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Research Article

Pricing Model for Dual Sales Channel with Promotion Effect Consideration

Chuiri Zhou

School of Management, University of Science & Technology of China, Hefei 230026, China

Correspondence should be addressed to Chuiri Zhou; zhoucr@ustc.edu.cn

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We focus on the pricing strategy of a dual sales channel member when his/her online retailer faces an upcoming overloaded express delivery service due to the sales peak of online shopping, especially referring to the occurring affairs in China. We characterize the pricing problem of the dual selling channel system as a two-period game. When the price discount is only provided by the online seller, we find that the prices of the traditional channel and the online channel in the two periods are higher while the overloaded degree of express delivery is lower and the overloaded delivery services can decrease the profits of both channels. When the price discounts are provided by both traditional and online sellers, we find that the derived Nash price equilibrium of both channels includes five possible combinations of prices. Both traditional and online sellers will choose their price strategies, respectively, according to their cost advantages which are affected by the overloaded degree of express delivery.

1. Introduction

Usually, the heavy sales promotion period occurs on important festivals. For example, in China, November 11 has been jokingly called the "Singles Day" by many young people, and during the "Double Eleven," many online stores hold a 24-hour madness sales, which makes the sales increase suddenly. "Singles Day" has grown into China's, or possibly the world's, busiest online shopping day. Only on the day of November 11, 2015, the online shopping platform http://taobao.com/ had a massive sales reaching 14.3 billion dollars and hundreds of millions of parcels of goods streamed in express delivery companies. As a result, the overloaded express logistics system forces the online orders to be postponed and eventually decreases the consumer's valuation for the products. Another example is the Christmas holidays, during which both the traditional and online retailers would like to offer a heavy discount.

As the same products can be sold by both channels, traditional and online sellers can be rivals. In heavy promotion period, online retailers offer high price discount and the resulting logistics is overloaded, so each side must decide his/her price strategy. This type of situation may be modeled by a dual sales channel as described in our study.

In this paper, we consider a dual-channel system in which traditional channel and online channel sell the same products during two sales periods, normal sales period and promotion sales period. This situation can be found in many brand products which are sold by both channels. When the promotion sales period comes, the overloaded delivery services will decline the consumer's satisfaction about the online purchase and the online seller has to decrease the price to compensate the consumer for the delay of parcel delivery. Each seller should choose his/her price policy to maximize his/her profit. We analyze the maximum profit policy in each promotion period in dual sales channel and conclude the conditions of Nash equilibrium.

To the best of our knowledge, the problem described in this paper has rarely been studied before. In particular, we investigate the interaction of price policy between normal sales period and promotion sale period. For the problem described above, we characterize the overload degree of the delivery service into dual-channel sales model and discuss its effects on the whole system equilibrium. We provide insights into the optimal policy structure. We also present a numerical analysis for the overloaded degree of express delivery.

The rest of the paper is organized as follows. Section 2 reviews the related literature and explains our contributions.

Section 3 gives a detailed description of the problem and a benchmark case. In Section 4, we formulate a scenario in which only online seller launches promotion sales. Then we formulate the model in which both online seller and traditional seller launch promotion sales in the second period in Section 5. We conclude with a brief summary of our findings and possible future research extensions in Section 6. All proofs are in the Appendix.

2. Related Literature Review

Our paper is mainly related to the research of dual sales channel supply chain. Generally speaking, dual-channels supply chain is both traditional retailer channel and online channel. Dual-channel mode can directly lead to conflict and competition between online channel and traditional channel [1]. Scholars have done a lot of research and have achieved lots of contributions concerned with how to realize supply chain performance [2, 3]. Dual sales channel has attracted considerable attention in recent years [4–6]. The literature of dual sales channel is mainly focused on three aspects, channel choice, channel coordination, and pricing policy. We will briefly discuss these aspects, respectively, in the following paragraphs.

The first is channel choice in supply chain. It is always a hot topic concerned with choosing which channel to maximize profit when facing two channels. Chiang et al. [7] found that the manufacturer preferred online channel to control the retailer and increase his/her profit when the consumer had higher acceptance of online channel even if the online channel was inefficient. After discussing the influence of manufacturer's online seller on traditional retailer, Arya et al. [8] demonstrated the retailer benefited from the manufacturer's online channel in some circumstances as the manufacturer decreased the wholesale price. Zhang et al. [9] showed that all the members of dual-channel tended to be leaders in the Stackelberg game mode and there was no one power structure in which all the members of supply chain were beneficial.

Chun et al. [10] argued that the manufacturer chose single traditional retailer channel when the consumers were more heterogeneous in the market or the retailer provided more services. It is necessary to implement online sales if the manufacturer is monopolistic [11]. When online sales cost is lower, a manufacturer will still sell by online channel even if there is channel conflict [12].

The second is dual-channel coordination. There must be conflicts in the dual-channel supply chain. Most traditional coordination policies, such as wholesale price policy, buyback policy, and revenue sharing policy, cannot coordinate the dual-channel supply chain because of inventory competition [13, 14]. There are many factors influencing the conflicts and coordination in dual-channel. Kurata et al. [15] manifested that only wholesale price policy could not achieve coordination of dual-channel supply chain when brand competition and channel competition coexisted. The product cycle is also a significant factor affecting dual-channel conflict [1]. When a manufacturer permits a retailer to provide added value to consumer in traditional channel, conflict will emerge if the cost of added value is higher than some threshold value and the retailer conceals the cost information [16].

Because of the conflict in dual-channel supply chain, how to coordinate the whole chain is meaningful affair and some valuable literature focuses on it. Cai et al. [17] argued the effect of discount contract and other pricing policies on the efficiency of supply chain. They suggested that the direct sale channel may ease channel conflict if the price of direct sale channel is consistent with the price of traditional retailer channel. Vertical and horizontal competition in decentralized dual-channel decrease the efficiency of whole supply chain, and Xu et al. [18] suggested that dual-revenue sharing contract might realize the supply chain coordination. At the same time, Ryan et al. [19] demonstrated that revenue sharing contract could make the manufacturer take price discrimination and then realized supply chain coordination. Yan and Ghose [20] gave the same conclusion and presented concrete implementation method as profit bargaining model.

Lastly, we present the pricing policy in dual sales channel. Because of competition between two channels, there are lots of literature examining pricing policy in dual-channel supply chain. When more consumers buy a product by online shopping, the price of both channels will reduce and the price of online channel is lower than the price of retailer channel. In the end, the profit of retailer channel decreases while the profit and the efficiency of online channel increase [21]. The price policy of dual-channel is concerned with each member's inventory policy [22]. Many other factors, such as the consumer loyalty to traditional channel and the service level of retailers, will affect the whole pricing strategy of dual-channel [23].

The most similar literature with our paper is Chen et al. [12], in which delivery time reflects service level and it discusses pricing policy. But differently, we present a decentralized dual-channel model in which overloaded express delivery services affect the consumer's purchasing willingness.

3. Model

Let us consider a dual-channel system consisting of a traditional retailer with physical stores and an online retailer, both of which sell the same product. We assume that the product is durable, and each buyer purchases one unit at most during the buying cycle including the normal sales period and promotion sales period. This assumption implies that if a consumer has bought the product in the first period, he/she would not buy it again in the second period even though the price in the latter is lower. Furthermore, we assume that consumers in the normal sales period are not sure whether the sellers offer discount prices or not, and thus consumers would not wait for the lower prices to purchase once they have positive surplus.

This paper focuses on two common scenarios. In the first scenario only the online seller provides a price discount in the second period. This setting is motivated by the online shopping promotion on "Double Eleven" or "Double Twelve" in China, during which many online stores launch sales promotion. However, most traditional sellers do not make sales

promotions because such festivals are nonholiday festivals and are popular only with younger people who are main online shoppers. In the second scenario, both the sellers of two channels adopt promotion strategies with discount prices to stimulate consumer demand. This case can be often seen during the important traditional holidays, such as Christmas holidays and Chinese Spring Festival, during which people have enough leisure time to go to brick-and-mortar stores for shopping.

Before examining the above two common scenarios, we firstly present the benchmark case in which there is only online retailer in the market.

Let us consider a benchmark case that there is only online retailer in the market. An example in practice is Private Beauty Adviser, a Chinese cosmetics company which sells the products only by online shops on Tmall, China's largest online shopping platforms.

We suppose that the consumer basic valuation of the product is v and the customer acceptance of the online channel is θ_1. The value of the parameter θ_1 is called the customer acceptance of the online channel, which usually satisfies $0 < \theta_1 < 1$. Let p_{Di} be the selling price in Period i, $i = 1, 2$. All consumers whose valuations satisfy $\theta_1 v - p_{D1} \geq 0$ would buy in the normal sales period. In the promotion sales period, the customer acceptance of the online channel changes from θ_1 to θ_2 due to the overloaded delivery services. The peak of online shopping and busy period of parcel express force the customers to wait more time for delivery; thus, we assume $\theta_1 \geq \theta_2$. The busier the express logistics system is, the smaller the value of θ_2 is. Similar to the normal period, consumers whose valuations satisfy $\theta_2 v - p_{D2} \geq 0$ would buy.

Assuming that the valuation of the consumers v is uniformly distributed within the consumer population from 0 to 1, with a density of 1, the demands of the two periods are, respectively, $S_1 = 1 - p_{D1}/\theta_1$ and $S_2 = p_{D1}/\theta_1 - p_{D2}/\theta_2$. The corresponding online seller's profits are then $\pi_{D1} = S_1 p_{D1}$ and $\pi_{D2} = S_2 p_{D2}$.

Straightforward algebra shows that, in the unique equilibrium, the seller will set prices $p_{D1} = 2\theta_1^2/(4\theta_1 - \theta_2)$ and $p_{D2} = \theta_1\theta_2/(4\theta_1 - \theta_2)$, and the corresponding maximum profits for the online shop are $\pi_{D1} = 2\theta_1^2(2\theta_1 - \theta_2)/(4\theta_1 - \theta_2)^2$ and $\pi_{D2} = \theta_1^2\theta_2/(4\theta_1 - \theta_2)^2$.

Noticing a larger θ_2 implies a lower overload degree of logistics in the promotion sales period. The comparative statics with respect to θ_2, $\partial p_{D1}/\partial\theta_2 > 0$, $\partial p_{D2}/\partial\theta_2 > 0$, $\partial S_{D1}/\partial\theta_2 < 0$, $\partial S_{D1}/\partial\theta_2 > 0$, $\partial\pi_{D1}/\partial\theta_2 < 0$, $\partial\pi_{D2}/\partial\theta_2 > 0$, and $\partial(\pi_{D1} + \pi_{D2})/\partial\theta_2 > 0$ show that the overloaded delivery services reduce the selling prices in the two periods and hurt the online shop's profits. Furthermore, as the express logistics system becomes busier, the online shop will have a stronger motivation to transfer consumers to Period 1 from Period 2.

4. Scenario I: The Sales Promotion Provided by Only Online Channel

As mentioned by previous sections, the first scenario is that only the online seller provides price discount in the second period. In this section, we first derive the demands of both channels in the two periods. We then analyze the optimal pricing decisions of the traditional seller and the online seller and study the effects of the express logistics services on the pricing.

4.1. The Demand Functions of the Two Channels. Suppose each consumer has a basic valuation v on the product when he/she makes a purchase decision. We assume that v is a random variable with a cumulative distribution function F implying that consumers are heterogeneous in the valuation. Let p_{ji} be the selling price of jth channel charged in Period i, where $i = 1, 2$, and $j = R, D$ denote the traditional channel and online channel, respectively.

In the normal sales period, if the product is sold in the traditional channel, the consumer utility surplus would be $v - p_{R1}$. A customer would buy an available product only if he/she has nonnegative utility surplus, so all consumers whose valuations satisfy $v - p_{R1} \geq 0$ will consider buying from the traditional channel.

If the product is sold through the online channel, consumers only get a virtual description of the product and typically are asked to wait several days for delivery, therefore reducing the expectation of consumption value θ_1. All consumers whose valuations satisfy $\theta_1 v - p_{D1} \geq 0$ will consider buying from the online channel.

If consumer can buy from either channel, they would prefer to buy from the channel where they can get more surplus. Specifically, if $v - p_{R1} > \theta_1 v - p_{D1}$, the traditional channel would be preferred to the online channel and vice versa.

