Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 963282, 13 pages
http://dx.doi.org/10.1155/2015/963282

Research Article

Distributed H_∞ Consensus Control of Networked Control Systems with Time-Triggered Protocol

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Received 11 June 2015; Revised 2 October 2015; Accepted 7 October 2015

Academic Editor: Jason Gu

The analysis and design of traditional networked control systems focused on single closed-loop scenario. This paper introduces a distributed control approach for the networked control systems (NCSs) with multiple subsystems based on a time-triggered network protocol. Firstly, some basic ideas of the time-triggered protocol are introduced and a time schedule scheme is employed for the NCS. Then, a novel model is proposed to the NCS regarding the network-induced delay. The resulting closed-loop system is time-delay linear system considering a distributed control law. A sufficient condition to H_∞ consensus control is present based on the Lyapunov-Krasovskii function. Also, the controller design approach towards the given H_∞ performance index is given by a cone complement linearization and iterative algorithm. Finally, numerical examples are given to validate the approach.

1. Introduction

Networked control systems (NCSs) are a class of closed-loop control systems in which sensors, controllers, and actuators are connected over network (see [1]). In recent years, NCSs have received increasing attention due to the broad application in industrial areas. The induced network brings about many advantages, such as low installation and maintenance costs, high reliability, and increased system flexibility. But, simultaneously, network-induced imperfections, such as time delays, packet losses and disorder, time-varying packet transmission/sampling intervals, and competition of multiple nodes accessing network, will decrease the performance of NCSs (see [2]). More seriously, some imperfections may cause instability. During the past decades, many researchers have studied the NCSs, and various methodologies have been proposed on the modeling (see [3, 4]), scheduling (see [5, 6]), analysis, and control design (see [7–11]).

When the plant is multiple-input-multiple-output (MIMO), the NCS is called MIMO NCSs. Because the nodes should compete to access not only outside nodes, but also other nodes inside, the research of MIMO NCSs is a more challenging job when compared with the so-called single-input-single-output (SISO) NCSs. Yan et al. [12] presented a continuous time model of MIMO NCSs with distributed time delays and uncertainties and gave delay-dependent stability criteria in terms of linear matrix inequalities (LMIs). Xia et al. [13] presented a discrete-time model of MIMO NCSs with multiple time-varying delays, and the design of output feedback controllers is proposed in terms of matrix inequalities, together with an iterative algorithm. Okajima et al. [14] proposed a design method for feedback-type dynamic quantization in a MIMO NCS, which is extended from SISO NCSs. Guan et al. [15] studied the optimal tracking performance for MIMO LTI discrete-time control systems with communication constraints in feedback path and how the bandwidth and AWGN of the communication channel affected the tracking capability. Jiang et al. [16] studied the optimal tracking performance of MIMO NCSs with AWGN channel between the controller and the plant and concluded that the optimal tracking performance was closely dependent on nonminimum phase zeros, unstable poles of the plant, and characteristics of the signals and channel. Cao et al. [17] presented delay dependant stability criteria for MIMO NCSs with nonlinear perturbation and delay, which gave much less conservative maximum allowable delay bound. Li et al. [18] modeled the MIMO NCS with multichannel packet disordering, packet dropout, and bounded time-varying transmission delay, as a jump linear system subject to Markovian chains, and a real-time controller was proposed such that the cost

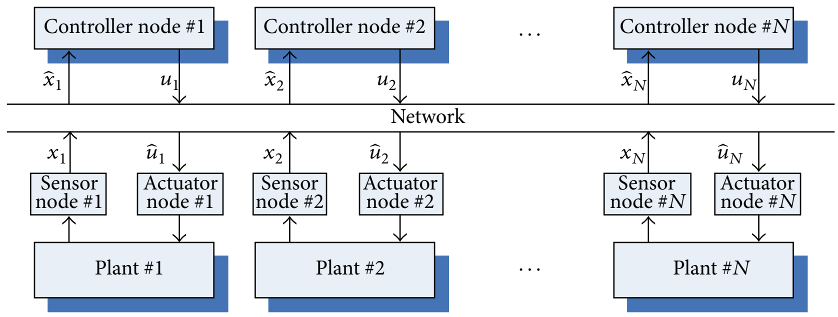

Figure 1: Structure of the networked control systems.

Figure 2: The Baggage Handling System.

function value is lower than a specified upper bound. Du et al. [19] modeled MIMO NCSs as unknown switched sequence and proposed a sufficient condition to be asymptotically stable in terms of a set of bilinear matrix inequalities. In the above reference, only one controller node is in the NCS, which is not a good choice for many real applications.

Actually, the distributed control using multiple controller nodes is a more interesting topic on the MIMO NCSs. But, until now, there are few articles published. Hirche et al. [20] introduced a novel distributed controller approach for NCS to achieve finite gain L2 stability independent of constant time delay, which consisted two parts. One was a local controller designed without network; the other was a remote part to compensate the network-induced delay to keep stability. But, actually, here the controller was divided into two separated parts, which was not so-called distributed control. In this paper, we consider a class of MIMO networked control systems with multiple subsystems, multiple sensor nodes, controller nodes, and actuator nodes, whose subsystems exchange information through network. Figure 1 gives the common structure of the NCSs. It is clear that the NCS is a distributed system over the network channel. This kind of system can be easily found, such as Baggage Handling Systems (BHS, as in Figure 2), product line systems, and Multi-joint Robots. For these systems, all subsystems are essentially required to be stable. Furthermore, there are usually some special requirements. Take BHS as an example; one BHS often has a few branches, which contain dozens of motors. While transplanting baggage, the velocity of motors in each branch should be consistent. Otherwise, the baggage may collide because of different velocity. For more details, the consensus control in NCSs means consensus not only in steady state, but also in transient response; that is, when the reference signal changes or disturbance occurs, the output signal of the subsystems is required to change simultaneously.

The consensus control of NCSs is an interesting problem and is different from that in multivehicle cooperative control, because the dynamic of each subsystem is different from the others. Some effective conclusions in multivehicle cooperative control cannot be used directly in this situation. Conditions to consensus control need to be investigated.

In this paper, we focus on the modeling and consensus control of the NCSs with time-triggered protocol and distributed control law. The contributions are as follows:

(i) Firstly, the time-triggered protocol is introduced and employed to the NCSs; a scheduling scenario which reduces the network-induced delay within each subsystem is introduced.

(ii) Secondly, a model for the NCSs with time-triggered protocol and short time-varying network-induced delays is proposed, while the distributed controllers which use the feedback information from the subsystem neighbors are used.

(iii) Thirdly, the sufficient conditions for asymptotical stability and H_∞ consensus control of the NCSs are obtained by Lyapunov-Krasovskii function. The conditions guarantee all subsystems reach consensus while satisfying the desired H_∞ performance on the fixed time-triggered protocol. Also, an iterative algorithm is given for distributed controller gain matrix.

The rest of this paper is organized as follows. Section 2 introduces the protocol of the NCSs, the feature of the network-induced delay, and the mathematical model. Section 3 deals with H_∞ consensus control problem for NCSs. Some numerical examples are given in Section 4 to demonstrate the effectiveness of the proposed design technique. The conclusion is provided in Section 5.

