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Research Article

ℓ_1-Induced State-Feedback Controller Design for Positive Fuzzy Systems

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The problem of ℓ_1-induced state-feedback controller design is investigated for positive Takagi-Sugeno (T-S) fuzzy systems with the use of linear Lyapunov function. First, a novel performance characterization is established to guarantee the asymptotic stability of the closed-loop system with ℓ_1-induced performance. Then, the sufficient conditions are presented to design the required fuzzy controllers and iterative convex optimization approaches are developed to solve the conditions. Finally, one example is presented to show the effectiveness of the derived theoretical results.

1. Introduction

In the real world, positive systems exist in many branches of science and technology such as industrial engineering and ecology [1, 2]. In many applications, the state variables of positive systems are used to denote the concentrations of material, thus taking nonnegative values. Many previous approaches used for general systems are no longer applicable to positive systems because of the special structures of positive systems. Positive systems are special systems and they are defined on cones instead of linear spaces. Therefore, some new problems of positive systems appear and they have been studied extensively [3–9]. For example, controllability and reachability for positive systems have been studied in [10, 11]. The design of state-feedback controllers guaranteeing the closed-loop system to be asymptotically stable has been studied by the linear programming approach in [12]. The positive state-space representation for a given transfer function has been proposed in [13]. For nonnegative and time-delay compartmental dynamic systems, stability has been thoroughly studied in [14–17]. Research on 2D positive systems has appeared in [18]. Moreover, the problem of controller design for positive systems has been investigated by the linear matrix inequality (LMI) approach in [19].

In addition, it is noted that existing research has been conducted mainly for positive linear systems and there have been few results on positive nonlinear systems. The reason is that nonlinearity is difficult to tackle and many methods derived for positive linear systems cannot be directly used for positive nonlinear systems. It is well known that T-S fuzzy model can approximate a nonlinear system arbitrarily well over a compact domain. Such a modeling approach provides an efficient method to tackle some problems for nonlinear systems [20–22]. With the modeling approach, some research methods applicable to linear systems can be used for nonlinear systems. Consequently, many important results on fuzzy systems have appeared; for example, a novel approach was proposed in [23] for stability analysis and stabilization of discrete-time T-S fuzzy delay systems. The problem of stability analysis and stabilization for 2D discrete fuzzy systems was investigated in [24]. As a result of the novel idea of delay partitioning technique, the proposed stability conditions in [23] were much less conservative than most of the existing results. A fuzzy filter design approach was established for fuzzy stochastic systems in [25], and the proposed approach can be used for fault detection problem due to its strong robustness. In recent years, many researchers have focused their interest on positive T-S fuzzy systems [26]. In detail, sufficient conditions of asymptotic stability and stabilization for discrete-time positive T-S systems were proposed in [26]. In [27], the problem of stability and constrained control was addressed for discrete-time positive T-S fuzzy systems with time-varying delays.

Recently, in comparison with the existing results derived with the quadratic Lyapunov function, a novel approach is derived for the synthesis problem of positive systems by using a linear Lyapunov function. The reason why we use the novel approach is that the variables of positive systems are nonnegative and therefore a linear Lyapunov function becomes valid. Many researchers have proposed some novel results based on the linear Lyapunov function [28–32]. Compared with quadratic Lyapunov function based results, the new results in terms of linear programming are more amenable to analysis and computation. Moreover, the applications of the so-called linear Lyapunov functions in the analysis of positive linear systems naturally lead to a variety of results based on a linear setting, which stimulates the use of L_1-gain as a performance index for positive linear systems. It is noted that some frequently used costs such as H_∞ norm are based on the L_2 signal space and these costs are not very natural to describe some of the features of practical physical systems. By contrast, 1-norm gives the sum of the values of the components and it can provide a more natural description for positive systems. It is more appropriate, for example, if the values denote the number of animals in a species or the amount of material. Based on the above discussion, it is noted that the synthesis problems for positive T-S fuzzy systems have not been fully investigated, especially with ℓ_1-induced performance and linear Lyapunov functions. This motivates our study.

In this paper, we investigate the state-feedback controller design problem for positive T-S fuzzy systems with the use of linear Lyapunov function. The main contributions of the paper are as follows: (1) an ℓ_1-induced performance index is explicitly presented for positive fuzzy systems and analytically characterized under a linear Lyapunov function framework; (2) the desired state-feedback controllers are derived with which the asymptotic stability of the closed-loop system is guaranteed and the proposed performance is satisfied; (3) iterative convex optimization approach is developed to solve the conditions.

