Consider the following:

$$\dot{x}_i^r(t) = -x_i^r + \sum_{C_k \in N^r(i)} a_{ik}^r f\left(x_k^r\right) + \sum_{C_k \in N^r(i)} b_{ik}^r v_k^r + \sum_{C_k \in N^r(i)} s_{ik}^r {x_k}^r + I_i^r + u_i(t). \quad (7)$$

Problem 4. Synchronization of chaotic SC-CNN (1) with uncertain state template and different chaotic systems (3) with uncertain parameter.

Some assumptions and necessary lemmas are presented as follows.

Assumption 5. For convenience of presentation, we assume that all cells in chaotic SC-CNN have fully connected. It means that the cardinality of all neighborhood set equals n. Consider the following:

$$|N(i)| = \left|N^d(i)\right| = \left|N^r(i)\right| = n, \quad i = 1, 2, \ldots, n. \quad (8)$$

Assumption 6. It is assumed that the uncertain state template S and feedback matrix A of chaotic SC-CNN are operator norms bounded as follows:

$$\|S\|_1 = \max_{1 \le i \le n} \sum_{k=1}^{n} \left|s_{ik}\right| \le \lambda, \qquad \|A\|_1 = \max_{1 \le i \le n} \sum_{k=1}^{n} \left|a_{ik}\right| \le \gamma. \quad (9)$$

Lemma 7. *For $c_1, c_2, \ldots, c_n \in R$, the following inequality holds:*

$$\sum_{i=1}^{n} \sum_{k=1}^{n} c_i c_k \le n \sum_{i=1}^{n} {c_i}^2. \quad (10)$$

Proof. It is clear that $c_i c_k \le ({c_i}^2 + {c_k}^2)/2$. This leads to

$$\sum_{i=1}^{n} \sum_{k=1}^{n} c_i c_k \le \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{{c_i}^2 + {c_k}^2}{2}. \quad (11)$$

We have

$$\sum_{k=1}^{n} \frac{{c_i}^2 + {c_k}^2}{2} = \frac{n{c_i}^2}{2} + \frac{{c_1}^2 + {c_2}^2 + \cdots + {c_n}^2}{2}. \quad (12)$$

Thus,

$$\sum_{i=1}^{n}\sum_{k=1}^{n}\frac{c_i^2+c_k^2}{2}=\sum_{i=1}^{n}\left(\frac{nc_i^2}{2}+\frac{c_1^2+c_2^2+\cdots+c_n^2}{2}\right)$$
$$=\frac{n\left(c_1^2+c_2^2+\cdots+c_n^2\right)}{2}+\frac{n\left(c_1^2+c_2^2+\cdots+c_n^2\right)}{2}$$
$$=n\sum_{i=1}^{n}c_i^2. \quad (13)$$

Therefore, from (11), (12), and (13) we have

$$\sum_{i=1}^{n}\sum_{k=1}^{n}c_ic_k\le n\sum_{i=1}^{n}c_i^2. \quad (14)$$

Hence, the proof is complete. □

Lemma 8 (Barbalat Lemma). *If the differentiable function $f(t)$ has a finite limit as $t\to\infty$ and if $\dot{f}$ is uniformly continuous, then $\dot{f}(t)\to 0$ as $t\to\infty$.*

3. Main Results

3.1. Synchronization of Two Identical Chaotic SC-CNNs with Uncertain State Template. Subtracting (4) from (5), we have the error dynamics as follows:

$$\dot{e}_i=-e_i+\sum_{k=1}^{n}a_{ik}\left(f\left(x_k^r\right)-f\left(x_k^d\right)\right)+\sum_{k=1}^{n}\hat{s}_{ik}x_k^r-\sum_{k=1}^{n}s_{ik}x_k^d+u_i(t)$$
$$=-e_i+\sum_{k=1}^{n}a_{ik}\left(f\left(x_k^r\right)-f\left(x_k^d\right)\right)+\sum_{k=1}^{n}\hat{s}_{ik}x_k^r-\sum_{k=1}^{n}s_{ik}x_k^r+\sum_{k=1}^{n}s_{ik}x_k^r-\sum_{k=1}^{n}s_{ik}x_k^d+u_i(t)$$
$$=-e_i+\sum_{k=1}^{n}a_{ik}\left(f\left(x_k^r\right)-f\left(x_k^d\right)\right)+\sum_{k=1}^{n}\left(\hat{s}_{ik}-s_{ik}\right)x_k^r+\sum_{k=1}^{n}s_{ik}e_k+u_i(t), \quad (15)$$

where $(\hat{s}_{ik})$, $i=1,2,\ldots,n$, $k=1,2,\ldots,n$, is an estimation for uncertain state template matrix (s_{ik}), $\mathbf{e}(t)=\mathbf{x}^r(t)-\mathbf{x}^d(t)=(e_1(t),e_2(t),\ldots,e_n(t))^T$ is the synchronization error between response systems and drive ones, and $\mathbf{u}(t)=(u_1(t),u_2(t),\ldots,u_n(t))^T$ is the vector of control input.

