Appendix A. *Formal definitions and Proofs on Cloud model*

1 The formal definitions of Cloud mode

We cannot derive the formula or the formal definition of cloud model accurately and simply. In fact, it is the charm of cloud model. The cloud model is easier to be determined by the algorithm or the program than other theoretical models and tools. For the reference, the following is the Matlab function of the forward normal cloud transformation algorithm, as shown in Eq(9).

```
%Input: expected value Ex, entropy En and hyper-entropy He, the number of cloud drops N
%Output: the cloud drops with certainty degree (u,mu)
function drawCloud(Ex,En,He,n)
for i = 1:n
    Enn = randn(1)*He + En;
    u(i) = randn(1)*Enn + Ex;
    mu(i) = exp(- ( u(i) - Ex)^2 / (2*Enn^2));
 end
plot(u,mu,'.b');
```

Then, we can define the cloud model according to the above algorithm. Given two universe sets *U* and *V*, we define the random variable *X* satisfied $X \sim N(Ex, y^2)$, then the density function of *X* is,

$$f(x) = \frac{1}{y\sqrt{2\pi}} \mathrm{e}^{-\frac{(x-Ex)^2}{2y^2}}. \tag{A1}$$

Similarly, we define the random variable *Y* satisfied $X \sim N(En, He^2)$, then the density function of *Y* is,

$$\mathrm{f}(y) = \frac{1}{He\sqrt{2\pi}} \mathrm{e}^{-\frac{(y-En)^2}{2He^2}}. \tag{A2}$$

Furthermore, we define the random variable *Z* on $U \times V$,

$$Z\text{:}\ \ U \times V \to R. \tag{A3}$$

And then, the density function of *Z* is,

$$\mathrm{f}(z) = \int_{-\infty}^{\infty} \frac{1}{y\sqrt{2\pi}} \mathrm{e}^{-\frac{(z-Ex)^2}{2y^2}} \frac{1}{He\sqrt{2\pi}} \mathrm{e}^{-\frac{(y-En)^2}{2He^2}} \mathrm{d}y. \tag{A4}$$

Z can produce the cloud drops with certainty degree, and it can be as the theoretical definition of the cloud model, even though cloud model's author might not accept such a definition, whose viewpoint is that cloud model should be rarely concerned with precise mathematics.

2 Equation (11) derived from the cloud model

Z in (A4) can produce the cloud drops with certainty degree, and is as the definition of the cloud model.

$$E|Z-Ex| = \int_{-\infty}^{\infty}|z-Ex|f(z)dz = \frac{1}{2\pi He}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|z-Ex|\frac{1}{y}e^{-\frac{(z-Ex)^2}{2y^2}}e^{-\frac{(y-En)^2}{2He^2}}dydz$$
$$= \frac{1}{2\pi He}\int_{-\infty}^{\infty}e^{-\frac{(y-En)^2}{2He^2}}dy\int_{-\infty}^{\infty}\frac{|z-Ex|}{y}e^{-\frac{(z-Ex)^2}{2y^2}}dz$$

Since,

$$\int_{-\infty}^{\infty}\frac{|z-Ex|}{y}e^{-\frac{(z-Ex)^2}{2y^2}}dz = \int_{-\infty}^{Ex}\frac{Ex-z}{y}e^{-\frac{(z-Ex)^2}{2y^2}}dz + \int_{Ex}^{\infty}\frac{z-Ex}{y}e^{-\frac{(z-Ex)^2}{2y^2}}dz$$
$$= \int_{-\infty}^{0} -te^{-\frac{t^2}{2}}ydt + \int_{0}^{\infty} te^{-\frac{t^2}{2}}ydt = 2\int_{0}^{\infty} ye^{-\frac{t^2}{2}}d\left(\frac{t^2}{2}\right) = 2y$$

Thus,

$$E|Z-Ex| = \frac{1}{\pi He}\int_{-\infty}^{\infty}ye^{-\frac{(y-En)^2}{2He^2}}dy = \frac{\sqrt{2}}{\sqrt{\pi}}\frac{1}{\sqrt{2\pi}He}\int_{-\infty}^{\infty}ye^{-\frac{(y-En)^2}{2He^2}}dy = \frac{\sqrt{2}}{\sqrt{\pi}}En$$

For the background pixels, $E|Z-Ex| = \frac{1}{n}\sum_{i=1}^{n}|z_i - Ex|$

Therefore, $En = \sqrt{\frac{\pi}{2}} \times \frac{1}{n}\sum_{i=1}^{n}|z_i - Ex|$

For an image with a threshold t, z_i is as the grayscale value $I(x)$ of pixels, and $Ex_b\ (t)$ is the grayscale mean of background pixels, and $n=|B(t)|$ is the number of background pixels.

