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Research Article

Sliding Mode Control and Modified Generalized Projective Synchronization of a New Fractional-Order Chaotic System

Junbiao Guan[1] and Kaihua Wang[2]

[1] *School of Science, Hangzhou Dianzi University, Hangzhou, Zhejiang 310018, China*
[2] *School of Mathematics and Statistics, Hainan Normal University, Haikou, Hainan 571158, China*

Correspondence should be addressed to Junbiao Guan; junbiaoguan@gmail.com

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A new fractional-order chaotic system is addressed in this paper. By applying the continuous frequency distribution theory, the indirect Lyapunov stability of this system is investigated based on sliding mode control technique. The adaptive laws are designed to guarantee the stability of the system with the uncertainty and external disturbance. Moreover, the modified generalized projection synchronization (MGPS) of the fractional-order chaotic systems is discussed based on the stability theory of fractional-order system, which may provide potential applications in secure communication. Finally, some numerical simulations are presented to show the effectiveness of the theoretical results.

1. Introduction

Though the concept of fractional calculus has been established more than three hundred years, its potential applications are fully carried out in recent decades, especially in the fields of control, engineering, and physics; see [1–3] and reference therein. It has been found that many systems in nature display fractional phenomena, such as electromagnetic waves, dielectric polarization, the chemotaxi behavior, quantitative finance, and evolution in complex media. Especially, the memristor, which was predicted as the missing circuit element [4], is more likely to be linked to fractional calculus due to its inherent features. Therefore, it is of considerable importance to study some aspects of fractional calculus.

The stability of fractional differential equation (FDE) is one of the most important aspects in FDE's application of control process [5, 6], and Lyapunov stability method provides an efficient way to analyze the stability of FDE without explicitly solving the differential equations [7]. There are two approaches with respect to Lyapunov stability, namely, direct one and indirect one. The former one is intuitive and specific, while the latter is based on the continuous frequency distribution theory, which appears more abstract but effective. Meanwhile, in order to stabilize the chaotic FDE, many control mechanisms are presented so far, among which sliding mode control (SMC) technique is extensively adopted as it can be utilized to improve the control performance criteria such as the robustness and fast time response [8–10].

Another important aspect of FDE's application of control process lies in synchronization of fractional-order chaotic systems [11, 12], which attracts increasing attention in recent years due to its potential applications in secure communication. There are many synchronization methods for fractional-order chaotic systems, including the Pecora and Carroll method, the one-way coupling method, feedback control method, the active control method, and the active sliding mode control method. Amongst these methods, it is worth mentioning that feedback control method serves as a good tool to synchronize the master (drive) and slave (response) systems up to a constant scaling factor, which is known as projective synchronization (PC) [13]. This synchronization scheme can be applied to secure communication easily because it can obtain faster communication with its proportional characteristics. Based on this, recently, Wu et al. proposed the modified generalized projective synchronization (MGPS) where the drive and response systems could be synchronized to a constant scaling matrix [14]. Compared with PC, the unpredictability of the scaling matrix in MGPS can greatly strengthen the security of communications.

Motivated by the above discussions, in this paper, we are dedicated to the study of SMC of a new fractional-order chaotic system, together with its MGPS. The main purpose of this work is the integrated applications of the continuous frequency distribution theory and adaptive SMC technique to investigate indirect Lyapunov stability of a new fractional-order chaotic system. Moreover, the MGPS of the fractional-order chaotic systems is discussed based on the stability theory of fractional-order system, which may provide potential applications in secure communication. The remainder of this paper is organized as follows. In Section 2, some basic definitions and lemmas are given. In Section 3, a new fractional-order chaotic system is presented and the stability of sliding mode dynamics of this system is studied based on the continuous frequency distribution theory. Adaptive sliding mode control of the fractional-order chaotic system with the model uncertainty and external disturbance is also investigated. In Section 4, MGPS of the fractional-order chaotic systems is discussed under the framework of stability theory of FDE. In Section 5, three numerical examples are given to show the effectiveness of the theoretical results. Finally, some concluding remarks are drawn in Section 6.

2. Preliminaries

In this section, we present some basic definitions and lemmas which will be useful throughout this paper.

The commonly-used Riemann-Liouville definition of the αth-order derivative is given by

$$D^{\alpha}x(t)=\left(\frac{d}{dt}\right)^{n}J^{n-\alpha}x(t),\quad \alpha>0, \tag{1}$$

where n is the first integer which is not less than α, J^{β} is the βth-order Riemann-Liouville integral operator as described by

$$J^{\beta}x(t)=\frac{1}{\Gamma(\beta)}\int_0^t\frac{x(\tau)}{(t-\tau)^{1-\beta}}d\tau,\quad 0<\beta\le 1, \tag{2}$$

where $\Gamma(\beta)=\int_0^{\infty}t^{\beta-1}e^{-t}dt$ is the gamma function.

Correspondingly, the Riemann-Liouville definition of the αth-order integral is given by

$$D^{-\alpha}x(t)=\frac{1}{\Gamma(\alpha)}\int_0^t\frac{x(\tau)}{(t-\tau)^{1-\alpha}}d\tau,\quad \alpha>0. \tag{3}$$

Besides the definitions of fractional calculus presented above, the following lemmas provided in [7] are necessary for the analysis in the next sections.

