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Research Article

Disturbance Compensation Based Finite-Time Tracking Control of Rigid Manipulator

Mohamed Elamin Sahabi,[1,2,3] Guipu Li,[1,2] Xiangyu Wang,[1,2] and Shihua Li[1,2]

[1] *School of Automation, Southeast University, Nanjing 210096, China*
[2] *Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, China*
[3] *Faculty of Engineering, Blue Nile University, P.O. Box 143, Ar-Rossieres, Blue Nile, Sudan*

Correspondence should be addressed to Xiangyu Wang; w.x.y@seu.edu.cn

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The finite-time tracking control problem of rigid manipulator system with mismatched disturbances is investigated via a composite control method. The proposed composite controller is based on finite-time disturbance observer and adding a power integrator technique. First, a finite-time disturbance observer is designed which guarantees that the disturbances can be estimated in a finite time. Then, a composite controller is developed based on adding a power integrator approach and the estimates of the disturbances. Under the proposed composite controller, the manipulator position can track the desired position in a finite time. Simulation results show the effectiveness of the proposed control scheme.

1. Introduction

Due to some superior capacities, such as high accuracy, high stiffness, and high load-carrying [1], manipulator systems have been widely used in robots [2], parallels [3], and mechanical systems [4]. To achieve the control goals for manipulators and improve the performances of the closed-loop manipulator systems, PID control laws were given in [5–7], sliding mode control techniques were adopted in [8, 9], adaptive control laws were designed in [10–13], robust control algorithm was presented in [14], and neural network based control law was shown in [15].

From the point view of convergence rates, most of the existing results on control of rigid manipulators achieve asymptotic stability results for the closed-loop systems [5–15]. In other words, the system convergence rates are at best exponential. Compared with asymptotically stable systems, finite-time stable systems usually demonstrate faster convergence rates, better disturbance rejection properties, and robustness against uncertainties [16, 17]. Due to such nice features, recently, the finite-time control techniques have gained increasing attention from researchers [16–36]. In these literatures, finite-time control results for manipulators are shown in [32–36].

Note that the aforementioned control schemes for manipulators with disturbances work in a robust way, which implies that the disturbances attenuation is at the price of sacrificing their nominal control performances. To improve this problem, a feasible way is to use feedforward-feedback composite control rather than pure feedback control to solve the control problems of manipulators with disturbances. Disturbance observer based control (DOBC) is an effective composite control method, which is composed of disturbance observer (DO) design and nominal feedback controller design [37–40]. Compared with feedback control methods, DOBC has several superiorities, such as faster rejection of disturbances and recovery of the nominal performances. Due to such nice features, DOBC approaches have been adopted in [41–43] to reject disturbances for manipulator systems. However, the DOBC methods in these papers for the control of manipulators still have several aspects to be improved. On one hand, in the existing literatures, most DOBC techniques are only available for systems with matched disturbances: namely, the disturbances enter the system in the same channel with

control inputs. On the other hand, the estimated disturbances converge to the real disturbances as time goes to infinity. To reject disturbances in a shorter time, a disturbance observer which provides a faster convergence rate is desired. The sliding mode differentiator in [44, 45] is such an observer and it has been utilized by our research group to solve the control problem of manipulators with matched disturbances in [46]. In practice, there are mismatched disturbances in the manipulators. However, up to now, there are still no finite-time control results through DOBC methods for manipulators in the presence of mismatched disturbances.

In this paper, a composite control scheme is developed for manipulator systems with mismatched disturbances. The composite control algorithm is designed based on finite-time disturbance observer (FTDO) and adding a power integrator control method. Under the proposed composite controller, in the presence of mismatched disturbances, the manipulator position can track the desired position in a finite time.

The remainder of this paper is organized as follows. Section 2 presents useful definitions and lemmas. The manipulator system model is given in Section 3. The control design is presented in Section 4. Section 5 shows the simulation results and the conclusions of this paper are drawn in Section 6.

2. Preliminaries

Consider the following nonlinear autonomous system:

$$\dot{x} = f(x), \quad x \subseteq \mathbb{R}^n, \ f(0) = 0, \tag{1}$$

where $f : D \to \mathbb{R}^n$ satisfies the locally Lipschitz continuous condition. Under this condition, the definition of finite-time stability can be described as follows.

Definition 1 (finite-time stability [16, 18]). The equilibrium $x = 0$ of system (1) is finite-time convergent if there are an open neighbourhood U of the origin and a function $T_x : U \setminus \{0\} \to (0, \infty)$, such that every solution trajectory $x(t, x_0)$ of system (1) starting from the initial point $x_0 \in U \setminus \{0\}$ is well-defined and unique in forward time for $t \in [0, T_x(x_0))$, and $\lim_{t \to T_x(x_0)} x(t, x_0) = 0$. Here $T_x(x_0)$ is called the convergence time (with respect to the initial state x_0). The equilibrium of system (1) is finite-time stable if it is Lyapunov stable and finite-time convergent. If $U = D = \mathbb{R}^n$, the origin is a globally finite-time stable equilibrium.

Definition 2 (homogeneity [18]). $V(x) : \mathbb{R}^n \to \mathbb{R}$ is a continuous function. $V(x)$ is homogeneous of degree $k > 0$ with respect to the dilation $(r_1, \ldots, r_n)$, if

$$V(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^k V(x), \quad \forall \varepsilon \in \mathbb{R}, \ x \in \mathbb{R}^n. \tag{2}$$

Let $(r_1, \ldots, r_n) \in \mathbb{R}^n$ with $r_i > 0$, $i = 1, \ldots, n$, and let $f(x) : \mathbb{R}^n \to \mathbb{R}$ be a continuous vector field. $f(x)$ is said to be homogeneous of degree $k \in \mathbb{R}$ with respect to the dilation $(r_1, \ldots, r_n)$, if for any given $\varepsilon > 0$,

$$f_i(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^{k+r_i} f_i(x), \quad i = 1, \ldots, n, \ \forall x \in \mathbb{R}^n, \tag{3}$$

where $k \geq -\max\{r_i, \ i = 1, \ldots, n\}$.

Lemma 3 (see [16, 18]). *Considering system (1), suppose that there exists a continuous function $V(x) : U \to \mathbb{R}$ such that the following conditions hold.*

(i) *$V(x)$ is positive definite.*

(ii) *There exist real numbers $c > 0$, $\alpha \in (0, 1)$ and an open neighborhood $U_0 \subset U$ of the origin such that*

$$\dot{V}(x) + cV^{\alpha}(x) \leq 0, \quad x \in U_0 \setminus \{0\}. \tag{4}$$

Then the origin is a finite-time stable equilibrium of system (1). If $U = U_0 = \mathbb{R}^n$, the origin is a globally finite-time stable equilibrium of system (1).

Lemma 4 (see [20]). *Let c, d be positive real numbers and let $\gamma(x, y) > 0$ be a real-valued function. Then,*

$$|x|^c |y|^d \leq \frac{c\gamma(x, y)|x|^{c+d}}{c+d} + \frac{d\gamma^{-c/d}(x, y)|y|^{c+d}}{c+d}. \tag{5}$$

Lemma 5 (see [20]). *For any real numbers x_i, $i = 1, \ldots, n$, and $0 < a \leq 1$, the following inequality holds:*

$$(|x_1| + \cdots + |x_n|)^a \leq |x_1|^a + \cdots + |x_n|^a. \tag{6}$$

When $a = p/q \leq 1$, where $p > 0$ and $q > 0$ are odd integers, another inequality holds:

$$|x^a - y^a| \leq 2^{1-a} |x - y|^a. \tag{7}$$

3. Problem Formulation

The dynamics of a n link rigid manipulator can be written as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + d(t), \quad q \in \mathbb{R}^n, \tag{8}$$

where $q = [q_1, \ldots, q_n]^T$, $\dot{q}$ and $\ddot{q} \in \mathbb{R}^n$ denote the link position, velocity, and acceleration, respectively, $\tau = [\tau_1, \ldots, \tau_n]^T$ is the control input vector, $M(q) \in \mathbb{R}^{n \times n}$ is the positive-definite symmetric inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ denotes the Centrifugal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the influence of gravity, and $d(t) \in \mathbb{R}^n$ is the bounded input disturbance vector. For simplicity, matrices $M(q)$, $C(q, \dot{q})$, $G(q)$ are denoted by M, C, G for short in the following.

Let $x_1 = q$ and x_2 denote manipulator velocity. Consider mismatched disturbances and system parameter uncertainties: namely, $\dot{x}_1 = x_2 + d_1$, $M = M_0 + \Delta M$, $C = C_0(x_1, x_2 + d_1) + \Delta C$, and $G = G_0 + \Delta G$, where M_0, $C_0(x_1, x_2 + d_1)$, G_0 are the nominal values of M, C, G, respectively, and ΔM, ΔG, ΔC are the uncertain parts of M, C, G, respectively. Then, the dynamics of the manipulator can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1, \\ \dot{x}_2 &= M_0^{-1}(\tau - G_0) + d_2, \end{aligned} \tag{9}$$

where $d_1(t) \in \mathbb{R}^n$ denotes the mismatched disturbances and $d_2(t) = M_0^{-1}[d - \Delta M(\dot{x}_2 + \dot{d}_1) - M_0\dot{d}_1 - (C_0(x_1, x_2 + d_1) + \Delta C)(x_2 + d_1) - \Delta G] \in \mathbb{R}^n$ denotes the matched lumped disturbances.