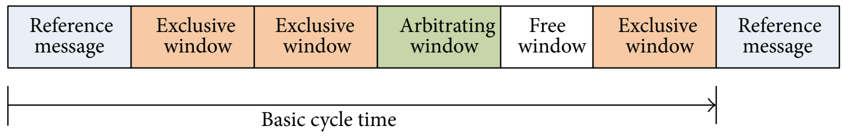

FIGURE 3: Basic cycle of TTCAN.

2. Modeling of the NCSs

2.1. Protocol of the Network. Consider the NCSs with multiple subsystems in Figure 1. The sensor nodes and the controller nodes will access the network after sampling or calculation, respectively. As we know, their authorities to access network depend on the protocol. In addition, the features of network-induced delay and data loss also depend on the protocol.

Network protocols can be classified into time-triggered protocol or event-triggered protocol. Time-triggered protocol allows the node to access network in certain time slot, such as Ether-CAT, FlexRay, and Time-Triggered CAN, while the nodes in event-triggered protocol access the network whenever they are ready for transmission, such as TCP/IP, CAN. Most communication networks adopt event-triggered protocol, because the protocol is efficient while nodes join and quit frequently. But the situation is different in control systems. Few nodes in control systems will join or exit frequently while working, unless the node is down or crashed. Moreover, event-triggered protocol brings about many uncertainties to the system, because of the random access. In [21], it is concluded that, compared with event-triggered protocol, the time-triggered protocol brought about more convenience to design and analysis. So we discuss the NCSs based on the time-triggered protocol. In order to model the NCSs, we introduce some important features of time-triggered protocol.

Usually, a basic cycle exists in time-triggered protocol network, which means that a basic period for all important nodes has at least one chance to transfer data. For example, a basic cycle of TTCAN [22] is shown in Figure 3. A basic cycle begins with a reference message, which is sent by a special node and can be identified by all participants. A basic cycle usually consists of several time windows (or slots) of different length and offers the necessary time for the message to be transmitted. The exclusive window is a time slot for periodic messages, while the arbitrating window is for aperiodic messages. Free window is reserved for further extensions. An exclusive window allows only one node to send a frame. In the arbitrating windows, these nodes that need to send frames are allowed to compete for network access as in event-triggered protocol. The end of an arbitrating window is always predictable. Thus, the advantages of event-triggered communication can be combined with those of time-triggered communication.

Of course, the sequence of these windows in a basic cycle can be designed according to scheduling strategy. For example, the sequence can be designed as in Figure 4 to reduce network-induced delay within each subsystem when it is used for NCS with four subsystems.

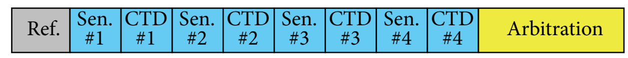

FIGURE 4: Basic cycle of NCS with four subsystems based on TTCAN.

Furthermore, the reference message also gives some important information, including a global time stamp, where participants can achieve a synchronization accuracy of 1 μs. That means the time jitter between all nodes can be negligible, unless the main time constant of the system is shorter than microseconds.

2.2. Modeling of the NCSs. According to the facts in Section 2.1, we can give the following reasonable assumptions.

Assumption 1. The sensor nodes and controller nodes are all time triggered.

Remark 2. According to the time-triggered protocol, the intelligent nodes access the network in appointed time slots. So it is reasonable to set sensor nodes and controller nodes to be time triggered. And they should be idle at rest time to reduce power consumption. The actuator nodes are either time triggered or event triggered, because the data packets from controller nodes arrive at almost the same time in each basic cycle.

Remark 3. For time-triggered protocol, little conflictions occur during transmission. It is reasonable to assume little data loss. So, we do not consider data loss in this paper.

By Assumption 1, the network-induced time delay $\tau_i = \tau_{i,sc} + \tau_{i,ca}$ is constant, where $\tau_{i,sc}$ is time delay between the sensor node and controller node of the ith subsystem and $\tau_{i,ca}$ is time delay between the controller node and actuator node. Also, we have $\tau_i < T$, where T is the basic cycle time.

Suppose that the plant of any subsystem is LTI and is described as space state equation:

$$\begin{aligned} \dot{x}_i(t) &= A_{pi} x_i(t) + B_{pi} \hat{u}_i(t) + \omega_i(t), \\ y_i(t) &= C_{pi} x_i(t), \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbf{R}^n$ is the state vector of the ith plant, $i = 1, \ldots, N$. $\hat{u}_i(t) \in \mathbf{R}^m$ is the control input vector, $y_i(t) \in \mathbf{R}^r$ is the output vector, $\omega_i(t) \in \mathbf{R}^q$ is the external disturbance, and $A_{pi} \in \mathbf{R}^{n\times n}$, $B_{pi} \in \mathbf{R}^{n\times m}$, and $C_{pi} \in \mathbf{R}^{r\times n}$ are known real constant matrices.

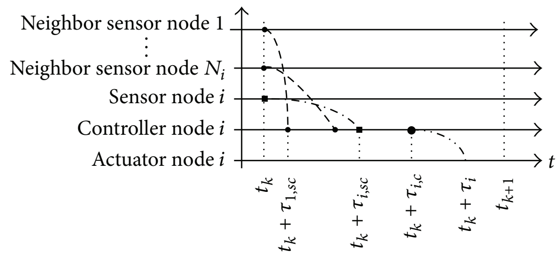

FIGURE 5: Data flow of ith subsystem.

We use $\hat{x}_i$ to denote the sampled data in the receiver of the ith controller node; u_i denotes the control variable calculated by the ith controller node.

By (1), the NCS can be described in discrete time as

$$\begin{aligned} x_i(t_{k+1}) &= A_i x_i(t_k) + B_{i1}\hat{u}_i(t_k) + B_{i2}\hat{u}_i(t_k+\tau_i) \\ &\quad + \omega_i(t_k), \\ y_i(t_k) &= C_i x_i(t_k), \end{aligned} \tag{2}$$

where $A_i = e^{A_{pi}T}$, $B_{i1} = \int_0^{\tau_i} e^{A_{pi}s} ds B_{pi}$, and $B_{i2} = \int_{\tau_i}^{T} e^{A_{pi}s} ds B_{pi}$. A_i, B_{ij}, $i = 1, \ldots, N$, $j = 1, 2$, are known matrices for τ_i is constant.

Since NCSs are usually large scale, it is not advisable to employ centralized control. In this paper, we employ a distributed control law as (3) for the NCS:

$$u_i = K_i \sum_{j\in N_i} a_{ij}\left(\hat{x}_i - \hat{x}_j\right), \tag{3}$$

where $K_i \in \mathbf{R}^{m\times n}$ is gain of ith controller node. The control law (3) means each subsystem controller uses both its own feedback and also datum from its neighbors.

The data flow of the ith subsystem is shown in Figure 5; $t_k + \tau_{i,c}$ is the moment the controller node calculates control variable. So the plant input is

$$\hat{u}_i(t) = \begin{cases} K_i \sum_{j\in N_i} a_{ij}\left(x_i(t_k) - x_j(t_k)\right), & t \in (t_k+\tau_i, t_{k+1}] \\ \hat{u}_i(t_k), & t \in (t_k, t_k+\tau_i], \end{cases} \tag{4}$$

where t_k, t_{k+1} are the kth and $(k+1)$th sampling time, $t_{k+1} - t_k = T$. Suppose the cycle time in Figure 4 is used in the NCS; when the ith subsystem calculates the control variable, $(k+1)$th$, \ldots, N$th subsystems have not sent their data packets. That means the neighbor of the ith subsystem is $N_i = \{v_l, v_j \in E,\ j < i\}$.