Lemma 1. *Let $h(t)$ be the impulse response of a linear system and let $\mu(\omega)$ be the diffusive representation (or frequency weighting function) of the impulse response $h(t)$, $h(t)$ and $\mu(\omega)$ verify the pseudo-Laplace transform definition*

$$h(t)=\int_0^{\infty}\mu(\omega)e^{-\omega t}d\omega. \tag{4}$$

Lemma 2. *Consider the nonlinear fractional-order differential equation (FDE) $D^{\alpha}(x)=f(x)$. Owing to the continuous frequency distributed model of the fractional integrator, the nonlinear system can be expressed as*

$$\begin{aligned}\frac{\partial z(\omega,t)}{\partial t}&=-\omega z(\omega,t)+f(x(t)),\\ x(t)&=\int_0^{\infty}\mu(\omega)z(\omega,t)d\omega\end{aligned} \tag{5}$$

with

$$\mu(\omega)=\frac{\sin\alpha\pi}{\pi}\omega^{-n}. \tag{6}$$

3. Sliding Mode Control of the New Fractional-Order Chaotic System

A new fractional-order chaotic system is described by

$$\begin{aligned}D^{q_1}x(t)&=ax(t)+dy(t)z(t)+gy^2(t),\\ D^{q_2}y(t)&=by(t)+ex(t)z(t)+hz(t),\\ D^{q_3}z(t)&=cz(t)+fx(t)y(t),\end{aligned} \tag{7}$$

where q_i denotes fractional-order and $0<q_i<1$, $(i=1,2,3)$, $a,b,c,d,e,f,g,h\in R$ are parameters. If one of its Lyapunov exponents is positive, then the system exhibits chaotic dynamical behavior. Specially, system (7) is chaotic when $a=-3$, $b=5$, $c=-10$, $d=1$, $e=-1$, $f=1$, $g=1$, $h=16$, $q_1=0.995$, $q_2=0.997$, $q_3=0.998$, as illustrated in Figure 1. When $q_1=q_2=q_3=1$, the corresponding integer-order differential equation has been proposed and its dynamics have been studied in [15].

In order to apply the nonlinear feedback control, we consider the controlled system described by

$$\begin{aligned}D^{q_1}x(t)&=ax(t)+dy(t)z(t)+gy^2(t),\\ D^{q_2}y(t)&=by(t)+ex(t)z(t)+hz(t)+u(t),\\ D^{q_3}z(t)&=cz(t)+fx(t)y(t).\end{aligned} \tag{8}$$

Choose the switching surface $s(t)$ as

$$s(t)=D^{q_2-1}y(t)+\int_0^t\phi(\tau)d\tau, \tag{9}$$

where $\phi(t)$ is described by $\phi(t)=(d+f)xz+gxy-(b-k)y$ with $k>0$.

When reaching the switching surface, we have

$$\begin{aligned}s(t)&=D^{q_2-1}y(t)+\int_0^t\phi(\tau)d\tau=0,\\ \dot{s}(t)&=D^{q_2}y(t)+\phi(t)=0.\end{aligned} \tag{10}$$

Thus the corresponding control law is obtained

$$u(t)=-(d+e+f)xz-gxy-hz-ky. \tag{11}$$

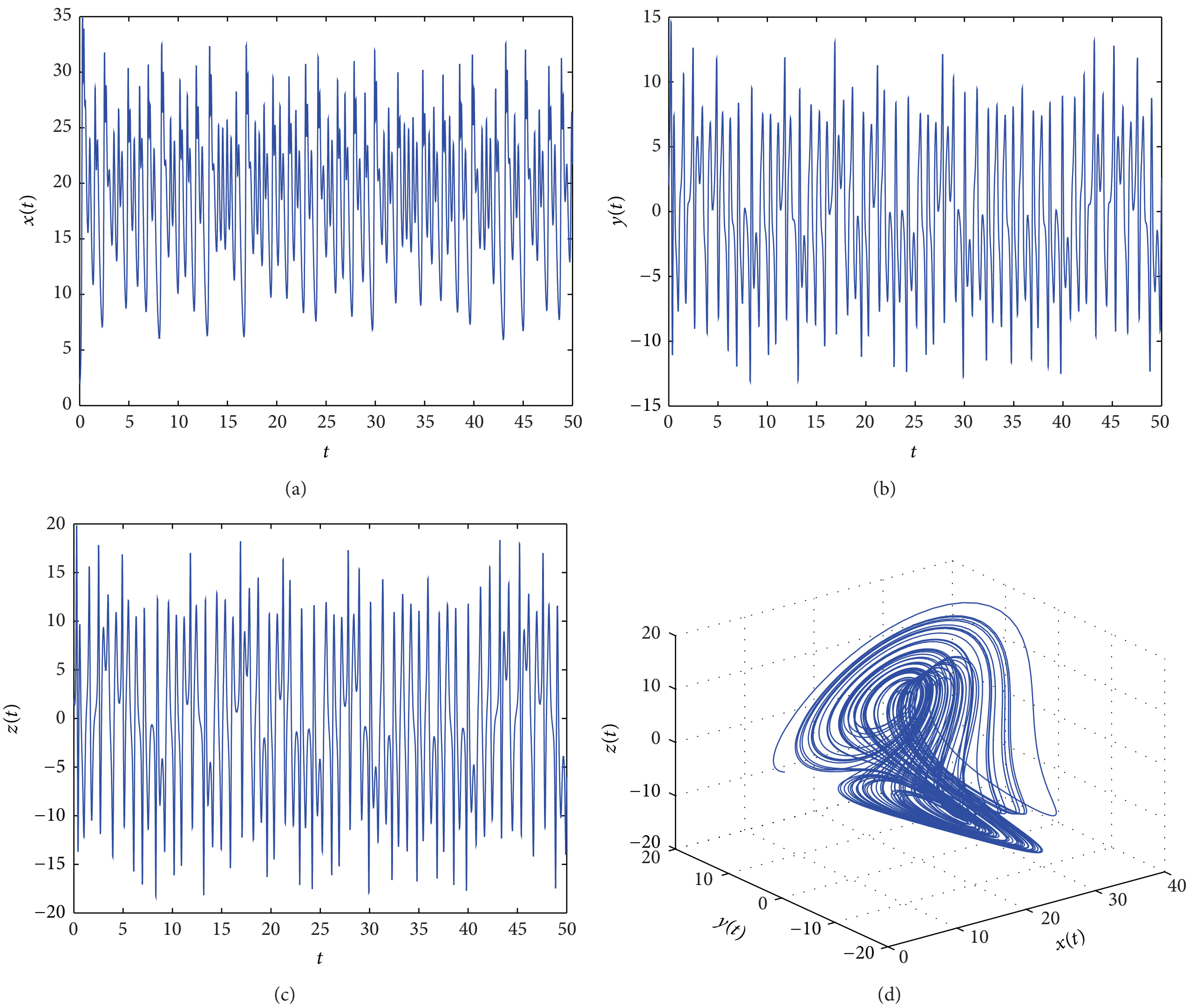

Figure 1: Chaotic attractor of system (7). (a–c) Time responses of x, y, and z. (d) Phase portrait.