The rest of this paper is organized as follows. In Section 2, some important preliminaries about positive T-S fuzzy systems are introduced. In Section 3, the state-feedback controller is designed for positive T-S fuzzy systems. One example is provided in Section 4 to show the applicability of the theoretical results. The results are finally concluded in Section 5.

2. Preliminaries

In this section, we introduce notations and several results concerning positive T-S fuzzy systems.

Let $\mathbb{R}$ denote the set of real numbers; $\mathbb{R}^n$ is the set of n-column real vectors; $\mathbb{R}^{n\times m}$ denotes the set of all real matrices of dimension $n \times m$. $A \geq\geq 0$ (resp., $A \gg 0$) means that, for all i and j, $[A]_{ij} \geq 0$ (resp., $[A]_{ij} > 0$). The notation $A \geq\geq B$ (resp., $A \gg B$) means that the matrix $A - B \geq\geq 0$ (resp., $A - B \gg 0$). Let $\overline{\mathbb{R}}_+^n$ denote the nonnegative orthants of $\mathbb{R}^n$. The superscript "T" represents matrix transpose. $\|\cdot\|$ denotes the Euclidean norm for vectors. $A_{i(\mu\nu)}$ is the element of A_i located at the μth row and the νth column. $A_{i(\nu)}$ denotes the νth column of A_i. The 1-norm of a vector $x(k) = (x_1(k), x_2(k), \dots, x_n(k))$ is defined as $\|x(k)\|_1 \triangleq \sum_{i=1}^n |x_i(k)|$. The ℓ_1-norm of an infinite sequence x is defined as $\|x\|_{\ell_1} \triangleq \sum_{k=0}^\infty \|x(k)\|_1$. The space of all vector-valued functions defined on $\overline{\mathbb{R}}_+^n$ with finite ℓ_1-norm is denoted by $\ell_1(\overline{\mathbb{R}}_+^n)$. If the dimensions of matrices are not explicitly stated, it is assumed that the matrices have compatible dimensions for algebraic operations. Vector $\mathbf{1} = [1, 1, \dots, 1]^T$.

Consider the following fuzzy system described by the ith rule.

Model Rule i. IF $\theta_1(k)$ is M_{i1}, $\theta_2(k)$ is M_{i2}, ..., and $\theta_g(k)$ is M_{ig}, THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_{wi} w(k), \\ y(k) &= C_i x(k) + D_{wi} w(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^m$, and $y(k) \in \mathbb{R}^\varrho$ are the system state, disturbance input, and controlled output, respectively. The index $i \in \{1, 2, \dots, r\}$ gives the rule number. $\theta_1(k), \theta_2(k), \dots, \theta_g(k)$ are the premise variables and $\theta(k) = [\theta_1(k), \theta_2(k), \dots, \theta_g(k)]$ is the premise variable vector. M_{ie} $(i = 1, 2, \dots, r;\ e = 1, 2, \dots, g)$ represents the fuzzy sets. Then, we have the fuzzy system:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^r h_i(\theta(k)) \left(A_i x(k) + B_{wi} w(k)\right), \\ y(k) &= \sum_{i=1}^r h_i(\theta(k)) \left(C_i x(k) + D_{wi} w(k)\right), \end{aligned} \tag{2}$$

where

$$\begin{aligned} h_i(\theta(k)) &= \frac{\mu_i(\theta(k))}{\sum_{i=1}^r \mu_i(\theta(k))}, \\ \mu_i(\theta(k)) &= \prod_{e=1}^g M_{ie}(\theta_e(k)), \end{aligned} \tag{3}$$

and $M_{ie}(\theta_e(k)) \in [0, 1]$ represents the grade of membership of $\theta_e(k)$ in M_{ie}. For all k, we have

$$\sum_{i=1}^r h_i(\theta(k)) = 1, \quad h_i(\theta(k)) \geq 0,\ i = 1, 2, \dots, r. \tag{4}$$

Here, the following definition is given, which will be used in the sequel.

Definition 1. System (2) is a discrete-time positive system if, for all $x(0) \geq\geq 0$ and input $w(k) \geq\geq 0$, one has $x(k) \geq\geq 0$ and $y(k) \geq\geq 0$ for $k \in \mathbb{N}$.