In order to guarantee the stability of the synchronization error system (15), a suitable control law is proposed as follows:

$$u_i(t)=-\sum_{k=1}^{n}a_{ik}\left(f\left(x_k^r\right)-f\left(x_k^d\right)\right)-\mu_ie_i, \quad (16)$$

where μ_i, $i=1,2,\ldots,n$, is the feedback strength. It is adapted by the following laws:

$$\dot{\mu}_i=e_i^2. \quad (17)$$

Appropriate update laws for uncertain state template are introduced as follows:

$$\dot{\hat{s}}_{ik}=-e_ix_k^r,\quad i=1,2,\ldots,n;\ k=1,2,\ldots,n. \quad (18)$$

Theorem 9. *The error system (15) is controlled by the controller (16) with the adaptive feedback strength (17) and update laws in (18). Then the trajectories of error system converge to zero. In other words, the drive system (4) and response system (5) are synchronized.*

Proof. We select Lyapunov function as follows:

$$V(t)=\frac{1}{2}\sum_{i=1}^{n}\left(e_i^2+\sum_{k=1}^{n}\left(\hat{s}_{ik}-s_{ik}\right)^2+\left(\mu_i-\phi\right)^2\right), \quad (19)$$

where ϕ is a constant, which will be given in the following. The time derivative of $V(t)$ is

$$\dot{V}(t)=\sum_{i=1}^{n}\left(e_i\dot{e}_i+\sum_{k=1}^{n}\left(\hat{s}_{ik}-s_{ik}\right)\dot{\hat{s}}_{ik}+\left(\mu_i-\phi\right)\dot{\mu}_i\right). \quad (20)$$

With control law $u_i(t)$ (16), the error dynamics (15) become

$$\dot{e}_i=-e_i+\sum_{k=1}^{n}\left(\hat{s}_{ik}-s_{ik}\right)x_k^r+\sum_{k=1}^{n}s_{ik}e_k-\mu_ie_i. \quad (21)$$

Inserting $\dot{e}_i$ from (21), adaptive feedback strength from (17), and state-template update laws from (18) into the right-hand side of (20), one has

$$\dot{V}(t)=\sum_{i=1}^{n}e_i\left(-e_i+\sum_{k=1}^{n}\left(\hat{s}_{ik}-s_{ik}\right)x_k^r+\sum_{k=1}^{n}s_{ik}e_k-\mu_ie_i\right)+\sum_{i=1}^{n}\sum_{k=1}^{n}\left(\hat{s}_{ik}-s_{ik}\right)\left(-e_ix_k^r\right)+\sum_{i=1}^{n}\left(\mu_i-\phi\right)e_i^2$$
$$=\sum_{i=1}^{n}-e_i^2+\sum_{i=1}^{n}\sum_{k=1}^{n}s_{ik}e_ie_k-\sum_{i=1}^{n}\phi e_i^2. \quad (22)$$

Using Assumption 6 and Lemma 7 we have

$$\sum_{i=1}^{n}\sum_{k=1}^{n}s_{ik}e_ie_k\le\sum_{i=1}^{n}\sum_{k=1}^{n}\lambda e_ie_k\le\lambda n\sum_{i=1}^{n}e_i^2. \quad (23)$$

Therefore, from (22) and inequality (23) we obtain the following inequality:

$$\dot{V}(t)\le\left(\lambda n-\phi-1\right)\sum_{i=1}^{n}e_i^2. \quad (24)$$

And now, we choose ϕ satisfying the following condition:

$$\phi\ge\lambda n-1. \quad (25)$$

Thus,

$$\dot{V}(t) \le 0. \tag{26}$$

According to Lyapunov theory, the inequality $\dot{V}(t) < 0$ points out that $V(t)$ converges to zero and it is bounded for all time. From the definition of $V(t)$ in (19), we have $\mathbf{e}(t) \in L_\infty$. Inequality (26) implies that the square of $\mathbf{e}(t)$ is integrable; that is, $\mathbf{e}(t) \in L_2$. Since the trajectories of chaotic systems are always bounded then (21) leads to $\dot{\mathbf{e}}(t) \in L_\infty$. According to Lemma 8, $\mathbf{e}(t) \in L_\infty \cap L_2$ and $\dot{\mathbf{e}}(t) \in L_\infty$; then $\mathbf{e}(t) \to 0$ as $t \to \infty$. It can be concluded that the error system (21) achieves global and asymptotical stability. In other words, the controller (16) and state-template estimation update laws (18) guarantee global and asymptotical synchronization of two identical SC-CNN systems (4) and (5). □

3.2. Synchronization of Two Nonidentical Chaotic SC-CNNs with Uncertain State Template. Consider the problem of synchronization of two nonidentical chaotic SC-CNNs with uncertain state templates, described by (6) and (7). The error dynamic system in this case is

$$\dot{e}_i = -e_i + h_i^r - h_i^d + \sum_{i=1}^{n} s_{ik}^r x_k^r - \sum_{i=1}^{n} s_{ik}^d x_k^d + u_i(t), \tag{27}$$

where

$$h_i^r = \sum_{k=1}^{n} a_{ik}^r f(x_k^r) + \sum_{k=1}^{n} b_{ik}^r v_k^r + I_i^r;$$
$$h_i^d = \sum_{k=1}^{n} a_{ik}^d f(x_k^d) + \sum_{k=1}^{n} b_{ik}^d v_k^d + I_i^d, \tag{28}$$
$$i = 1, 2, \ldots, n.$$