4. Controller Design and Stability Analysis

Controller design is mainly composed of two parts, that is, disturbance observer design and composite controller design.

4.1. Finite-Time Disturbance Observer Design

Assumption 6. $d_1(t)$ and $d_2(t)$ in (9) are second-order differentiable and $\ddot{d}_1$, $\ddot{d}_2$ have Lipschitz constant vectors $L_1, L_2 \in \mathbb{R}^n$, respectively.

Remark 7. In practice, lots of disturbances satisfy Assumption 6, such as constant disturbances, ramp disturbances, and harmonic disturbances.

Under Assumption 6, to estimate the disturbances and their all-order derivatives, finite time disturbance observers will be designed in this subsection. Before proceeding, denote $\zeta = M_0^{-1}(\tau - G_0) = [\zeta_1, \ldots, \zeta_n]^T$. According to [44, 45], a disturbance observer for the ith ($i = 1, \ldots, n$) link of system (9) can be designed as

$$\begin{aligned}
\dot{\hat{x}}_{1,i} &= x_{2,i} + v_{1,i}^0, \\
v_{1,i}^0 &= -\lambda_{1,i}^0 L_{1,i}^{1/3} \left|\hat{x}_{1,i} - x_{1,i}\right|^{2/3} \operatorname{sgn}\left(\hat{x}_{1,i} - x_{1,i}\right) + \hat{d}_{1,i}, \\
\dot{\hat{d}}_{1,i} &= v_{1,i}^1 \\
&= -\lambda_{1,i}^1 L_{1,i}^{1/2} \left|\hat{d}_{1,i} - v_{1,i}^0\right|^{1/2} \operatorname{sgn}\left(\hat{d}_{1,i} - v_{1,i}^0\right) + \dot{\hat{d}}_{1,i}, \\
\dot{\hat{d}}_{1,i} &= v_{1,i}^2 = -\lambda_{1,i}^2 L_{1,i} \operatorname{sgn}\left(\dot{\hat{d}}_{1,i} - v_{1,i}^1\right), \\
\dot{\hat{x}}_{2,i} &= v_{2,i}^0 + \zeta_i, \\
v_{2,i}^0 &= -\lambda_{2,i}^0 L_{2,i}^{1/3} \left|\hat{x}_{2,i} - x_{2,i}\right|^{2/3} \operatorname{sgn}\left(\hat{x}_{2,i} - x_{2,i}\right) + \hat{d}_{2,i}, \\
\dot{\hat{d}}_{2,i} &= v_{2,i}^1 \\
&= -\lambda_{2,i}^1 L_{2,i}^{1/2} \left|\hat{d}_{2,i} - v_{2,i}^0\right|^{1/2} \operatorname{sgn}\left(\hat{d}_{2,i} - v_{2,i}^0\right) + \dot{\hat{d}}_{2,i}, \\
\dot{\hat{d}}_{2,i} &= v_{2,i}^2 = -\lambda_{2,i}^2 L_{2,i} \operatorname{sgn}\left(\dot{\hat{d}}_{2,i} - v_{2,i}^1\right),
\end{aligned} \tag{10}$$

where $\lambda_{1,i}^0, \lambda_{1,i}^1, \lambda_{1,i}^2, \lambda_{2,i}^0, \lambda_{2,i}^1, \lambda_{2,i}^2$ are the observer coefficients to be designed and $\hat{x}_{1,i}, \hat{x}_{2,i}, \hat{d}_{1,i}, \dot{\hat{d}}_{1,i}, \hat{d}_{2,i}, \dot{\hat{d}}_{2,i}$ are the estimates of $x_{1,i}, x_{2,i}, d_{1,i}, \dot{d}_{1,i}, d_{2,i}, \dot{d}_{2,i}$, respectively.

By letting $w_{1,i}^0 = \hat{x}_{1,i} - x_{1,i}$, $w_{2,i}^0 = \hat{x}_{2,i} - x_{2,i}$, $w_{1,i}^1 = \hat{d}_{1,i} - d_{1,i}$, $w_{2,i}^1 = \hat{d}_{2,i} - d_{2,i}$, $w_{1,i}^2 = \dot{\hat{d}}_{1,i} - \dot{d}_{1,i}$, $w_{2,i}^2 = \dot{\hat{d}}_{2,i} - \dot{d}_{2,i}$, the observation error system of the ith link can be obtained as follows:

$$\begin{aligned}
\dot{w}_{1,i}^0 &= -\lambda_{1,i}^0 L_{1,i}^{1/3} \left|w_{1,i}^0\right|^{2/3} \operatorname{sgn}\left(w_{1,i}^0\right) + w_{1,i}^1, \\
\dot{w}_{1,i}^1 &= -\lambda_{1,i}^1 L_{1,i}^{1/2} \left|w_{1,i}^1 - \dot{w}_{1,i}^0\right|^{1/2} \operatorname{sgn}\left(w_{1,i}^1 - \dot{w}_{1,i}^0\right) + w_{1,i}^2, \\
\dot{w}_{1,i}^2 &= -\lambda_{1,i}^2 L_{1,i} \operatorname{sgn}\left(w_{1,i}^2 - \dot{w}_{1,i}^1\right) - \ddot{d}_{1,i}, \\
\dot{w}_{2,i}^0 &= -\lambda_{2,i}^0 L_{2,i}^{1/3} \left|w_{2,i}^0\right|^{2/3} \operatorname{sgn}\left(w_{2,i}^0\right) + w_{2,i}^1, \\
\dot{w}_{2,i}^1 &= -\lambda_{2,i}^1 L_{2,i}^{1/2} \left|w_{2,i}^1 - \dot{w}_{2,i}^0\right|^{1/2} \operatorname{sgn}\left(w_{2,i}^1 - \dot{w}_{2,i}^0\right) + w_{2,i}^2, \\
\dot{w}_{2,i}^2 &= -\lambda_{2,i}^2 L_{2,i} \operatorname{sgn}\left(w_{2,i}^2 - \dot{w}_{2,i}^1\right) - \ddot{d}_{2,i}.
\end{aligned} \tag{11}$$

From [44, 45], it can be obtained that disturbance observer error system (11) is finite-time stable, that is, there exists a finite time $t_{1,i}$ such that $w_{1,i}^0 = 0$, $w_{2,i}^0 = 0$, $w_{1,i}^1 = 0$, $w_{2,i}^1 = 0$, $w_{1,i}^2 = 0$, $w_{2,i}^2 = 0$ for $t \geq t_{1,i}$. Then there is a finite time t_1 such that disturbance observation errors of n links equal to zero while $t \geq t_1$.

4.2. Finite-Time Controller Design

Theorem 8. *Assume that the desired position vector $q_d = [q_{d1}, \ldots, q_{dn}]^T$ is twice differentiable. For system (9), if Assumption 6 holds and controller τ is designed as*

$$\tau = M_0 \left\{\ddot{q}_d - \dot{\hat{d}}_1 - \hat{d}_2 - u\right\} + G_0, \tag{12}$$

the manipulator position x_1 will track the desired position q_d in a finite time, where $1 < r = r_1/r_2 < 2$, r_1, r_2 are positive odd integers, $k_1 = [k_{1,1}, \ldots, k_{1,n}]^T$, $k_2 = [k_{2,1}, \ldots, k_{2,n}]^T$, $k_{1,i} > 0$, $k_{2,i} > \{(2^{1-1/r}/(1+r))[2^{3-1/r} r/(1+r)k_{1,i}]^r + (1/k_{1,i}) + (r/(1+r))[4/(1+r)k_{1,i}]^{1/r}\}$, $i = 1, \ldots, n$.

Proof. Defining the tracking errors as $e_1 = [e_{1,1}, \ldots, e_{1,n}]^T = q_d - x_1$, $e_2 = [e_{2,1}, \ldots, e_{2,n}]^T = \dot{q}_d - x_2$, then the following tracking error system can be derived:

$$\begin{aligned}
\dot{e}_1 &= e_2 - d_1, \\
\dot{e}_2 &= \ddot{q}_d - M_0^{-1}(\tau - G_0) - d_2.
\end{aligned} \tag{13}$$

Let $\tilde{e}_1 = e_1$, $\tilde{e}_2 = e_2 - \hat{d}_1$. Then system (13) can be written as

$$\begin{aligned}
\dot{\tilde{e}}_1 &= \tilde{e}_2 + \hat{d}_1 - d_1, \\
\dot{\tilde{e}}_2 &= \ddot{q}_d - M_0^{-1}(\tau - G_0) - \hat{d}_2 - \dot{\hat{d}}_1 + \hat{d}_2 - d_2.
\end{aligned} \tag{14}$$

The stability analysis for closed-loop system (12) and (14) is shown in the following.

The stability analysis can be divided into two steps. In Step 1, system states of (12) and (14) will not escape to infinity for $t < t_1$. In Step 2, system (12) and (14) is finite time stable when $t \geq t_1$.