Next, some useful results are introduced.

Lemma 2 (see [26]). *The discrete-time system (2) is positive if and only if*

$$\begin{aligned} A_i &\geq\geq 0, \\ B_{wi} &\geq\geq 0, \\ C_{zi} &\geq\geq 0, \\ D_{zwi} &\geq\geq 0, \\ C_i &\geq\geq 0, \\ & i = 1, 2, \ldots, r. \end{aligned} \tag{5}$$

Lemma 3 (see [33]). *Positive system (2) with input $w(k) = 0$ is asymptotically stable if there exists a vector $p_i \geq\geq 0$ (or $p_i \gg 0$) satisfying*

$$p_i^T A_j - p_j^T \ll 0, \tag{6}$$

where $i, j = 1, 2, \ldots, r$.

Here, the definition of ℓ_1-induced performance is introduced. We say that a stable positive system (2) has ℓ_1-induced performance at the level γ if, under zero initial conditions,

$$\sup_{w\neq 0, w\in \ell_1(\overline{\mathbb{R}}_+^n)} \frac{\|y\|_{\ell_1}}{\|w\|_{\ell_1}} < \gamma, \tag{7}$$

where $\gamma > 0$ is a given scalar.

The following result serves as a characterization on the asymptotic stability of system (2) with the ℓ_1-induced performance in (7).

Lemma 4 (see [33]). *The positive fuzzy system (2) is asymptotically stable and satisfies $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}$ if there exists a vector $p_i \geq\geq 0$ satisfying*

$$\begin{aligned} \mathbf{1}^T C_j + p_i^T A_j - p_j^T &\ll 0, \\ \mathbf{1}^T D_{wj} + p_i^T B_{wj} - \gamma \mathbf{1}^T &\ll 0, \end{aligned} \tag{8}$$

where $i, j = 1, 2, \ldots, r$.

3. State-Feedback Controller Synthesis for Positive Fuzzy Systems

In this section, the ℓ_1-induced state-feedback controller synthesis problem is formulated. Based on the stability and performance conditions, a state-feedback controller is designed for positive fuzzy systems. Finally, an iterative convex optimization approach is developed to solve the conditions accordingly.

Here, we deal with the stabilization problem for the following positive fuzzy systems:

$$\begin{aligned} & x(k+1) \\ &\quad = \sum_{i=1}^{r} h_i(\theta(k)) \left(A_i x(k) + B_i u(k) + B_{wi} w(k)\right), \\ & y(k) = \sum_{i=1}^{r} h_i(\theta(k)) \left(C_i x(k) + D_i u(k) + D_{wi} w(k)\right), \end{aligned} \tag{9}$$

where $x(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^m$, $u(k) \in \mathbb{R}^l$, and $y(k) \in \mathbb{R}^\varrho$ are the system state, disturbance input, control input, and controlled output, respectively.

For the positive fuzzy system in (9), we construct the following parallel distributed compensation (PDC) fuzzy controller.

Control Rule i. IF $\theta_1(k)$ is M_{i1}, $\theta_2(k)$ is M_{i2}, …, and $\theta_g(k)$ is M_{ig}, THEN

$$u(k) = K_i x(k). \tag{10}$$

The overall fuzzy controller is represented by

$$u(k) = \sum_{i=1}^{r} h_i(\theta(k)) K_i x(k). \tag{11}$$

Then, the closed-loop system with the PDC fuzzy controller (11) is given by

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \\ &\quad \cdot \left(\left(A_i + B_i K_j\right) x(k) + B_{wi} w(k)\right), \\ y(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k)) h_j(\theta(k)) \\ &\quad \cdot \left(\left(C_i + D_i K_j\right) x(k) + D_{wi} w(k)\right). \end{aligned} \tag{12}$$

The problem to be addressed in this paper is described as follows.

Problem PPSFCD (Positivity-Preserving ℓ_1-Induced State-Feedback Controller Design). Given the fuzzy positive system (9), the control objective is to find controller (11) such that the closed-loop system (12) is positive and asymptotically stable and satisfies the ℓ_1-induced performance $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}$ under zero initial conditions.

First, the stability characterization of the closed-loop system (12) is proposed as follows.