Assuming the valuation of the consumers v is uniformly distributed within the consumer population from 0 to 1, with a density of 1, the first-period demand functions for the traditional and online sellers can be expressed, respectively, as

$$S_{R1} = \int_{\Omega_{R1}} dF(v) = \begin{cases} 1 - p_{R1} & 0 \leq p_{R1} < \dfrac{p_{D1}}{\theta_1} \\ 1 - \dfrac{p_{R1} - p_{D1}}{1 - \theta_1} & \dfrac{p_{D1}}{\theta_1} \leq p_{R1} < 1 - \theta_1 + p_{D1} \\ 0 & (1 - \theta_1) + p_{D1} \leq p_{R1} \leq 1, \end{cases} \tag{1}$$

$$S_{D1} = \int_{\Omega_{D1}} dF(v) = \begin{cases} 0 & 0 \leq p_{R1} < \dfrac{p_{D1}}{\theta_1} \\ \dfrac{\theta_1 p_{R1} - p_{D1}}{\theta_1(1 - \theta_1)} & \dfrac{p_{D1}}{\theta_1} \leq p_{R1} < 1 - \theta_1 + p_{D1} \\ 1 - \dfrac{p_{D1}}{\theta_1} & (1 - \theta_1) + p_{D1} \leq p_{R1} \leq 1, \end{cases} \tag{2}$$

where $\Omega_{R1} = \{v \mid v - p_{R1} \geq 0, v - p_{R1} \geq \theta_1 v - p_{D1}, 0 \leq v \leq 1\}$, $\Omega_{D1} = \{v \mid v - p_{D1} \geq 0, v - p_{R1} \leq \theta_1 v - p_{D1}, 0 \leq v \leq 1\}$.

Equation (1) shows that when the price of the traditional seller exceeds p_{D1}/θ_1, the demand of the traditional seller

becomes more price elastic ($|\partial[1-(p_{R1}-p_{D1})/(1-\theta_1)]/\partial p_{R1}| > \partial(1-p_{R1})/\partial p_{R1}$) because the traditional seller can lose consumers to the online shop. The value p_{D1}/θ_1 is the "real" price in the online channel, correcting for the diminished benefit because of the product delivery. When the price of the traditional seller is high, some of the consumers will find that the online channel is the best choice even though they have to lose a part of the value of the product, $(1-\theta_1)v$.

We can easily obtain that the consumers whose valuations are larger than $\min(p_{R1}, p_{D1}/\theta_1)$ would buy the product in the normal sales period, and they would not buy again in the promotion period even though the prices are lower; then the potential consumers of the promotion period are those whose valuations are less than $\min(p_{R1}, p_{D1}/\theta_1)$. We define $\bar{v} = \min(p_{R1}, p_{D1}/\theta_1)$.

In the promotion sales period, the traditional seller does not do promotions and his/her second-period price is still p_{R1}, making his/her second-period sales volume zero. The online seller provides consumers with a discount price p_{D2} to stimulate demand. The customer acceptance of the online channel changes from θ_1 to θ_2. The peak of online shopping and busy period of parcel deliveries force customers to wait more time for delivery; thus, we assume $\theta_1 \geq \theta_2$.

Similar to the normal period, consumers whose valuations satisfy $\theta_2 v - p_{D2} \geq 0$ would buy from the online seller. The demand function for the online channel in Period 2 is

$$S_{D2} = \bar{v} - \frac{p_{D2}}{\theta_2}. \tag{3}$$

4.2. The Optimization Problem and the Nash Equilibrium. In our model, the traditional channel and the online channel are independent decision-makers, and each focuses on its own profit. Let c_R and c_D be marginal costs for the product sold by the traditional seller and online seller, respectively. We would expect that $c_R > c_D$ because the traditional seller pays for shipping and handling in a typical market; however, the online seller does not need to pay for the high site use fee. For analytic simplicity, we normalize the marginal cost of the online seller to zero and use c to denote the marginal cost of the traditional seller. The total profits for traditional channel and online channel π_R and π_D, respectively, are equal to

$$\pi_R = (p_{R1} - c) S_{R1}, \tag{4}$$

$$\pi_D = p_{D1} S_{D1} + p_{D2} S_{D2}. \tag{5}$$

To find the equilibrium solution for both firms, we can use the concept of subgame-perfect Nash equilibrium which leads to a backward induction procedure. Following this solution procedure, we firstly solve for the online seller's second-period price.

Given the first-period price of the traditional seller and the online seller, we get the second-period price and second-period profit of the online seller:

$$\begin{aligned} p_{D2}(p_{R1}, p_{D1}) &= \frac{\bar{v}\theta_2}{2}, \\ \pi_{D2}(p_{R1}, p_{D1}) &= \frac{\bar{v}^2\theta_2}{4}. \end{aligned} \tag{6}$$

Substituting them into (5), we get

$$\pi_D = p_{D1} S_{D1} + \frac{\bar{v}^2\theta_2}{4}. \tag{7}$$

Given the two sellers' profits as defined by (4) and (7), the best response price of firm i, $p_i(p_j)$, for any given price p_j of firm j is given by the following lemma.

Lemma 1. *(i) Given price p_{D1} of online channel, the best response price of the traditional channel is*

$$p_{R1}(p_{D1}) = \begin{cases} \text{Null} & \text{if } 0 \leq p_{D1} < \theta_1 + c - 1 \\ \dfrac{p_{D1} + c + 1 - \theta_1}{2} & \text{if } \theta_1 + c - 1 \leq p_{D1} < \dfrac{\theta_1(1-\theta_1+c)}{2-\theta_1} \\ \dfrac{p_{D1}}{\theta_1} & \text{if } \dfrac{\theta_1(1-\theta_1+c)}{2-\theta_1} \leq p_{D1} < \dfrac{(c+1)\theta_1}{2} \\ \dfrac{c+1}{2} & \text{if } p_{D1} \geq \dfrac{(c+1)\theta_1}{2}. \end{cases} \tag{8}$$

(ii) Given price p_{R1} of traditional channel, the best response price of the online channel is

$$p_{D1}(p_{R1}) = \begin{cases} \dfrac{2p_{R1}\theta_1^2}{4\theta_1 + \theta_1\theta_2 - \theta_2} & \text{if } c \leq p_{R1} < \dfrac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1^2} \\ p_{R1} + \theta_1 - 1 & \text{if } \dfrac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1^2} \leq p_{R1} < \dfrac{2\theta_1^2}{4\theta_1 - \theta_2} + (1-\theta_1) \\ \dfrac{2\theta_1^2}{4\theta_1 - \theta_2} & \text{if } p_{R1} \geq \dfrac{2\theta_1^2}{4\theta_1 - \theta_2} + (1-\theta_1). \end{cases} \tag{9}$$

The first part of Lemma 1 illustrates the traditional channel's best pricing strategy. When faced with a low online seller's price, the traditional seller will focus on increasing the marginal profits by charging a high first-period price that $p_{R1} \geq p_{D1}/\theta_1$. Note that the traditional seller will drop out of the market when the online seller's price is extremely low.

From (9), we observe that $p_{D1}/\theta_1 \leq p_{R1}$ always holds, implying that, for the online channel, it is optimal for him/her to set a lower price than the traditional seller.

Using (8) and (9), we solve for $p_{R1}{}^*$ and $p_{D1}{}^*$ simultaneously and then check the solution feasibility. Following this approach, we get price equilibrium $(p_{R1}{}^*, p_{D1}{}^*)$ as follows:

$$p_{R1}{}^* = \begin{cases} \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)(1 - \theta_1 + c)}{2(4\theta_1 + \theta_1\theta_2 - \theta_2 - \theta_1{}^2)} & \text{if } 0 \leq c < \dfrac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1{}^2} \\ c & \text{if } \dfrac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1{}^2} \leq c < \dfrac{2\theta_1{}^2}{4\theta_1 - \theta_2} + (1 - \theta_1) \\ \text{Null} & \text{if } \dfrac{2\theta_1{}^2}{4\theta_1 - \theta_2} + (1 - \theta_1) \leq c \leq 1, \end{cases}$$

$$p_{D1}{}^* = \begin{cases} \dfrac{(1 - \theta_1 + c)\theta_1{}^2}{4\theta_1 + \theta_1\theta_2 - \theta_2 - \theta_1{}^2} & \text{if } 0 \leq c < \dfrac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1{}^2} \\ c + \theta_1 - 1 & \text{if } \dfrac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1{}^2} \leq c < \dfrac{2\theta_1{}^2}{4\theta_1 - \theta_2} + (1 - \theta_1) \\ \dfrac{2\theta_1{}^2}{4\theta_1 - \theta_2} & \text{if } \dfrac{2\theta_1{}^2}{4\theta_1 - \theta_2} + (1 - \theta_1) \leq c \leq 1. \end{cases} \tag{10}$$

The derived Nash price equilibrium includes three possible combinations of prices.

(i) Pricing Strategy I. Consider $p_{R1}{}^* = \text{Null}$, $p_{D1}{}^* = 2\theta_1{}^2/(4\theta_1 - \theta_2)$, and $p_{D2}{}^* = \theta_1\theta_2/(4\theta_1 - \theta_2)$. Here, the traditional seller will drop out of the market and the pricing decision of the online seller is the same as the benchmark case.

(ii) Pricing Strategy II. Consider $p_{R1}{}^* = c$, $p_{D1}{}^* = c + \theta_1 - 1$, and $p_{D2}{}^* = (c + \theta_1 - 1)\theta_2/(2\theta_1)$. Here, the traditional seller coexists with the online seller in the market, but his/her sale volume and profit are zero.

(iii) Pricing Strategy III. Consider $p_{R1}{}^* = (4\theta_1 - \theta_2 + \theta_1\theta_2)(1 - \theta_1 + c)/2(4\theta_1 + \theta_1\theta_2 - \theta_2 - \theta_1{}^2)$, $p_{D1}{}^* = (1 - \theta_1 + c)\theta_1{}^2/(4\theta_1 + \theta_1\theta_2 - \theta_2 - \theta_1{}^2)$, and $p_{D2}{}^* = \theta_1\theta_2(1 - \theta_1 + c)/[2(4\theta_1 + \theta_1\theta_2 - \theta_2 - \theta_1{}^2)]$. Here, both of the two sellers have positive sales volumes and profits. We summarize the Nash price equilibrium and three possible combinations in Table 1.

Proposition 2. *There exist two cost thresholds C_A and C_B, which are defined as*

$$C_A = \frac{(1-\theta_1)(4\theta_1 + \theta_1\theta_2 - \theta_2)}{4\theta_1 + \theta_1\theta_2 - \theta_2 - 2\theta_1{}^2}, \qquad C_B = \frac{2\theta_1{}^2}{4\theta_1 - \theta_2} + (1 - \theta_1), \tag{11}$$

such that when $0 \leq c < C_A$, Pricing Strategy III is selected by the two sellers, and when $C_A \leq c < C_B$, Pricing Strategy II is adopted; otherwise, when $c \geq C_B$, the traditional seller will drop out of the market and Price Strategy I is adopted by the online seller.

Proposition 2 implies that when the online shop has a small cost advantage, Pricing Strategy III is adopted and the market is shared by the two sellers. When the online shop has a medium cost advantage, Pricing Strategy II is adopted. In this case, although the traditional seller has zero sales and profits, his/her pricing decisions will affect the pricing of the online seller that the online seller would arrange prices so that nothing is sold from the traditional channel in the normal sales period. When the online shop has a large cost advantage, the traditional seller provides no threat to the online seller.

Figure 1 illustrates the effect of the overloaded degree of express delivery (θ_2) on the pricing strategies. Noticing that a larger θ_2 implies a lower overloaded degree of logistics in the promotion period, we show that as θ_2 increases, the two important thresholds, C_A and C_B, increase, implying that the lower overloaded degree of logistics will lead to a smaller probability of adopting Pricing Strategy I and larger probabilities of adopting Pricing Strategies II and III. This occurs because as the degree of logistics overloading decreases, the online channel has stronger incentive to transfer more consumers to the promotion sales period from the normal sales period, mitigating the competition between both two channels in the normal sales period. As a result, the traditional seller is more likely to coexist with the online seller in the market with Pricing Strategy II or III.

Next we examine how the equilibrium outcome will change with θ_2.

Proposition 3. *(i) The selling prices of the traditional channel and the online channel in the two periods are higher when the overloaded degree of logistics is lower (with a higher θ_2). (ii) The overloaded delivery services can decline the profits of both two channels.*

Intuitively, for the online channel, a decrease in the overloaded degree of logistics leads to an increase in the consumer's willingness-to-pay for the products in the second

Table 1: Equilibrium strategies when price discounts are provided only by online channel ("+" denotes the positive value).