Step 1. For the ith ($i = 1, \ldots, n$) link, an energy function is defined as

$$\begin{aligned}
B_i\left(x_i, \hat{x}_i, \hat{d}_i, \dot{\hat{d}}_i, \tilde{e}_i\right) = \frac{1}{2}\Big(&x_{1,i}^2 + x_{2,i}^2 + \hat{x}_{1,i}^2 + \hat{x}_{2,i}^2 + \hat{d}_{1,i}^2 \\
&+ \hat{d}_{2,i}^2 + \dot{\hat{d}}_{1,i}^2 + \dot{\hat{d}}_{2,i}^2 + \tilde{e}_{1,i}^2 + \tilde{e}_{2,i}^2\Big).
\end{aligned} \tag{15}$$

Since $d_{1,i}$ and $d_{2,i}$ are estimated by $\widehat{d}_{1,i}$ and $\widehat{d}_{2,i}$ in finite time t_1, respectively, then $|d_{1,i}-\widehat{d}_{1,i}|$ and $|d_{2,i}-\widehat{d}_{2,i}|$ are bounded for $t<t_1$. Thus, there exist constants $w_{d_{1,i}}^{\max}$ and $w_{d_{2,i}}^{\max}$ such that $|d_{1,i}-\widehat{d}_{1,i}|\le w_{d_{1,i}}^{\max}$, $|d_{2,i}-\widehat{d}_{2,i}|\le w_{d_{2,i}}^{\max}$, and $|\dot{\widehat{d}}_{1,i}-\dot{\widehat{d}}_{1,i}|\le w_{d_{1,i}}^{\max}$. Moreover, $x_{1,i}x_{2,i}\le(1/2)(x_{1,i}^2+x_{2,i}^2)$, $|x_{1,i}|^\alpha<1+|x_{1,i}|$ for any $0<\alpha<1$. Denote $K_{1,i}=k_{1,i}$, $K_{2,i}=(2-1/r)2^{1-1/r}k_{1,i}^{1+r}k_{2,i}$. Then the following inequalities can be established:

$$
\begin{aligned}
x_{1,i}\dot{x}_{1,i} &= x_{1,i}\left(x_{2,i}+\widehat{d}_{1,i}+d_{1,i}-\widehat{d}_{1,i}\right)\le\frac{1}{2}\left(x_{1,i}^2+x_{2,i}^2\right)\\
&\quad+\frac{1}{2}\left(x_{1,i}^2+\widehat{d}_{1,i}^2\right)+\frac{1}{2}\left(\left(w_{d_{1,i}}^{\max}\right)^2+x_{1,i}^2\right),\\
x_{2,i}\dot{x}_{2,i} &= x_{2,i}\left(\ddot{q}_{di}+K_{2,i}\left(\widetilde{e}_{2,i}^r+K_{1,i}^r\widetilde{e}_{1,i}\right)^{2/r-1}-\dot{\widehat{d}}_{1,i}\right.\\
&\quad\left.+d_{2,i}-\widehat{d}_{2,i}\right)\\
&\le\frac{1}{2}\left(\left(\ddot{q}_{di}+K_{2,i}+K_{2,i}K_{1,i}^{2-r}+w_{d_{2,i}}^{\max}\right)^2+x_{2,i}^2\right)+\frac{1}{2}\\
&\quad\cdot K_{2,i}\left(x_{2,i}^2+\widetilde{e}_{2,i}^2\right)+\frac{1}{2}K_{2,i}K_{1,i}^{2-r}\left(x_{2,i}^2+\widetilde{e}_{1,i}^2\right)\\
&\quad+\frac{1}{2}\left(x_{2,i}^2+\dot{\widehat{d}}_{1,i}^2\right),\\
\widehat{x}_{1,i}\dot{\widehat{x}}_{1,i} &= \widehat{x}_{1,i}\left(x_{2,i}\right.\\
&\quad\left.-\lambda_{1,i}^0L_{1,i}^{1/3}\left|\widehat{x}_{1,i}-x_{1,i}\right|^{2/3}\operatorname{sgn}\left(\widehat{x}_{1,i}-x_{1,i}\right)+\widehat{d}_{1,i}\right)\\
&\le\frac{1}{2}\left(\widehat{x}_{1,i}^2+x_{2,i}^2\right)+\lambda_{1,i}^0L_{1,i}^{1/3}\widehat{x}_{1,i}^2+\frac{1}{2}\lambda_{1,i}^0L_{1,i}^{1/3}\left(\widehat{x}_{1,i}^2\right.\\
&\quad\left.+x_{1,i}^2\right)+\frac{1}{2}\lambda_{1,i}^0L_{1,i}^{1/3}\left(\widehat{x}_{1,i}^2+\widehat{d}_{1,i}^2\right)+\frac{1}{2}\left(\left(\lambda_{1,i}^0L_{1,i}^{1/3}\right)^2\right.\\
&\quad\left.+\widehat{x}_{1,i}^2\right),\\
\widehat{x}_{2,i}\dot{\widehat{x}}_{2,i} &= \widehat{x}_{2,i}\left(-\lambda_{2,i}^0L_{2,i}^{1/3}\left|\widehat{x}_{2,i}-x_{2,i}\right|^{2/3}\operatorname{sgn}\left(\widehat{x}_{2,i}-x_{2,i}\right)\right.\\
&\quad\left.+\ddot{q}_{di}+K_{2,i}\left(\widetilde{e}_{2,i}^r+K_{1,i}^r\widetilde{e}_{1,i}\right)^{2/r-1}-\dot{\widehat{d}}_{1,i}\right)\\
&\le\lambda_{2,i}^0L_{2,i}^{1/3}\left(\widehat{x}_{2,i}^2+\frac{1}{2}\left(\widehat{x}_{2,i}^2+x_{2,i}^2\right)\right)+\frac{1}{2}\left(\widehat{x}_{2,i}^2+\dot{\widehat{d}}_{1,i}^2\right)\\
&\quad+\frac{1}{2}K_{2,i}\left(\widehat{x}_{2,i}^2+\widetilde{e}_{2,i}^2\right)+\frac{1}{2}K_{2,i}K_{1,i}^{2-r}\left(\widehat{x}_{2,i}^2+\widetilde{e}_{1,i}^2\right)\\
&\quad+\frac{1}{2}\left(\left(\ddot{q}_{di}+\lambda_{2,i}^0L_{2,i}^{1/3}+K_{2,i}+K_{2,i}K_{1,i}^{2-r}\right)^2+\widehat{x}_{2,i}^2\right),\\
\widehat{d}_{1,i}\dot{\widehat{d}}_{1,i} &= \widehat{d}_{1,i}\left(-\lambda_{1,i}^1L_{1,i}^{1/2}\left|\widehat{d}_{1,i}-v_{1,i}^0\right|^{1/2}\operatorname{sgn}\left(\widehat{d}_{1,i}-v_{1,i}^0\right)\right.\\
&\quad\left.+\dot{\widehat{d}}_{1,i}\right)\le2\lambda_{1,i}^1L_{1,i}^{1/2}\widehat{d}_{1,i}^2+\frac{1}{2}\lambda_{1,i}^1L_{1,i}^{1/2}\lambda_{1,i}^0L_{1,i}^{1/3}\left(2\widehat{d}_{1,i}^2\right.\\
&\quad\left.+\widehat{x}_{1,i}^2+x_{1,i}^2\right)+\frac{1}{2}\left(\widehat{d}_{1,i}^2+\dot{\widehat{d}}_{1,i}^2\right)\\
&\quad+\frac{1}{2}\left(\left(\lambda_{1,i}^1L_{1,i}^{1/2}+\lambda_{1,i}^1L_{1,i}^{1/2}\lambda_{1,i}^0L_{1,i}^{1/3}\right)^2+\widehat{d}_{1,i}^2\right),\\
\widehat{d}_{2,i}\dot{\widehat{d}}_{2,i} &= \widehat{d}_{2,i}\left(-\lambda_{2,i}^1L_{2,i}^{1/2}\left|\widehat{d}_{2,i}-v_{2,i}^0\right|^{1/2}\operatorname{sgn}\left(\widehat{d}_{2,i}-v_{2,i}^0\right)\right.\\
&\quad\left.+\dot{\widehat{d}}_{2,i}\right)\le2\lambda_{2,i}^1L_{2,i}^{1/2}\widehat{d}_{2,i}^2+\frac{1}{2}\lambda_{2,i}^1L_{2,i}^{1/2}\lambda_{2,i}^0L_{2,i}^{1/3}\left(2\widehat{d}_{2,i}^2\right.\\
&\quad\left.+\widehat{x}_{2,i}^2+x_{2,i}^2\right)+\frac{1}{2}\left(\widehat{d}_{2,i}^2+\dot{\widehat{d}}_{2,i}^2\right)\\
&\quad+\frac{1}{2}\left(\left(\lambda_{2,i}^1L_{2,i}^{1/2}+\lambda_{2,i}^1L_{2,i}^{1/2}\lambda_{2,i}^0L_{2,i}^{1/3}\right)^2+\widehat{d}_{2,i}^2\right),\\
\widehat{d}_{1,i}\dot{\widehat{d}}_{1,i} &= \widehat{d}_{1,i}\left(-\lambda_{1,i}^2L_{1,i}\operatorname{sgn}\left(\widehat{d}_{1,i}-v_{1,i}^1\right)\right)\\
&\le\frac{1}{2}\left(\left(\lambda_{1,i}^2L_{1,i}\right)^2+\dot{\widehat{d}}_{1,i}^2\right),\\
\widehat{d}_{2,i}\dot{\widehat{d}}_{2,i} &= \widehat{d}_{2,i}\left(-\lambda_{2,i}^2L_{2,i}\operatorname{sgn}\left(\widehat{d}_{2,i}-v_{2,i}^1\right)\right)\\
&\le\frac{1}{2}\left(\left(\lambda_{2,i}^2L_{2,i}\right)^2+\dot{\widehat{d}}_{2,i}^2\right),\\
\widetilde{e}_{1,i}\dot{\widetilde{e}}_{1,i} &= \widetilde{e}_{1,i}\left(\widetilde{e}_{2,i}+\widehat{d}_{1,i}-d_{1,i}\right)\le\frac{1}{2}\left(\widetilde{e}_{1,i}^2+\widetilde{e}_{2,i}^2\right)\\
&\quad+\frac{1}{2}\left(\left(w_{d_{1,i}}^{\max}\right)^2+\widetilde{e}_{1,i}^2\right),\\
\widetilde{e}_{2,i}\dot{\widetilde{e}}_{2,i} &= \widetilde{e}_{2,i}\left(-K_{2,i}\left(\widetilde{e}_{2,i}^r+K_{1,i}^r\widetilde{e}_{1,i}\right)^{2/r-1}+\dot{\widehat{d}}_{1,i}-\dot{\widehat{d}}_{1,i}\right.\\
&\quad\left.+\widehat{d}_{2,i}-d_{2,i}\right)\le K_{2,i}\widetilde{e}_{2,i}^2+\frac{1}{2}K_{1,i}^{2-r}K_{2,i}\left(\widetilde{e}_{1,i}^2+\widetilde{e}_{2,i}^2\right)\\
&\quad+\frac{1}{2}\left(\left(K_{2,i}+K_{1,i}^{2-r}K_{2,i}+w_{d_{2,i}}^{\max}+w_{d_{1,i}}^{\max}\right)^2+\widetilde{e}_{2,i}^2\right).
\end{aligned}
\tag{16}
$$