Hence we have

$$D^{q_1}x(t) = ax(t) + dy(t)z(t) + gy^2(t),$$
$$D^{q_2}y(t) = (b-k)y(t) - (d+f)x(t)z(t) - gx(t)y(t),$$
$$D^{q_3}z(t) = cz(t) + fx(t)y(t). \tag{12}$$

Theorem 3. *The FDE sliding mode dynamics described in (12) are globally asymptotically stable if $a<0$, $b-k<0$, $c<0$.*

Proof. Owing to the continuous frequency distributed model of the fractional integrator, the nonlinear system can be rewritten as

$$\frac{\partial z_1(\omega,t)}{\partial t} = -\omega z_1(\omega,t) + ax(t) + dy(t)z(t) + gy^2(t),$$
$$x(t) = \int_0^\infty \mu_1(\omega) z_1(\omega,t)\,d\omega,$$
$$\frac{\partial z_2(\omega,t)}{\partial t} = -\omega z_2(\omega,t) + (b-k)y(t) - (d+f)x(t)z(t) - gx(t)y(t),$$
$$y(t) = \int_0^\infty \mu_2(\omega) z_2(\omega,t)\,d\omega,$$
$$\frac{\partial z_3(\omega,t)}{\partial t} = -\omega z_3(\omega,t) + cz(t) + fx(t)y(t),$$
$$z(t) = \int_0^\infty \mu_3(\omega) z_3(\omega,t)\,d\omega, \tag{13}$$

where $\mu_i(\omega) = (\sin q_i\pi/\pi)\omega^{-q_i}$, $i=1,2,3$.

Let us define two Lyapunov functions as follows.

(1) $v_i(\omega,t)$ $(i=1,2,3)$ are the monochromatic Lyapunov functions corresponding to the elementary frequency ω.

(2) $V_i(t)$ $(i = 1, 2, 3)$ are the Lyapunov functions summing all the monochromatic $v_i(\omega, t)$ with the weighting function $\mu_i(\omega)$.

Let $v_i(\omega, t) = (1/2)z_i^2(\omega, t)$ and let $V_i(t) = \int_0^\infty \mu_i(\omega)z_i(\omega, t)d\omega, i = 1, 2, 3$. Note that $v_i(\omega, t)$ $(i = 1, 2, 3)$ are positive, and $\mu_i(\omega)$ $(i = 1, 2, 3)$ are positive for all ω. Thus $V_i(t)$ $(i = 1, 2, 3)$ are positive Lyapunov functions. The time derivative of $V_i(t)$ yields

$$\frac{dV_i(t)}{dt} = \int_0^\infty \mu_i(\omega) \frac{\partial v_i(\omega, t)}{\partial t} d\omega = \int_0^\infty \mu_i(\omega) z_i \frac{\partial z_i}{\partial t} d\omega, \quad i = 1, 2, 3. \tag{14}$$

Combining (13) with (14), we have

$$\begin{aligned}
\frac{dV_1(t)}{dt} &= \int_0^\infty \mu_1(\omega) \frac{\partial v_1(\omega, t)}{\partial t} d\omega \\
&= \int_0^\infty \mu_1(\omega) z_1 \left[-\omega z_1 + ax + dyz + gy^2\right] d\omega \\
&= -\int_0^\infty \omega\mu_1(\omega) z_1^2 d\omega + x\left(ax + dyz + gy^2\right), \\
\frac{dV_2(t)}{dt} &= \int_0^\infty \mu_2(\omega) \frac{\partial v_2(\omega, t)}{\partial t} d\omega \\
&= \int_0^\infty \mu_2(\omega) z_2 \left[-\omega z_2 + (b-k) y \right.\\
&\qquad \left. - (d+f) xz - gxy\right] d\omega \\
&= -\int_0^\infty \omega\mu_2(\omega) z_2^2 d\omega \\
&\quad + y\left[(b-k) y - (d+f) xz - gxy\right], \\
\frac{dV_3(t)}{dt} & \\
&= \int_0^\infty \mu_3(\omega) \frac{\partial v_3(\omega, t)}{\partial t} d\omega \\
&= \int_0^\infty \mu_3(\omega) z_3 \left[-\omega z_3 + cz + fxy\right] d\omega \\
&= -\int_0^\infty \omega\mu_3(\omega) z_3^2 d\omega + z\left(cz + fxy\right).
\end{aligned} \tag{15}$$

Let us define $V(t) = V_1(t) + V_2(t) + V_3(t)$; then

$$\begin{aligned}
\frac{dV(t)}{dt} = \sum_{i=1}^{3} \frac{dV_i(t)}{dt} = &-\sum_{i=1}^{3} \int_0^\infty \omega\mu_i(\omega) z_i^2 d\omega \\
&+ \left[ax^2 + (b-k) y^2 + cz^2\right].
\end{aligned} \tag{16}$$