Using (2) and (4), the closed-loop system can be described as

$$\begin{aligned} x_i(t_{k+1}) &= \left(A_i + B_{i2}K_i \sum_{j\in N_i} a_{ij}\right) x_i(t_k) \\ &\quad + B_{i1}K_i \sum_{j\in N_i} a_{ij} x_i(t_{k-1}) \\ &\quad - B_{i2}K_i \sum_{j\in N_i} a_{ij} x_j(t_k) \\ &\quad - B_{i1}K_i \sum_{j\in N_i} a_{ij} x_j(t_{k-1}) + \omega_i(k). \end{aligned} \tag{5}$$

Considering that the consensus control is also an objective, we define a novel controlled output as follows:

$$z_i(t_k) = x_{i+1}(t_k) - x_1(t_k), \quad i = 2, \ldots, N. \tag{6}$$

Let $\xi(k) = (x_1^T(t_k), \ldots, x_N^T(t_k))^T \in \mathbf{R}^{Nn}$, $z(k) = (z_1^T(t_k), \ldots, z_N^T(t_k))^T$; we have the following model by (5):

$$\begin{aligned} \xi(k+1) &= \left(\overline{A} + \overline{B}_2\overline{K}L\right)\xi(k) + \overline{B}_1\overline{K}L\xi(k-1) \\ &\quad + \omega(k), \\ z(k) &= H\xi(k), \end{aligned} \tag{7}$$

where $\overline{A} = \operatorname{diag}(A_1, \ldots, A_N)$, $\overline{B}_i = \operatorname{diag}(B_{1i}, \ldots, B_{Ni})$, $i = 1, 2$, $H = \begin{pmatrix} -I & I & & 0 \\ \vdots & & \ddots & \\ -I & 0 & & I \end{pmatrix}$, $\overline{K} = \operatorname{diag}(K_1, \ldots, K_N)$, and $L = \lfloor l_{ij} \rfloor$ is the Laplacian matrix, $l_{ij} = \{-a_{ij},\ j \neq i;\ \sum_{j\in N_i} a_{ij},\ j = i\}$.

From the above, the objective is to design distributed controller (3) such that

(1) the control law asymptotically solves the consensus problem; that is, the states of subsystems satisfy

$$\lim_{k\to\infty}\left(x_i(k) - x_j(k)\right) = 0; \tag{8}$$

(2) the closed-loop system satisfies the following dissipation inequality:

$$\begin{aligned} \frac{\|z(k)\|_2^2}{\|\omega(k)\|_2^2} &= \frac{\sum_{k=0}^{\infty} z^T(k) z(k)}{\sum_{k=0}^{\infty} \omega^T(k)\omega(k)} < \gamma^2, \\ &\qquad \omega(k) \in L^2[0,\infty). \end{aligned} \tag{9}$$

3. Main Results

Firstly, some useful lemmas are given.

Lemma 4 (see [23]). *Assume $x(k) \in \mathbf{R}^n$; then, for any matrices $X > 0$, $M_1, M_2 \in \mathbf{R}^{n\times n}$ and a scalar function $h := h(k) \geq 0$, the following inequality holds:*

$$\begin{aligned} &-\sum_{i=k-h}^{k-1} \eta^T(i) X \eta(i) \\ &\leq \zeta^T(k)\left(\Lambda + h\begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} X^{-1} \begin{bmatrix} M_1 & M_2 \end{bmatrix}\right)\zeta(k), \end{aligned} \tag{10}$$

where $\Lambda = \begin{bmatrix} M_1^T+M_1 & -M_1^T+M_2 \\ * & -M_2^T-M_2 \end{bmatrix}$, $\zeta(k) = \begin{bmatrix} x(k) \\ x(k-h) \end{bmatrix}$, *and* $\eta(i) = x(i+1) - x(i)$.

Using Lemma 4, we have the following lemma.

Lemma 5. *Assume* $\omega(k) = 0$, *if there exist symmetric positive definite matrices* P, Q, *and* Z *and matrices* M_1, M_2, *and* $\overline{K}$ *such that*

$$\begin{bmatrix} -P+Q+M_1^T+M_1 & -M_1^T+M_2 & M_1^T & \left(\overline{A}+\overline{B}_2\overline{K}L-I\right)^T & \left(\overline{A}+\overline{B}_2\overline{K}L\right)^T \\ * & -Q-M_2^T-M_2 & M_2^T & \left(\overline{B}_1\overline{K}L\right)^T & \left(\overline{B}_1\overline{K}L\right)^T \\ * & * & -Z & 0 & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{bmatrix} < 0. \tag{11}$$

The NCS (7) is asymptotically stable.

Proof. For system (7), define $y(k) = \xi(k) - \xi(k-1)$. Choose Lyapunov-Krasovskii function as follows:

$$\begin{aligned} V(k) &= \xi^T(k)P\xi(k) + \xi^T(k-1)Q\xi(k-1) \\ &\quad + y^T(k)Zy(k). \end{aligned} \tag{12}$$

Then, the difference of $V(k)$ is

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \xi^T(k+1)P\xi(k+1) + \xi^T(k)Q\xi(k) \\ &\quad + y^T(k+1)Zy(k+1) - \xi^T(k)P\xi(k) \\ &\quad - \xi^T(k-1)Q\xi(k-1) - y^T(k)Zy(k) \\ &= \xi^T(k)\left[\left(\overline{A}+\overline{B}_2\overline{K}L\right)^T P\left(\overline{A}+\overline{B}_2\overline{K}L\right)\right]\xi(k) \\ &\quad + 2\xi^T(k)\left[\left(\overline{A}+\overline{B}_2\overline{K}L\right)^T P\left(\overline{B}_1\overline{K}L\right)\right]\xi(k-1) \\ &\quad + \xi^T(k-1)\left[\left(\overline{B}_1\overline{K}L\right)^T P\left(\overline{B}_1\overline{K}L\right)\right]\xi(k-1) \\ &\quad + \xi^T(k)Q\xi(k) + y^T(k+1)Zy(k+1) \\ &\quad - \xi^T(k)P\xi(k) - \xi^T(k-1)Q\xi(k-1) \\ &\quad - y^T(k)Zy(k). \end{aligned} \tag{13}$$

Aimed at the term $y^T(k+1)Zy(k+1)$, using (7),

$$\begin{aligned} & y^T(k+1)Zy(k+1) \\ &= (\xi(k+1)-\xi(k))^T Z(\xi(k+1)-\xi(k)) \\ &= \xi^T(k)\left[\left(\overline{A}+\overline{B}_2\overline{K}L-I\right)^T Z\left(\overline{A}+\overline{B}_2\overline{K}L-I\right)\right]\xi(k) \\ &\quad + 2\xi^T(k)\left[\left(\overline{A}+\overline{B}_2\overline{K}L-I\right)^T Z\left(\overline{B}_1\overline{K}L\right)\right]\xi(k-1) \\ &\quad + \xi^T(k-1)\left[\left(\overline{B}_1\overline{K}L\right)^T Z\left(\overline{B}_1\overline{K}L\right)\right]\xi(k-1). \end{aligned} \tag{14}$$