Pricing strategies	Conditions	Prices			Sales			Profits		
		p_{R1}	p_{D1}	p_{D2}	S_{R1}	S_{D1}	S_{D2}	π_{R1}	π_{D1}	π_{D2}
Pricing Strategy I	$C_B \le c \le 1$	Null	$\frac{2\theta_1^2}{4\theta_1 - \theta_2}$	$\frac{\theta_1\theta_2}{4\theta_1 - \theta_2}$	Null	+	+	Null	+	+
Pricing Strategy II	$C_A \le c < C_B$	c	$c + \theta_1 - 1$	$\frac{(c + \theta_1 - 1)\theta_2}{2\theta_1}$	Zero	+	+	Zero	+	+
Pricing Strategy III	$0 \le c < C_A$	$\frac{(4\theta_1 - \theta_2 + \theta_1\theta_2)(1 - \theta_1 + c)}{2(4\theta_1 - \theta_1^2 - \theta_2 + \theta_1\theta_2)}$	$\frac{\theta_1^2(1 - \theta_1 + c)}{4\theta_1 - \theta_1^2 - \theta_2 + \theta_1\theta_2}$	$\frac{\theta_1\theta_2(1 - \theta_1 + c)}{2(4\theta_1 + \theta_1\theta_2 - \theta_2 - \theta_1^2)}$	+	+	+	+	+	+

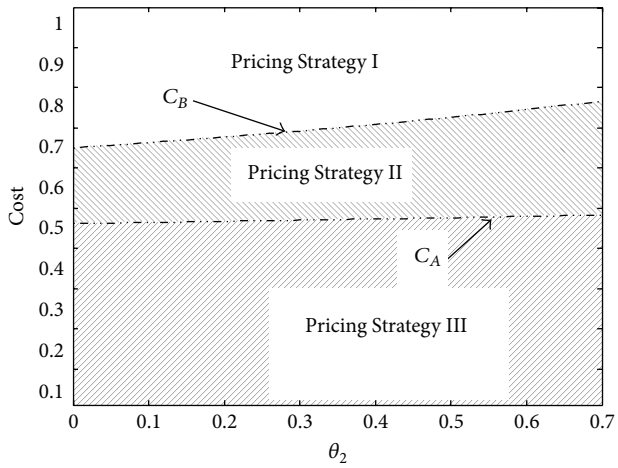

Figure 1: The impact of the overloaded degree of logistics service on equilibrium strategies.

period. Thus, the firm will charge a higher second-period price to get larger marginal profits. A practice example is that, in the Lefeng, one of the largest online shopping cosmetics mall in China, during the busy period such as "Singles Day," consumers are provided with discounts through the way that they choose the slow delivery modes. As the overloaded degree of logistics increases, the first-period price of the online channel decreases since the online channel will have a stronger motivation to transfer consumers to Period 1 from Period 2. This implies that, with coming overloaded express logistics services, the online channel may provide price discounts in advance. For the traditional channel, although the quality of delivery service has no direct impact on it, it indirectly affects its prices.

5. Scenario II: The Sales Promotion Provided by Both Channels

In this scenario, both channels adopt promotion strategies with discount prices to stimulate consumer demand. This case can be often seen during the important traditional holidays, such as Christmas holidays and Chinese Spring Festival, where people have enough leisure time to go shopping.

5.1. The Demand Functions of the Two Channels. Similar to Scenario I, the first-period demand functions for the traditional and online channels can be expressed, respectively, as

$$S_{R1} = \int_{\Omega_{R1}} dF(v) = \begin{cases} 1 - p_{R1} & 0 \le p_{R1} < \dfrac{p_{D1}}{\theta_1} \\ 1 - \dfrac{p_{R1} - p_{D1}}{1 - \theta_1} & \dfrac{p_{D1}}{\theta_1} \le p_{R1} < 1 - \theta_1 + p_{D1} \\ 0 & 1 - \theta_1 + p_{D1} \le p_{R1} \le 1, \end{cases} \tag{12}$$

$$S_{D1} = \int_{\Omega_{D1}} dF(v) = \begin{cases} 0 & 0 \le p_{R1} < \dfrac{p_{D1}}{\theta_1} \\ \dfrac{\theta_1 p_{R1} - p_{D1}}{\theta_1(1 - \theta_1)} & \dfrac{p_{D1}}{\theta_1} \le p_{R1} < 1 - \theta_1 + p_{D1} \\ 1 - \dfrac{p_{D1}}{\theta_1} & 1 - \theta_1 + p_{D1} \le p_{R1} \le 1, \end{cases} \tag{13}$$

where $\Omega_{R1} = \{v \mid v - p_{R1} \ge 0, v - p_{R1} \ge \theta_1 v - p_{D1}, 0 \le v \le 1\}$, $\Omega_{D1} = \{v \mid v - p_{D1} \ge 0, v - p_{R1} \le \theta_1 v - p_{D1}, 0 \le v \le 1\}$.

The customer acceptance of the online channel changes from θ_1 to θ_2 in the promotion period. To simplify the problem, we assume that $\overline{v} \ge p_{R2} \ge c$ and $\overline{v} \ge p_{D2}/\theta_2$. This assumption reflects that each channel would not set high price to hold his/her market share. Furthermore, if $\overline{v} \le c$, this scenario will reduce to Scenario I, where only the online channel provides the sales promotion.

The demand functions for the traditional and online channels in the second period, respectively, can be expressed as

$$S_{R2} = \int_{\Omega_{R2}} dF(v) = \begin{cases} \overline{v} - p_{R2} & 0 \le p_{R2} < \dfrac{p_{D2}}{\theta_2} \\ \overline{v} - \dfrac{p_{R2} - p_{D2}}{1 - \theta_2} & \dfrac{p_{D2}}{\theta_2} \le p_{R2} < (1 - \theta_2)\overline{v} + p_{D2} \\ 0 & (1 - \theta_2)\overline{v} + p_{D2} \le p_{R2} \le \overline{v}, \end{cases} \tag{14}$$

$$S_{D2} = \int_{\Omega_{D2}} dF(v) = \begin{cases} 0 & 0 \le p_{R2} < \dfrac{p_{D2}}{\theta_2} \\ \dfrac{\theta_2 p_{R2} - p_{D2}}{\theta_2(1 - \theta_2)} & \dfrac{p_{D2}}{\theta_2} \le p_{R2} < (1 - \theta_2)\overline{v} + p_{D2} \\ \overline{v} - \dfrac{p_{D2}}{\theta_2} & (1 - \theta_2)\overline{v} + p_{D2} \le p_{R2} \le \overline{v}, \end{cases} \tag{15}$$

where $\Omega_{R2} = \{v \mid v - p_{R2} \ge 0, v - p_{R2} \ge \theta_1 v - p_{D2}, 0 \le v \le \overline{v}\}$, $\Omega_{D2} = \{v \mid v - p_{D2} \ge 0, v - p_{R2} \le \theta_1 v - p_{D2}, 0 \le v \le \overline{v}\}$.

Equations (14) and (15) show that the demand functions of both channels are continuous. We find that the consumer whose valuations are in the interval $[\min(p_{R2}, p_{D2}/\theta_2), \overline{v}]$ would buy the product in the promotion sales period and the consumer whose valuation is in the interval $[0, \min(p_{R2}, p_{D2}/\theta_2)]$ declines to buy in either period.

The total profits for traditional channel and online channel π_R and π_D, respectively, are equal to

$$\pi_R = (p_{R1} - c) S_{R1} + (p_{R2} - c) S_{R2}, \tag{16}$$

$$\pi_D = p_{D1} S_{D1} + p_{D2} S_{D2}. \tag{17}$$

To maximize their profits, each seller optimizes their price. We initially solve for equilibrium prices of two channels in the second period given p_{R1} and p_{D1}.

5.2. Analysis for the Promotion Sales Period. In the promotion period, the problem of the traditional channel can be stated as follows:

$$\operatorname*{Max}_{p_{R2}} \pi_{R2}(p_{R2}) = (p_{R2} - c) S_{R2}. \tag{18}$$

The problem of the online channel in this period is similar:

$$\operatorname{Max} \pi_D = p_{D2} S_{D2}. \tag{19}$$

To solve problems (18) and (19), we must take into account the piecewise-liner demands S_{R2} and S_{D2} in (14) and (15).

According to (14) and (15), the profit functions of the two channels shown as (18) and (19) are rewritten as

$$\pi_{R2} = \begin{cases} (p_{R2} - c)(\bar{v} - p_{R2}) & c \le p_{R2} < \dfrac{p_{D2}}{\theta_2} \\ (p_{R2} - c)\left(\bar{v} - \dfrac{p_{R2} - p_{D2}}{1 - \theta_2}\right) & \dfrac{p_{D2}}{\theta_2} \le p_{R2} < (1 - \theta_2)\bar{v} + p_{D2} \\ 0 & (1 - \theta_2)\bar{v} + p_{D2} \le p_{R2} \le \bar{v}, \end{cases} \tag{20}$$

$$\pi_{D2} = \begin{cases} p_{D2}\left(\bar{v} - \dfrac{p_{D2}}{\theta_2}\right) & 0 \le p_{D2} < p_{R2} - (1 - \theta_2)\bar{v} \\ p_{D2}\dfrac{\theta_2 p_{R2} - p_{D2}}{\theta_2(1 - \theta_2)} & p_{R2} - (1 - \theta_2)\bar{v} \le p_{D2} < \theta_2 p_{R2} \\ 0 & \theta_2 p_{R2} \le p_{D2} \le \bar{v}\theta_2. \end{cases} \tag{21}$$

We give the best pricing strategy for each channel in promotion sales period with given values of $\bar{v}$ and his/her competitor's price in the following lemma.

Lemma 4. *(i) Given price p_{D2} of online channel and $\bar{v}$, the best response price of the traditional channel is*

$$p_{R2}^*(p_{D1}, \bar{v}) = \begin{cases} Null & \text{if } 0 \le p_{D2} < c - \bar{v} + \bar{v}\theta_2 \\ \dfrac{\bar{v} - \bar{v}\theta_2 + p_{D2} + c}{2} & \text{if } c - \bar{v} + \bar{v}\theta_2 \le p_{D2} < \dfrac{\theta_2(\bar{v} - \bar{v}\theta_2 + c)}{2 - \theta_2} \\ \dfrac{p_{D2}}{\theta_2} & \text{if } \dfrac{\theta_2(\bar{v} - \bar{v}\theta_2 + c)}{2 - \theta_2} \le p_{D2} < \dfrac{(\bar{v} + c)\theta_2}{2} \\ \dfrac{\bar{v} + c}{2} & \text{if } \dfrac{(\bar{v} + c)\theta_2}{2} \le p_{D2} < \bar{v}\theta_2. \end{cases} \tag{22}$$

(ii) Given price p_{R2} of traditional channel and $\bar{v}$, the best response price of the online channel is

$$p_{D2}^*(p_{R2}, \bar{v}) = \begin{cases} \dfrac{\theta_2 p_{R2}}{2} & \text{if } c \le p_{R2} < \dfrac{2(1 - \theta_2)\bar{v}}{2 - \theta_2} \\ p_{R2} - (1 - \theta_2)\bar{v} & \text{if } \dfrac{2(1 - \theta_2)\bar{v}}{2 - \theta_2} \le p_{R2} < \bar{v} - \dfrac{\bar{v}\theta_2}{2} \\ \dfrac{\theta_2 \bar{v}}{2} & \text{if } \bar{v} - \dfrac{\bar{v}\theta_2}{2} \le p_{R2} \le \bar{v}. \end{cases} \tag{23}$$

Lemma 4 illustrates one's best pricing strategies in the promotion period under his/her rival's decisions. In the second period where price discounts are provided by both two channels, the traditional seller will (i) exit the market when the online seller's second-period price is extremely low, (ii) charge a high second-period price that $p_{R2}^*(p_{D1}, \bar{v}) \ge p_{D2}/\theta_2$ to increase his/her marginal profits when the online seller's second-period price is low, and (iii) charge a low second-period price that $p_{R2}^*(p_{D1}, \bar{v}) \le p_{D2}/\theta_2$ to capture more market share. For the online channel, it is optimal for him/her to set a lower price than the traditional seller that $p_{D2}^*(p_{R2}, \bar{v}) \le \theta_2 p_{R2}$.

Using (22) and (23), we solve for $p_{R2}^*(\bar{v})$ and $p_{D2}^*(\bar{v})$ simultaneously and then get Proposition 5.