Theorem 10. *The problem of synchronization of two nonidentical chaotic SC-CNNs with all uncertain state templates (6) and (7) is solved by controller and state-template update laws are proposed as follows:*

$$u_i(t) = h_i^d - h_i^r + \sum_{k=1}^{n} \hat{s}_{ik}^d x_k^d - \sum_{k=1}^{n} \hat{s}_{ik}^r x_k^r, \tag{29}$$

$$\dot{\hat{s}}_{ik}^d = -e_i x_k^d; \qquad \dot{\hat{s}}_{ik}^r = e_i x_k^r;$$
$$i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, n. \tag{30}$$

Proof. With control law (30), the error dynamic (27) can be rewritten as follows:

$$\dot{e}_i = -e_i - \sum_{i=1}^{n} (\hat{s}_{ik}^r - s_{ik}^r) x_k^r + \sum_{i=1}^{n} (\hat{s}_{ik}^d - s_{ik}^d) x_k^d. \tag{31}$$

The Lyapunov function is chosen as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{n} e_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik}^d - s_{ik}^d)^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik}^r - s_{ik}^r)^2. \tag{32}$$

The time derivative of $V(t)$ is

$$\dot{V}(t) = \sum_{i=1}^{n} e_i \dot{e}_i + \sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik}^d - s_{ik}^d) \dot{\hat{s}}_{ik}^d + \sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik}^r - s_{ik}^r) \dot{\hat{s}}_{ik}^r. \tag{33}$$

Inserting $\dot{e}_i$ from (31) and update laws from (30) into the above equation, one obtains

$$\begin{aligned}\dot{V}(t) = &-\sum_{i=1}^{n} e_i^2 - \sum_{i}^{n} e_i \sum_{k=1}^{n} (\hat{s}_{ik}^r - s_{ik}^r) x_k^r \\ &+ \sum_{i}^{n} e_i \sum_{i=1}^{n} (\hat{s}_{ik}^d - s_{ik}^d) x_k^d \\ &+ \sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik}^d - s_{ik}^d)(-e_i x_k^d) + \sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik}^r - s_{ik}^r) e_i x_k^r.\end{aligned} \tag{34}$$

After removing the opposite terms, one has

$$\dot{V}(t) = -\sum_{i=1}^{n} e_i^2 \le 0. \tag{35}$$

From (35), $V(t)$ is a decreasing monotonic and lower bounded function. So $V(t)$ has a finite limit as $t \to \infty$. From the definition of $V(t)$ in (32) and afore-mentioned property, one has $\mathbf{e}$, $\mathbf{S}^d - \widehat{\mathbf{S}}^d$ and $\mathbf{S}^r - \widehat{\mathbf{S}}^r$ being bounded. Since the trajectories of chaotic systems are always bounded and from (29), $\mathbf{u}(t)$ is bounded, so that $\dot{\mathbf{x}}^r = \mathbf{x}^r + \mathbf{u}$ is bounded. Finally, $\dot{\mathbf{e}} = \dot{\mathbf{x}}^r - \dot{\mathbf{x}}^d$ is bounded too.

Consider $\ddot{V}(t) = -2\sum_{i=1}^{n} e_i \dot{e}_i$, in which the above results infer that $\ddot{V}(t)$ is bounded. Then $\dot{V}(t)$ is a uniformly continuous function. Applying Lemma 8, one obtains

$$\lim_{t\to\infty} \dot{V}(t) = \lim_{t\to\infty} \left(-\sum_{i=1}^{n} e_i^2\right) = 0 \tag{36}$$

or $\mathbf{e}(t) \to 0$ as $t \to \infty$. Thus, the proof is achieved completely. □

Remark 11. We can also use the following controller for synchronization Problem 3. Consider the following:

$$u_i(t) = -\operatorname{sign}(e_i)(\gamma^d + \gamma^r) + \sum_{k=1}^{n} \hat{s}_{ik}^d x_k^d - \sum_{k=1}^{n} \hat{s}_{ik}^r x_k^r, \tag{37}$$

where γ^d, γ^r are the bounded operator norms of feedback $\mathbf{A}^d$ and $\mathbf{A}^r$, respectively; sign(·) is sign function and update laws in (30).

Proof. With Lyapunov function (32), we have

$$\begin{aligned}\dot{V}(t) &= -\sum_{i=1}^{n} e_i^{\,2} + \sum_{i=1}^{n}\sum_{k=1}^{n} e_i\left(a_{ik}^{r} f\left(x_k^{r}\right) - a_{ik}^{d} f\left(x_k^{d}\right)\right) \\ &\quad - \sum_{i=1}^{n} e_i \operatorname{sign}\left(e_i\right)\left(\gamma^{r} + \gamma^{d}\right) \\ &\le -\sum_{i=1}^{n} e_i^{\,2} + \sum_{i=1}^{n}\sum_{k=1}^{n} \left|e_i\right|\left(\left|a_{ik}^{r}\right|\left|f\left(x_k^{r}\right)\right| + \left|a_{ik}^{d}\right|\left|f\left(x_k^{d}\right)\right|\right) \\ &\quad - \sum_{i=1}^{n} e_i \operatorname{sign}\left(e_i\right)\left(\gamma^{r} + \gamma^{d}\right).\end{aligned} \tag{38}$$

Using Assumption 6, we have

$$\sum_{k=1}^{n}\left|a_{ik}^{r}\right| \le \gamma^{r}, \qquad \sum_{k=1}^{n}\left|a_{ik}^{d}\right| \le \gamma^{d}. \tag{39}$$

Note that function (2) satisfies $|f(\cdot)| \le 1$ and the sign function satisfying $z\operatorname{sign}(z) = |z|$; it can be concluded that $\dot{V}(t) \le -\sum_{i=1}^{n} e_i^{\,2} \le 0$. □

Remark 12. Theorem 10 is also valid for Problem 2.