Denote by $W_i = -\int_0^\infty \omega\mu_i(\omega)z_i^2 d\omega$ $(i = 1, 2, 3)$ and $W = \sum_{i=1}^3 W_i$. The frequency discretization of W_i gives

$$W_i = -\sum_{j=1}^{J} \omega_{ij}\mu_i\left(\omega_{ij}\right) z_i^2\left(\omega_{ij}, t\right) \Delta\omega_{ij} = \sum_{j=1}^{J} \omega_{ij} c_{ij} z_i^2\left(\omega_{ij}, t\right), \tag{17}$$

where $c_{ij} = -\mu_i(\omega_{ij})\Delta\omega_{ij} < 0$. Hence $W_i < 0$ and $W = \sum_{i=1}^3 W_i < 0$. On the other hand, if $a < 0$, $b - k < 0$, $c < 0$, then $ax^2 + (b-k)y^2 + cz^2 < 0$. According to the above analysis we have $dV(t)/dt < 0$, which implies that system (12) is asymptotically stable. □

It is to be noted that the above slide mode control of the fractional-order chaotic system is conducted under the condition that no model uncertainty or external disturbance exists. But in reality, these factors are ubiquitous. In view of this, in what follows we consider this fractional-order chaotic system with the model uncertainty and external disturbance described by

$$\begin{aligned}
D^{q_1} x(t) &= ax(t) + dy(t) z(t) + gy^2(t), \\
D^{q_2} y(t) &= by(t) + ex(t) z(t) + hz(t) \\
&\quad + \Delta h\left(x, y, z\right) + \Delta w(t) + u(t), \\
D^{q_3} z(t) &= cz(t) + fx(t) y(t),
\end{aligned} \tag{18}$$

where $\Delta h(x, y, z)$, $\Delta w(t)$ are the model uncertainty and the external disturbance, respectively. Note that they are often bounded. Therefore we give the following assumption.

Assumption 4. The model uncertainty and the external disturbance are assumed to be bounded; that is, there exist unknown positive constants ψ, ω such that $|\Delta h(x, y, z)| < \psi$, $|\Delta w(t)| < \omega$.

In order to give an estimation of the two constants ψ and ω, we present the adaptive rules defined as follows:

$$\dot{\tilde{\psi}} = \lambda_1 |s|, \qquad \dot{\tilde{\omega}} = \lambda_2 |s|, \tag{19}$$

where $\tilde{\psi}$, $\tilde{\omega}$ are estimations for ψ, ω, respectively, and λ_1, λ_2 are positive constants.

The controller is designed as follows:

$$\begin{aligned}
u(t) = &-\left(d + e + f\right) xz - gxy - hz \\
&- ky - \rho \operatorname{sign}(s) - \left(\tilde{\psi} + \tilde{\omega}\right) \operatorname{sign}(s),
\end{aligned} \tag{20}$$

where ρ is a positive constant. Then we reach the following theorem.

Theorem 5. *The FDE with the unknown bounded uncertainty and the external disturbance described in (18) will converge to the sliding surface $s(t) = 0$ under the controller (20).*

Proof. Selecting a Lyapunov candidate as $V(t) = (1/2)s^2 + (1/2\lambda_1)(\widetilde{\psi} - \psi)^2 + (1/2\lambda_2)(\widetilde{\omega} - \omega)^2$ and taking the time derivative of both sides, one obtains

$$\begin{aligned}\dot{V} &= s\dot{s} + \frac{1}{\lambda_1}(\widetilde{\psi} - \psi)\dot{\widetilde{\psi}} + \frac{1}{\lambda_2}(\widetilde{\omega} - \omega)\dot{\widetilde{\omega}} \\ &= s(\Delta h(x,y,z) + \Delta w(t)) \\ &\quad - s(\widetilde{\psi} + \widetilde{\omega})\operatorname{sign}(s) - s\rho \operatorname{sign}(s) \\ &\quad + (\widetilde{\psi} - \psi)|s| + (\widetilde{\omega} - \omega)|s| \\ &\le |s|(\psi + \omega) - |s|(\widetilde{\psi} + \widetilde{\omega}) - \rho|s| \\ &\quad + (\widetilde{\psi} - \psi)|s| + (\widetilde{\omega} - \omega)|s| \\ &= -\rho|s| \le 0.\end{aligned} \tag{21}$$

Integrating (21) from zero to t yields

$$\int_0^t \rho|s|\,dt \le V(0) - V(t), \tag{22}$$

which implies $\lim_{t\to\infty} s = 0$. □

4. MGPS of the New Fractional-Order Chaotic Systems

In this section, we investigate MGPS of the new fractional-order chaotic systems. First of all, it is necessary to give the definition of MGPS [14].

Definition 6. The drive system $D^\alpha(X_m) = F(X_m)$ and the response system $D^\alpha(X_s) = F(X_s) + U$ are said to achieve modified general projection synchronization (MGPS) if there exists a controller U such that $\lim_{t\to\infty}\|X_s - \Lambda X_m\| = 0$, where $\Lambda = \operatorname{diag}(d_1, d_2, \ldots, d_n)$, $d_i \ne 0$, $i = 1, 2, \ldots, n$ and $X = (x_1, x_2, \ldots, x_n)$, $U = (u_1, u_2, \ldots, u_n)$.

We consider the drive and the response fractional-order chaotic systems described as follows:

$$\begin{aligned}\begin{pmatrix} D^\alpha x_m \\ D^\alpha y_m \\ D^\alpha z_m \end{pmatrix} &= \begin{bmatrix} a & 0 & 0 \\ 0 & b & h \\ 0 & 0 & c \end{bmatrix}\begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix} + \begin{pmatrix} dy_m z_m + gy_m^2 \\ ex_m z_m \\ fx_m y_m \end{pmatrix}, \\ \begin{pmatrix} D^\alpha x_s \\ D^\alpha y_s \\ D^\alpha z_s \end{pmatrix} &= \begin{bmatrix} a & 0 & 0 \\ 0 & b & h \\ 0 & 0 & c \end{bmatrix}\begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} \\ &\quad + \begin{pmatrix} dy_s z_s + gy_s^2 \\ ex_s z_s \\ fx_s y_s \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}.\end{aligned} \tag{23}$$

Define the synchronization errors by

$$\begin{aligned} e_1 &= x_s - d_1 x_m, \\ e_2 &= y_s - d_2 y_m, \\ e_3 &= z_s - d_3 z_m, \end{aligned} \tag{24}$$

where $\Lambda = (d_1, d_2, d_3)$ and d_i $(i = 1, 2, 3)$ are the nonzero scaling factors.