By Lemma 4, the inequality holds:

$$\begin{aligned} & -y^T(k)Zy(k) \le \left[\xi^T(k) \;\; \xi^T(k-1)\right]\Bigg\{\Lambda \\ & + \begin{bmatrix} M_1^T \\ M_1^T \end{bmatrix} Z^{-1}\left[M_1 \;\; M_2\right]\Bigg\}\begin{bmatrix} \xi(k) \\ \xi(k-1) \end{bmatrix}, \end{aligned} \tag{15}$$

where $\Lambda = \begin{bmatrix} M_1^T+M_1 & -M_1^T+M_2 \\ * & -M_2^T-M_2 \end{bmatrix}$. Then, we have

$$\begin{aligned} \Delta V(k) &\le \xi^T(k)\Big[\left(\overline{A}+\overline{B}_2\overline{K}L\right)^T P\left(\overline{A}+\overline{B}_2\overline{K}L\right) \\ &\quad + \left(\overline{A}+\overline{B}_2\overline{K}L-I\right)^T Z\left(\overline{A}+\overline{B}_2\overline{K}L-I\right) - P + Q\Big] \\ &\quad \cdot \xi(k) + 2\xi^T(k)\Big[\left(\overline{A}+\overline{B}_2\overline{K}L\right)^T P\left(\overline{B}_1\overline{K}L\right) \\ &\quad + \left(\overline{A}+\overline{B}_2\overline{K}L-I\right)^T Z\left(\overline{B}_1\overline{K}L\right)\Big]\xi(k-1) + \xi^T(k \\ &\quad - 1)\Big[\left(\overline{B}_1\overline{K}L\right)^T P\left(\overline{B}_1\overline{K}L\right) + \left(\overline{B}_1\overline{K}L\right)^T Z\left(\overline{B}_1\overline{K}L\right) \\ &\quad - Q\Big]\xi(k-1) + \left[\xi^T(k) \;\; \xi^T(k-1)\right]\Bigg\{\Lambda \\ &\quad + \begin{bmatrix} M_1^T \\ M_1^T \end{bmatrix} Z^{-1}\left[M_1 \;\; M_2\right]\Bigg\}\begin{bmatrix} \xi(k) \\ \xi(k-1) \end{bmatrix} \\ &= \left[\xi^T(k) \;\; \xi^T(k-1)\right]\Bigg\{\Phi + \Lambda \\ &\quad + \begin{bmatrix} M_1^T \\ M_1^T \end{bmatrix} Z^{-1}\left[M_1 \;\; M_2\right]\Bigg\}\begin{bmatrix} \xi(k) \\ \xi(k-1) \end{bmatrix}, \end{aligned} \tag{16}$$

where $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12}^T & \phi_{22} \end{bmatrix}$, and

$$\begin{aligned}
\phi_{11} &= \left(\overline{A} + \overline{B}_2\overline{K}L\right)^T P \left(\overline{A} + \overline{B}_2\overline{K}L\right) \\
&\quad + \left(\overline{A} + \overline{B}_2\overline{K}L - I\right)^T Z \left(\overline{A} + \overline{B}_2\overline{K}L - I\right) - P \\
&\quad + Q, \\
\phi_{12} &= \left(\overline{A} + \overline{B}_2\overline{K}L\right)^T P \left(\overline{B}_1\overline{K}L\right) \\
&\quad + \left(\overline{A} + \overline{B}_2\overline{K}L - I\right)^T Z \left(\overline{B}_1\overline{K}L\right), \\
\phi_{22} &= \left(\overline{B}_1\overline{K}L\right)^T P \left(\overline{B}_1\overline{K}L\right) + \left(\overline{B}_1\overline{K}L\right)^T Z \left(\overline{B}_1\overline{K}L\right) \\
&\quad - Q.
\end{aligned} \tag{17}$$

Hence, (7) is stable if the following matrix inequality holds:

$$\Phi + \Lambda + \begin{bmatrix} M_1^T \\ M_1^T \end{bmatrix} Z^{-1} \begin{bmatrix} M_1 & M_2 \end{bmatrix} < 0. \tag{18}$$

By Schur complements, inequality (18) is equivalent to

$$\begin{bmatrix} \phi_{11} + M_1^T + M_1 & \phi_{12} - M_1^T + M_2 & M_1^T \\ * & \phi_{22} - M_2^T - M_2 & M_2^T \\ * & * & -Z \end{bmatrix} < 0. \tag{19}$$

That is equivalent to

$$\begin{gathered}
\begin{bmatrix} \left(\overline{A} + \overline{B}_2\overline{K}L\right)^T P \left(\overline{A} + \overline{B}_2\overline{K}L\right) - P + Q + M_1^T + M_1 & \left(\overline{A} + \overline{B}_2\overline{K}L\right)^T P \left(\overline{B}_1\overline{K}L\right) - M_1^T + M_2 & M_1^T \\ * & \left(\overline{B}_1\overline{K}L\right)^T P \left(\overline{B}_1\overline{K}L\right) - Q - M_2^T - M_2 & M_2^T \\ * & * & -Z \end{bmatrix} \\
+ \begin{bmatrix} \left(\overline{A} + \overline{B}_2\overline{K}L - I\right)^T \\ \left(\overline{B}_1\overline{K}L\right)^T \\ 0 \end{bmatrix} Z \left[\left(\overline{A} + \overline{B}_2\overline{K}L - I\right) \ \left(\overline{B}_1\overline{K}L\right) \ 0\right] < 0.
\end{gathered} \tag{20}$$

By Schur complements, we have

$$\begin{gathered}
\begin{bmatrix} -P + Q + M_1^T + M_1 & -M_1^T + M_2 & M_1^T & \left(\overline{A} + \overline{B}_2\overline{K}L - I\right)^T \\ * & -Q - M_2^T - M_2 & M_2^T & \left(\overline{B}_1\overline{K}L\right)^T \\ * & * & -Z & 0 \\ * & * & * & -Z^{-1} \end{bmatrix} \\
+ \begin{bmatrix} \left(\overline{A} + \overline{B}_2\overline{K}L\right)^T \\ \left(\overline{B}_1\overline{K}L\right)^T \\ 0 \\ 0 \end{bmatrix} P \left[\left(\overline{A} + \overline{B}_2\overline{K}L\right) \ \left(\overline{B}_1\overline{K}L\right) \ 0 \ 0\right] < 0.
\end{gathered} \tag{21}$$

Then, we have inequality (11). This completes the proof. □

Theorem 6. *With the distributed control law (3), the NCS (7) achieves consensus with a given H_∞ disturbance attenuation index γ, if there exist symmetric positive definite matrices P, Q, and Z and matrices M_1, M_2, and $\overline{K}$, such that*

$$\begin{bmatrix} -P + Q + M_1^T + M_1 + H^TH & -M_1^T + M_2 & 0 & M_1^T & \left(\overline{A} + \overline{B}_2\overline{K}L - I\right)^T & \left(\overline{A} + \overline{B}_2\overline{K}L\right)^T \\ * & -Q - M_2^T - M_2 & 0 & M_2^T & \left(\overline{B}_1\overline{K}L\right)^T & \left(\overline{B}_1\overline{K}L\right)^T \\ * & * & -\gamma^2 I & 0 & I & I \\ * & * & * & -Z & 0 & 0 \\ * & * & * & * & -Z^{-1} & 0 \\ * & * & * & * & * & -P^{-1} \end{bmatrix} < 0. \tag{22}$$

And if the matrix inequality is feasible, the feedback matrix of the consensus protocol is $\overline{K}$.