Hence, Eq.(11) in the manuscript is achieved.

3 *Measurement of He derived from the cloud model*

He^2 is as the variance of variance, and it is the quantification on how a distribution deviate the Gaussian distribution. For comparison, Wang construct a random variable, whose central moments are as close as possible to those of the cloud model. And the mean, the variance and the third central moment of the constructed variable are equal to cloud model. Therefore, the quantification of the difference between cloud model and Gaussian distribution is achieved in some extents. Accordingly, Wang provide an accurate quantity on the deviation measure, $6He^4+12He^2En^2$, from the point of view of statistical characteristics, especially the fourth central moment.

The expectation of the cloud model Z is Ex, that is,

$$E\{Z\} = Ex \tag{A5}$$

Proof:

$$E\{Z\} = \int_{-\infty}^{\infty} zf(z)dz \tag{A6}$$

Substituting $f(z)$, then,

$$E\{Z\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} z\frac{1}{y\sqrt{2\pi}}e^{-\frac{(z-Ex)^2}{2y^2}}dz\right]\frac{1}{He\sqrt{2\pi}}e^{-\frac{(y-En)^2}{2He^2}}dy$$
$$= \int_{-\infty}^{\infty} Ex\frac{1}{He\sqrt{2\pi}}e^{-\frac{(x-En)^2}{2He^2}}dy \ = Ex.$$

The variance of the cloud model Z is,

$$Var\{Z\} = E\{[Z - E(Z)]^2\} = En^2 + He^2 \tag{A7}$$

Proof: $Var\{Z\} = \int_{-\infty}^{\infty} [z - E(Z)]^2 f(z) dz$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} (z - Ex)^2 \frac{1}{y\sqrt{2\pi}} e^{-\frac{(z-Ex)^2}{2y^2}} dz \right] \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= \int_{-\infty}^{\infty} y^2 \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= E\{Y^2\} = He^2 + En^2.$$

The third central moment of the cloud model Z is 0, that is,

$$E\{[Z - E(Z)]^3\} = 0 \tag{A8}$$

Proof: $E\{[Z - E(Z)]^3\} = \int_{-\infty}^{\infty} [z - E(Z)]^3 f(z) dz$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} (z - Ex)^3 \frac{1}{y\sqrt{2\pi}} e^{-\frac{(z-Ex)^2}{2y^2}} dz \right] \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= \int_{-\infty}^{\infty} 0 \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy = 0.$$

The fourth central moment of the cloud model Z is,

$$E\{[Z - E(Z)]^4\} = 9He^4 + 18He^2En^2 + 3En^4 \tag{A9}$$

Proof: Considering the Gaussian random variable X with zeros mean and variance σ^2, we have $E\{X^4\} = 3\sigma^4$, then

$$E\{[Z - E(Z)]^4\} = \int_{-\infty}^{\infty} [z - Ex]^4 f(z) dz$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} (z - Ex)^4 \frac{1}{y\sqrt{2\pi}} e^{-\frac{(z-Ex)^2}{2y^2}} dz \right] \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= \int_{-\infty}^{\infty} 3y^4 \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= 3 \int_{-\infty}^{\infty} [(y - En) + En]^4 \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= 3 \int_{-\infty}^{\infty} \begin{bmatrix} (y - En)^4 + 4(y - En)^3 En + \\ 6(y - En)^2 En^2 + 4(y - En) En^3 + En^4 \end{bmatrix} \frac{1}{He\sqrt{2\pi}} e^{-\frac{(y-En)^2}{2He^2}} dy$$

$$= 3(3He^4 + 6He^2En^2 + En^4).$$

Thus, one can construct the Gaussian random variable X', whose central moments are as close as possible to those of the cloud model Z. Therefore, the quantification of the difference between cloud model and Gaussian distribution is achieved in some extents. We define the probability density function of X', that is,

$$f(x') = \frac{1}{\sqrt{2\pi(En^2 + He^2)}} e^{-\frac{(x\prime-Ex)^2}{2(En^2+He^2)}}$$

The mean of X' is Ex, the variance of X' is En^2+He^2, and the third central moment of X' is 0. These statistics characteristics of X' are the same with those of cloud model Z. While the fourth central moment of X' is,

$$E\{(X' - Ex)^4\} = 3(En^2 + He^2)^2 = 3He^4 + 6He^2En^2 + 3En^4 \quad \text{(A10)}$$

He is the uncertain measurement of entropy, which is determined by randomness and fuzziness of entropy *En*. Conceptually, *He* of cloud model *Z* is a deviation measure from a normal distribution. Furthermore, one can make a comparison between Eq(A9) and Eq(A10), and provide an accurate quantity, $6He^4+12He^2En^2$, for the deviation from the point of view of statistical characteristics, especially the fourth central moment.