3.3. Synchronization of Chaotic SC-CNN with Uncertain State Template and Different Uncertain Parameter Chaotic System. Consider the problem synchronization of two different chaotic systems.

Drive System. General chaotic system:

$$\dot{y}_i = g_i\left(y_1, y_2, \ldots, y_n\right) + G_i\left(y_1, y_2, \ldots, y_n\right)\theta, \tag{40}$$

with uncertain parameter vector $\theta = (\theta_1, \theta_2, \ldots, \theta_p)^T$.

Response System. Chaotic SC-CNN:

$$\begin{aligned}\dot{x}_i &= -x_i + \sum_{k=1}^{n} a_{ik} f\left(x_k\right) + \sum_{k=1}^{n} b_{ik} v_k \\ &\quad + \sum_{k=1}^{n} s_{ik} x_k + I_i + u_i(t),\end{aligned} \tag{41}$$

with uncertain state template matrix $\mathbf{S} = (s_{ik})_{n\times n}$. $G_i(y)$ is ith row of an $n \times p$ matrix $\mathbf{G}(y)$. When needed, we can rewrite them as the following details:

$$\begin{gathered} G_i(y) = \left(G_{i1}(y), G_{i2}(y), \ldots, G_{ip}(y)\right), \\ \mathbf{G}(y) = \left(G_{ij}(y)\right), \\ i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, p. \end{gathered} \tag{42}$$

Subtracting (40) from (41), we have error dynamic system as follows:

$$\begin{aligned}\dot{e}_i &= \left(-x_i + \sum_{k=1}^{n} a_{ik} f\left(x_k\right) + \sum_{k=1}^{n} b_{ik} v_k + I_i\right) \\ &\quad - g_i\left(y_1, y_2, \ldots, y_n\right) + \sum_{k=1}^{n} s_{ik} x_k \\ &\quad - G_i\left(y_1, y_2, \ldots, y_n\right)\theta + u_i(t) \\ &= h_i^{r}(x) - h_i^{d}(y) + \sum_{k=1}^{n} s_{ik} x_k \\ &\quad - G_i\left(y_1, y_2, \ldots, y_n\right)\theta + u_i(t),\end{aligned} \tag{43}$$

where

$$\begin{gathered} h_i^{r}(x) = -x_i + \sum_{k=1}^{n} a_{ik} f\left(x_k\right) + \sum_{k=1}^{n} b_{ik} v_k + I_i, \\ h_i^{d}(y) = g_i\left(y_1, y_2, \ldots, y_n\right). \end{gathered} \tag{44}$$

In order to guarantee the stability of the synchronization error system (43), a suitable control law is proposed as follows:

$$\begin{aligned} u_i(t) &= -e_i - h_i^{r}(x) + h_i^{d}(y) + G_i\left(y_1, y_2, \ldots, y_n\right)\hat{\theta} \\ &\quad - \sum_{k=1}^{n} \hat{s}_{ik} x_k, \end{aligned} \tag{45}$$

where $\hat{\theta}$ and $\hat{\mathbf{S}} = (\hat{s}_{ik})$ are an estimation for θ and $\mathbf{S} = (s_{ik})$, respectively. The update laws for them are introduced as follows:

$$\begin{gathered} \dot{\hat{\theta}}_j = -\sum_{i=1}^{n} G_{ij}(y)\, e_i, \quad j = 1, 2, \ldots, p, \\ \dot{\hat{s}}_{ik} = e_i x_k, \quad i = 1, 2, \ldots, n; \; k = 1, 2, \ldots, n. \end{gathered} \tag{46}$$

Theorem 13. *The error system (43) is controlled by the controller (45) and update laws in (46). Then the trajectories of error system (43) converge to zero; hence the drive system (40) and response system (41) are synchronized.*

Proof. The Lyapunov function is selected as follows:

$$\begin{aligned} V(t) &= \frac{1}{2}\sum_{i=1}^{n} e_i^{\,2} + \frac{1}{2}\sum_{j=1}^{p}\left(\hat{\theta}_j - \theta_j\right)^2 \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}\left(\hat{s}_{ik} - s_{ik}\right)^2. \end{aligned} \tag{47}$$

The time derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{n} e_i \dot{e}_i + \sum_{j=1}^{p}\left(\hat{\theta}_j - \theta_j\right)\dot{\hat{\theta}}_j \\ &\quad + \sum_{i=1}^{n}\sum_{k=1}^{n}\left(\hat{s}_{ik} - s_{ik}\right)\dot{\hat{s}}_{ik}. \end{aligned} \tag{48}$$

Inserting control laws $u(t)$ in (45) into right-hand side of (43), one has

$$\dot{e}_i = -e_i + G_i(y)(\hat{\theta} - \theta) - \sum_{k=1}^{n} (\hat{s}_{ik} - s_{ik}) x_k. \quad (49)$$