Taking the first derivative of $B_i(x_i,\widehat{x}_i,\widehat{d}_i,\dot{\widehat{d}}_i,\widetilde{e}_i)$ yields

$$\dot{B}_i\le K_iB_i+C_i, \tag{17}$$

where

$$
\begin{aligned}
K_i &= \max\left\{\frac{3}{2}+\frac{1}{2}\lambda_{1,i}^0L_{1,i}^{1/3}+\frac{1}{2}\lambda_{1,i}^1L_{1,i}^{1/2}\lambda_{1,i}^0L_{1,i}^{1/3},2+\frac{1}{2}\right.\\
&\quad\cdot K_{2,i}+\frac{1}{2}K_{2,i}K_{1,i}^{2-r}+\frac{1}{2}\lambda_{2,i}^0L_{2,i}^{1/3}+\frac{1}{2}\lambda_{2,i}^1L_{2,i}^{1/2}\lambda_{2,i}^0L_{2,i}^{1/3},\\
&\quad1+2\lambda_{1,i}^0L_{1,i}^{1/3}+\frac{1}{2}\lambda_{1,i}^1L_{1,i}^{1/2}\lambda_{1,i}^0L_{1,i}^{1/3},1+\frac{1}{2}K_{2,i}+\frac{1}{2}\\
&\quad\cdot K_{2,i}K_{1,i}^{2-r}+\frac{3}{2}\lambda_{2,i}^0L_{2,i}^{1/3}+\frac{1}{2}\lambda_{2,i}^1L_{2,i}^{1/2}\lambda_{2,i}^0L_{2,i}^{1/3},\frac{3}{2}+\frac{1}{2}\\
&\quad\cdot\lambda_{1,i}^0L_{1,i}^{1/3}+2\lambda_{1,i}^1L_{1,i}^{1/2}+\lambda_{1,i}^1L_{1,i}^{1/2}\lambda_{1,i}^0L_{1,i}^{1/3},1\\
&\quad+2\lambda_{2,i}^1L_{2,i}^{1/2}+\lambda_{2,i}^1L_{2,i}^{1/2}\lambda_{2,i}^0L_{2,i}^{1/3},2,1,1+\frac{3}{2}K_{2,i}K_{1,i}^{2-r},1\\
&\quad\left.+2K_{2,i}+\frac{1}{2}K_{2,i}K_{1,i}^{2-r}\right\},
\end{aligned}
$$

$$C = \max\left\{\frac{1}{2}\left(w_{d_{1,i}}^{\max}\right)^2,\right.$$
$$\frac{1}{2}\left(\ddot{q}_{di} + K_{2,i} + K_{2,i}K_{1,i}^{2-r} + w_{d_{2,i}}^{\max}\right)^2, \frac{1}{2}\left(\lambda_{1,i}^0 L_{1,i}^{1/3}\right)^2,$$
$$\frac{1}{2}\left(\ddot{q}_{di} + \lambda_{2,i}^0 L_{2,i}^{1/3} + K_{2,i} + K_{2,i}K_{1,i}^{2-r}\right)^2,$$
$$\frac{1}{2}\left(\lambda_{1,i}^1 L_{1,i}^{1/2} + \lambda_{1,i}^1 L_{1,i}^{1/2}\lambda_{1,i}^0 L_{1,i}^{1/3}\right)^2,$$
$$\frac{1}{2}\left(\lambda_{2,i}^1 L_{2,i}^{1/2} + \lambda_{2,i}^1 L_{2,i}^{1/2}\lambda_{2,i}^0 L_{2,i}^{1/3}\right)^2, \frac{1}{2}\left(\lambda_{1,i}^2 L_{1,i}\right)^2,$$
$$\left.\frac{1}{2}\left(\lambda_{2,i}^2 L_{2,i}\right)^2, \frac{1}{2}\left(K_{2,i} + K_{1,i}^{2-r}K_{2,i} + w_{d_{2,i}}^{\max} + w_{d_{1,i}}^{\max}\right)^2\right\}. \tag{18}$$

Solving inequality (17), it can be obtained that $B_i(x_i, \hat{x}_i, \hat{d}_i, \hat{\hat{d}}_i, \tilde{e}_i) \le (B_i(x_i(0), \hat{x}_i(0), \hat{d}_i(0), \hat{\hat{d}}_i(0), \tilde{e}_i(0)) + C_i/K_i)e^{K_i t} - C_i/K_i$. Hence, system states $\tilde{e}_{1,i}, \tilde{e}_{2,i}$ are bounded when $t < t_1$.

Step 2. It will be shown that closed-loop system (12) and (14) is finite time stable when $t \ge t_1$. Before proceeding, denote $u = [u_1, \ldots, u_n]^T = \ddot{q}_d - M_0^{-1}(\tau - G_0) - \hat{d}_2 - \hat{\hat{d}}_1$. While $t \ge t_1$, system (14) reduces to

$$\begin{aligned}\dot{\tilde{e}}_1 &= \tilde{e}_2,\\ \dot{\tilde{e}}_2 &= u.\end{aligned} \tag{19}$$

For the ith ($i = 1, \ldots, n$) link, choose a Lyapunov function $V_1(\tilde{e}_{1,i}) = \tilde{e}_{1,i}^2/2$. Taking the derivative of $V_1(\tilde{e}_{1,i})$ yields $\dot{V}_1(\tilde{e}_{1,i}) = \tilde{e}_{1,i}\tilde{e}_{2,i} = \tilde{e}_{1,i}(\tilde{e}_{2,i} - \tilde{e}_{2,i}^*) + \tilde{e}_{1,i}\tilde{e}_{2,i}^*$, where $\tilde{e}_{2,i}^*$ is a virtual control law which can be defined as $\tilde{e}_{2,i}^* = -k_{1,i}\tilde{e}_{1,i}^{1/r}$. Then the following inequality can be derived:

$$\dot{V}_1(\tilde{e}_{1,i}) \le -k_{1,i}\tilde{e}_{1,i}^{1+1/r} + \tilde{e}_{1,i}\left(\tilde{e}_{2,i} - \tilde{e}_{2,i}^*\right). \tag{20}$$

Define $\xi_{2,i} = \tilde{e}_{2,i}^r - \tilde{e}_{2,i}^{*r}$. Choose the Lyapunov function:

$$V_2(\tilde{e}_{1,i}, \tilde{e}_{2,i}) = V_1(\tilde{e}_{1,i}) + \frac{\int_{\tilde{e}_{2,i}^*}^{\tilde{e}_{2,i}}\left(s^r - \tilde{e}_{2,i}^{*r}\right)^{2-1/r} ds}{(2 - 1/r)\, 2^{1-1/r} k_{1,i}^{1+r}}. \tag{21}$$

By noting that $\partial\tilde{e}_{2,i}^{*r}/\partial\tilde{e}_{1,i} = -k_{1,i}^r$, the derivative of $V_2(\tilde{e}_{1,i}, \tilde{e}_{2,i})$ along system (19) satisfies

$$\begin{aligned}\dot{V}_2(\tilde{e}_{1,i}, \tilde{e}_{2,i}) \le{}& -k_{1,i}\tilde{e}_{1,i}^{1+1/r} + \tilde{e}_{1,i}\left(\tilde{e}_{2,i} - \tilde{e}_{2,i}^*\right)\\ &+ \frac{\tilde{e}_{2,i}}{2^{1-1/r}k_{1,i}}\int_{\tilde{e}_{2,i}^*}^{\tilde{e}_{2,i}}\left(s^r - \tilde{e}_{2,i}^{*r}\right)^{1-1/r} ds\\ &+ \frac{\xi_{2,i}^{2-1/r}u_i}{(2 - 1/r)\, 2^{1-1/r}k_{1,i}^{1+r}}.\end{aligned} \tag{22}$$