Proposition 5. *Given the first-period pricing decisions of the both channels, p_{R1} and p_{D1}, the second-period prices for the traditional and online seller are*

$$p_{R2}(\bar{v}) = \begin{cases} Null & \text{if } c \le \bar{v} < \dfrac{2c}{2 - \theta_2} \\ c & \text{if } \dfrac{2c}{2 - \theta_2} \le \bar{v} < \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)} \\ \dfrac{2(\bar{v} - \bar{v}\theta_2 + c)}{4 - \theta_2} & \text{if } \bar{v} \ge \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)}, \end{cases}$$

$$p_{D2}(\bar{v}) = \begin{cases} \dfrac{\theta_2 \bar{v}}{2} & \text{if } c \le \bar{v} < \dfrac{2c}{2 - \theta_2} \\ \bar{v}\theta_2 - \bar{v} + c & \text{if } \dfrac{2c}{2 - \theta_2} \le \bar{v} < \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)} \\ \dfrac{\theta_2(\bar{v} - \bar{v}\theta_2 + c)}{4 - \theta_2} & \text{if } \bar{v} \ge \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)}. \end{cases} \tag{24}$$

Proposition 5 shows that the traditional seller will exit the market due to the cost disadvantage if potential size of the market is small. If the potential size of the market is medium, although the online seller has a cost advantage, he/she cannot ignore the impact of the traditional seller. If the potential size of the market is large, the market is shared by both channels. Their prices are both increasing in the potential size of the market.

Corollary 6. *The corresponding profit functions for the traditional and online seller in the second period are, respectively,*

$$\pi_{R2}(\bar{v}) = \begin{cases} 0 & \text{if } c \le \bar{v} < \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)} \\ \dfrac{(2\bar{v} - 2\bar{v}\theta_2 - 2c + c\theta_2)^2}{(4 - \theta_2)^2(1 - \theta_2)} & \text{if } \bar{v} \ge \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)}, \end{cases} \tag{25}$$

$$\pi_{D2}(\bar{v}) = \begin{cases} \dfrac{\theta_2 \bar{v}^2}{4} & \text{if } c \le \bar{v} < \dfrac{2c}{2 - \theta_2} \\ \dfrac{(\bar{v}\theta_2 - \bar{v} + c)(\bar{v} - c)}{\theta_2} & \text{if } \dfrac{2c}{2 - \theta_2} \le \bar{v} < \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)} \\ \dfrac{\theta_2(\bar{v} - \bar{v}\theta_2 + c)^2}{(4 - \theta_2)^2(1 - \theta_2)} & \text{if } \bar{v} \ge \dfrac{(2 - \theta_2)c}{2(1 - \theta_2)}. \end{cases} \tag{26}$$

5.3. Analysis for the Normal Sales Period

5.3.1. The Traditional Channel's Pricing Problem. Given the first-period price of the online channel, the traditional channel has three pricing strategy options: (i) low price strategy: $p_{R1} < p_{D1}/\theta_1$ and $\bar{v} = p_{R1}$; (ii) medium price strategy: $p_{D1}/\theta_1 \le p_{R1} < 1-\theta_1+p_{D1}$ and $\bar{v} = p_{D1}/\theta_1$; (iii) high price strategy: $1-\theta_1+p_{D1} \le p_{R1} \le 1$ and $\bar{v} = p_{D1}/\theta_1$. Which pricing strategy should the traditional seller choose? We first analyze the first one.

Under the pricing strategy where $p_{R1} < p_{D1}/\theta_1$, the total profit of the traditional seller over the two periods is

$$\pi_R(p_{R1} \mid p_{D1}) = \begin{cases} (p_{R1}-c)(1-p_{R1}) & \text{if } c \le p_{R1} < \dfrac{(2-\theta_2)c}{2(1-\theta_2)} \\ (p_{R1}-c)(1-p_{R1}) + \dfrac{(2p_{R1}-2p_{R1}\theta_2-2c+c\theta_2)^2}{(4-\theta_2)^2(1-\theta_2)} & \text{if } p_{R1} \ge \dfrac{(2-\theta_2)c}{2(1-\theta_2)}. \end{cases} \tag{27}$$

Maximizing $\pi_R(p_{R1} \mid p_{D1})$ by choosing an optimal p_{R1} with condition that $p_{R1} < p_{D1}/\theta_1$, we get Lemma 7.

Lemma 7. *Given the online channel's decision in the first period, p_{D1}, the optimal price for the traditional channel under low price strategy in the first period is as follows: (i) when $c < 1-\theta_2$,*

$$p_{R1}{}^*(p_{D1}) = \begin{cases} \dfrac{p_{D1}}{\theta_1} & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1\left[16-8\theta_2+\theta_2{}^2+\left(8-4\theta_2+\theta_2{}^2\right)c\right]}{2\left(12-4\theta_2+\theta_2{}^2\right)} \\ \dfrac{16-8\theta_2+\theta_2{}^2+\left(8-4\theta_2+\theta_2{}^2\right)c}{2\left(12-4\theta_2+\theta_2{}^2\right)} & \text{if } p_{D1} \ge \dfrac{\theta_1\left[16-8\theta_2+\theta_2{}^2+\left(8-4\theta_2+\theta_2{}^2\right)c\right]}{2\left(12-4\theta_2+\theta_2{}^2\right)} \end{cases} \tag{28}$$

and (ii) when $c \ge 1-\theta_2$,

$$p_{R1}{}^*(p_{D1}) = \begin{cases} \dfrac{p_{D1}}{\theta_1} & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(c+1)}{2} \\ \dfrac{c+1}{2} & \text{if } p_{D1} \ge \dfrac{\theta_1(c+1)}{2}. \end{cases} \tag{29}$$

From (25) and Lemma 7, we know that when the online seller has a large cost advantage that may make up the disadvantage of overloaded express logistics service, that is, $c \ge 1-\theta_2$, the traditional seller with low price strategy would have no profit in the second period since $\bar{v} = p_{R1}{}^*(p_{D1}) \le (c+1)/2 \le (2-\theta_2)c/[2(1-\theta_2)]$.

Next, we consider the other two pricing strategy options for the traditional seller that $p_{R1} \ge p_{D1}/\theta_1$. Under such pricing strategies, since $\bar{v} = p_{D1}/\theta_1$, the second-period profit of the traditional seller is independent of p_{R1}. The first-period profit of the traditional seller is positive under medium price strategy where $p_{D1}/\theta_1 \le p_{R1} \le 1-\theta_1+p_{D1}$ and it is zero under high price strategy where $1-\theta_1+p_{D1} \le p_{R1} \le 1$; thus, the medium price strategy in which $p_{D1}/\theta_1 \le p_{R1} \le 1-\theta_1+p_{D1}$ is better than the high price strategy in which $1-\theta_1+p_{D1} \le p_{R1} \le 1$.

Under the medium price strategy, the total profit over the two periods is

$$\pi_R(p_{R1} \mid p_{D1}) = (p_{R1}-c)\left(1-\frac{p_{R1}-p_{D1}}{1-\theta_1}\right) + \pi_{R2}(p_{R1} \mid \bar{v}), \tag{30}$$

where $\bar{v} = p_{D1}/\theta_1$. Recalling that the second-period profit function $\pi_{R2}(p_{R1} \mid \bar{v})$ is independent of p_{R1}, we get the following optimization problem:

$$\begin{aligned} \max_{p_{R1}} \quad & (p_{R1}-c)\left(1-\frac{p_{R1}-p_{D1}}{1-\theta_1}\right), \\ \text{s.t.} \quad & \frac{p_{D1}}{\theta_1} \le p_{R1} \le 1-\theta_1+p_{D1}. \end{aligned} \tag{31}$$

Lemma 8. *Given the online channel's decision in the first period, p_{D1}, the optimal price for the traditional channel under medium price strategy in the first period is*

$$p_{R1}{}^*(p_{D1}) = \begin{cases} \dfrac{p_{D1}+1+c-\theta_1}{2} & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ \dfrac{p_{D1}}{\theta_1} & \text{if } p_{D1} \le \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1}. \end{cases} \tag{32}$$

Furthermore, the medium price strategy is always better than the high price strategy for the online channel in the first period.

Comparing these pricing strategy options, we derive the optimal one for the traditional seller, which is illustrated in Proposition 9.

Proposition 9. *When price discounts are provided by both channels, if the traditional seller faces a low price set by*

his/her online competitor, he/she will adopt a price strategy that $p_{R1}{}^*(p_{D1}) \geq p_{D1}/\theta$*; otherwise, he/she will use a strategy that* $p_{R1}{}^*(p_{D1}) < p_{D1}/\theta$*. Specifically, (i) when* $0 \leq c < 1-\theta_2$*,*

$$p_{R1}{}^*(p_{D1}) = \begin{cases} \dfrac{p_{D1}+1+c-\theta_1}{2} & \text{if } \theta_1 c \leq p_{D1} < \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ \dfrac{p_{D1}}{\theta_1} & \text{if } \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \leq p_{D1} < \dfrac{\theta_1\left(16-8\theta_2+\theta_2{}^2+\left(8-4\theta_2+\theta_2{}^2\right)c\right)}{2\left(12-4\theta_2+\theta_2{}^2\right)} \\ \dfrac{16-8\theta_2+\theta_2{}^2+\left(8-4\theta_2+\theta_2{}^2\right)c}{2\left(12-4\theta_2+\theta_2{}^2\right)} & \text{if } p_{D1} \geq \dfrac{\theta_1\left(16-8\theta_2+\theta_2{}^2+\left(8-4\theta_2+\theta_2{}^2\right)c\right)}{2\left(12-4\theta_2+\theta_2{}^2\right)}, \end{cases} \tag{33}$$

and (ii) when $1-\theta_2 \leq c \leq 1$*,*

$$p_{R1}{}^*(p_{D1}) = \begin{cases} \dfrac{p_{D1}+1+c-\theta_1}{2} & \text{if } \theta_1 c \leq p_{D1} < \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ \dfrac{p_{D1}}{\theta_1} & \text{if } \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \leq p_{D1} < \dfrac{\theta_1(1+c)}{2} \\ \dfrac{1+c}{2} & \text{if } p_{D1} \geq \dfrac{\theta_1(1+c)}{2}. \end{cases} \tag{34}$$

Proposition 9 illustrates effect of the online channel on traditional channel's first-period price. If he/she anticipates a low first-period price of online channel, he/she will charge a relative high price $p_{R1} > p_{D1}/\theta_1$ to get more marginal profits, and if he/she anticipates a high first-period price of online channel, he/she will charge a relative low price that $p_{R1} < p_{D1}/\theta_1$ to capture more market share; otherwise, if he/she anticipates a medium price from his/her online competitor, he/she will give equal weight to the market share and marginal profits by charging a price that $p_{R1} = p_{D1}/\theta_1$.

5.3.2. The Online Channel's Pricing Problem. Given the first-period price of traditional seller, the total profit of the online channel is

$$\begin{aligned} \pi_D(p_{D1} \mid p_{R1}) &= \pi_{D1}(p_{D1} \mid p_{R1}) + \pi_{D2}(p_{D1} \mid p_{R1}) \\ &= \begin{cases} \left(1-\dfrac{p_{D1}}{\theta_1}\right)p_{D1} + \pi_{D2}\left(p_{D1} \mid \bar{v} = \dfrac{p_{D1}}{\theta_1}\right) & \theta_1 c \leq p_{D1} < p_{R1}+\theta_1-1 \\ \dfrac{(\theta_1 p_{R1}-p_{D1})\,p_{D1}}{\theta_1(1-\theta_1)} + \pi_{D2}\left(p_{D1} \mid \bar{v} = \dfrac{p_{D1}}{\theta_1}\right) & p_{R1}+\theta_1-1 \leq p_{D1} \leq p_{R1}\theta_1 \\ 0 + \pi_{D2}(p_{D1} \mid \bar{v} = p_{R1}) & p_{D1} > p_{R1}\theta_1, \end{cases} \end{aligned} \tag{35}$$

where

$$\begin{aligned} \pi_{D2}\left(p_{D1} \mid \bar{v} = \frac{p_{D1}}{\theta_1}\right) &= \begin{cases} \dfrac{\theta_2 p_{D1}{}^2}{4\theta_1{}^2} & \text{if } \theta_1 c \leq p_{D1} < \dfrac{2\theta_1 c}{2-\theta_2} \\ \dfrac{(p_{D1}\theta_2 - p_{D1} + \theta_1 c)(p_{D1}-\theta_1 c)}{\theta_1{}^2\theta_2} & \text{if } \dfrac{2\theta_1 c}{2-\theta_2} \leq p_{D1} < \dfrac{(2-\theta_2)\theta_1 c}{2(1-\theta_2)} \\ \dfrac{\theta_2(p_{D1}-p_{D1}\theta_2+\theta_1 c)^2}{\theta_1{}^2(4-\theta_2)^2(1-\theta_2)} & \text{if } p_{D1} \geq \dfrac{(2-\theta_2)\theta_1 c}{2(1-\theta_2)}, \end{cases} \\ \pi_{D2}(p_{D1} \mid \bar{v} = p_{R1}) &= \begin{cases} \dfrac{\theta_2 p_{R1}{}^2}{4} & \text{if } c \leq p_{R1} < \dfrac{2c}{2-\theta_2} \\ \dfrac{(p_{R1}\theta_2 - p_{R1} + c)(p_{R1}-c)}{\theta_2} & \text{if } \dfrac{2c}{2-\theta_2} \leq p_{R1} < \dfrac{(2-\theta_2)c}{2(1-\theta_2)} \\ \dfrac{\theta_2(p_{R1}-p_{R1}\theta_2+c)^2}{(4-\theta_2)^2(1-\theta_2)} & \text{if } p_{R1} \geq \dfrac{(2-\theta_2)c}{2(1-\theta_2)}. \end{cases} \end{aligned} \tag{36}$$

Since $\pi_D(p_{D1} \mid p_{R1})$ is nonquasiconcave, we cannot ascertain the existence and uniqueness of a Nash equilibrium. Our objective is to study the feasible regions for the existence of Nash equilibrium prices. In order to obtain the feasible regions associated with a given value for the parameters c, θ_1, and θ_2 in which a Nash price equilibrium exists, we first derive the necessary and sufficient conditions on those variables in which an equilibrium exists, and, secondly, we calculate all equilibria in prices for each firm and each critical interval.