Inserting $e_i(t)$ in (49) and update laws in (46) into the right-hand side of (48), one obtains

$$\begin{aligned}\dot{V}(t) = &-\sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} e_i G_i(y)(\hat{\theta} - \theta) \\ &- \sum_{i=1}^{n}\sum_{k=1}^{n} e_i (\hat{s}_{ik} - s_{ik}) x_k - \sum_{j=1}^{p} (\hat{\theta}_j - \theta_j) \sum_{i=1}^{n} G_{ij}(y) e_i \\ &+ \sum_{i=1}^{n}\sum_{k=1}^{n} (\hat{s}_{ik} - s_{ik}) e_i x_k.\end{aligned} \quad (50)$$

It is apparent that

$$\begin{aligned}\sum_{j=1}^{p} (\hat{\theta}_j - \theta_j) \sum_{i=1}^{n} G_{ij}(y) e_i &= \sum_{j=1}^{p}\sum_{i=1}^{n} e_i G_{ij}(y)(\hat{\theta}_j - \theta_j) \\ &= \sum_{i=1}^{n} e_i \sum_{j=1}^{p} G_{ij}(y)(\hat{\theta}_j - \theta_j) \\ &= \sum_{i=1}^{n} e_i G_i(y)(\hat{\theta} - \theta).\end{aligned} \quad (51)$$

Therefore, we have

$$\dot{V}(t) = -\sum_{i=1}^{n} e_i^2 \leq 0. \quad (52)$$

From inequality (52), with similar arguments above, it can be concluded that the trajectories of error system (43) converge to zero. □

4. Illustrative Examples

In this section, some illustrative examples for the above problems are given. We used Matlab tool for simulating these examples.

Example 1. Synchronization of two identical chaotic SC-CNNs with uncertain state template is as follows.

In this example, the synchronization problem of two following chaotic SC-CNNs in [5] is considered. Therefore,

$$\begin{aligned}\dot{x}_1 &= -x_1 + a_{11} f(x_1) + s_{12} x_1 + s_{13} x_3, \\ \dot{x}_2 &= -x_2 + s_{22} x_2 + s_{23} x_3, \\ \dot{x}_3 &= -x_3 + -0.3143 x_1 + s_{32} x_2 + s_{33} x_3,\end{aligned} \quad (53)$$

with uncertain state template

$$\hat{\mathbf{S}} = \begin{pmatrix} \hat{s}_{11} & 0 & \hat{s}_{13} \\ 0 & \hat{s}_{22} & \hat{s}_{23} \\ -0.3143 & \hat{s}_{32} & \hat{s}_{33} \end{pmatrix}. \quad (54)$$

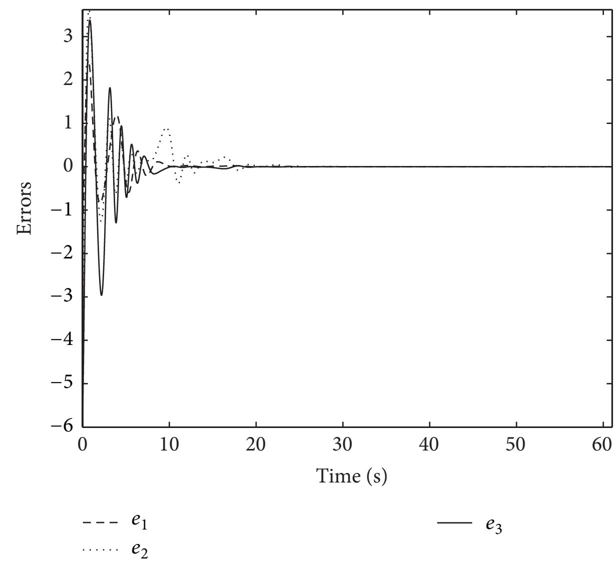

Figure 1: Synchronization errors in Example 1.

Exact state template of chaotic SC-CNN (53) is

$$\mathbf{S} = \begin{pmatrix} -1.2418 & 0 & 0.3050 \\ 0 & 1.4725 & -1 \\ -0.3143 & 0.3143 & 0.6857 \end{pmatrix} \quad (55)$$

and $a_{11} = 2.2754$. The initial value of update matrix $\hat{\mathbf{S}}$ is

$$\hat{\mathbf{S}}(0) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ -0.3143 & 2 & 1 \end{pmatrix}. \quad (56)$$

The initial values of drive and response systems are chosen as

$$\begin{aligned}\mathbf{x}^d(0) &= (1, 2, 3)^T, \\ \mathbf{x}^r(0) &= (-1, -2, -3)^T.\end{aligned} \quad (57)$$

Figure 1 shows the synchronization errors of two identical chaotic SC-CNNs (53). The time response of adaptive state template $\hat{\mathbf{S}}$ is illustrated in Figures 2 and 3. It can be seen that the synchronization errors converge to zero and estimated state template converges to the exact state template.

Example 2. Synchronization of two nonidentical chaotic SC-CNNs with uncertain state template is as follows.