From Lemma 5, it can be obtained that $|\tilde{e}_{2,i} - \tilde{e}_{2,i}^*| \le 2^{1-1/r}|\xi_{2,i}|^{1/r}$. It follows from (22) that

$$\begin{aligned}\dot{V}_2(\tilde{e}_{1,i}, \tilde{e}_{2,i}) \le{}& -k_{1,i}\tilde{e}_{1,i}^{1+1/r} + 2^{1-1/r}\left|\tilde{e}_{1,i}\right|\left|\xi_{2,i}\right|^{1/r}\\ &+ \frac{\left|\tilde{e}_{2,i}\right|\left|\xi_{2,i}\right|}{k_{1,i}} + \frac{\xi_{2,i}^{2-1/r}u_i}{(2 - 1/r)\, 2^{1-1/r}k_{1,i}^{1+r}}.\end{aligned} \tag{23}$$

From Lemma 4, it can be obtained that

$$2^{1-1/r}\left|\tilde{e}_{1,i}\right|\left|\xi_{2,i}\right|^{1/r} \le \frac{2^{1-1/r}r}{1+r}\gamma\tilde{e}_{1,i}^{1+1/r} + \frac{2^{1-1/r}}{1+r}\gamma^{-r}\xi_{2,i}^{1+1/r}, \tag{24}$$

where γ can be any positive constant. Letting $(2^{1-1/r}/(1 + r))\gamma = k_{1,i}/4$ yields

$$\begin{aligned}2^{1-1/r}\left|\tilde{e}_{1,i}\right|\left|\xi_{2,i}\right|^{1/r} \le{}& \frac{k_{1,i}}{4}\tilde{e}_{1,i}^{1+1/r}\\ &+ \frac{2^{1-1/r}}{1+r}\left[\frac{2^{3-1/r}r}{(1+r)\,k_{1,i}}\right]^r \xi_{2,i}^{1+1/r}.\end{aligned} \tag{25}$$

Note that $\tilde{e}_{2,i} = (\xi_{2,i} + \tilde{e}_{2,i}^{*r})^{1/r}$. Using Lemma 5, it can be obtained that $|\tilde{e}_{2,i}| \le |\xi_{2,i}|^{1/r} + |\tilde{e}_{2,i}^{*r}| = |\xi_{2,i}|^{1/r} + k_{1,i}|\tilde{e}_{1,i}|^{1/r}$; thus, $|\tilde{e}_{2,i}\xi_{2,i}|/k_{1,i} \le \xi_{2,i}^{1+1/r}/k_{1,i} + |\tilde{e}_{1,i}|^{1/r}|\xi_{2,i}|$. It follows from Lemma 4 that

$$\left|\tilde{e}_{1,i}\right|^{1/r}\left|\xi_{2,i}\right| \le \frac{\gamma}{1+r}\tilde{e}_{1,i}^{1+1/r} + \frac{r}{1+r}\gamma^{-1/r}\xi_{2,i}^{1+1/r}, \tag{26}$$

where γ can be any positive constant. By letting $\gamma/(1 + r) = k_{1,i}/4$, it follows that

$$\begin{aligned}\frac{\left|\tilde{e}_{2,i}\xi_{2,i}\right|}{k_{1,i}} \le{}& \frac{\xi_{2,i}^{1+1/r}}{k_{1,i}} + \frac{k_{1,i}}{4}\tilde{e}_{1,i}^{1+1/r}\\ &+ \frac{r}{1+r}\left[\frac{4}{(1+r)\,k_{1,i}}\right]^{1/r}\xi_{2,i}^{1+1/r}.\end{aligned} \tag{27}$$

Substituting (24) and (27) into (28) yields

$$\begin{aligned}\dot{V}_2(\tilde{e}_{1,i}, \tilde{e}_{2,i}) \le{}& -\frac{k_{1,i}}{2}\tilde{e}_{1,i}^{1+1/r} + \delta\xi_{2,i}^{1+1/r}\\ &+ \frac{\xi_{2,i}^{2-1/r}u_i}{(2 - 1/r)\, 2^{1-1/r}k_{1,i}^{1+r}},\end{aligned} \tag{28}$$

where

$$\begin{aligned}\delta = \left\{\frac{2^{1-1/r}}{1+r}\left[\frac{2^{3-1/r}r}{(1+r)\,k_{1,i}}\right]^r + \frac{1}{k_{1,i}}\right.\\ \left.+ \frac{r}{1+r}\left[\frac{4}{(1+r)\,k_{1,i}}\right]^{1/r}\right\}.\end{aligned} \tag{29}$$

Controller u_i can be designed as

$$u_i = -\left(2 - \frac{1}{r}\right)2^{1-1/r}k_{1,i}^{1+r}k_{2,i}\left(\tilde{e}_{2,i}^r + k_{1,i}^r\tilde{e}_{1,i}\right)^{2/r-1}, \tag{30}$$

where $k_{2,i} > \delta$. Substituting (30) into (28) yields

$$\dot{V}_2(\tilde{e}_{1,i}, \tilde{e}_{2,i}) \le -\frac{k_{1,i}}{2}\tilde{e}_{1,i}^{1+1/r} - k_{3,i}\xi_{2,i}^{1+1/r}, \tag{31}$$

where $k_{3,i} = k_{2,i} - \delta > 0$. By using Lemma 5, it follows that $\dot{V}_2 + cV_2^{(1+r)/2r} \le -k_i(\tilde{e}_{1,i}^{1+1/r} + \xi_{2,i}^{1+1/r})/2 \le 0$, where $k_i = \max\{k_{1,i}/2 - c(1/2)^{(1+r)/2r}, k_{3,i} - c(1/(2-1/r)k_{1,i}^{1+r})^{(1+r)/2r}\}$. According to Lemma 3, it can be concluded that closed-loop system (19) and (30) is finite-time stable. Then system (12) and (14) is finite-time stable; that is, manipulator position q can track the desired trajectory q_d in a finite time. This completes the proof. □

Remark 9. Under the condition without disturbance observer, with adding a power integrator method, a controller for manipulator system (9) can be designed as

$$\tau = M_0(\ddot{q}_d - \bar{u}) + G_0, \tag{32}$$

where $\bar{u} = [\bar{u}_1, \ldots, \bar{u}_2]^T$, $\bar{u}_i = -(2 - 1/\bar{r})2^{1-1/\bar{r}}\bar{k}_{1,i}^{1+\bar{r}}\bar{k}_{2,i}[(\dot{q}_{di} - x_{2,i})^{\bar{r}} + \bar{k}_{1,i}^{\bar{r}}(q_{di} - x_{1,i})]^{2/\bar{r}-1}$, $1 < \bar{r} = \bar{r}_1/\bar{r}_2 < 2$, $\bar{r}_1, \bar{r}_2$ are positive odd integers, $\bar{k}_{1,i} > 0$, $\bar{k}_{2,i} > \{(2^{1-1/\bar{r}}/(1+\bar{r}))[2^{3-1/\bar{r}}\bar{r}/(1+\bar{r})\bar{k}_{1,i}]^{\bar{r}} + 1/\bar{k}_{1,i} + (\bar{r}/(1+\bar{r}))[4/(1+\bar{r})\bar{k}_{1,i}]^{1/\bar{r}}\}$, $i = 1, \ldots, n$. Under controller (32), manipulator position q can not track the desired trajectory q_d in any long time, which will be shown in simulations.

5. Numerical Simulations

Simulations are conducted on a two-link rigid robot manipulator. The cases without/with output noises are considered in the simulations. The manipulator model is shown in Figure 1. The dynamic of the manipulator is

$$\begin{bmatrix} a_{11}(q_2) & a_{12}(q_2) \\ a_{12}(q_2) & a_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -\beta_{12}(q_2)\dot{q}_1^2 - 2\beta_{12}(q_2)\dot{q}_1\dot{q}_2 \\ \beta_{12}(q_2)\dot{q}_2^2 \end{bmatrix} + \begin{bmatrix} \gamma_1(q_1, q_2)g \\ \gamma_2(q_1, q_2)g \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \tag{33}$$

where

$$\begin{aligned} a_{11}(q_2) &= (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) + J_1, \\ a_{12}(q_2) &= m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2), \\ a_{22} &= m_2 l_2^2 + J_2, \\ \beta_{12}(q_2) &= m_2 l_1 l_2 \sin(q_2), \\ \gamma_1(q_1, q_2) &= (m_1 + m_2)l_1 \cos(q_2) + m_2 l_2 \cos(q_1 + q_2), \\ \gamma_2(q_1, q_2) &= m_2 l_2 \cos(q_1 + q_2). \end{aligned} \tag{34}$$

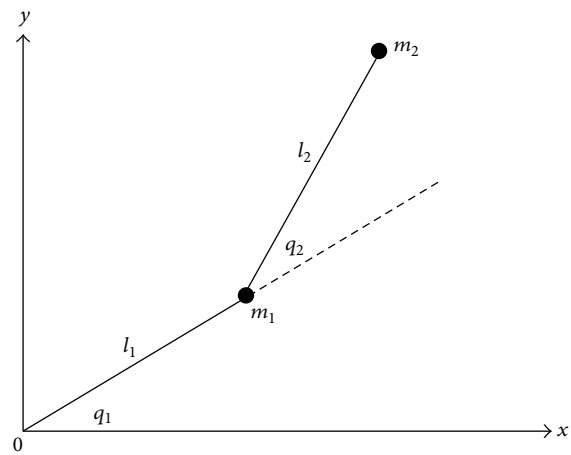

Figure 1: The two-link robot manipulator model.