Similar to the traditional channel, the online channel also has three pricing strategy options when his/her rival's first-period price is given as follows: (i) high price strategy: $p_{D1} > p_{R1}\theta_1$ and $\bar{v} = p_{R1}$; (ii) medium price strategy: $p_{R1} + \theta_1 - 1 \le p_{D1} \le p_{R1}\theta_1$ and $\bar{v} = p_{D1}/\theta_1$; (iii) low price strategy: $p_{D1} \le p_{R1} + \theta_1 - 1$ and $\bar{v} = p_{D1}/\theta_1$.

Notice that the total profit function in (35) is continuous and $\partial\pi_D(p_{D1} \mid p_{R1})/\partial p_{D1}$ is negative at the left of the point $p_{D1} = p_{R1}\theta_1$ and is zero at the right, implying that $\pi_D(p_{D1} \mid p_{R1})$ is nonincreasing at the point $p_{D1} = p_{R1}\theta_1$. Thus, for the online seller, it is never optimal for him/her to use a high price strategy that $p_{D1} > p_{R1}\theta_1$. This implies that anticipating a peak period for parcel deliveries, the online seller will set a low first-period price to transfer more consumers to the normal sales period from the promotion sales period.

We can also prove that, in a Nash price equilibrium, $p_{R1} < \theta_1 c + 1 - \theta_1$. In a Nash price equilibrium $(p_{R1}{}^*, p_{D1}{}^*)$, $p_{R1}{}^*$ and $p_{D1}{}^*$ should satisfy traditional seller's response function proposed in Proposition 9. Since $p_{R1}{}^* = (p_{D1}{}^* + 1 + c - \theta_1)/2 \le \theta_1 c + 1 - \theta_1$, if $p_{R1}{}^* \ge \theta_1 c + 1 - \theta_1$, then $p_{R1}{}^* \le p_{D1}{}^*/\theta_1$. Recall that for the online seller it is never optimal to set such price that $p_{R1}{}^* \le p_{D1}{}^*/\theta_1$. Thus, in a Nash price equilibrium, the first-period price should not be higher than $\theta_1 c + 1 - \theta_1$.

Now we consider the case $p_{R1} < \theta_1 c + 1 - \theta_1$, and the profit function is changed to

$$\begin{aligned}\pi_D(p_{D1} \mid p_{R1}) &= \pi_{D1}(p_{D1} \mid p_{R1}) + \pi_{D2}(p_{D1} \mid p_{R1}) \\ &= \frac{p_{D1}(\theta_1 p_{R1} - p_{D1})}{\theta_1(1-\theta_1)} + \begin{cases} \dfrac{\theta_2 p_{D1}{}^2}{4\theta_1{}^2} & \text{if } \theta_1 c \le p_{D1} < \dfrac{2\theta_1 c}{2-\theta_2} \\ \dfrac{(p_{D1}\theta_2 - p_{D1} + \theta_1 c)(p_{D1} - \theta_1 c)}{\theta_1{}^2\theta_2} & \text{if } \dfrac{2\theta_1 c}{2-\theta_2} \le p_{D1} < \dfrac{(2-\theta_2)\theta_1 c}{2(1-\theta_2)} \\ \dfrac{\theta_2(p_{D1} - p_{D1}\theta_2 + \theta_1 c)^2}{\theta_1{}^2(4-\theta_2)^2(1-\theta_2)} & \text{if } p_{D1} \ge \dfrac{(2-\theta_2)\theta_1 c}{2(1-\theta_2)}. \end{cases}\end{aligned} \tag{37}$$

The optimization problem of online channel is given by (37) subject to $\theta_1 c \le p_{D1} \le p_{R1}\theta_1$.

Lemma 10. *Given the traditional channel's first-period price p_{R1} and $p_{R1} < \theta_1 c + 1 - \theta_1$, the optimal price for the online channel is*

$$p_{D1}{}^*(p_{R1}) = \begin{cases} \theta_1 c & \text{if } c \le p_{R1} < \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)c}{2\theta_1} \\ \dfrac{2\theta_1{}^2 p_{R1}}{4\theta_1 - \theta_2 + \theta_1\theta_2} & \text{if } \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)c}{2\theta_1} \le p_{R1} < \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)c}{(2-\theta_2)\theta_1} \\ \dfrac{\theta_1{}^2\theta_2 p_{R1} + (2-\theta_2)(1-\theta_1)\theta_1 c}{2(2\theta_1\theta_2 + 1 - \theta_1 - \theta_2)} & \text{if } \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)c}{(2-\theta_2)\theta_1} \le p_{R1} < \overline{PR} \\ \dfrac{\theta_1{}^2(4-\theta_2)^2 p_{R1} + \theta_1\theta_2(1-\theta_1)c}{2(16\theta_1 - 7\theta_1\theta_2 - \theta_2 + \theta_2{}^2)} & \text{if } \overline{PR} \le p_{R1}, \end{cases} \tag{38}$$

where

$$\overline{PR} = \frac{\left(32\theta_1 - 28\theta_1\theta_2 - 4\theta_2 + 3\theta_1\theta_2{}^2 + 5\theta_2{}^2 - \theta_2{}^3 + \theta_2\sqrt{\theta_2(16\theta_1 - 7\theta_1\theta_2 - \theta_2 + \theta_2{}^2)(2\theta_1\theta_2 + 1 - \theta_1 - \theta_2)}\right)c}{2(8 - 4\theta_2 + \theta_2{}^2)(1-\theta_2)\theta_1}. \tag{39}$$

5.3.3. The Equilibrium Prices for the Two Channels. Solving (33), (34), and (38) simultaneously, we can get the following lemma.

Lemma 11. *There are Nash price equilibria for the traditional channel and online channel. Specifically,*

$$p_{R1}{}^{*} = \begin{cases} \dfrac{2(1-\theta_1)\left(16\theta_1 - \theta_2{}^2 + \theta_2 + 7\theta_1\theta_2\right) + 2G_1 c}{G} & \text{if } 0 \le c < C_E \\ \dfrac{2(1-\theta_1)(2\theta_1\theta_2 + 1 - \theta_1 - \theta_2) + \left(2 + 3\theta_1\theta_2 - 2\theta_1{}^2 - 2\theta_2 + \theta_1{}^2\theta_2\right)c}{\theta_1{}^2\theta_2 + 8\theta_1\theta_2 + 4 - 4\theta_1 - 4\theta_2} & \text{if } C_D \le c < C_F \\ \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)(1 - \theta_1 + c)}{2\left(4\theta_1 - \theta_1{}^2 - \theta_2 + \theta_1\theta_2\right)} & \text{if } C_F \le c < \widehat{C} \\ \dfrac{1 - \theta_1 + (1+\theta_1)c}{2} & \text{if } \widehat{C} \le c \le 1, \end{cases}$$

$$p_{D1}{}^{*} = \begin{cases} \dfrac{\theta_1{}^2(1-\theta_1)(4-\theta_2)^2 + G_2\theta_1 c}{G} & \text{if } 0 \le c < C_E \\ \dfrac{(1-\theta_1)\theta_1{}^2\theta_2 + (4 + 3\theta_1\theta_2 - 4\theta_1 - 2\theta_2)c}{\theta_1{}^2\theta_2 + 8\theta_1\theta_2 + 4 - 4\theta_1 - 4\theta_2} & \text{if } C_D \le c < C_F \\ \dfrac{\theta_1{}^2(1 - \theta_1 + c)}{4\theta_1 - \theta_1{}^2 - \theta_2 + \theta_1\theta_2} & \text{if } C_F \le c < \widehat{C} \\ \theta_1 c & \text{if } \widehat{C} \le c \le 1, \end{cases} \tag{40}$$

where

$$\begin{aligned} & C_D \le C_E, \\ & C_D = \frac{4\left(8 - 4\theta_2 + \theta_2{}^2\right)\theta_1(1-\theta_1)(1-\theta_2)\sqrt{2\theta_1\theta_2 + 1 - \theta_1 - \theta_2}}{\left(\theta_1{}^2\theta_2 + 8\theta_1\theta_2 + 4 - 4\theta_1 - 4\theta_2\right)\theta_2\sqrt{\theta_2\left(16\theta_1 - 7\theta_1\theta_2 - \theta_2 + \theta_2{}^2\right)} + G_3\sqrt{2\theta_1\theta_2 + 1 - \theta_1 - \theta_2}}, \\ & C_E = \frac{4\left(8 - 4\theta_2 + \theta_2{}^2\right)\theta_1(1-\theta_1)(1-\theta_2)\sqrt{16\theta_1 - 7\theta_1\theta_2 - \theta_2 + \theta_2{}^2}}{\left(G\theta_2\sqrt{\theta_2(2\theta_1\theta_2 + 1 - \theta_1 - \theta_2)}\right) + G_3\sqrt{16\theta_1 - 7\theta_1\theta_2 - \theta_2 + \theta_2{}^2}}, \\ & C_F = \frac{(1-\theta_1)(2-\theta_2)\theta_1}{6\theta_1 + 3\theta_1\theta_2 - 2\theta_2 - 2\theta_1{}^2}, \\ & \widehat{C} = \frac{\theta_1(1-\theta_1)}{3\theta_1 - \theta_1{}^2 - \theta_2 + \theta_1\theta_2}, \\ & G = \left(4 - \theta_1{}^2\right)\theta_2{}^2 - \left(28\theta_1 - 8\theta_1{}^2 + 4\right)\theta_2 - 16\theta_1{}^2 + 64\theta_1 > 0, \\ & G_1 = 16\theta_1 + \theta_2{}^2 - 6\theta_1\theta_2 - \theta_2 - \theta_1{}^2\theta_2 > 0, \\ & G_2 = 16\theta_1 - 12\theta_1\theta_2 + 4\theta_2 + \theta_1\theta_2{}^2 > 0, \\ & G_3 = (2-\theta_1)^2\theta_2{}^3 + 4\left(5 - 2\theta_1{}^2 - 2\theta_1\right)\theta_2{}^2 + 16\left(2\theta_1{}^2 - 4\theta_1 - 1\right)\theta_2 - 32\theta_1{}^2 + 96\theta_1 \ge 0. \end{aligned} \tag{41}$$

From Lemma 11, we note that when $\widehat{C} \le c \le 1$, the online seller's price in the first period is $\theta_1 c$. It may be caused by the assumption that $\overline{v} = \min(p_{R1}, p_{D1}/\theta_1) \ge c$. Since $p_{R1} \ge c$, whether $\overline{v} \ge c$ or not depends on the size relation between p_{D1} and $\theta_1 c$. For the online channel, it may be optimal for him/her to set a price that $\overline{v} \le c$ to capture the entire market. If $\overline{v} \le c$, Scenario II will reduce to Scenario I, where only the online channel provides the sales promotion. By profits

comparing, we find that, regardless of traditional channels' decisions, when $c \geq \widehat{C}$, it is optimal for the online channel to set a low price smaller than $\theta_1 c$ to cover the entire market. Thus, we get the following lemma.