In this example, the drive system is selected as chaotic SC-CNN in [5] and the response system is chosen as chaotic SC-CNN in [19] as follows:

$$\begin{aligned}\dot{x}_1^r &= -x_1^r + a_{11}^r f(x_1^r) + s_{11}^r x_1^r + s_{12}^r x_2^r, \\ \dot{x}_2^r &= -x_2^r + x_1^r + x_3^r, \\ \dot{x}_3^r &= -x_3^r + s_{32}^r x_2^r + x_3^r.\end{aligned} \quad (58)$$

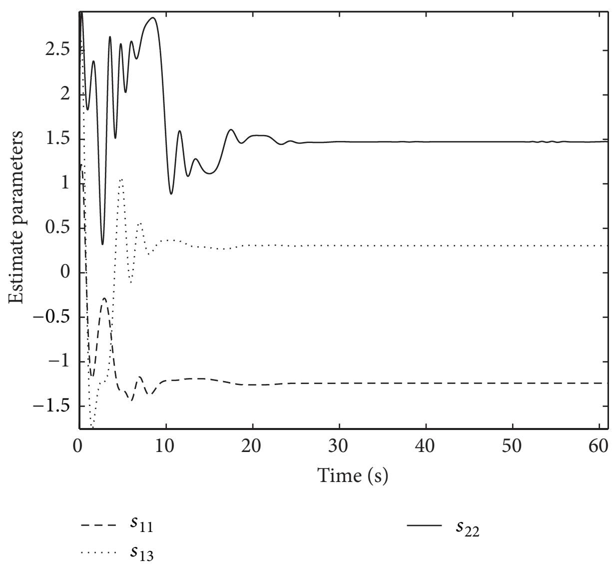

Figure 2: Time responses of the adaptive state template $\widehat{\mathbf{S}}$ in Example 1.

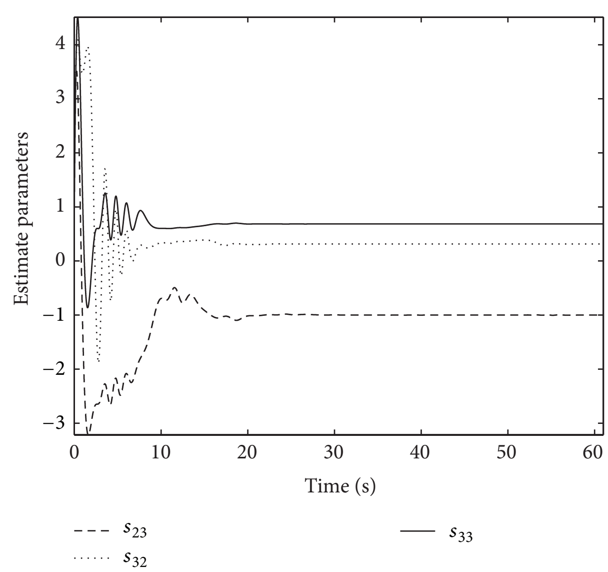

Figure 3: Time responses of the adaptive state template $\widehat{\mathbf{S}}$ in Example 1.

With drive system, we assume that state template has five uncertain elements as follows:

$$\widehat{\mathbf{S}}^d = \begin{pmatrix} \widehat{s}_{11}^d & 0 & 0.3050 \\ 0 & \widehat{s}_{21} & \widehat{s}_{23} \\ -0.3143 & \widehat{s}_{32} & \widehat{s}_{33} \end{pmatrix}. \quad (59)$$

The uncertain state template of response system is

$$\widehat{\mathbf{S}}^r = \begin{pmatrix} \widehat{s}_{11}^r & \widehat{s}_{12}^r & 0 \\ 1 & 0 & 1 \\ 0 & \widehat{s}_{32}^r & 1 \end{pmatrix}. \quad (60)$$

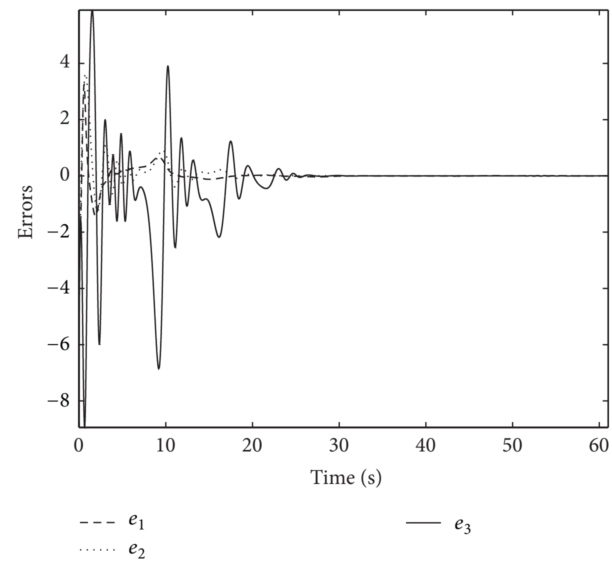

Figure 4: Synchronization errors in Example 2.

a_{11}^d and exact state template of drive system have similar values as Example 1. $a_{11}^r = 3.86$ and exact state template of response system is given as follows:

$$\mathbf{S}^r = \begin{pmatrix} -1.55 & 8.98 & 0 \\ 1 & 0 & 1 \\ 0 & -14.26 & 1 \end{pmatrix}. \quad (61)$$

The initial values of update matrixes $\widehat{\mathbf{S}}^d$ and $\widehat{\mathbf{S}}^r$ are selected as follows:

$$\widehat{\mathbf{S}}^d(0) = \begin{pmatrix} 1 & 0 & 0.3050 \\ 0 & 2 & 2 \\ -0.3143 & 2 & 1 \end{pmatrix},$$
$$\widehat{\mathbf{S}}^r(0) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \quad (62)$$

Vectors $\mathbf{x}^d(0) = (1, 2, 3)^T$, $\mathbf{x}^r(0) = (-1, -2, -3)^T$ are chosen as the initial values of drive and response systems, respectively.