Then the error dynamics are obtained

$$\begin{aligned} D^\alpha e_1 &= ae_1 + dy_s z_s + gy_s^2 - d_1 dy_m z_m - d_1 gy_m^2 + u_1, \\ D^\alpha e_2 &= be_2 + hz_s + ex_s z_s - d_2 hz_m - d_2 ex_m z_m + u_2, \\ D^\alpha e_3 &= ce_3 + fx_s y_s - d_3 fx_m y_m + u_3. \end{aligned} \tag{25}$$

We design the controllers u_i $(i = 1, 2, 3)$ for the response system as follows:

$$\begin{aligned} u_1 &= -dy_s z_s - gy_s^2 + d_1 dy_m z_m + d_1 gy_m^2 - k_1 e_1, \\ u_2 &= -hz_s - ex_s z_s + d_2 hz_m + d_2 ex_m z_m - k_2 e_2, \\ u_3 &= -fx_s y_s + d_3 fx_m y_m - k_3 e_3, \end{aligned} \tag{26}$$

where the control gains matrix $K = \operatorname{diag}(k_1, k_2, k_3)$.

Thus the fractional-order error dynamics can be rewritten as follows:

$$\begin{pmatrix} D^\alpha e_1 \\ D^\alpha e_2 \\ D^\alpha e_3 \end{pmatrix} = \begin{pmatrix} a - k_1 & 0 & 0 \\ 0 & b - k_2 & 0 \\ 0 & 0 & c - k_3 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = Ae, \tag{27}$$

where

$$A = \begin{pmatrix} a - k_1 & 0 & 0 \\ 0 & b - k_2 & 0 \\ 0 & 0 & c - k_3 \end{pmatrix}. \tag{28}$$

According the stability theory of fractional-order system [16], straightforwardly we have the following theorem.

Theorem 7. *Consider the drive and the response system described in (23), if the controllers in (26) are employed and $|\arg(\lambda(A))| > \alpha\pi/2$ is satisfied, where $\lambda(A)$ denotes the eigenvalue of the matrix A given in (28), then MGPS between the drive system and the response system is achieved.*

It is worth mentioning that the above MGPS can be easily utilized in secure communication. More specifically, the original signal can be modulated into the chaotic signal generated by the new fractional-order chaotic system and then forms a combined signal. The combined signal can be injected into one of three variables. The receiver can obtain and decode the combined signal via the MGPS and the scaling factor, respectively.

5. Three Illustrative Examples

In this section, we will perform some numerical examples to verify the theoretical analysis.