Lemma 12. *When price discounts are provided by both channels, there are Nash price equilibria for the whole market. Specifically,*

$$p_{R1}{}^* = \begin{cases} \dfrac{2(1-\theta_1)\left(16\theta_1 - \theta_2{}^2 + \theta_2 + 7\theta_1\theta_2\right) + 2G_1 c}{G} & if\ 0 \leq c < C_E \\ \dfrac{2(1-\theta_1)(2\theta_1\theta_2 + 1 - \theta_1 - \theta_2) + \left(2 + 3\theta_1\theta_2 - 2\theta_1{}^2 - 2\theta_2 + \theta_1{}^2\theta_2\right)c}{\theta_1{}^2\theta_2 + 8\theta_1\theta_2 + 4 - 4\theta_1 - 4\theta_2} & if\ D_D \leq c < C_F \\ \dfrac{(4\theta_1 - \theta_2 + \theta_1\theta_2)(1 - \theta_1 + c)}{2\left(4\theta_1 - \theta_1{}^2 - \theta_2 + \theta_1\theta_2\right)} & if\ C_F \leq c < C_A \\ c & if\ C_A \leq c < C_B \\ Null & if\ C_B \leq c \leq 1, \end{cases}$$

$$p_{D1}{}^* = \begin{cases} \dfrac{\theta_1{}^2(1-\theta_1)(4-\theta_2)^2 + G_2\theta_1 c}{G} & if\ 0 \leq c < C_E \\ \dfrac{(1-\theta_1)\theta_1{}^2\theta_2 + (4 + 3\theta_1\theta_2 - 4\theta_1 - 2\theta_2)c}{\theta_1{}^2\theta_2 + 8\theta_1\theta_2 + 4 - 4\theta_1 - 4\theta_2} & if\ C_D \leq c < C_F \\ \dfrac{\theta_1{}^2(1-\theta_1+c)}{4\theta_1 - \theta_1{}^2 - \theta_2 + \theta_1\theta_2} & if\ C_F \leq c < C_A \\ c + \theta_1 - 1 & if\ C_A \leq c < C_B \\ \dfrac{2\theta_1{}^2}{4\theta_1 - \theta_2} & if\ C_B \leq c \leq 1, \end{cases} \tag{42}$$

where $C_D, C_E, C_F, G, G_1, G_2, G_3$ are shown in Lemma 11 and C_A and C_B are shown in Proposition 2.

From Lemma 12, we find that when $C_D \leq c \leq C_E$, there are two *Nash price equilibria*. However, in our numerical analysis, we find multiple equilibria in only a very small region associated with c. Thus, although multiple equilibria may occur, it appears that such cases are rare.

Using $p_{R1}{}^*$ and $p_{D1}{}^*$, we can get the second-period prices, as well as the sales and profits of the two channels. The outcomes of the price-setting game between traditional channel and online channel are stated in the following proposition.

Proposition 13. *When price discounts are provided by both channels, the derived Nash price equilibrium includes five possible combinations of prices, which can be shown in Table 2.*

Our results show that (i) when the cost advantage of the online channel is very large, the traditional seller provides no threat to the online channel. The online seller can effectively ignore the potential cannibalization of consumers by the traditional seller. In this case, the traditional seller will drop out of the market, and the online channel will adopt Pricing Strategy I, in which the pricing decisions are the same as the benchmark case; (ii) when the cost advantage of the online channel is large, Pricing Strategy II is adopted. Under this strategy, although the traditional seller has zero sales and profits in the first period, his/her pricing decisions will affect the pricing of the online seller that the online seller would arrange prices so that nothing is sold from the traditional channel in the normal sales period. In the second period, the online channel will ignore the threat from traditional channel and the traditional channel will exit the market; (iii) when the cost advantage of the online channel is medium, Pricing Strategy III is adopted. In this case, the traditional channel has positive sales and profits in the first period, but he/she exits the market in the second period. The online channel has positive sales and profits in both periods and his/her price decisions are the same as the scenario where only the online channel provides price discounts; (iv) when the cost advantage of the online channel is small, the equilibrium strategy is Strategy IV, under which the traditional channel coexists with the online channel. In the first period, the two channels both have positive sales and profits. In the second period, the traditional channel has zero sales and profit, but his/her existence can lower the price of the online channel; (v) when the cost advantage of the online channel is very small, Pricing Strategy V is adopted and the market is shared by the two sellers in both two periods.

5.3.4. Numerical Analysis. We now present numerical analysis to demonstrate the impact of the overloaded delivery services.

Table 2: Equilibrium strategies when price discounts are provided by both channels ("+" denotes the positive value).

Pricing strategies	Conditions	Prices				Sales				Profits			
		p_{R1}	p_{D1}	p_{R2}	p_{D2}	S_{R1}	S_{D1}	S_{R2}	S_{D2}	π_{R1}	π_{D1}	π_{R2}	π_{D2}
Pricing Strategy I	$C_B \le c \le 1$	Null	$\frac{2\theta_1^2}{4\theta_1-\theta_2}$	Null	$\frac{\theta_1\theta_2}{4\theta_1-\theta_2}$	Null	+	Null	+	Null	+	Null	+
Pricing Strategy II	$C_A \le c < C_B$	c	$c+\theta_1-1$	Null	$\frac{(c+\theta_1-1)\theta_2}{2\theta_1}$	Zero	+	Null	+	Zero	+	Null	+
Pricing Strategy III	$C_F \le c < C_A$	$\frac{(4\theta_1-\theta_2+\theta_1\theta_2)(1-\theta_1+c)}{2(4\theta_1-\theta_1^2-\theta_2+\theta_1\theta_2)}$	$\frac{\theta_1^2(1-\theta_1+c)}{4\theta_1-\theta_1^2-\theta_2+\theta_1\theta_2}$	Null	$\frac{\theta_1\theta_2(1-\theta_1+c)}{2(4\theta_1+\theta_1\theta_2-\theta_2-\theta_1^2)}$	+	+	Null	+	+	+	Null	+
Pricing Strategy IV	$C_D \le c < C_F$	$\frac{2(1-\theta_1)(2\theta_1\theta_2+1-\theta_1-\theta_2)}{\theta_1^2\theta_2+8\theta_1\theta_2+4-4\theta_1-4\theta_2}+\frac{(2+3\theta_1\theta_2-2\theta_1^2-2\theta_2+\theta_1^2\theta_2)c}{\theta_1^2\theta_2+8\theta_1\theta_2+4-4\theta_1-4\theta_2}$	$\frac{(1-\theta_1)\theta_1^2\theta_2}{\theta_1^2\theta_2+8\theta_1\theta_2+4-4\theta_1-4\theta_2}+\frac{(4+3\theta_1\theta_2-4\theta_1-2\theta_2)c}{\theta_1^2\theta_2+8\theta_1\theta_2+4-4\theta_1-4\theta_2}$	c	$\frac{-(1-\theta_1)(1-\theta_2)\theta_1\theta_2}{\theta_1^2\theta_2+8\theta_1\theta_2+4-4\theta_1-4\theta_2}+\frac{(2+\theta_1-2\theta_2+3\theta_1\theta_2-\theta_1^2)c}{\theta_1^2\theta_2+8\theta_1\theta_2+4-4\theta_1-4\theta_2}$	+	+	Zero	+	+	+	Zero	+
Pricing Strategy V	$0 \le c < C_E$	$\frac{2(1-\theta_1)(16\theta_1-\theta_2^2+\theta_2+7\theta_1\theta_2)}{G}+\frac{2G_1c}{G}$	$\frac{\theta_1^2(1-\theta_1)(4-\theta_2)^2+G_2\theta_1c}{G}$	$\frac{2\theta_1(1-\theta_1)(1-\theta_2)(4-\theta_2)}{G}+\frac{2\theta_1(4-\theta_2)(5-\theta_1-\theta_2)c}{G}$	$\frac{\theta_1\theta_2(1-\theta_1)(1-\theta_2)(4-\theta_2)}{G}+\frac{\theta_1\theta_2(4-\theta_2)(5-\theta_1-\theta_2)c}{G}$	+	+	+	+	+	+	+	+

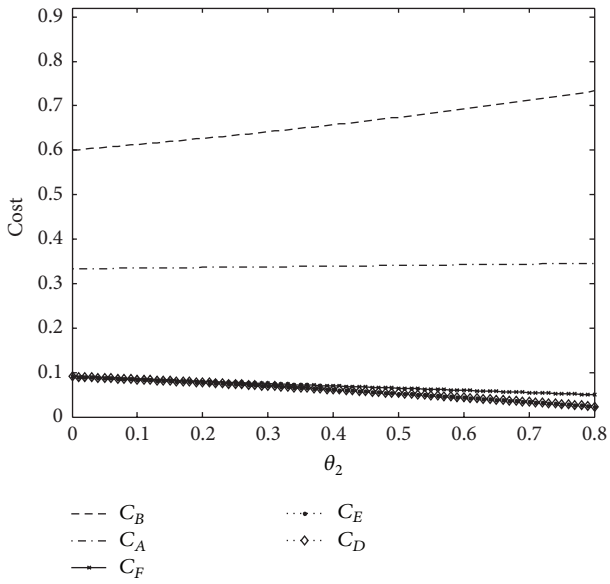

Figure 2: The impact of the overloaded degree of logistics service on equilibrium strategies.

Figure 2 illustrates how the overloaded degree of logistics (θ_2) affects the two sellers' pricing strategies. We can observe that (i) the cost threshold C_D approximately equals the cost threshold C_E, suggesting that the region for multiple equilibria is very small. Thus, although multiple equilibria may occur, it appears that such cases are very rare; (ii) as θ_2 increases, the probabilities of adopting Pricing Strategies I and V decrease while those for Pricing Strategies II, III, and IV increase. This is because, with more satisfying logistics services in the promotion sales period, the online channel has less incentive to transfer more consumers to the normal sales period from the promotion sales period. This would mitigate the channel competition in the normal sales period and intensify the competition in the promotion sales period. As a result, the traditional seller has a larger probability to coexist with online channel in the first period and is more likely to have no sales and even exit the market in the second period.

In the benchmark case and Scenario I, we have examined the impact of θ_2 on equilibrium decisions and profits under Pricing Strategies I, II, and III. We have obtained that as θ_2 increases, the selling prices of both channels in the two periods increase, as well as their profits. What is the impact of θ_2 under Pricing Strategies IV and V? We will answer the question in the following numerical examples.

Figures 3 and 4 show the prices and the profits of the traditional channel and online channel as θ_2 varies. We use parameter value $\theta_1 = 0.6$. To satisfy the cost conditions, we let $c = 0.12$ for Figure 3 (Pricing Strategy IV) and $c = 0.08$ for Figure 4 (Pricing Strategy V). Similar to Pricing Strategies I, II, and III, under Pricing Strategies IV and V, the first-period prices of the two channels, p_{R1} and p_{D1}, both increase in θ_2, as well as the second-period price of the online channel p_{D2} and their profits π_R and π_D. The second-period price for the traditional channel is nonincreasing in θ_2 under Pricing Strategy V. This happens because the consumer's willingness-to-pay for the products from the online channel

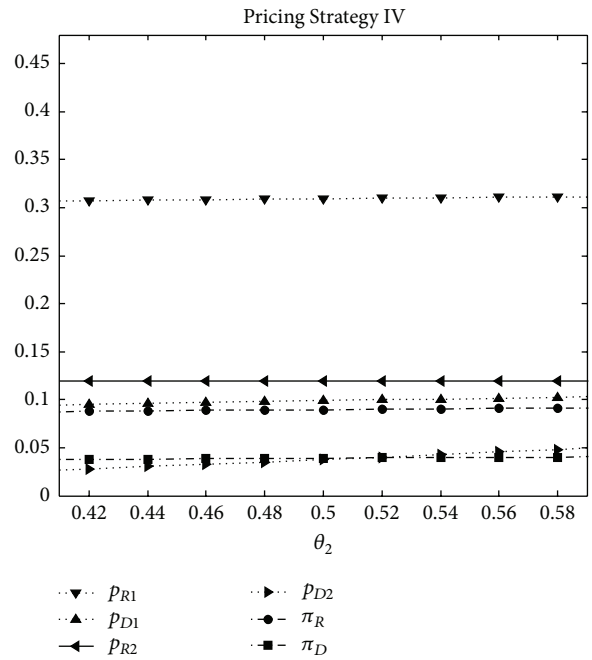

Figure 3: The impact of the overloaded degree of logistics service.