Proof. Let Lyapunov-Krasovskii function as (12). Using (7) and Lemmas 4 and 5, we have

$$\Delta V(k) \le \left[\xi^T(k) \;\; \xi^T(k-1) \;\; \omega^T(k)\right] \widehat{\Phi} \begin{bmatrix} \xi(k) \\ \xi(k-1) \\ \omega(k) \end{bmatrix},$$

$$\widehat{\Phi} = \begin{bmatrix} \phi_{11} + M_1^T + M_1 + M_1^T Z^{-1} M_1 & \phi_{12} - M_1^T + M_2 + M_2^T Z^{-1} M_1 & \phi_{13} \\ * & \phi_{22} - M_2^T - M_2 + M_2^T Z^{-1} M_2 & \phi_{23} \\ * & * & P + Z \end{bmatrix}, \quad (23)$$

$$\phi_{13} = \left(\overline{A} + \overline{B}_2 \overline{K} L\right)^T P + \left(\overline{A} + \overline{B}_2 \overline{K} L - I\right)^T Z,$$

$$\phi_{23} = \left(\overline{B}_1 \overline{K} L\right)^T (P + Z).$$

Firstly, from condition (22), we have (11), so the NCS (7) is asymptotically stable. Then, we have

$$\lim_{k\to\infty} \left(x_i(k) - x_j(k)\right) = 0. \quad (24)$$

Then, we find H_∞ performance index.

For any $k > 0$, consider the following cost function:

$$J = \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 \omega^T(k) \omega(k)\right]. \quad (25)$$

By the zero initial condition ($V(0) = 0$), we have

$$J = \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(k)\right]$$
$$- V(\infty) + V(0)$$
$$\le \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(k)\right], \quad (26)$$

for

$$z^T(k) z(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(k)$$
$$= \left[\xi^T(k) \;\; \xi^T(k-1) \;\; \omega^T(k)\right] \Theta \begin{bmatrix} \xi(k) \\ \xi(k-1) \\ \omega(k) \end{bmatrix}, \quad (27)$$

where

$$\Theta = \begin{bmatrix} \phi_{11} + M_1^T + M_1 + M_1^T Z^{-1} M_1 + H^T H & \phi_{12} - M_1^T + M_2 + M_2^T Z^{-1} M_1 & \phi_{13} \\ * & \phi_{22} - M_2^T - M_2 + M_2^T Z^{-1} M_2 & \phi_{23} \\ * & * & P + Z - \gamma^2 I \end{bmatrix}. \quad (28)$$

According to Schur complements, condition (22) is equivalent to $\Theta < 0$. That is,

$$\sum_{k=0}^{\infty} z^T(k) z(k) < \gamma^2 \sum_{k=0}^{\infty} \omega^T(k) \omega(k), \quad (29)$$

so the robust H_∞ consensus control is achieved. This completes the proof. □

Note that condition (22) is nonconvex as it contains P, P^{-1}, Z, and Z^{-1}. Using cone complement linearization [24], we have the following corollary.

Corollary 7. *With the distributed control law (3), the NCS (7) achieves consensus with a given H_∞ disturbance attenuation index γ, if there exist symmetric positive definite matrices P, Q, Z, S, and T and matrices M_1, M_2, and $\overline{K}$, such that*

$$\min_{P,Q,M_1,M_2,\overline{K}} \quad [\operatorname{tr}(ZS + PT)]$$

$$s.t. \quad \begin{bmatrix} -P + Q + M_1^T + M_1 + H^T H & -M_1^T + M_2 & 0 & M_1^T & \left(\overline{A} + \overline{B}_2 \overline{K} L - I\right)^T & \left(\overline{A} + \overline{B}_2 \overline{K} L\right)^T \\ * & -Q - M_2^T - M_2 & 0 & M_2^T & \left(\overline{B}_1 \overline{K} L\right)^T & \left(\overline{B}_1 \overline{K} L\right)^T \\ * & * & -\gamma^2 I & 0 & I & I \\ * & * & * & -Z & 0 & 0 \\ * & * & * & * & -S & 0 \\ * & * & * & * & * & -T \end{bmatrix} < 0, \quad (30)$$

$$\begin{bmatrix} Z & I \\ I & S \end{bmatrix} \geq 0, \tag{31}$$

$$\begin{bmatrix} P & I \\ I & T \end{bmatrix} \geq 0. \tag{32}$$

If the matrix inequality is feasible, then the feedback matrix of the consensus protocol is $\overline{K}$.

The proof is omitted.

In order to design distributed control law (3), we present an iterative algorithm as follows.

Algorithm 8. (1) For (30)–(32), find a feasible solution: P^0, Q^0, $Z^0, M_1^0, M_2^0, \overline{K}^0, S^0$, and T^0, and let $h = 0$.

(2) Set $P^{h+1} = P^h$, $Z^{h+1} = Z^h$, $S^{h+1} = S^h$, and $T^{h+1} = T^h$, and solve the following optimal problem:

$$\begin{aligned} \min \quad & \left[\operatorname{tr}\left(Z^h S + S^h Z + P^h T + T^h P\right)\right] \\ \text{subject to} \quad & (30), (31), (32). \end{aligned} \tag{33}$$

(3) If a stopping criterion given in advance is satisfied, the iteration ends.

Otherwise, go to Step (2).

Remark 9. A simple stopping criterion is that P^h, Z^h provide a feasible solution to inequality (30). And using LMI toolbox, it is easily confirmed.

4. Numerical Examples

Example 1. We use the DC motors of [25] as the plants of the NCS in Figure 1, where the transfer function of the DC motor is

$$G(s) = \frac{2029.826}{(s + 26.29)(s + 2.296)}. \tag{34}$$

The DC motor model is rewritten in state space as

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -60.3756 & -28.586 \end{bmatrix} x(t) \\ &\quad + \begin{bmatrix} 0 \\ 2029.826 \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{aligned} \tag{35}$$

And, in this example, the basic cycle time is chosen as $T = 0.005$ s, there are three DC motors connected by the Time-Triggered CAN, and the total network-induced delay of each closed loop is $\tau_1 = 0.001$ s, $\tau_2 = 0.002$ s, and $\tau_3 = 0.003$ s.