Proposition 5. *The closed-loop system in (12) with input $w(k) = 0$ is positive and asymptotically stable if there exist* *vectors* $p_i \gg 0$, $y_{ijt(1)}, \ldots, y_{ijt(1)} \in \mathbb{R}^{1\times\varrho}$ *satisfying the following LPs:*

$$p_i^T A_j + \sum_{\sigma=1}^{l} y_{ijt(\sigma)} - p_j^T \ll 0, \quad (13)$$

$$\frac{1}{l} A_{i(\mu\nu)} \sum_{h=1}^{n} p_{\xi(h)} B_{t(hs)} + B_{i(\mu s)} y_{jt\xi(s)} \geq 0, \quad (14)$$

$$\frac{1}{l} C_{i(\alpha\nu)} \sum_{h=1}^{n} p_{\xi(h)} B_{t(hs)} + D_{i(\alpha s)} y_{jt\xi(s)} \geq 0, \quad (15)$$

with $i, j, t, \xi = 1, 2, \ldots, r$, $\mu, \nu = 1, 2, \ldots, n$, $s = 1, 2, \ldots, l$, $\alpha = 1, 2, \ldots, q$, *and*

$$K_t = \begin{bmatrix} \dfrac{y_{ijt(1)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(h1)}} \\ \dfrac{y_{ijt(2)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(h2)}} \\ \vdots \\ \dfrac{y_{ijt(l)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(hl)}} \end{bmatrix}. \quad (16)$$

Proof. We have

$$\begin{aligned} \left[A_j + B_j K_t\right]_{\mu\nu} &= A_{j(\mu\nu)} + \sum_{s=1}^{l} B_{j(\mu s)} \frac{y_{ijt(s)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(hs)}} \\ &= \sum_{s=1}^{l} \left[\frac{1}{l} A_{j(\mu\nu)} + B_{j(\mu s)} \frac{y_{ijt(s)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(hs)}} \right]. \end{aligned} \quad (17)$$

It is noted that $(1/l)A_{j(\mu\nu)} + B_{j(\mu s)}(y_{ijt(s)}/\sum_{h=1}^{n} p_{i(h)} B_{j(hs)}) \geq 0$ for $i, j, t, \xi = 1, 2, \ldots, r$ implies $A_j + B_j K_t \geq\geq 0$.

Moreover, the following equation holds:

$$\begin{aligned} \left[C_j + D_j K_t\right]_{\alpha\nu} &= C_{j(\alpha\nu)} + \sum_{s=1}^{l} D_{j(\alpha s)} \frac{y_{ijt(s)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(hs)}} \\ &= \sum_{s=1}^{l} \left[\frac{1}{l} C_{j(\alpha\nu)} + D_{j(\alpha s)} \frac{y_{ijt(s)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(hs)}} \right]. \end{aligned} \quad (18)$$

We note that $(1/l)C_{j(\alpha\nu)} + D_{j(\alpha s)}(y_{ijt(s)}/\sum_{h=1}^{n} p_{i(h)} B_{j(hs)}) \geq 0$ for $i, j, t, \xi = 1, 2, \ldots, r$ implies $C_j + D_j K_t \geq\geq 0$. Therefore, from (14) and (15), we have that the closed-loop system (12) is positive.

On the other hand, consider the Lyapunov function $V(x(k)) = (\sum_{i=1}^{r} h_i(\theta(k)) p_i)^T x(k)$ and we have

$$\begin{aligned} \Delta V(x(k)) &= \left(\sum_{i=1}^{r} h_i(\theta(k+1)) p_i\right)^T x(k+1) - \left(\sum_{i=1}^{r} h_i(\theta(k)) p_i\right)^T x(k) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{t=1}^{r}\sum_{\xi=1}^{r} h_i(\theta(k+1)) h_j(\theta(k)) h_t(\theta(k)) h_\xi(\theta(k)) \left(p_i^T\left(A_j + B_j K_t C_\xi\right) - p_j^T\right) x(k). \end{aligned} \quad (19)$$

For $x(k) \neq 0$, we have that $p_i^T(A_j + B_j K_t) - p_j^T \ll 0$ implies $\Delta V(x(k)) < 0$. Therefore, system (12) is asymptotically stable.