The synchronization errors of two nonidentical chaotic SC-CNNs (53) and (58) are shown in Figure 4. It is obvious that the synchronization errors converge to zero. The time response of adaptive state template $\widehat{\mathbf{S}}^d$ is illustrated in Figure 5. Figure 6 shows the time response of adaptive state template $\widehat{\mathbf{S}}^r$. It is clear that the adaptive state templates converge to some constants.

Example 3. Synchronization of chaotic SC-CNN with uncertain state template and different uncertain parameter chaotic systems is as follows.

In this example, the Lorenz system is selected as drive system and response system is the chaotic SC-CNN.

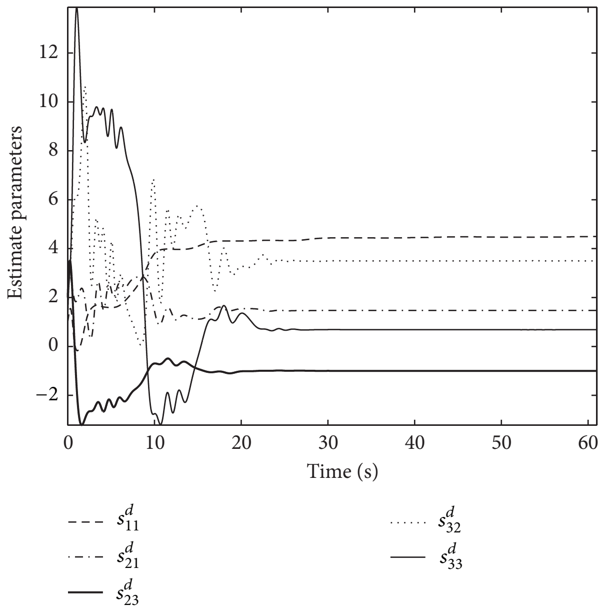

Figure 5: Time responses of the adaptive state template $\widehat{\mathbf{S}}^d$ in Example 2.

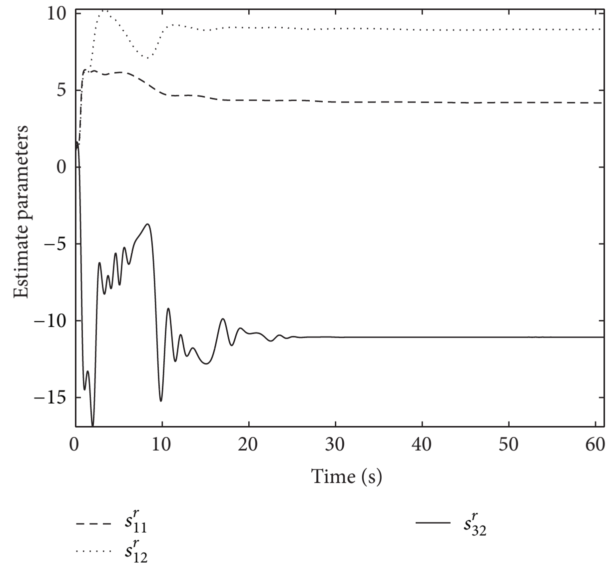

Figure 6: Time responses of the adaptive state template $\widehat{\mathbf{S}}^r$ in Example 2.

Lorenz System. Consider the following:

$$\begin{aligned}\dot{y}_1 &= 10\left(y_2 - y_1\right),\\ \dot{y}_2 &= 28y_1 - y_2 - y_1y_3,\\ \dot{y}_3 &= y_1y_2 - \frac{8}{3}y_3,\end{aligned} \tag{63}$$

or in the form of (3) as

$$\begin{aligned}\dot{y}_1 &= 0 + \left(y_2 - y_1, 0, 0\right)\left(10, 28, \frac{8}{3}\right)^T,\\ \dot{y}_2 &= -y_2 - y_1y_3 + \left(0, y_1, 0\right)\left(10, 28, \frac{8}{3}\right)^T,\\ \dot{y}_3 &= y_1y_2 + \left(0, 0, -y_3\right)\left(10, 28, \frac{8}{3}\right)^T,\end{aligned} \tag{64}$$

with parameter vector $\theta = (10, 28, 8/3)^T$. Therefore,

$$\begin{aligned}\mathbf{g} &= \left(0, -y_2 - y_1y_3, y_1y_2\right),\\ \mathbf{G} &= \begin{pmatrix} y_2 - y_1 & 0 & 0\\ 0 & y_1 & 0\\ 0 & 0 & -y_3\end{pmatrix}.\end{aligned} \tag{65}$$

Response chaotic SC-CNN [19]:

$$\begin{aligned}\dot{x}_1 &= -x_1 + a_{11}f\left(x_1\right) + s_{11}x_1 + s_{12}x_2\\ \dot{x}_2 &= -x_2 + x_1 + x_3\\ \dot{x}_3 &= -x_3 + s_{32}x_2 + x_3.\end{aligned} \tag{66}$$

The uncertain state template of response system is

$$\widehat{\mathbf{S}} = \begin{pmatrix} \widehat{s}_{11} & \widehat{s}_{12} & 0\\ 1 & 0 & 1\\ 0 & \widehat{s}_{32} & 1\end{pmatrix}. \tag{67}$$

$a_{11} = -7.717$ and exact state template of response system is given as follows:

$$\mathbf{S} = \begin{pmatrix} 1.3443 & -4.925 & 0\\ 1 & 0 & 1\\ 0 & 3.649 & 1\end{pmatrix}. \tag{68}$$

The initial values of update parameters $\widehat{\theta}$ and $\widehat{\mathbf{S}}$ are selected as follows:

$$\begin{aligned}\theta\left(0\right) &= \left(1, 2, 2\right)^T,\\ \widehat{\mathbf{S}} &= \begin{pmatrix} 2 & 2 & 0\\ 1 & 0 & 1\\ 0 & 2 & 1\end{pmatrix}.\end{aligned} \tag{69}$$

The initial values of drive and response systems are chosen as $\mathbf{y}(0) = (1, 2, 3)$, $\mathbf{x}(0) = (-1, -2, -3)$, respectively.