Then, we have the parameters in (7) as follows:

$$\overline{A} = \begin{bmatrix} 0.9993 & 0.004658 & & & & \\ -0.2812 & 0.8861 & & & & \\ & & 0.9993 & 0.004658 & & \\ & & -0.2812 & 0.8861 & & \\ & & & & 0.9993 & 0.004658 \\ & & & & -0.2812 & 0.8861 \end{bmatrix},$$

$$\overline{B}_1 = \begin{bmatrix} 0.001 & & \\ 2.001 & & \\ & 0.0040 & \\ & 3.946 & \\ & & 0.0089 \\ & & 5.836 \end{bmatrix},$$

$$\overline{B}_2 = \begin{bmatrix} 0.0231 & & \\ 7.454 & & \\ & 0.202 & \\ & 5.509 & \\ & & 0.0153 \\ & & 3.6190 \end{bmatrix},$$

$$L = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ -1 & & 1 & & & \\ & -1 & & 1 & & \\ & & -1 & & 1 & \\ & & & -1 & & 1 \end{bmatrix}. \tag{36}$$

Here, we just consider the stability of the whole NCS; then using the matrix inequality in Lemma 5 together with Algorithm 8, the following feasible solution can be obtained by MATLAB LMI toolbox:

$$P = \begin{bmatrix} 0.7331 & 0.0011 & -0.0022 & 0.0111 & -0.0001 & -0.0001 \\ 0.0011 & 0.8764 & 0.0037 & -0.0140 & 0.0001 & 0.0002 \\ -0.0022 & 0.0037 & 0.7252 & 0.0253 & -0.0017 & 0.0064 \\ 0.0111 & -0.0140 & 0.0253 & 0.8117 & 0.0024 & -0.0097 \\ -0.0001 & 0.0001 & -0.0017 & 0.0024 & 0.7239 & 0.0301 \\ -0.0001 & 0.0002 & 0.0064 & -0.0097 & 0.0301 & 0.7989 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.4566 & 0.0114 & -0.0121 & 0.0142 & 0.0001 & -0.0007 \\ 0.0114 & 0.5985 & 0.0164 & -0.0211 & -0.0001 & 0.0010 \\ -0.0121 & 0.0164 & 0.4484 & 0.0223 & -0.0114 & 0.0134 \\ 0.0142 & -0.0211 & 0.0223 & 0.5772 & 0.0150 & -0.0187 \\ 0.0001 & -0.0001 & -0.0114 & 0.0150 & 0.4522 & 0.0184 \\ -0.0007 & 0.0010 & 0.0134 & -0.0187 & 0.0184 & 0.5796 \end{bmatrix},$$

$$Z = \begin{bmatrix} 1.0164 & 0.0003 & 0.0002 & -0.0002 & 0.0000 & 0.0000 \\ 0.0003 & 0.9573 & -0.0003 & 0.0156 & -0.0000 & 0.0003 \\ 0.0002 & -0.0003 & 1.0174 & -0.0011 & 0.0000 & -0.0000 \\ -0.0002 & 0.0156 & -0.0011 & 0.9825 & -0.0001 & 0.0053 \\ 0.0000 & -0.0000 & 0.0000 & -0.0001 & 1.0178 & -0.0016 \\ 0.0000 & 0.0003 & -0.0000 & 0.0053 & -0.0016 & 0.9887 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -0.9176 & -0.0010 & -0.0035 & 0.0049 & 0.0000 & -0.0001 \\ -0.0001 & -0.8679 & 0.0047 & -0.0191 & 0.0000 & -0.0001 \\ 0.0036 & -0.0050 & -0.9193 & 0.0017 & -0.0043 & 0.0058 \\ -0.0051 & -0.0056 & 0.0019 & -0.8903 & 0.0057 & -0.0120 \\ -0.0000 & 0.0000 & 0.0054 & -0.0072 & -0.9205 & 0.0040 \\ 0.0001 & -0.0004 & -0.0072 & 0.0052 & 0.0029 & -0.8969 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.8879 & -0.0005 & 0.0045 & -0.0060 & 0.0000 & 0.0001 \\ -0.0011 & 0.8353 & -0.0061 & 0.0200 & -0.0000 & 0.0001 \\ -0.0037 & 0.0050 & 0.8897 & -0.0032 & 0.0055 & -0.0071 \\ 0.0053 & 0.0047 & -0.0033 & 0.8570 & -0.0072 & 0.0134 \\ 0.0000 & 0.0000 & -0.0057 & 0.0075 & 0.8899 & -0.0041 \\ -0.0001 & 0.0003 & 0.0078 & -0.0060 & -0.0031 & 0.8618 \end{bmatrix},$$

$$\overline{K} = \begin{bmatrix} 0.0229 & -0.0341 & & & & \\ & & 0.0117 & -0.0164 & & \\ & & & & 0.0108 & -0.0146 \end{bmatrix}. \tag{37}$$

Using above $\overline{K}$, we have the state response curves shown in Figure 6(a), where the initial state $x_i^T(0) = [-1 \;\; 1]$, $x_i^T(-t) = [0 \;\; 0]$, $i = 1, 2, 3$. It is clear that the system is asymptotically stable.

When we set $L = \left[\begin{smallmatrix} 1 & & & & & \\ & 1 & & & & \\ -1 & & 1 & & & \\ & -1 & & 1 & & \\ -1 & & 0 & & 1 & \\ & -1 & & 0 & & 1 \end{smallmatrix}\right]$, the control gain matrix is $\overline{K} = \left[\begin{smallmatrix} 0.0228 & -0.0340 & & & & \\ & & 0.0107 & -0.0150 & & \\ & & & & 0.0105 & -0.0144 \end{smallmatrix}\right]$. The resulting state response curve is shown in Figure 6(b), with the same initial condition as in Figure 6(a).

For we are concerned with the stability only, the controlled output $z(t)$ cannot converge to zero. That means consensus is not achieved with above control gains, as shown in Figure 7, where the input signal is $r(t) = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, $t > 0$.

Example 2. Each subsystem of the NCS in Figure 1 is described as follows (see [1]):

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \hat{u}(t), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{aligned} \tag{38}$$

The basic cycle time is chosen as $T = 1$ s; there are three subsystems in the NCS. The network protocol is Time-Triggered CAN. And the total network-induced delay of each subsystem is $\tau_1 = 0.1$ s, $\tau_2 = 0.2$ s, and $\tau_3 = 0.3$ s.

We have the parameters in (7) as follows:

$$\overline{A} = \begin{bmatrix} 1 & 0.9516 & & & & \\ 0 & 0.9048 & & & & \\ & & 1 & 0.9516 & & \\ & & 0 & 0.9048 & & \\ & & & & 1 & 0.9516 \\ & & & & 0 & 0.9048 \end{bmatrix},$$

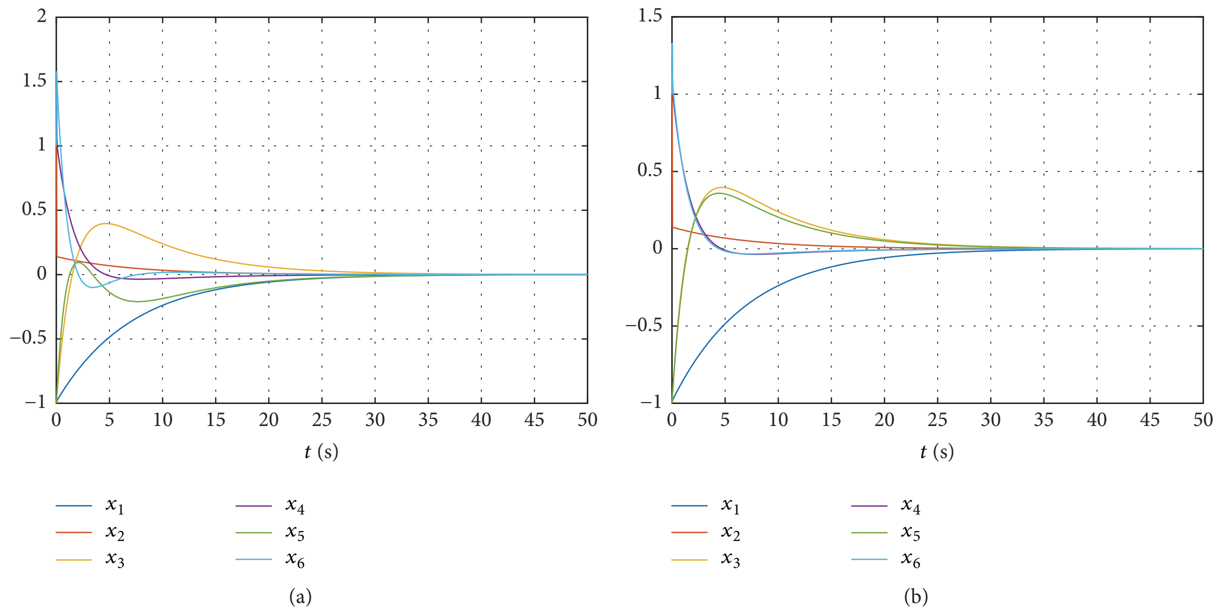

Figure 6: The state response curve.