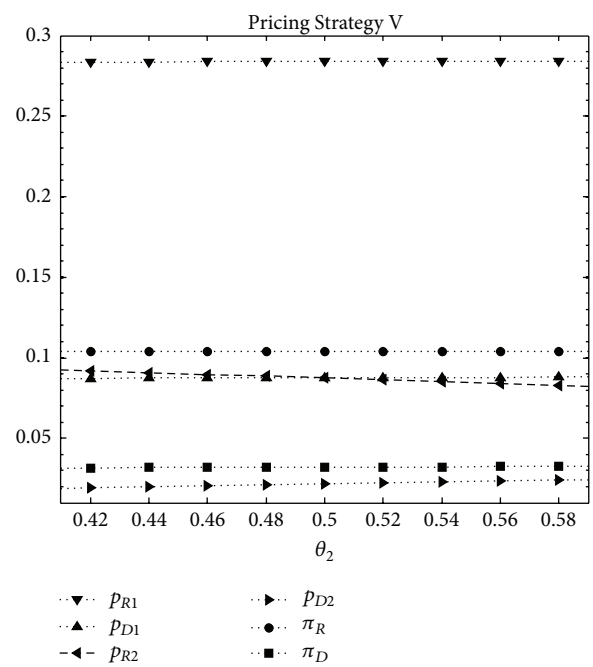

Figure 4: The impact of the overloaded degree of logistics service.

in the second period increases as θ_2 increases, and then the traditional channel reduces his/her price to compete with online channel. Furthermore, due to cost advantage, the online channel's prices are always lower than those of the traditional channel, which can usually be observed in practice.

6. Conclusion

Dual sales channel supply chain management is a hot research topic while online shopping is more and more

popular. The current literature focuses on the coordination of dual-channel and each member's pricing strategy. In this paper, we model the factor of overloaded logistics into dual sales channel and discuss each channel's optimal pricing strategy as well as how the express delivery services affect the price of the two channels. We summarize the main findings and discuss their managerial insights as follows.

To begin with, we formulate a scenario in which only online seller launches promotion sale. As the degree of logistics overloading increases, the first-period price of the online channel decreases since the online channel will have a stronger motivation to transfer consumers to Period 1 from Period 2. This implies that, with coming overloaded express logistics services, the online channel may provide price discounts in advance. For the traditional channel, although the overloaded degree of express delivery has no direct impact on him/her, it indirectly affects his/her prices through the online channel. Moreover, the higher logistics pressure exerted on the online channel in the promotion sales period leads to fiercer competitions between two channels in the normal sales period which will hurt the traditional channel's profit.

Then we formulate the model in which both online seller and traditional seller launch promotion sales in the second period. We find the following results. (i) When the cost advantage of the online channel is very large, the traditional seller provides no threat to the online channel. (ii) When the cost advantage of the online channel is large, the traditional seller's pricing decisions will affect the pricing of the online seller. (iii) When the cost advantage of the online channel is medium, the traditional channel has positive sales and profits in the first period, but he/she exits the market in the second period. (iv) When the cost advantage of the online channel is small, the traditional channel coexists with the online channel. (v) When the cost advantage of the online channel is very small, the market is shared by the two sellers in both periods.

We close the paper with discussions about its limitations and highlight possible avenues for future research. Our paper only formulates the overloaded degree of express delivery as a parameter into our model and analyzes the Nash equilibrium results. However, there are other factors influencing the Nash equilibrium, such as consumer's loyalty and traditional retailer's added value service. As we know, the online shopping is developing more and more quickly and many people are used to purchasing from online shops. Online sellers often launch promotion sales on special days or festival days. Future research may be directed toward online seller's policy to change consumer's evaluation of overloaded logistics service or online seller's strategy to avoid overloaded logistics service.

Appendix

Proof of Lemma 1. (i) With a given p_{D1}, the traditional seller's profit function is

$$\pi_R(p_{D1}) = \begin{cases} (p_{R1}-c)(1-p_{R1}) & p_{R1} < \dfrac{p_{D1}}{\theta_1} \\ (p_{R1}-c)\left(1-\dfrac{p_{R1}-p_{D1}}{1-\theta_1}\right) & \dfrac{p_{D1}}{\theta_1} \le p_{R1} < 1-\theta_1+p_{D1} \\ 0 & (1-\theta_1)+p_{D1} \le p_{R1} \le 1. \end{cases} \tag{A.1}$$

Using the first-order conditions for the profit function, we get two interior solutions as follows (denoted by Sol_i, $i = 1, 2$):

$$\begin{aligned} \mathrm{Sol}_1 &= \frac{(c+1)}{2} \ge c, \\ \mathrm{Sol}_2 &= \frac{(p_{D1}+c+1-\theta_1)}{2}. \end{aligned} \tag{A.2}$$

We can easily verify that the second-order conditions with respect to p_{R1} are nonpositive. Comparing the two solutions with boundary points p_{D1}/θ_1 and $1-\theta_1+p_{D1}$, we find

$$\begin{aligned} & c \le \mathrm{Sol}_1 \le \frac{p_{D1}}{\theta_1}, \\ & \mathrm{Sol}_2 \le \frac{p_{D1}}{\theta_1}, \\ & \qquad\qquad \text{when } p_{D1} \ge \frac{(c+1)\theta_1}{2}, \\ & \mathrm{Sol}_1 \ge \frac{p_{D1}}{\theta_1}, \\ & c \le \mathrm{Sol}_2 \le \frac{p_{D1}}{\theta_1}, \\ & \qquad \text{when } \frac{\theta_1(1-\theta_1+c)}{(2-\theta_1)} \le p_{D1} \le \frac{(c+1)\theta_1}{2}, \\ & \mathrm{Sol}_1 \ge \frac{p_{D1}}{\theta_1}, \\ & \mathrm{Sol}_2 \ge \max\left(\frac{p_{D1}}{\theta_1}, c\right), \\ & \qquad \text{when } \theta_1+c-1 < p_{D1} \le \frac{\theta_1(1-\theta_1+c)}{(2-\theta_1)}, \\ & \mathrm{Sol}_1 \ge \frac{p_{D1}}{\theta_1} > c, \\ & \mathrm{Sol}_2 \le c, \\ & \qquad\qquad \text{when } p_{D1} \le \theta_1+c-1. \end{aligned} \tag{A.3}$$

Notice that when $p_{D1} \le \theta_1 + c - 1$, the traditional seller's sales are zero with the condition $p_{R1} \ge c$, implying that the traditional channel cannot get any profit and will exit the market. From (A.3), we then directly get the corresponding conditions where the interior solutions Sol_1 and Sol_2 and the corner solutions $\mathrm{Sol}_3 = p_{D1}/\theta_1$ are optimal.

(ii) Given p_{R1}, we have rewritten the online seller's profit function as

$$\pi_D(p_{R1}) = \begin{cases} p_{D1}\left(1-\dfrac{p_{D1}}{\theta_1}\right)+\dfrac{(p_{D1})^2\theta_2}{4\theta_1^2} & 0 \le p_{D1} < p_{R1}+\theta_1-1 \\ \dfrac{p_{D1}(\theta_1 p_{R1}-p_{D1})}{\theta_1(1-\theta_1)}+\dfrac{(p_{D1})^2\theta_2}{4\theta_1^2} & p_{R1}+\theta_1-1 \le p_{D1} < \theta_1 p_{R1} \\ \dfrac{p_{R1}^2\theta_2}{4} & p_{D1} \ge \theta_1 p_{R1}. \end{cases} \tag{A.4}$$

The first-order conditions yield two interior solutions as follows (denoted by Sol_i, $i = 1, 2$):

$$\mathrm{Sol}_1 = \frac{2\theta_1^2}{(4\theta_1-\theta_2)}, \qquad \mathrm{Sol}_2 = \frac{2p_{R1}\theta_1^2}{(4\theta_1+\theta_1\theta_2-\theta_2)}. \tag{A.5}$$

We can easily verify that the second-order conditions with respect to p_{D1} are nonpositive. Comparing these two solutions with the boundary points $p_{R1}+\theta_1-1$ and $\theta_1 p_{R1}$, we find

$$\begin{aligned} &\mathrm{Sol}_1 \le p_{R1}+\theta_1-1, \\ &\mathrm{Sol}_2 \le p_{R1}+\theta_1-1, \\ &\qquad\qquad \text{when } p_{R1} \ge \frac{2\theta_1^2}{4\theta_1-\theta_2}+(1-\theta_1), \\ &\mathrm{Sol}_1 > p_{R1}+\theta_1-1, \\ &\mathrm{Sol}_2 < p_{R1}+\theta_1-1, \\ &\text{when } \frac{(1-\theta_1)(4\theta_1+\theta_1\theta_2-\theta_2)}{4\theta_1+\theta_1\theta_2-\theta_2-2\theta_1^2} < p_{R1} < \frac{2\theta_1^2}{4\theta_1-\theta_2}+(1-\theta_1), \\ &\mathrm{Sol}_1 > p_{R1}+\theta_1-1, \\ &p_{R1}+\theta_1-1 \le \mathrm{Sol}_2 \le \theta_1 p_{R1}, \\ &\qquad\qquad \text{when } p_{R1} \ge \frac{(1-\theta_1)(4\theta_1+\theta_1\theta_2-\theta_2)}{4\theta_1+\theta_1\theta_2-\theta_2-2\theta_1^2}. \end{aligned} \tag{A.6}$$

From (A.6), we then directly get the corresponding conditions where the interior solutions, Sol_1 and Sol_2, and the corner solutions $\mathrm{Sol}_3 = p_{R1}+\theta_1-1$ are optimal. □

Proof of Proposition 3. It can be easy to verify that $\partial p_{R1}^*/\partial\theta_2 \ge 0$, $\partial p_{D1}^*/\partial\theta_2 \ge 0$, $\partial p_{D2}^*/\partial\theta_2 \ge 0$, $\partial\pi_R^*/\partial\theta_2 > 0$, and $\partial\pi_D^*/\partial\theta_2 > 0$. □

Proof of Lemma 4. (i) Using the first-order conditions for the profit function in (20), we get two interior solutions as follows (denoted by PR_i, $i = 1, 2$):

$$PR_1 = \frac{\overline{v}+c}{2}, \qquad PR_2 = \frac{\overline{v}-\overline{v}\theta_2+p_{D2}+c}{2}. \tag{A.7}$$

We can easily verify that the second-order conditions with respect to p_{R2} are nonpositive. Similar to the proof of Lemma 1, we get the corresponding conditions where these interior solutions and boundary points are the equilibrium prices by comparing the values of interior solutions with the boundary points. By comparing, we find that when $0 \le p_{D2} < c-\overline{v}+\overline{v}\theta_2$, $\pi_{R2}(p_{R2})$ always increases in p_{R2} until it reaches the point $((1-\theta_2)\overline{v}+p_{D2}, 0)$ and then remains unchanged at zero. This implies that if the online channel sets a very low price, the traditional channel cannot get any profit and will exit the market.

(ii) Using the first-order conditions for the profit function in (21), we get two interior solutions as follows (denoted by PD_i, $i = 1, 2$): $PD_1 = \theta_2\overline{v}/2$, $PD_2 = \theta_2 p_{R2}/2$. We can easily verify that the second-order conditions with respect to p_{D2} are nonpositive. Similar to the proof of Lemma 1, we get the corresponding conditions where these interior solutions and boundary points are the equilibrium prices by comparing the values of interior solutions with the boundary points. □

Proof of Lemma 7. Using the first-order conditions for the profit function in (27), we get two interior solutions as follows (denoted by PT_i, $i = 1, 2$):

$$PT_1 = \frac{1+c}{2} \ge c, \qquad PT_2 = \frac{16-8\theta_2+\theta_2^2+(8-4\theta_2+\theta_2^2)c}{2(12-4\theta_2+\theta_2^2)}. \tag{A.8}$$

We can easily verify that the second-order conditions with respect to p_{R1} are negative. Comparing the values of the two interior solutions with the boundary points $(2-\theta_2)c/(2(1-\theta_2))$, we find that if $c < 1-\theta_2$, then $PT_1 > (2-\theta_2)c/(2(1-\theta_2))$ and $PT_2 > (2-\theta_2)c/(2(1-\theta_2))$; otherwise, $PT_1 \le (2-\theta_2)c/(2(1-\theta_2))$ and $PT_2 \le (2-\theta_2)c/(2(1-\theta_2))$. In the former case, $\pi_R(p_{R1} \mid p_{D1})$ reaches its maximum value at point $p_{R1}(p_{D1}) = PT_2$. Considering the condition $p_{R1} < p_{D1}/\theta_1$, if $PT_2 < p_{D1}/\theta_1$, the traditional seller's optimal decision is $p_{R1}(p_{D1}) = PT_2$ and if $PT_2 \ge p_{D1}/\theta_1$, the optimal decision for the traditional seller is $p_{R1}(p_{D1}) = p_{D1}/\theta_1$. In the latter case, the traditional seller's optimal decision is $p_{R1}(p_{D1}) = PT_1$ or $p_{R1}(p_{D1}) = p_{D1}/\theta_1$, depending on whether $PT_1 < p_{D1}/\theta_1$ or not. □

Proof of Lemma 8. Using the first-order condition for the profit function in (31), we get the optimal price $p_{R1}{}^*(p_{D1}) = (p_{D1}+1+c-\theta_1)/2$, but only if this price is between p_{D1}/θ_1 and $p_{D1}+1-\theta_1$. Otherwise, the optimal price for the traditional seller is $p_{R1}{}^*(p_{D1}) = p_{D1}/\theta_1$. □