Now, let $K_t = \begin{bmatrix} K_{t(1)} \\ K_{t(2)} \\ \vdots \\ K_{t(l)} \end{bmatrix}$, where $K_{t(\sigma)}$ are vectors in $\mathbb{R}^{\varrho}$; one has

$$p_i^T B_j K_t = \sum_{\sigma=1}^{l} \left[\sum_{h=1}^{n} p_{i(h)} B_{j(h\sigma)}\right] K_{t(\sigma)} = \sum_{\sigma=1}^{l} y_{ijt(\sigma)}, \quad (20)$$

with $y_{ijt(\sigma)} = [\sum_{h=1}^{n} p_{i(h)} B_{j(h\sigma)}] K_{t(\sigma)}$. Consequently, inequality $p_i^T(A_j + B_j K_t) - p_j^T \ll 0$ can be written as

$$p_i^T A_j + \sum_{\sigma=1}^{l} y_{ijt(\sigma)} - p_j^T \ll 0,$$

$$K_t = \begin{bmatrix} \dfrac{y_{ijt(1)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(h1)}} \\ \dfrac{y_{ijt(2)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(h2)}} \\ \vdots \\ \dfrac{y_{ijt(l)}}{\sum_{h=1}^{n} p_{i(h)} B_{j(hl)}} \end{bmatrix}. \quad (21)$$

□

Next, a sufficient condition for the existence of a solution to Problem PPSFCD is obtained.

Theorem 6. *The closed-loop system in (12) is positive and asymptotically stable and satisfies* $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}$ *if there exist matrices* K_i *and vectors* $p_i \gg 0$ *satisfying*

$$A_j + B_j K_t \geq\geq 0, \quad (22)$$

$$C_j + D_j K_t \geq\geq 0, \quad (23)$$

$$\mathbf{1}^T \left(C_j + D_j K_t\right) + p_i^T \left(A_j + B_j K_t\right) - p_t^T \ll 0, \quad (24)$$

$$\mathbf{1}^T D_{wj} + p_i^T B_{wj} - \gamma \mathbf{1}^T \ll 0, \quad (25)$$

where $i, j, t = 1, 2, \ldots, r$.

In the following, our aim is to derive a numerically tractable means to synthesize a derived controller. It is noted that when matrix K_t is fixed, (24) turns out to be linear with respect to the other variables. Therefore, a natural way is to fix K_t and solve (24)-(25) by linear programming.

Algorithm PPSFCD

Step 1. Set $\kappa = 1$. We use Proposition 5 to solve for K_i^1 such that the positive system (9) with (10) is positive and asymptotically stable.

Step 2. For fixed K_i^κ, solve the following optimization problem for p_i^κ and γ_κ.

OP1: minimize γ_κ subject to the following constraints:

$$\begin{aligned} \mathbf{1}^T \left(C_j + D_j K_t\right) + p_i^T \left(A_j + B_j K_t\right) - p_t^T &\ll 0, \\ \mathbf{1}^T D_{wj} + p_i^T B_{wj} - \gamma \mathbf{1}^T &\ll 0, \\ p_i^\kappa &\gg 0. \end{aligned} \quad (26)$$

Denote γ_κ^*, p_i^κ as the solution to the optimization problem. If $|(\gamma_\kappa^* - \gamma_{\kappa-1}^*)/\gamma_\kappa^*| \leq \varepsilon_1$, where ε_1 is a prescribed bound, then $K_i = K_i^\kappa$, $p_i = p_i^\kappa$. STOP.

Step 3. For fixed p_i^κ, solve the following optimization problem for K_i^κ.

OP2: minimize γ_κ subject to the following constraints:

$$\begin{aligned} A_j + B_j K_t &\geq\geq 0, \\ C_j + D_j K_t &\geq\geq 0, \\ \mathbf{1}^T \left(C_j + D_j K_t\right) + p_i^T \left(A_j + B_j K_t\right) - p_t^T &\ll 0, \\ \mathbf{1}^T D_{wj} + p_i^T B_{wj} - \gamma \mathbf{1}^T &\ll 0. \end{aligned} \quad (27)$$

Denote γ_κ^* as the solution to the optimization problem.

Step 4. If $|(\gamma_\kappa^* - \gamma_{\kappa-1}^*)/\gamma_\kappa^*| \leq \varepsilon_2$, where ε_2 is prescribed tolerance, STOP; else, set $\kappa = \kappa + 1$ and $K_i^\kappa = K_i^{\kappa-1}$, and then go to Step 2.