$$\overline{B}_1 = \begin{bmatrix} 0.0005 & & \\ 0.0100 & & \\ & 0.0020 & \\ & 0.0198 & \\ & & 0.0045 \\ & & 0.0296 \end{bmatrix},$$

$$\overline{B}_2 = \begin{bmatrix} 0.0479 & & \\ 0.0852 & & \\ & 0.0464 & \\ & 0.0754 & \\ & & 0.0439 \\ & & 0.0656 \end{bmatrix},$$

$$L = 0.5 * \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ -1 & & 1 & & & \\ & -1 & & 1 & & \\ & & -1 & & 1 & \\ & & & -1 & & 1 \end{bmatrix}. \tag{39}$$

Set $\gamma = 0.8$; then using Corollary 7 and solving LMIs (30), (31), and (32) with MATLAB YALMIP tools box, it is found that

$$P = \begin{bmatrix} 27.7529 & 60.1707 & 1.0187 & 8.0159 & -1.3427 & 0.3512 \\ 60.1707 & 151.6629 & 3.2651 & 9.8879 & -0.2402 & -2.0660 \\ 1.0187 & 3.2651 & 5.3291 & 11.7535 & -0.0931 & 0.0056 \\ 8.0159 & 9.8879 & 11.7535 & 40.4633 & -0.5335 & -2.2785 \\ -1.3427 & -0.2402 & -0.0931 & -0.5335 & 2.3633 & 2.4611 \\ 0.3512 & -2.0660 & 0.0056 & -2.2785 & 2.4611 & 11.1664 \end{bmatrix},$$

$$Q = \begin{bmatrix} 5.5460 & 13.3815 & 0.5242 & 1.8687 & -0.0157 & 0.0613 \\ 13.3815 & 34.2054 & 0.2486 & 1.1222 & -0.2377 & -0.8933 \\ 0.5242 & 0.2486 & 1.5205 & 4.8289 & -0.0560 & -0.2246 \\ 1.8687 & 1.1222 & 4.8289 & 16.9049 & -0.5001 & -2.0982 \\ -0.0157 & -0.2377 & -0.0560 & -0.5001 & 0.4603 & 1.4683 \\ 0.0613 & -0.8933 & -0.2246 & -2.0982 & 1.4683 & 6.3043 \end{bmatrix},$$

$$Z=\begin{bmatrix} 0.1821 & 0.0955 & 0.1454 & 0.0970 & 0.1446 & 0.0989\\ 0.0955 & 0.1005 & 0.0966 & 0.1004 & 0.0976 & 0.1003\\ 0.1454 & 0.0966 & 0.2006 & 0.0967 & 0.1235 & 0.0991\\ 0.0970 & 0.1004 & 0.0967 & 0.1004 & 0.0983 & 0.1003\\ 0.1446 & 0.0976 & 0.1235 & 0.0983 & 0.1997 & 0.0989\\ 0.0989 & 0.1003 & 0.0991 & 0.1003 & 0.0989 & 0.1002 \end{bmatrix},$$

$$T=\begin{bmatrix} 4.0623 & -0.9325 & 1.6962 & -0.8017 & 1.6925 & -0.6702\\ -0.9325 & 19.6255 & -0.6937 & -13.4188 & -0.6407 & 3.6191\\ 1.6962 & -0.6937 & 4.3974 & -0.9371 & 1.3467 & -0.6085\\ -0.8017 & -13.4188 & -0.9371 & 38.6762 & -0.6207 & -18.4034\\ 1.6925 & -0.6407 & 1.3467 & -0.6207 & 4.4193 & -0.6187\\ -0.6702 & 3.6191 & -0.6085 & -18.4034 & -0.6187 & 87.4037 \end{bmatrix},$$

$$S=\begin{bmatrix} 0.0010 & 0.0229 & -0.0001 & -0.0089 & -0.0002 & -0.0146\\ 0.0229 & 0.9120 & -0.0065 & -0.8100 & -0.0014 & -0.1140\\ -0.0001 & -0.0065 & 0.0004 & 0.0162 & -0.0000 & -0.0099\\ -0.0089 & -0.8100 & 0.0162 & 1.3595 & -0.0021 & -0.5577\\ -0.0002 & -0.0014 & -0.0000 & -0.0021 & 0.0002 & 0.0037\\ -0.0146 & -0.1140 & -0.0099 & -0.5577 & 0.0037 & 0.7012 \end{bmatrix}\times 10^{5},$$

$$M_1=\begin{bmatrix} -0.1821 & -0.0955 & -0.1454 & -0.0970 & -0.1446 & -0.0989\\ -0.0955 & -0.1005 & -0.0966 & -0.1004 & -0.0976 & -0.1003\\ -0.1454 & -0.0966 & -0.2006 & -0.0967 & -0.1235 & -0.0991\\ -0.0970 & -0.1004 & -0.0967 & -0.1004 & -0.0983 & -0.1003\\ -0.1446 & -0.0976 & -0.1235 & -0.0983 & -0.1997 & -0.0989\\ -0.0989 & -0.1003 & -0.0991 & -0.1003 & -0.0989 & -0.1002 \end{bmatrix},$$

$$M_2=\begin{bmatrix} 0.1821 & 0.0955 & 0.1454 & 0.0970 & 0.1446 & 0.0989\\ 0.0955 & 0.1005 & 0.0966 & 0.1004 & 0.0976 & 0.1003\\ 0.1454 & 0.0966 & 0.2006 & 0.0967 & 0.1235 & 0.0991\\ 0.0970 & 0.1004 & 0.0967 & 0.1004 & 0.0983 & 0.1003\\ 0.1446 & 0.0976 & 0.1235 & 0.0983 & 0.1997 & 0.0989\\ 0.0989 & 0.1003 & 0.0991 & 0.1003 & 0.0989 & 0.1002 \end{bmatrix},$$

$$\overline{K}=\begin{bmatrix} -4.2494 & -10.9620 & & & & \\ & & -1.98 & -6.7336 & & \\ & & & & -1.328 & -5.7161 \end{bmatrix}. \tag{40}$$

Using above $\overline{K}$, we have the state response curves shown in Figures 8 and 9.