The reference trajectories are [32]

$$\begin{aligned} q_{d1} &= 1.25 - 1.4e^{-t} + 0.35e^{-4t}, \quad t \ge 0, \\ q_{d2} &= 1.25 + e^{-t} - 0.25e^{-4t}, \quad t \ge 0. \end{aligned} \tag{35}$$

The system parameters and disturbances are selected as $l_1 = 1$ m, $l_2 = 0.8$ m, $J_1 = 5$ kgm, $J_2 = 5$ kgm, $m_1 = 0.4$ kg, $m_2 = 1.2$ kg, $\Delta m_1 = 0.004$ kg, $\Delta m_2 = 0.01$ kg, $d_1 = [0.03\cos(2t); 0.05\cos(2t)]$, $d = [3\sin(2t); 5\sin(2t)]$.

5.1. The Case without Output Measurement Noises. To validate the effectiveness of the proposed composite control algorithm, closed-loop system performances under composite controller (12) and under finite-time controller (32) will be compared in this part.

Taking the practical input saturation into consideration, the control inputs for both controller (12) and controller (32) are limited not to exceed 60 Nm. Under these limitations, efforts have been spent to make both closed-loop systems as good as possible. For composite controller (12), the parameters are chosen as $r = 15/13$, $k_1 = [0.8; 0.6]$, $k_2 = [4.8; 3.5]$. The observer gains and Lipschitz constants are chosen as $\lambda_1^0 = [12; 10]$, $\lambda_1^1 = [5; 5]$, $\lambda_1^2 = [2; 2]$, $\lambda_2^0 = [10; 8]$, $\lambda_2^1 = [2; 2]$, $\lambda_2^2 = [1; 0.5]$, $L_1 = [10; 20]$, $L_2 = [10; 20]$. For finite-time controller (32), the parameters are selected as $\bar{r} = 15/13$, $\bar{k}_1 = [0.8; 0.6]$, $\bar{k}_2 = [21; 23]$.

Simulation results are presented in Figures 2–4. Figures 2 and 3 show that observation errors of the disturbance observers converge to the origin in a finite time for both links of the manipulator. From Figure 4, it can be seen that manipulator positions can track the desired positions in a finite time under controller (12) while they cannot track the desired positions under controller (32).

5.2. The Case with Output Measurement Noises. In this part, in the presence of output measurement noises, the closed-loop system performances under composite controller (12) and under finite-time controller (32) are compared to validate the effectiveness of composite controller (12).

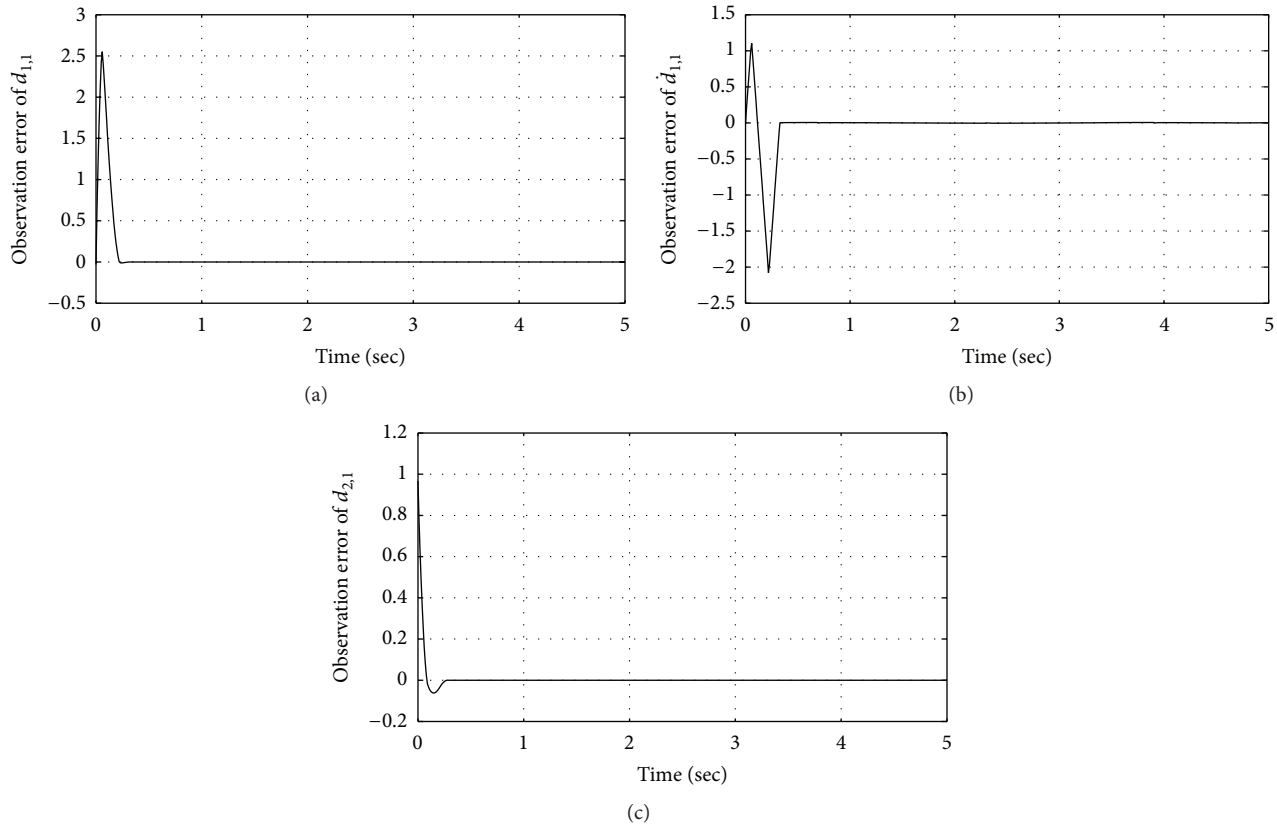

Figure 2: Disturbance observation errors of the 1st link. (a) Observation error of $d_{1,1}$. (b) Observation error of $\dot{d}_{1,1}$. (c) Observation error of $d_{2,1}$.

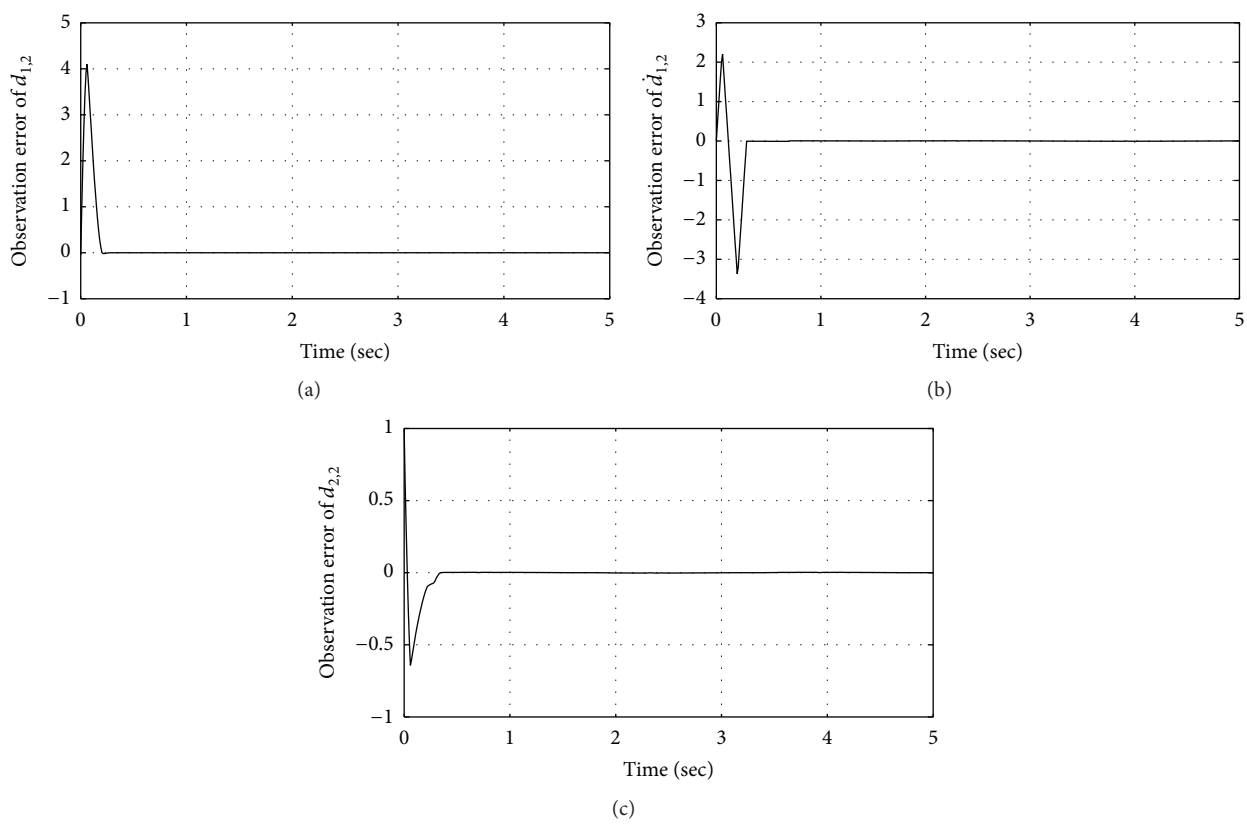

Figure 3: Disturbance observation errors of the 2nd link. (a) Observation error of $d_{1,2}$. (b) Observation error of $\dot{d}_{1,2}$. (c) Observation error of $d_{2,2}$.