Remark 7. The parameter γ can be optimized iteratively. Notice that $\gamma_{\kappa+1}^* \leq \gamma_\kappa^*$ since the corresponding parameters obtained in Step 3 will be utilized as the initial values in Step 2 to derive a smaller γ. Since the sequence $\{\gamma_\kappa^*\}$ is bounded from below, the convergence of the iterative process is naturally guaranteed.

4. Illustrative Example

In this section, one illustrative example is presented in the following to illustrate the effectiveness of the proposed controller design approach.

In this example, we consider the following Lotka-Volterra population model, which reflects various interactions between two species [34]:

$$\begin{aligned} x_1(k+1) &= e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)} x_1(k) + 1.5x_2(k) \\ &\quad + 0.1u(k) + 0.2w(k), \\ x_2(k+1) &= 0.2x_1(k) \\ &\quad + e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)} x_2(k) \\ &\quad + 0.2u(k) + 0.1w(k), \\ z(k) &= x_1(k) + x_2(k), \end{aligned} \quad (28)$$

where $x_i(k)$ is the density of population i at kth generation and $x(0) = \begin{bmatrix} 0.5 & 0.3 \end{bmatrix}^T$. The external disturbance $w(k)$ is assumed to be

$$w(k) = \begin{cases} 0.15, & 5 \leq k \leq 10, \\ 0, & \text{otherwise}. \end{cases} \quad (29)$$

Define $\theta_1(k) = e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)}$ and $\theta_2(k) = e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)}$. We calculate the minimum and maximum values of $\theta_1(k)$ and $\theta_2(k)$. They are obtained as follows:

$$\begin{aligned} \max \theta_1(k) &= 2.0437, \\ \min \theta_1(k) &= 0.3250, \\ \max \theta_2(k) &= 2.2034, \\ \min \theta_2(k) &= 0.7861. \end{aligned} \quad (30)$$

From the maximum and minimum values, $\theta_1(k)$ and $\theta_2(k)$ can be represented by

$$\begin{aligned} \theta_1(k) &= e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)} \\ &= M_1(\theta_1(k)) \cdot 2.0437 + M_2(\theta_1(k)) \cdot 0.3250, \\ \theta_2(k) &= e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)} \\ &= N_1(\theta_2(k)) \cdot 2.2034 + N_2(\theta_2(k)) \cdot 0.7861, \end{aligned} \quad (31)$$

with

$$\begin{aligned} M_1(\theta_1(k)) &= \frac{\theta_1(k) - 0.3250}{2.0437 - 0.3250}, \\ M_2(\theta_1(k)) &= \frac{2.0437 - \theta_1(k)}{2.0437 - 0.3250}, \\ N_1(\theta_2(k)) &= \frac{\theta_2(k) - 0.7861}{2.2034 - 0.7861}, \\ N_2(\theta_2(k)) &= \frac{2.2034 - \theta_2(k)}{2.2034 - 0.7861}. \end{aligned} \quad (32)$$

The membership functions $M_1(\theta_1(k))$ and $M_2(\theta_1(k))$ are named "Big" and "Small" for $\theta_1(k)$, respectively, and similarly for $N_1(\theta_2(k))$ and $N_2(\theta_2(k))$ for $\theta_2(k)$. Then, the following fuzzy rules are employed.

Rule 1. IF $\theta_1(k)$ is "Big" and $\theta_2(k)$ is "Big," THEN

$$\begin{aligned} x(k+1) &= A_1 x(k) + B_1 u(k) + B_{w1} w(k), \\ z(k) &= C_{z1} x(k). \end{aligned} \tag{33}$$

Rule 2. IF $\theta_1(k)$ is "Big" and $\theta_2(k)$ is "Small," THEN

$$\begin{aligned} x(k+1) &= A_2 x(k) + B_2 u(k) + B_{w2} w(k), \\ z(k) &= C_{z2} x(k). \end{aligned} \tag{34}$$

Rule 3. IF $\theta_1(k)$ is "Small" and $\theta_2(k)$ is "Big," THEN

$$\begin{aligned} x(k+1) &= A_3 x(k) + B_3 u(k) + B_{w3} w(k), \\ z(k) &= C_{z3} x(k). \end{aligned} \tag{35}$$