The synchronization errors of Lorenz system (63) and chaotic SC-CNN (66) are revealed in Figure 7. It is seen that the synchronization errors converge to zero. The time response of adaptive parameter $\widehat{\theta}$ is shown in Figure 8. The adaptive parameters are clearly bounded. The time response of adaptive state template $\widehat{\mathbf{S}}$ is illustrated in Figure 9. It can be seen that the adaptive state template converges to some constants.

The simulation results indicate that the proposed controllers can do synchronization of corresponding problems.

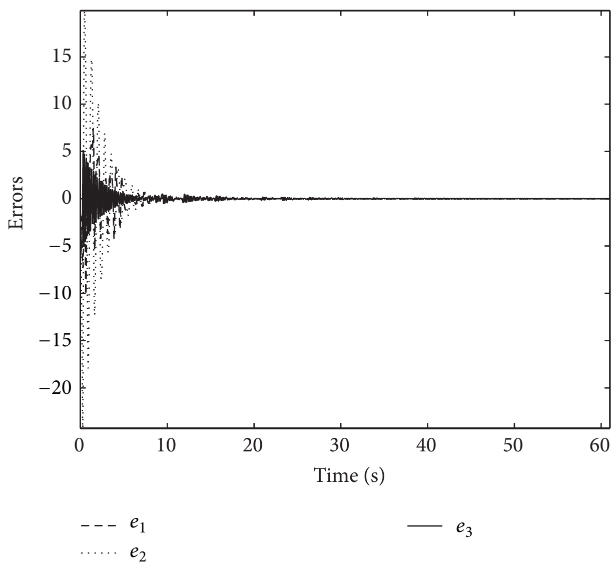

Figure 7: Synchronization errors in Example 3.

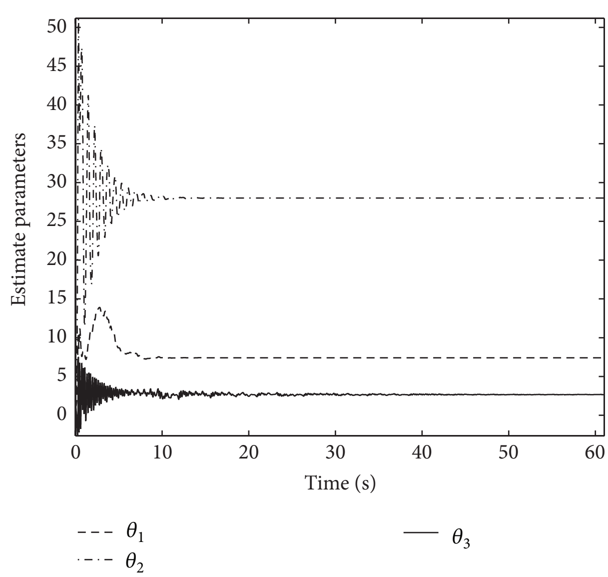

Figure 8: Time responses of the adaptive parameter $\widehat{\theta}$ in Example 3.

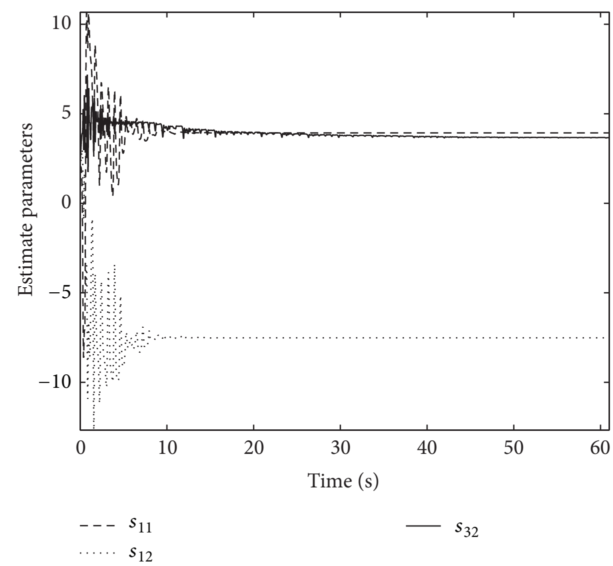

Figure 9: Time responses of the adaptive state template $\widehat{\mathbf{S}}$ in Example 3.

5. Conclusion

In this paper, three problems of synchronization of chaotic SC-CNN with uncertain state template are investigated. The adaptive controllers and suitable update laws are proposed to guarantee synchronization. These results can be applied to construct the image encryption scheme or secure communication based on chaos. However, the determination of finite time to achieve synchronization is still unsolved. In the future works, the finite time of synchronization will be interesting to solve as well as to construct specific applications.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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