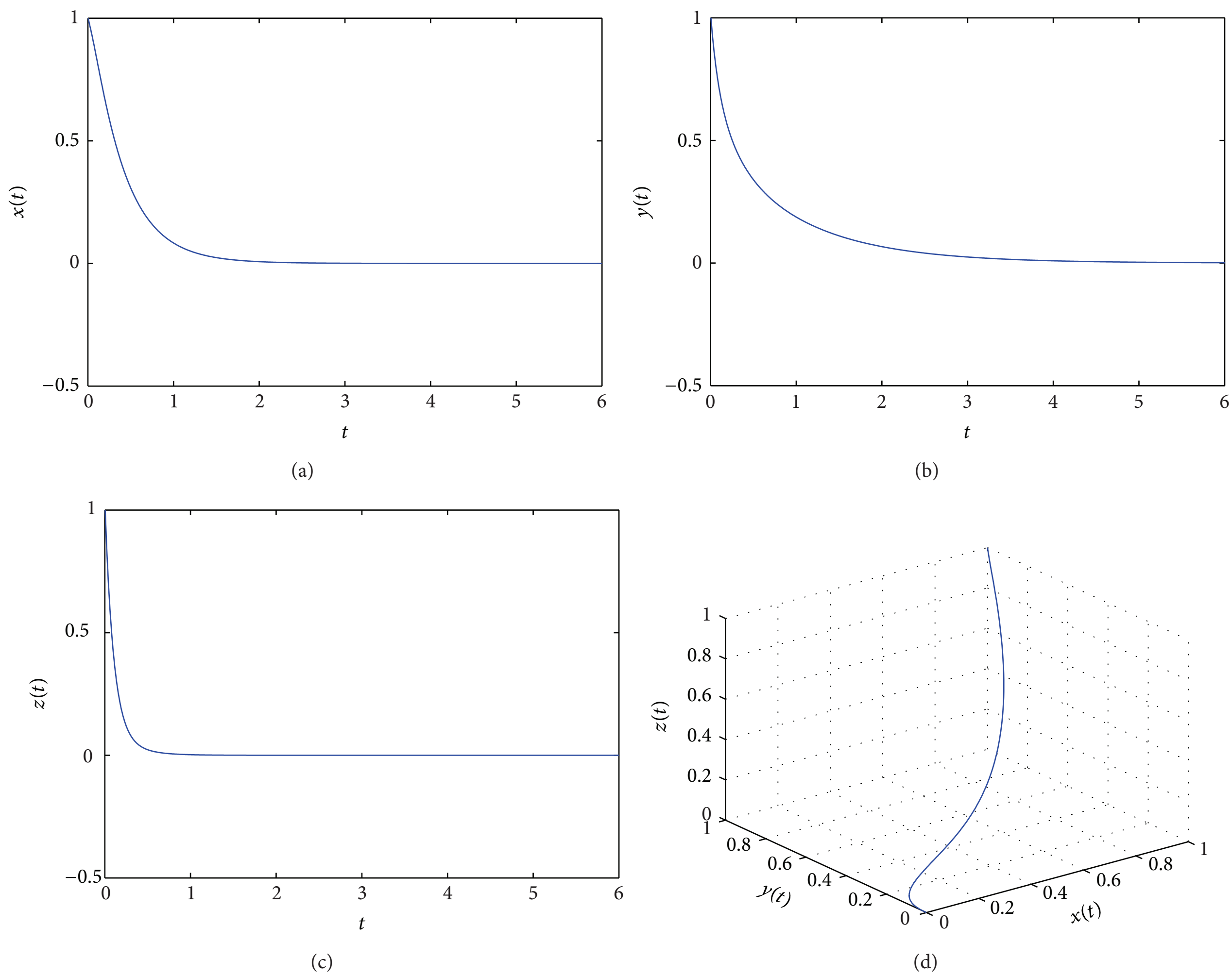

Figure 2: Sliding mode dynamics of system (12) are globally stable. (a–c) Time responses of x, y, and z. (d) Phase portrait.

Example 1. Consider the following FDE:

$$\begin{aligned} D^{q_1} x(t) &= ax(t) + dy(t) z(t) + gy^2(t), \\ D^{q_2} y(t) &= by(t) + ex(t) z(t) + hz(t) + u(t), \\ D^{q_3} z(t) &= cz(t) + fx(t) y(t), \end{aligned} \tag{29}$$

with $a = -3$, $b = 5$, $c = -10$, $d = 1$, $e = -1$, $f = 1$, $g = 1$, $h = 16$, $q_1 = 0.995$, $q_2 = 0.997$, $q_3 = 0.998$. If we choose the controller $u(t) = -xz - xy - 16z - 6y$, then from Theorem 3 we know that system (29) is asymptotically stable, as illustrated in Figure 2.

Example 2. Consider the following FDE with the model uncertainty and the external disturbance:

$$\begin{aligned} D^{q_1} x(t) &= ax(t) + dy(t) z(t) + gy^2(t), \\ D^{q_2} y(t) &= by(t) + ex(t) z(t) + hz(t) \\ &\quad + \Delta h(x, y, z) + \Delta w(t) + u(t), \\ D^{q_3} z(t) &= cz(t) + fx(t) y(t), \end{aligned} \tag{30}$$

where $a = -3$, $b = 5$, $c = -10$, $d = 1$, $e = -1$, $f = 1$, $g = 1$, $h = 16$, $q_1 = 0.995$, $q_2 = 0.997$, $q_3 = 0.998$, $\Delta h(x, y, z) = 0.1\sin(y)$, $\Delta w(t) = 0.1\sin(t)$. If we choose the controller $u(t) = -xz - xy - 16z - y - 2\,\mathrm{sign}(s) - (\tilde{\psi} + \tilde{\omega})\,\mathrm{sign}(s)$ with the adaptive laws $\dot{\tilde{\psi}} = 1.12|s|$, $\dot{\tilde{\omega}} = 0.52|s|$ and the switching surface $s(t) = D^{q_2 - 1} y + \int_0^t [2xz + xy - 4y]\, d\tau$, then according to Theorem 5 we know that system (30) converges to the sliding surface $s = 0$, which is illustrated in Figure 3.

Example 3. Consider the drive and the response system given by

$$\begin{pmatrix} D^\alpha x_m \\ D^\alpha y_m \\ D^\alpha z_m \end{pmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & h \\ 0 & 0 & c \end{bmatrix} \begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix} + \begin{pmatrix} dy_m z_m + gy_m^2 \\ ex_m z_m \\ fx_m y_m \end{pmatrix},$$

$$\begin{pmatrix} D^\alpha x_s \\ D^\alpha y_s \\ D^\alpha z_s \end{pmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & h \\ 0 & 0 & c \end{bmatrix} \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix}$$