Rule 4. IF $\theta_1(k)$ is "Small" and $\theta_2(k)$ is "Small," THEN

$$\begin{aligned} x(k+1) &= A_4 x(k) + B_4 u(k) + B_{w4} w(k), \\ z(k) &= C_{z4} x(k), \end{aligned} \tag{36}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 2.0437 & 1.5 \\ 0.2 & 2.2034 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 2.0437 & 1.5 \\ 0.2 & 0.7861 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0.3250 & 1.5 \\ 0.2 & 2.2034 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} 0.3250 & 1.5 \\ 0.2 & 0.7861 \end{bmatrix}, \\ B_1 &= B_2 = B_3 = B_4 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \\ B_{w1} &= B_{w2} = B_{w3} = B_{w4} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ C_{z1} &= C_{z2} = C_{z3} = C_{z4} = \begin{bmatrix} 1 & 1 \end{bmatrix}. \end{aligned} \tag{37}$$

We have the fuzzy model for the nonlinear system as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{4} h_i(\theta(k)) \left(A_i x(k) + B_i u(k) + B_{wi} w(k) \right), \\ z(k) &= \sum_{i=1}^{4} h_i(\theta(k)) C_{zi} x(k), \end{aligned} \tag{38}$$

with

$$\begin{aligned} h_1(\theta(k)) &= \frac{\left(e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)} - 0.3250\right)\left(e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)} - 0.7861\right)}{(2.0437-0.3250)(2.2034-0.7861)}, \\ h_2(\theta(k)) &= \frac{\left(e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)} - 0.3250\right)\left(2.2034 - e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)}\right)}{(2.0437-0.3250)(2.2034-0.7861)}, \\ h_3(\theta(k)) &= \frac{\left(2.0437 - e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)}\right)\left(e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)} - 0.7861\right)}{(2.0437-0.3250)(2.2034-0.7861)}, \\ h_4(\theta(k)) &= \frac{\left(2.0437 - e^{0.5-\sin(0.5\pi k)x_1(k)-0.2x_2(k)}\right)\left(2.2034 - e^{0.8-0.2\cos(1.5\pi k)x_1(k)-0.5x_2(k)}\right)}{(2.0437-0.3250)(2.2034-0.7861)}. \end{aligned} \tag{39}$$

By solving the conditions in Theorem 6, a feasible solution is achieved with

$$\begin{aligned} p_1 &= \begin{bmatrix} 0.2536 & 1.1746 \end{bmatrix}^T, \\ p_2 &= \begin{bmatrix} 0.2489 & 1.1673 \end{bmatrix}^T, \\ p_3 &= \begin{bmatrix} 0.2352 & 1.1582 \end{bmatrix}^T, \\ p_4 &= \begin{bmatrix} 0.2235 & 1.1299 \end{bmatrix}^T, \end{aligned} \tag{40}$$

which further yields the matrices of the state-feedback controller as

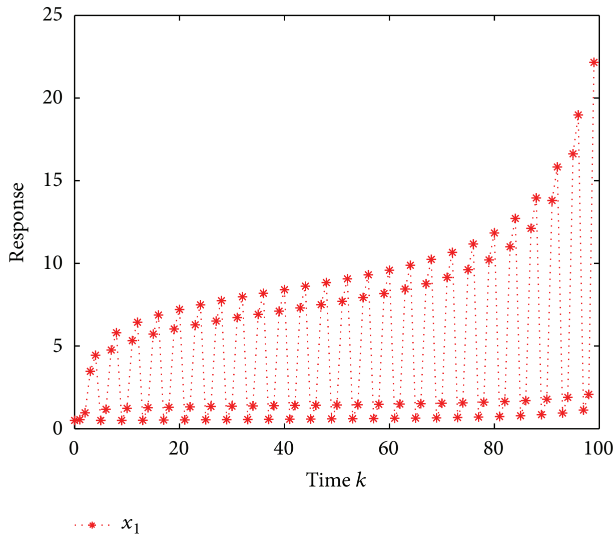

Figure 1: Time response x_1 of open-loop system.