In Figure 8, the disturbance signal $\omega(t) = 0$, and the initial state $x_i^T(0) = [-1 \;\; 1]$, $x_i^T(-t) = [0 \;\; 0]$, $i = 1,2,3$. It is clear that the system is asymptotically stable. And, in Figure 9, $\omega(t) = \{[1 \;\; 1]^T,\ t \in [10,20];\ 0,\ \text{others}\}$; we can see the state will converge to a certain value; H_∞ performance index is satisfied. In Figure 10, the controlled outputs converge to zero with $\omega(t) = \{[1 \;\; 1]^T,\ t \in [10,20];\ 0,\ \text{others}\}$. When $\omega(t) = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$, $t > 0$, the controlled

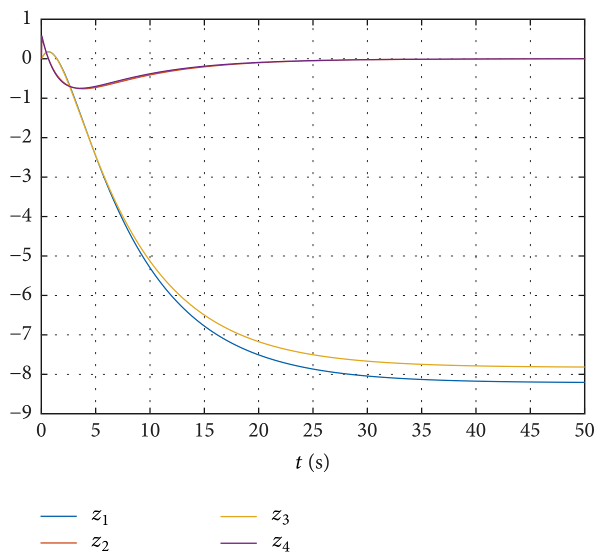

Figure 7: The consensus of Example 1.

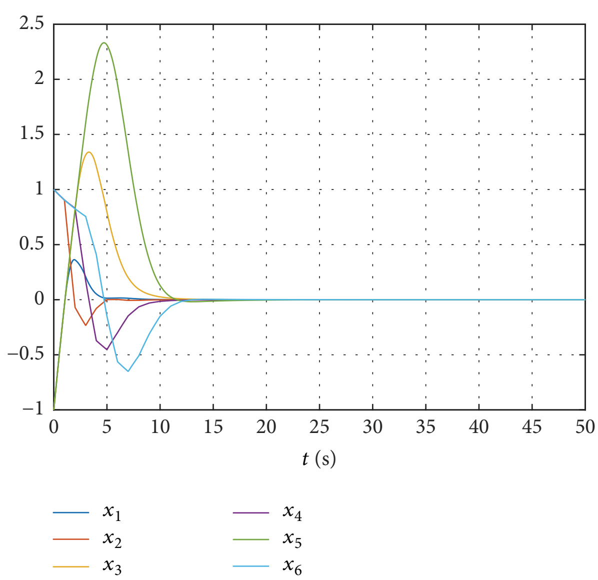

Figure 8: The state response curve without input signal.

output is shown in Figure 11, which means the consensus is achieved.

5. Conclusions

In this paper, a class of networked control systems with multiple subsystems is studied, including the modeling and stability analysis for the massive networked control systems with multiple subsystems and distributed control law. Firstly, the time-triggered protocol is introduced to the system, and the executive windows are scheduled to each intelligent node, which need to access network for real-time control. Secondly, while considering the delay, the model of the NCS

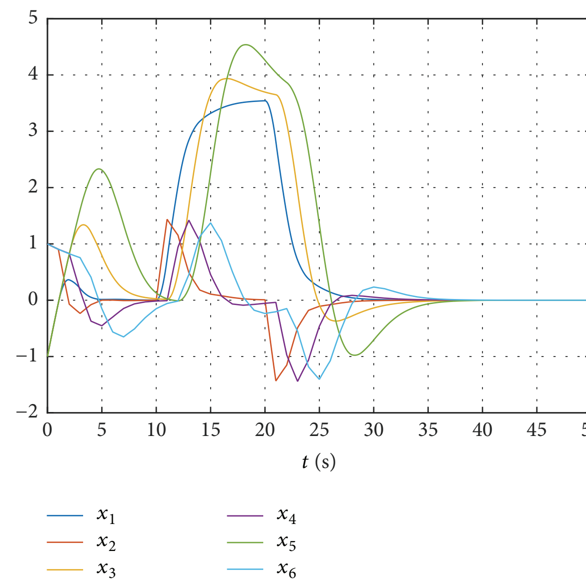

Figure 9: The state response curve with input signal.

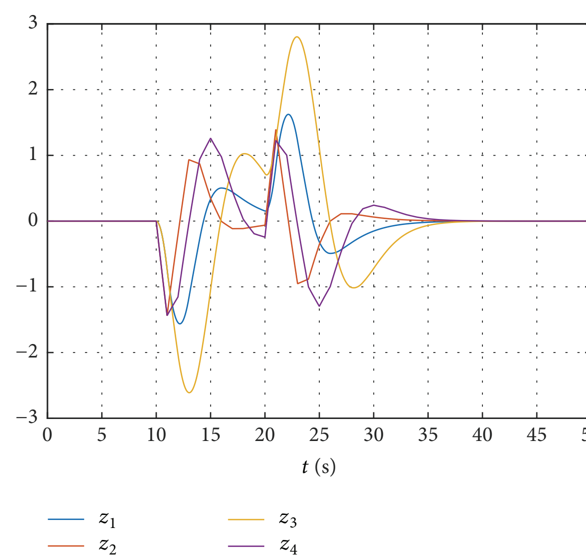

Figure 10: The consensus of Example 2.

with distributed controller is presented. Then, H_∞ consensus control problem is studied. With the Lyapunov-Krasovskii functional method, the consensus is analyzed, and the sufficient condition with matrix inequalities is given. Finally, simulations are given to validate the proposed approach. Our further work will focus on such problems as tracking control problems and synchronous coordinative problems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

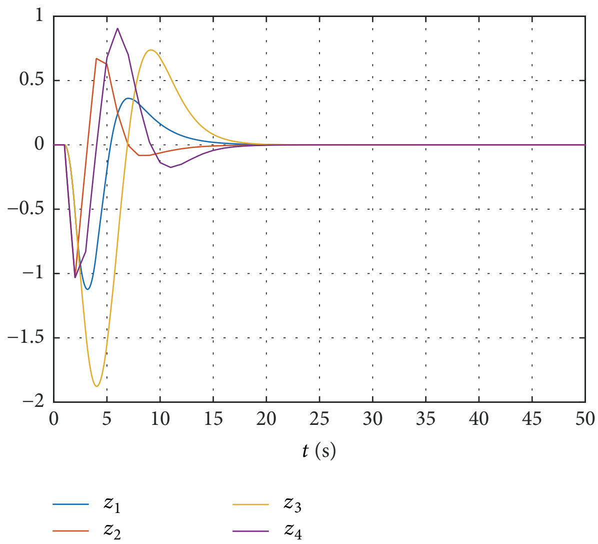

FIGURE 11: The consensus of Example 2 when input is not zeros.

Acknowledgments

This work was supported by NSFC (Grant no. 61074023), the Jiangsu Postdoctoral Fund, and the Jiangsu Overseas (Grant no. 1001003B) Research & Training Program for University Prominent Young & Middle-Aged Teachers and Presidents.

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