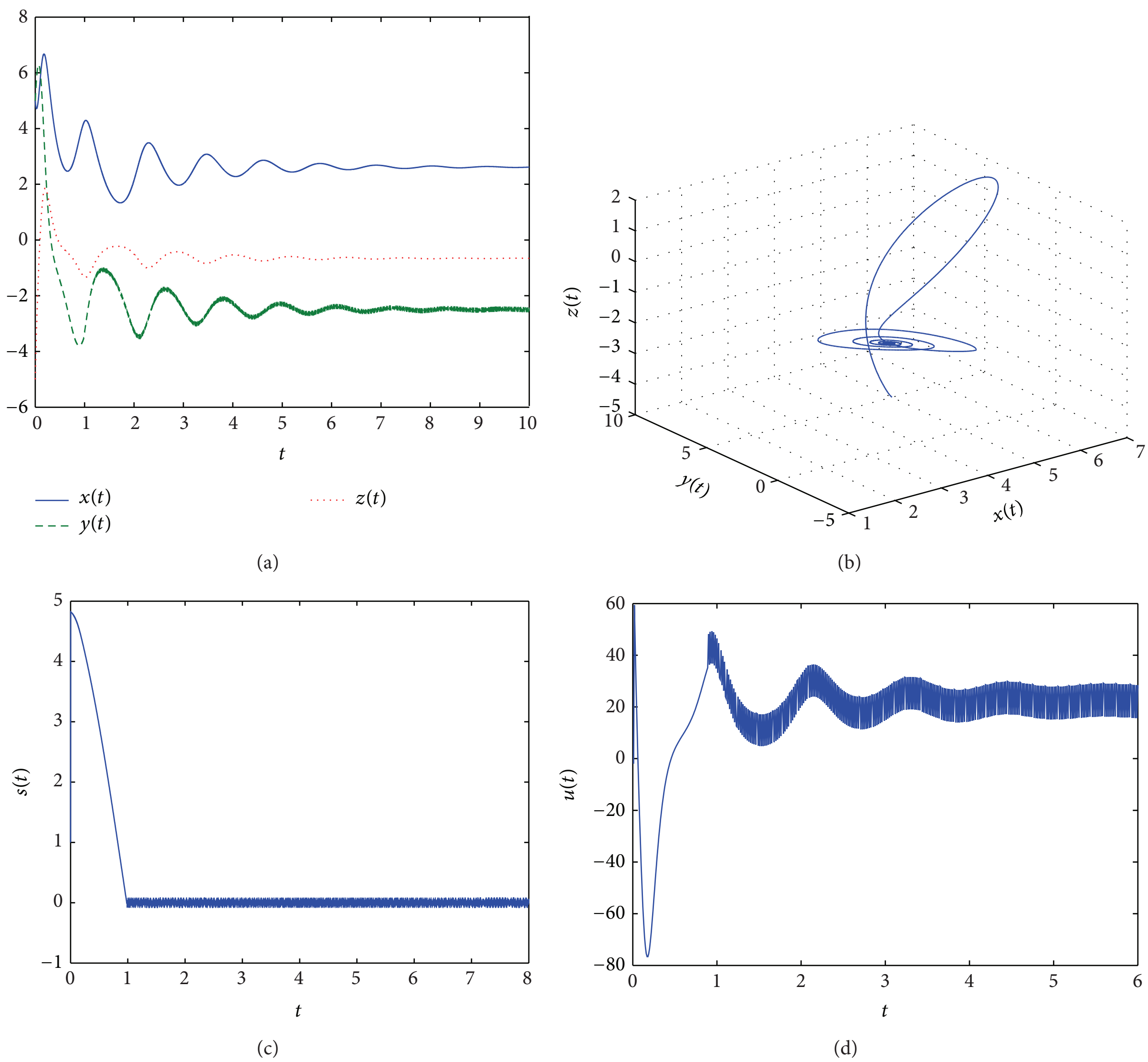

FIGURE 3: Adaptive sliding mode control of system (30). (a) Time responses of x, y, and z. (b) Phase portrait. (c) Time response of $s(t)$. (d) Time response of $u(t)$.

$$+\begin{pmatrix} dy_s z_s + g y_s^2 \\ e x_s z_s \\ f x_s y_s \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \tag{31}$$

where $a = -3$, $b = 5$, $c = -10$, $d = 1$, $e = -1$, $f = 1$, $g = 1$, $h = -16$, $\alpha = 0.995$, $d_1 = 0.5$, $d_2 = -1.2$, $d_3 = 3$. The controllers u_i $(i = 1, 2, 3)$ for the response system are designed as follows:

$$\begin{aligned} u_1 &= -y_s z_s - y_s^2 + 0.5 d y_m z_m + 0.5 g y_m^2 - k_1 e_1, \\ u_2 &= 16 z_s + x_s z_s + 19.2 z_m + 1.2 x_m z_m - k_2 e_2, \\ u_3 &= -x_s y_s + 3 x_m y_m - k_3 e_3, \end{aligned} \tag{32}$$

where the control gains matrix $K = \operatorname{diag}(k_1, k_2, k_3) = (0, 6, 0)$. According to Theorem 7 we know that MGPS between the drive system and the response system in (31) is achieved, as shown in Figure 4.

6. Concluding Remarks

A new fractional-order chaotic system has been addressed in this paper. The indirect Lyapunov stability of this system has been studied based on sliding mode control technique. Furthermore, the adaptive laws have been designed to guarantee the stability of this system with the uncertainty and external disturbance. By applying the stability theory of fractional-order system, MGPS of this chaotic systems has also been investigated, which may provide potential applications in secure communication. Finally, three illustrative numerical