Proof of Proposition 9. From Lemma 7, the profit for the traditional channel under *low price strategy* is as follows: (i) when $c < 1-\theta_2$,

$$\pi_R{}^*(p_{D1}) = \begin{cases} \dfrac{(\theta_1 - p_{D1})(p_{D1} - c\theta_1)}{\theta_1{}^2} & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \\ \dfrac{-(12-4\theta_2+\theta_2{}^2)p_{D1}{}^2}{\theta_1{}^2(4-\theta_2)^2} - \left(\dfrac{1}{\theta_1} + \dfrac{(8-4\theta_2+\theta_2{}^2)c}{\theta_1(4-\theta_2)^2}\right)p_{D1} - c + \dfrac{(2-\theta_2)^2c^2}{(1-\theta_2)(4-\theta_2)^2} & \text{if } \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \le p_{D1} < \dfrac{\theta_1\left[16-8\theta_2+\theta_2{}^2+(8-4\theta_2+\theta_2{}^2)c\right]}{2(12-4\theta_2+\theta_2{}^2)} \\ \dfrac{(16-\theta_2{}^3)(1-c)^2 + (5c^2-10c+9)\theta_2{}^2 - (16c^2-40c+24)\theta_2}{4(1-\theta_2)(12-4\theta_2+\theta_2{}^2)} & \text{if } p_{D1} \ge \dfrac{\theta_1\left[16-8\theta_2+\theta_2{}^2+(8-4\theta_2+\theta_2{}^2)c\right]}{2(12-4\theta_2+\theta_2{}^2)} \end{cases} \tag{A.9}$$

and (ii) when $c \ge 1-\theta_2$,

$$\pi_R{}^*(p_{D1}) = \begin{cases} \dfrac{(\theta_1 - p_{D1})(p_{D1} - c\theta_1)}{\theta_1{}^2} & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(c+1)}{2} \\ \dfrac{(1-c)^2}{4} & \text{if } p_{D1} \ge \dfrac{\theta_1(c+1)}{2}. \end{cases} \tag{A.10}$$

From Lemma 8, the profit for the traditional channel under *medium pricing strategy* is

$$\pi_R{}^*(p_{D1}) = \pi_{R1}{}^*(p_{D1}) + \pi_{R2}{}^*\left(\frac{p_{D1}}{\theta_1}\right), \tag{A.11}$$

where

$$\pi_{R1}{}^*(p_{D1}) = \begin{cases} \dfrac{(p_{D1}+1-c-\theta_1)^2}{4(1-\theta_1)} & \text{if } \theta_1 c \le p_{D1} \le \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ \dfrac{(\theta_1 - p_{D1})(p_{D1} - c\theta_1)}{\theta_1{}^2} & \text{if } p_{D1} > \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1}, \end{cases}$$
$$\pi_{R2}{}^*(p_{D1}) = \begin{cases} 0 & \text{if } c\theta_1 \le p_{D1} < \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \\ \dfrac{(2p_{D1} - 2p_{D1}\theta_2 - 2c\theta_1 + c\theta_1\theta_2)^2}{(4-\theta_2)^2(1-\theta_2)\theta_1{}^2} & \text{if } p_{D1} \ge \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)}. \end{cases} \tag{A.12}$$

Recall that the *medium price strategy* is always better than the *high price strategy* for the online channel in the first period. Thus, in the following, we only compare the *low price strategy* with the *medium price strategy*.

Comparing the values of $\theta_1 c$, $\theta_1(1+c-\theta_1)/(2-\theta_1)$, $[\theta_1(2-\theta_2)c]/[2(1-\theta_2)]$, $\theta_1[16-8\theta_2+\theta_2{}^2+(8-4\theta_2+\theta_2{}^2)c]/[2(12-4\theta_2+\theta_2{}^2)]$, and $\theta_1(c+1)/2$, we have

$$\begin{aligned} \theta_1 c &\le \frac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \le \frac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ &\le \frac{\theta_1\left[16-8\theta_2+\theta_2{}^2+(8-4\theta_2+\theta_2{}^2)c\right]}{2(12-4\theta_2+\theta_2{}^2)}, \\ &\qquad \text{if } 0 \le c < \frac{2(1-\theta_1)(1-\theta_2)}{2-2\theta_1+\theta_1\theta_2}, \\ \theta_1 c &\le \frac{\theta_1(1+c-\theta_1)}{2-\theta_1} \le \frac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \\ &\le \frac{\theta_1\left[16-8\theta_2+\theta_2{}^2+(8-4\theta_2+\theta_2{}^2)c\right]}{2(12-4\theta_2+\theta_2{}^2)}, \\ &\qquad \text{if } \frac{2(1-\theta_1)(1-\theta_2)}{2-2\theta_1+\theta_1\theta_2} \le c < 1-\theta_2, \\ \theta_1 c &\le \frac{\theta_1(1+c-\theta_1)}{2-\theta_1} \le \frac{\theta_1(c+1)}{2} \le \frac{\theta_1(2-\theta_2)c}{2(1-\theta_2)}, \\ &\qquad \text{if } 1-\theta_2 \le c \le 1. \end{aligned} \tag{A.13}$$

We use superscripts "l" to represent the low price strategy and "m" to represent the medium price strategy.

When $0 \le c < 2(1-\theta_1)(1-\theta_2)/(2-2\theta_1+\theta_1\theta_2)$,

$$\pi_R^{m*}(p_{D1}) - \pi_R^{l\,*}(p_{D1}) = \begin{cases} \ge 0 & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \\ \ge 0 & \text{if } \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} < p_{D1} \le \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ = 0 & \text{if } \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \le p_{D1} < \dfrac{\theta_1\left[16-8\theta_2+\theta_2^2+\left(8-4\theta_2+\theta_2^2\right)c\right]}{2\left(12-4\theta_2+\theta_2^2\right)} \\ \le 0 & \text{if } p_{D1} \ge \dfrac{\theta_1\left[16-8\theta_2+\theta_2^2+\left(8-4\theta_2+\theta_2^2\right)c\right]}{2\left(12-4\theta_2+\theta_2^2\right)}. \end{cases} \tag{A.14}$$

When $2(1-\theta_1)(1-\theta_2)/(2-2\theta_1+\theta_1\theta_2) \le c < 1-\theta_2$,

$$\pi_R^{m*}(p_{D1}) - \pi_R^{l\,*}(p_{D1}) = \begin{cases} \ge 0 & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ = 0 & \text{if } \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \le p_{D1} < \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \\ = 0 & \text{if } \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \le p_{D1} < \dfrac{\theta_1\left[16-8\theta_2+\theta_2^2+\left(8-4\theta_2+\theta_2^2\right)c\right]}{2\left(12-4\theta_2+\theta_2^2\right)} \\ \le 0 & \text{if } p_{D1} \ge \dfrac{\theta_1\left[16-8\theta_2+\theta_2^2+\left(8-4\theta_2+\theta_2^2\right)c\right]}{2\left(12-4\theta_2+\theta_2^2\right)}. \end{cases} \tag{A.15}$$

When $1-\theta_2 \le c \le 1$,

$$\pi_R^{m*}(p_{D1}) - \pi_R^{l\,*}(p_{D1}) = \begin{cases} \ge 0 & \text{if } \theta_1 c \le p_{D1} < \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \\ = 0 & \text{if } \dfrac{\theta_1(1+c-\theta_1)}{2-\theta_1} \le p_{D1} < \dfrac{\theta_1(c+1)}{2} \\ \le 0 & \text{if } \dfrac{\theta_1(c+1)}{2} \le p_{D1} < \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)} \\ \le 0 & \text{if } p_{D1} \ge \dfrac{\theta_1(2-\theta_2)c}{2(1-\theta_2)}. \end{cases} \tag{A.16}$$

From (A.14), (A.15), and (A.16), we get the prices as follows:

(i) If $0 \le c < 1-\theta_2$, the optimal first-period price for the traditional seller with a given p_{D1} is shown in (33).

(ii) If $1-\theta_2 \le c \le 1$, the optimal first-period price for the traditional seller with a given p_{D1} is shown in (34). □

Proof of Lemma 10. Using the first-order conditions for the profit function in (37), we get two interior solutions as follows (denoted by PO_i, $i = 1, 2$):

$$PO_1 = \frac{2\theta_1^2 p_{R1}}{4\theta_1 - \theta_2 + \theta_1\theta_2}, \tag{A.17}$$

$$PO_2 = \frac{\theta_1^2\theta_2 p_{R1} + (2-\theta_2)(1-\theta_1)\theta_1 c}{2(2\theta_1\theta_2 + 1 - \theta_1 - \theta_2)}, \tag{A.18}$$

$$PO_3 = \frac{\theta_1^2(4-\theta_2)^2 p_{R1} + \theta_1\theta_2(1-\theta_1)c}{2\left(16\theta_1 - 7\theta_1\theta_2 - \theta_2 + \theta_2^2\right)}. \tag{A.19}$$

We can easily verify that the second-order conditions with respect to p_{D1} are negative. Comparing the values of the two interior solutions with the boundary points $\mathrm{BP}_1 = \theta_1 c$, $\mathrm{BP}_2 = 2\theta_1 c/(2-\theta_2)$, and $\mathrm{BP}_3 = [(2-\theta_2)\theta_1 c]/[2(1-\theta_2)]$, we find that (i) if $c \le p_{R1} < [(4\theta_1 - \theta_2 + \theta_1\theta_2)c]/2\theta_1$, then $PO_1 \le \theta_1 c$, $PO_2 < \mathrm{BP}_2$, $PO_3 < \mathrm{BP}_3$, implying that the profit function in (37) decreases in p_{D1}, so the optimal price for the online channel is $p_{D1}{}^*(p_{R1}) = \theta_1 c$. (ii) If $[(4\theta_1 - \theta_2 + \theta_1\theta_2)c]/2\theta_1 \le p_{R1} < [(4\theta_1 - \theta_2 + \theta_1\theta_2)c]/[(2-\theta_2)\theta_1]$, then $\theta_1 c \le PO_1 \le \mathrm{BP}_2$, $PO_2 < \mathrm{BP}_2$, and $PO_3 < \mathrm{BP}_3$, and the optimal price for the online channel is $p_{D1}{}^*(p_{R1}) = PO_1$. (iii) If $[(4\theta_1 - \theta_2 + \theta_1\theta_2)c]/[(2-\theta_2)\theta_1]/2\theta_1 \le p_{R1} < [(8\theta_1 + \theta_2^2 - 5\theta_1\theta_2 - \theta_2)c]/[\theta_1(1-\theta_2)(4-\theta_2)]$,

then $PO_1 > \mathrm{BP}_2$, $\mathrm{BP}_2 \le PO_2 \le \mathrm{BP}_3$, and $PO_3 \le \mathrm{BP}_3$, and the optimal price for the online channel is $p_{D1}{}^*(p_{R1}) = PO_2$.
(iv) If $[(8\theta_1 + \theta_2{}^2 - 5\theta_1\theta_2 - \theta_2)c]/[\theta_1(1-\theta_2)(4-\theta_2)] \le p_{R1} < [(2-\theta_2)c/(1-\theta_2)]$, then $PO_1 > \mathrm{BP}_2$, $\mathrm{BP}_2 \le PO_2 \le \mathrm{BP}_3$, and $PO_3 \ge \mathrm{BP}_3$; in this case, the online seller's optimal decision is PO_2 or PO_3, depending on whether $\pi_D(PO_2 \mid p_{R1}) \ge \pi_D(PO_3 \mid p_{R1})$ or not. Some simple calculations show that $\pi_D(PO_2 \mid p_{R1}) - \pi_D(PO_3 \mid p_{R1}) \ge 0$ if $[(8\theta_1 + \theta_2{}^2 - 5\theta_1\theta_2 - \theta_2)c]/[\theta_1(1-\theta_2)(4-\theta_2)] \le p_{R1} < \overline{PR}$ holds and $\pi_D(PO_2 \mid p_{R1}) - \pi_D(PO_3 \mid p_{R1}) \le 0$ if $\overline{PR} \le p_{R1} < [(2-\theta_2)c/(1-\theta_2)]$.
(v) If $p_{R1} \ge [(2-\theta_2)c/(1-\theta_2)]$, then $PO_1 > \mathrm{BP}_2$, $PO_2 \ge \mathrm{BP}_3$, and $PO_3 \ge \mathrm{BP}_3$. The online seller's optimal decision in this case is $p_{D1}{}^*(p_{R1}) = PO_3$. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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