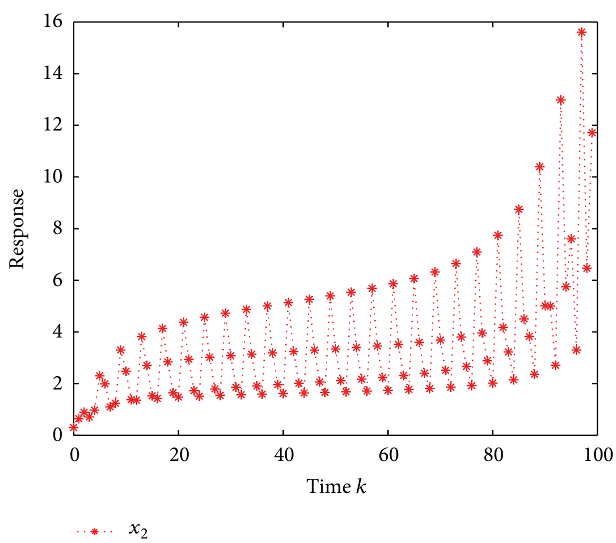

Figure 2: Time response x_2 of open-loop system.

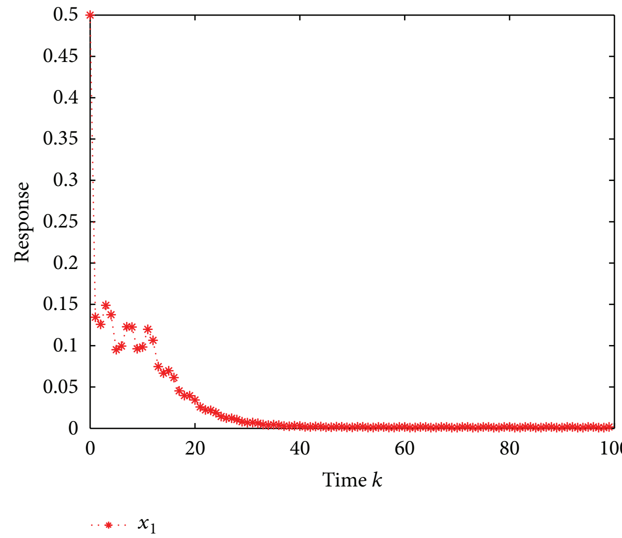

Figure 3: Time response x_1 of closed-loop system.

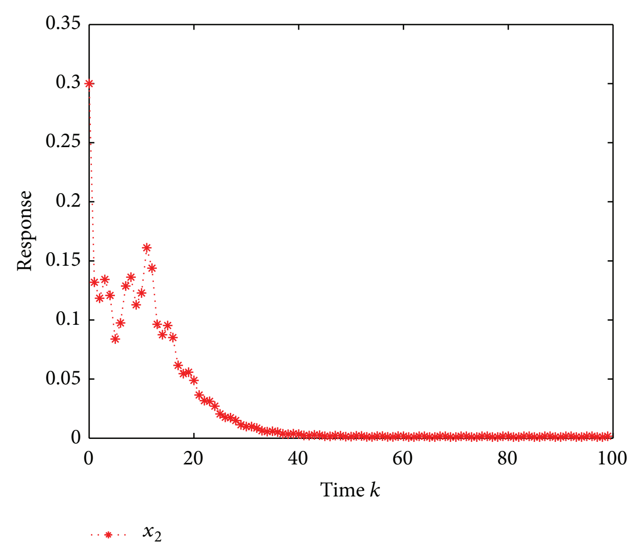

Figure 4: Time response x_2 of closed-loop system.

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.2380 & -0.3534 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.1630 & -0.2973 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -0.3472 & -0.4281 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} -0.4291 & -0.3723 \end{bmatrix}. \end{aligned} \tag{41}$$

The responses of the open-loop and the closed-loop system are shown in Figures 1–4, from which we can see that the system can be stabilized by the designed controller.

With this choice of initial condition, Figures 1 and 2 show the response of open-loop system. It can be seen that the open-loop system is not globally asymptotically stable. Figures 3 and 4 show the state response of the closed-loop system, from which we can see that the state of the closed-loop system converges to zero.

5. Conclusion

In this paper, the state-feedback controller synthesis problem for positive T-S fuzzy systems has been addressed under ℓ_1 performance. Novel performance characterization of the closed-loop system has been established. Based on the novel characterization, sufficient conditions have been developed for the existence of state-feedback controllers. Moreover, iterative convex optimization algorithms have been developed to solve the design conditions. Finally, one example has been presented to demonstrate the effectiveness of the proposed approach. Further research topics include the controller synthesis problem for positive fuzzy systems with time-varying or distributed delays with linear Lyapunov functions.

Competing Interests

The authors declare that they have no competing interests.

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