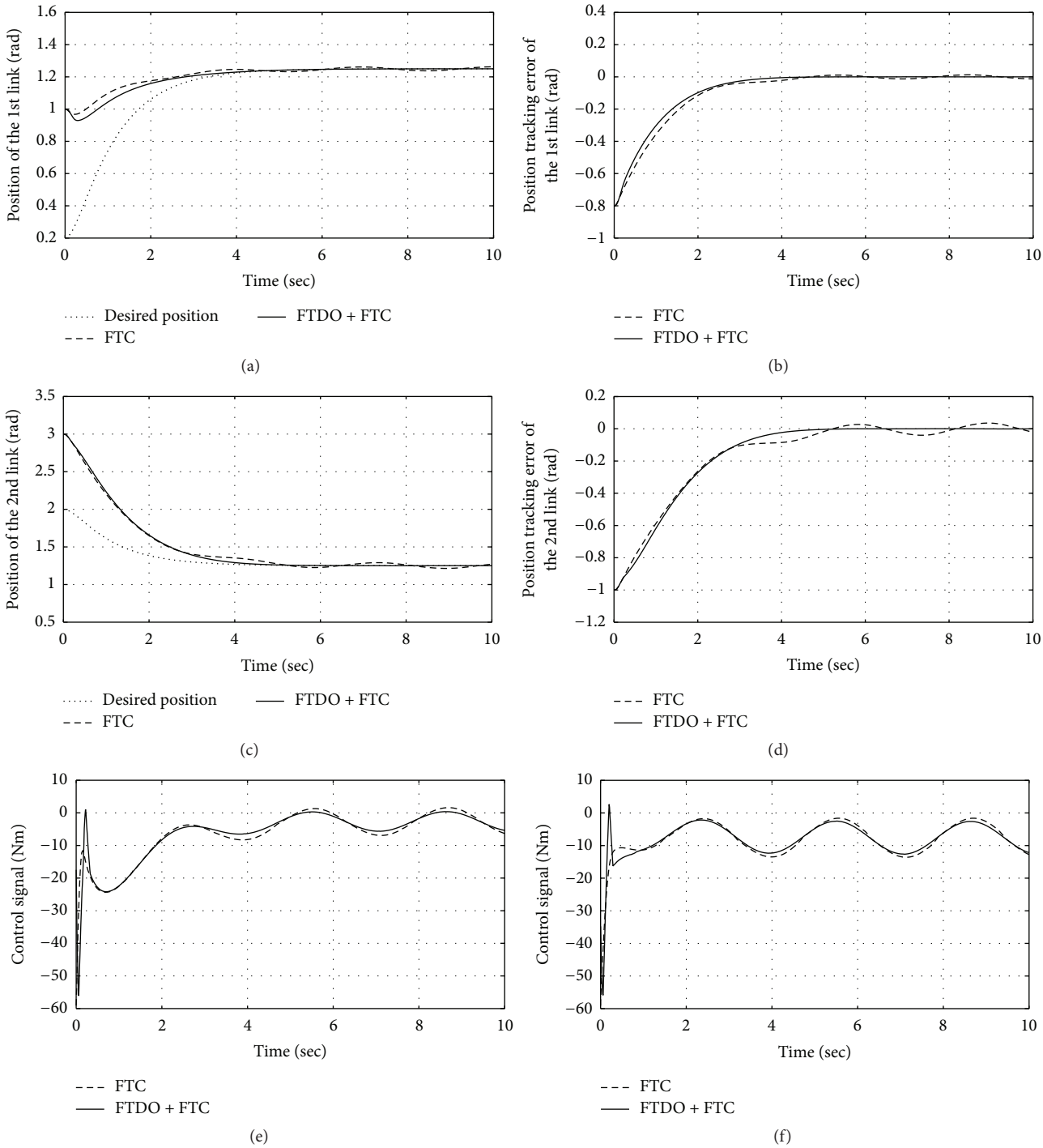

Figure 4: Response curves of system (9) under controllers (12) and (32). (a) Positions of the 1st link (rad). (b) Position tracking errors of the 1st link (rad). (c) Positions of the 2nd link (rad). (d) Position tracking errors of the 2nd link (rad). (e) Control signals for the 1st link (Nm). (f) Control signals for the 2nd link (Nm).

Generally speaking, in practice, higher-frequency measurement noises can be filtered by some filters, for example, Kalman filters. In this way, the measured states used by the controllers are usually signals with only lower-frequency noises. Hence, in simulations, only lower-frequency noises are considered and the output measurement noises are assumed to be $n = [0.01\sin(5t); 0.01\cos(5t)]$. In other words, the measured output is $y = x_1 + n$. Differentiating y yields $\dot{y} = x_2 + \overline{d}_1$, where $\overline{d}_1 = d_1 + \dot{n}$ (d_1 is defined in system (9)). By an observer almost the same as (10) (the only difference is that the observer estimates $\overline{d}_1$ rather than d_1 in system (9)), the mismatched disturbance $\overline{d}_1$ can still be estimated. Then based on the disturbances estimates, a composite controller almost the same as (12) (the only difference is the replacement

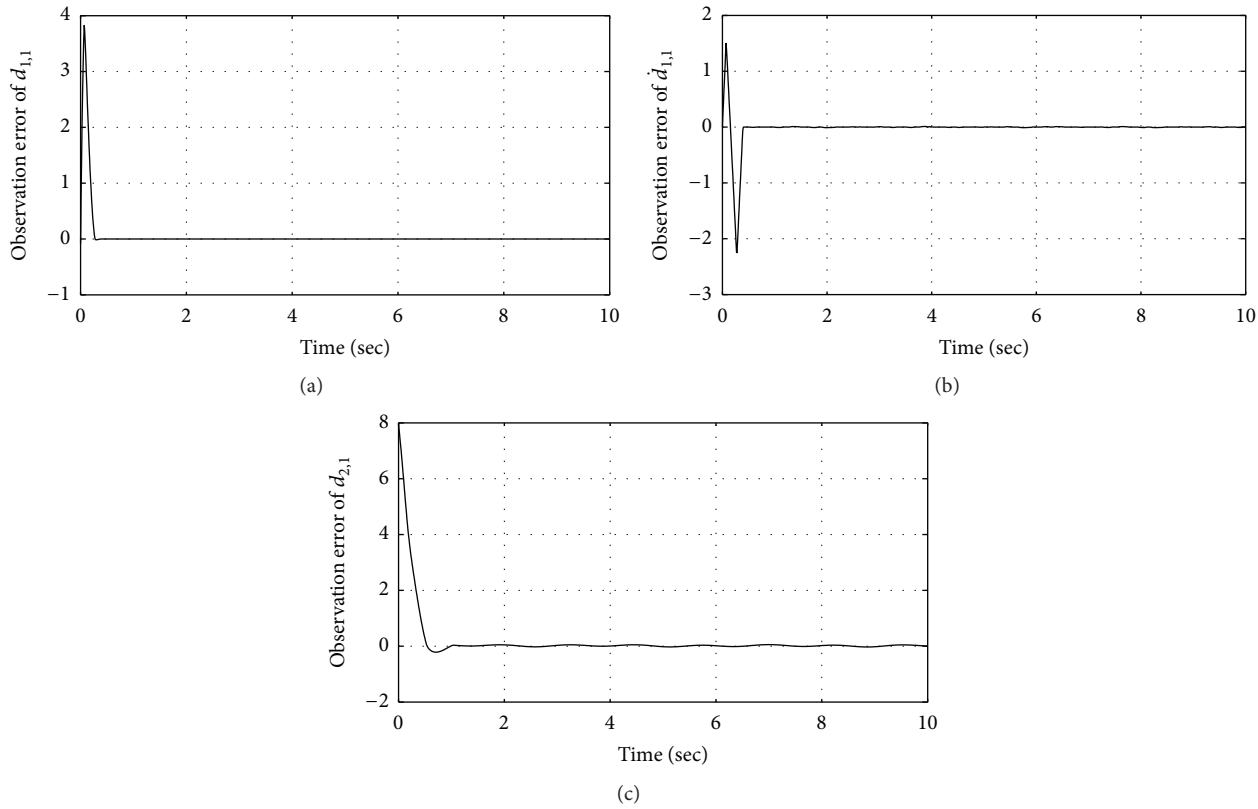

Figure 5: Disturbance observation errors of the 1st link in the presence of output measurement noises. (a) Observation error of $d_{1,1}$. (b) Observation error of $\dot{d}_{1,1}$. (c) Observation error of $d_{2,1}$.