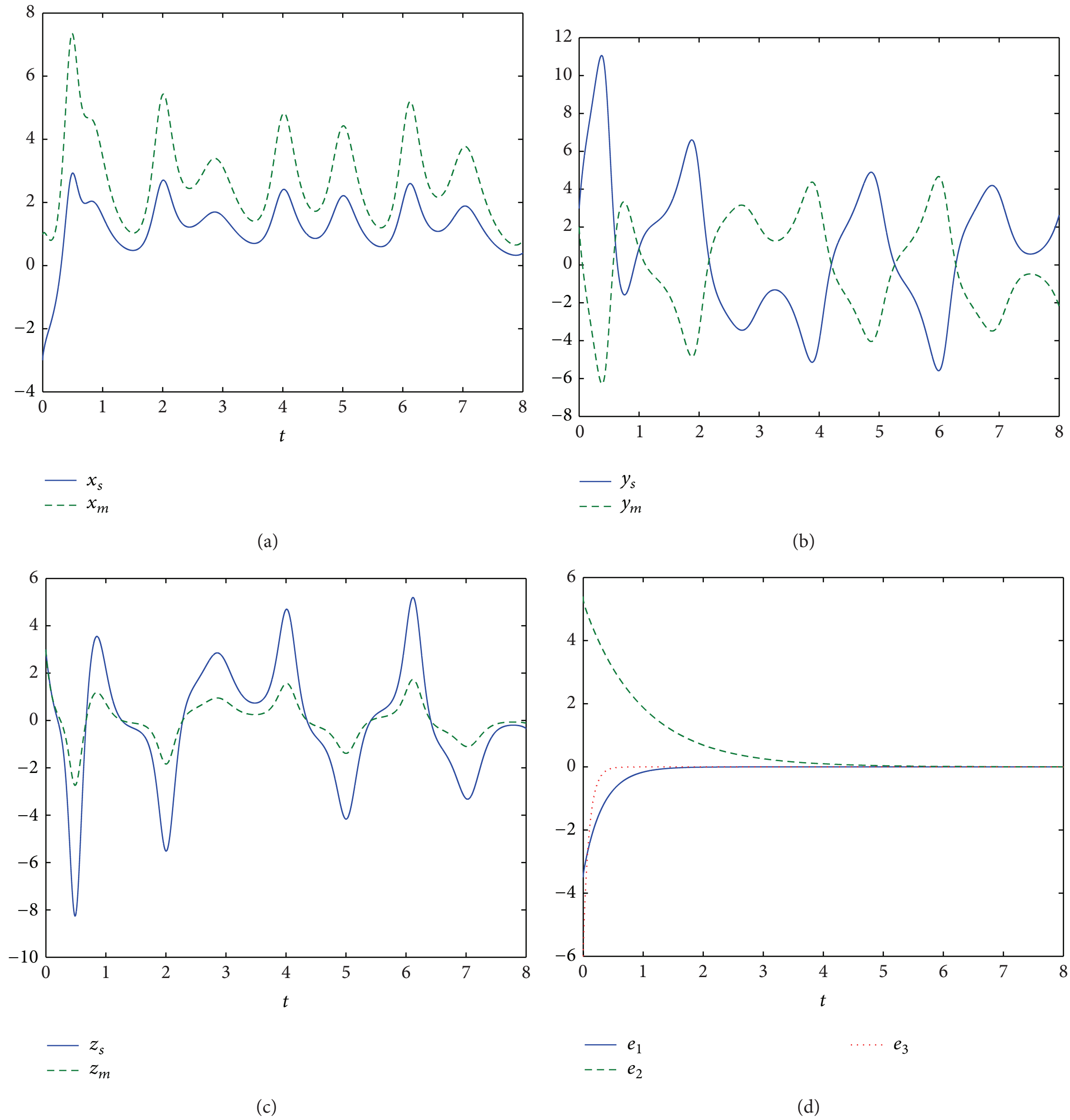

FIGURE 4: MGPS of system (31). (a–c) Time responses of x_m and x_s, y_m and y_s, and z_m and z_s. (d) Time response of synchronization errors.

examples have been presented to verify the effectiveness of the theoretical results.

Conflict of Interests

The authors declare that they have no conflict of interests.

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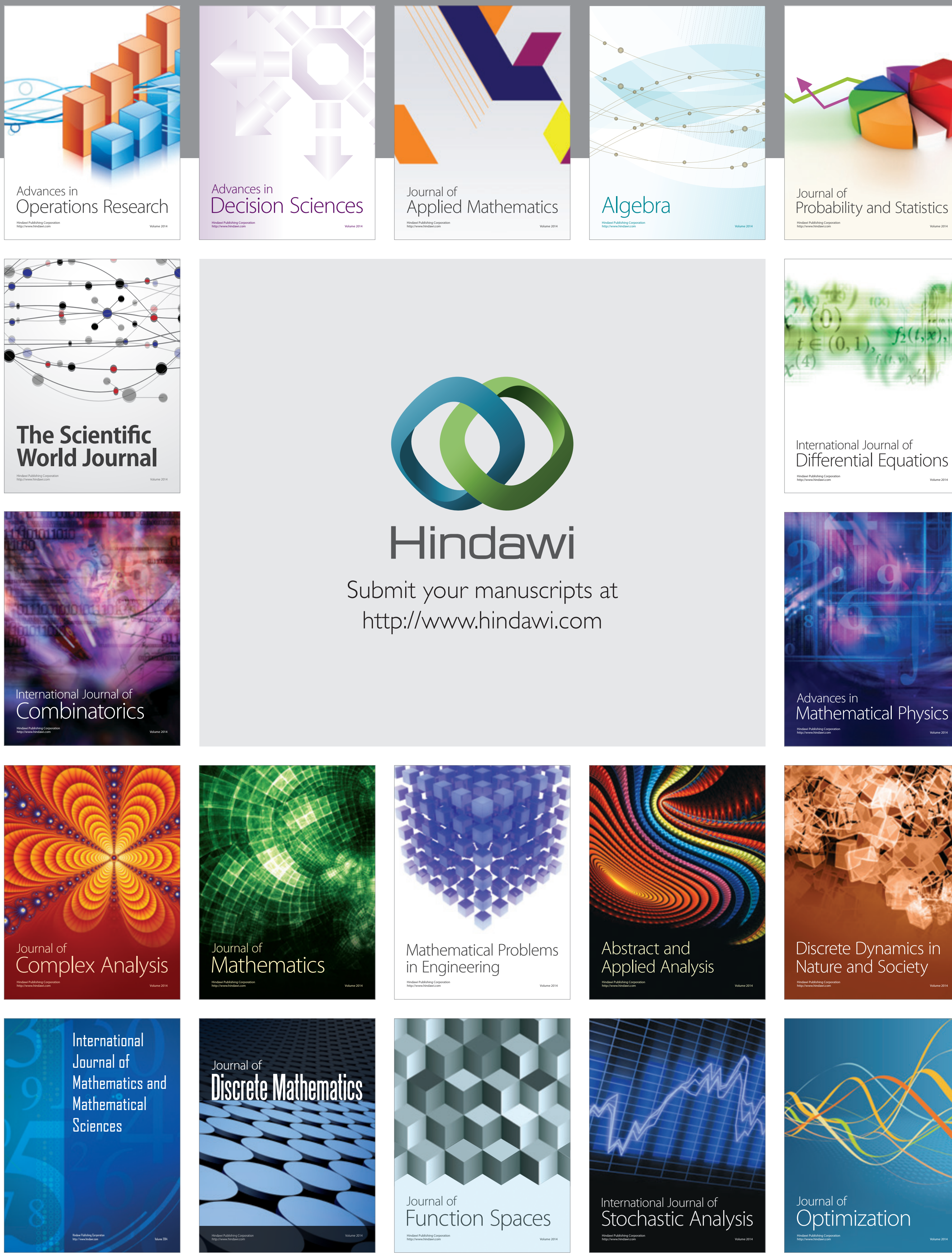
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