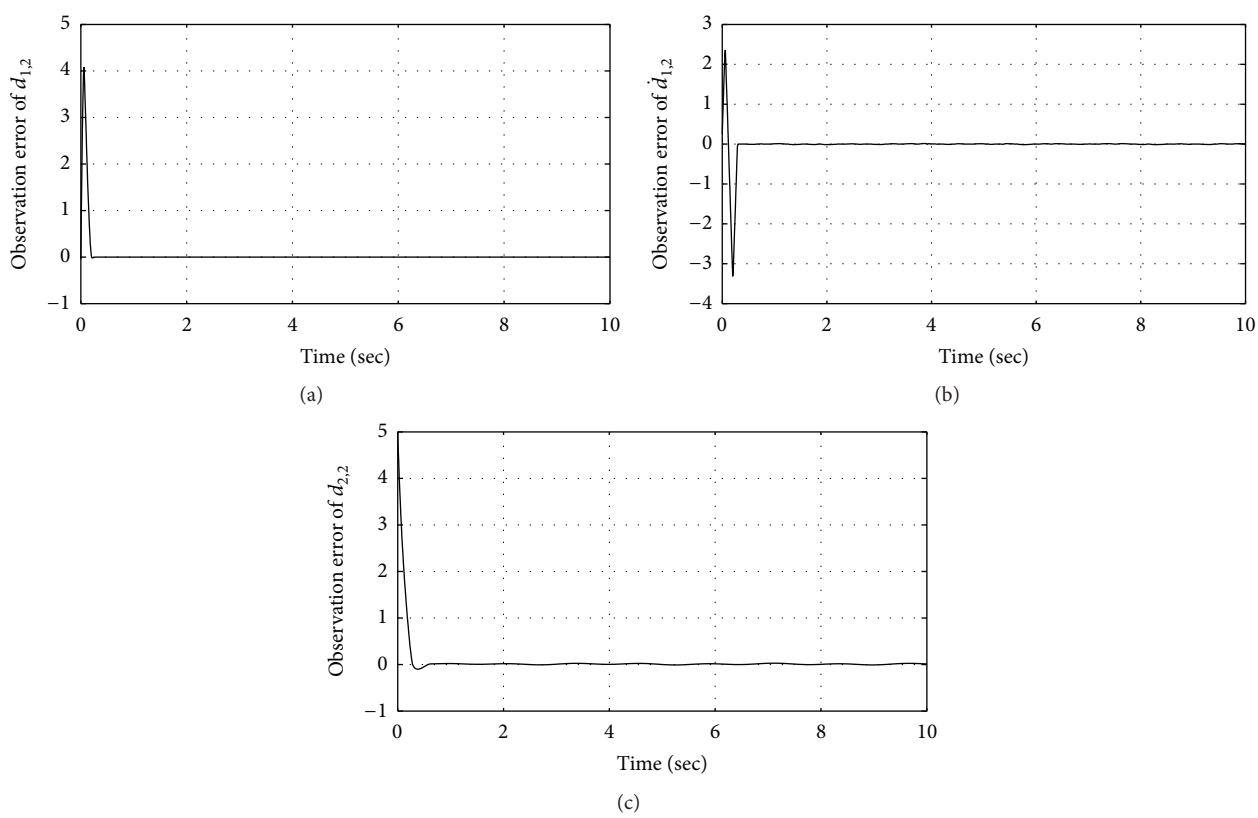

Figure 6: Disturbance observation errors of the 2nd link in the presence of output measurement noises. (a) Observation error of $d_{1,2}$. (b) Observation error of $\dot{d}_{1,2}$. (c) Observation error of $d_{2,2}$.

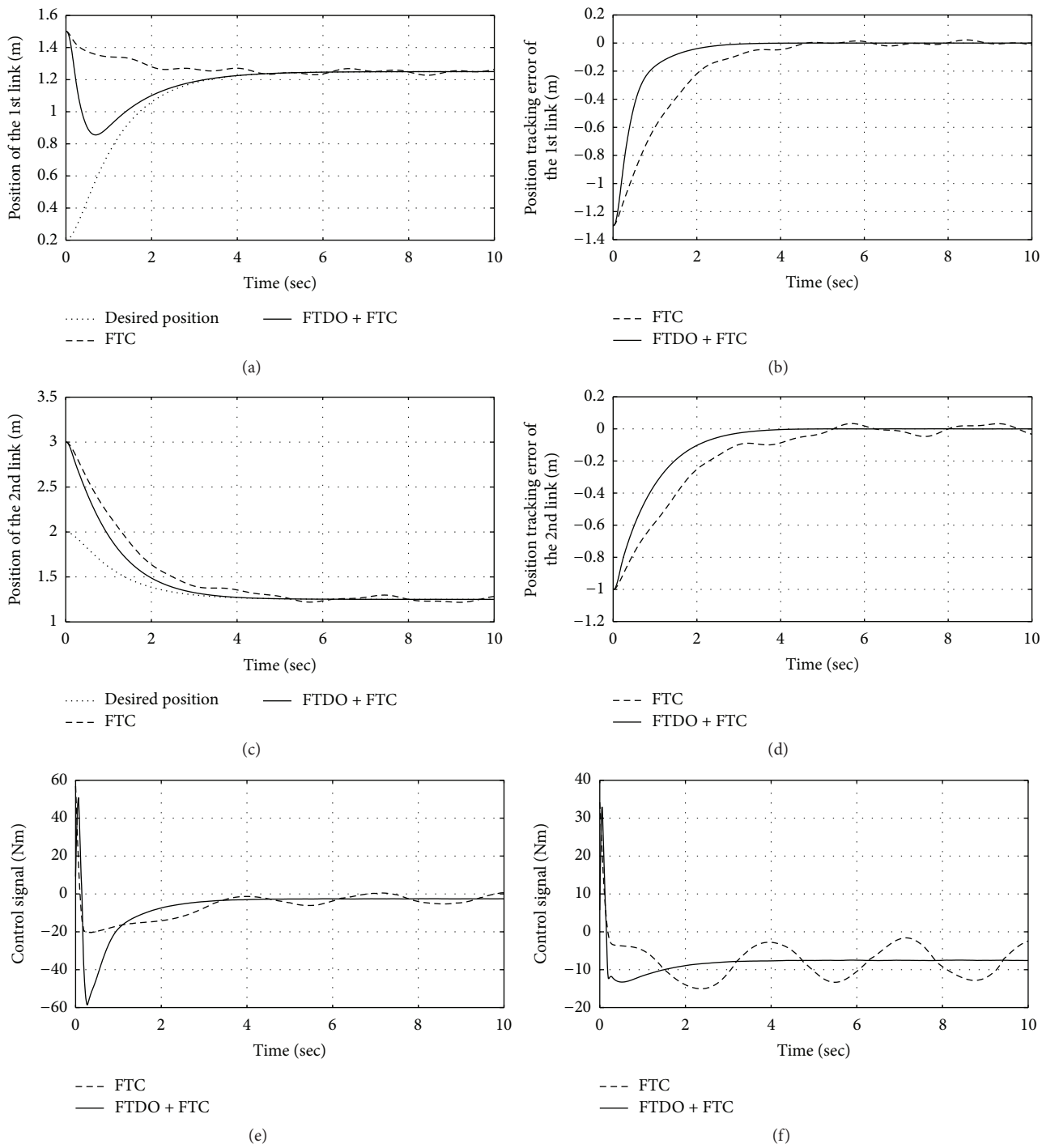

Figure 7: Response curves of system (9) under controllers (12) and (32) in the presence of output measurement noises. (a) Positions of the 1st link (rad). (b) Position tracking errors of the 1st link (rad). (c) Positions of the 2nd link (rad). (d) Position tracking errors of the 2nd link (rad). (e) Control signals for the 1st link (Nm). (f) Control signals for the 2nd link (Nm).

of $x_1, \hat{d}_1, \hat{\dot{d}}_1$ by $y = x_1 + n, \hat{\bar{d}}_1, \hat{\bar{\dot{d}}}_1$, resp.) can be designed. Under the same input saturation as the case without output noises, the parameters for disturbance observer (10) and composite controller (12) are selected as $r = 15/13$, $k_1 = [0.9; 0.7]$, $k_2 = [4.5; 3.5]$, $\lambda_1^0 = [12; 10]$, $\lambda_1^1 = [5; 5]$, $\lambda_1^2 = [2; 2]$, $\lambda_2^0 = [10; 8]$, $\lambda_2^1 = [2; 2]$, $\lambda_2^2 = [1; 0.5]$, $L_1 = [10; 20]$, $L_2 = [10; 20]$. For finite-time controller (32), the parameters are selected as $\bar{r} = 15/13$, $\bar{k}_1 = [0.8; 0.6]$, $\bar{k}_2 = [18; 25]$.

Simulation results are given in Figures 5–7. Figures 5 and 6 show that, even in the presence of lower-frequency noises, observer (10) still works well and provides accurate disturbances estimates in a fast way. Moreover, manipulator

positions can still track the desired positions in a finite time under controller (12) while controller (32) fails to do this.

6. Conclusions

This paper has studied the position tracking control problem of rigid manipulator system with mismatched disturbances. By using adding a power integrator technique and FTDO method, a composite control scheme has been developed. The proposed control method has realized that the manipulator positions tracked the desired positions in finite time and simulations have shown the effectiveness of the proposed composite control algorithm.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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