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*Research Article*

# Fixed Points and Exponential Stability for Impulsive Time-Delays BAM Neural Networks via LMI Approach and Contraction Mapping Principle

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The fixed point technique has been employed in the stability analysis of time-delays bidirectional associative memory (BAM) neural networks with impulse. By formulating a contraction mapping in a product space, a new LMI-based exponential stability criterion was derived. Lately, fixed point methods have educed various good results inspiring this work, but those criteria cannot be programmed by a computer. In this paper, LMI conditions of the obtained result can be applicable to computer Matlab LMI toolbox which meets the need of the large-scale calculation in real engineering. Moreover, a numerical example and a comparable table are presented to illustrate the effectiveness of the proposed methods.

## 1. Introduction

Bidirectional associative memory (BAM) neural networks model was originally introduced by Kosko [1, 2]:

$$\begin{aligned} \dot{x}_i &= -a_i x_i(t) + \sum_{j=1}^{p} w_{ji} g_j\left(y_j(t)\right) + I_i, \quad i = 1, 2, \ldots, n, \\ \dot{y}_j &= -b_j y_j(t) + \sum_{i=1}^{n} v_{ij} g_i\left(x_i(t)\right) + J_j, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{1}$$

Thanks to its generalization of the single-layer autoassociative Hebbian correlation to two-layer pattern-matched heteroassociative circuits, widespread applications have been found in various areas, such as automatic control, signal and image processing, pattern recognition, artificial intelligence, parallel computation, and optimization problems. Often, a stable equilibrium of BAM neural networks is the important precondition of the successful applications. There are many factors influencing the stability of neural networks, in which pulse and time delay are usually the main reasons [3–8]. So, the stability analysis of impulse or delays system has become a hot topic. All the time, the Lyapunov functional method is employed to deduce stability criteria [9–20]. But every method may have its limitations. During the recent decades, other techniques have been developed to investigate the stability, in which the fixed point method is always one of those alternatives [21–27]. For example, in 2015, Zhou utilized Brouwer's fixed point theorem to prove the existence and uniqueness of equilibrium of the hybrid BAM neural networks with proportional delays and finally constructed appropriate delay differential inequalities to derive the stability of equilibrium [28]. In [29], Banach fixed point theorem was applied to show the existence of the unique equilibrium of BAM neural networks with time-varying delays in the leakage terms, and then the Lyapunov functional method was for demonstrating the global exponential stability. Different from [28, 29], we shall use Banach fixed point theorem deriving straightway the stability criterion of impulsive time-delay BAM neural networks, in which LMI conditions facilitate computer programming. Finally, a numerical example is presented to illustrate the effectiveness of the proposed methods.

For the sake of convenience, we introduce the following standard notations [18]:

(i) $L = (l_{ij})_{n\times n} > 0(< 0)$: a positive (negative) definite matrix; that is, $y^T L y > 0(< 0)$ for any $0 \neq y \in R^n$.

(ii) $L = (l_{ij})_{n\times n} \geqslant 0(\leqslant 0)$: a semipositive (seminegative) definite matrix; that is, $y^T L y \geqslant 0(\leqslant 0)$ for any $y \in R^n$.

(iii) $L_1 \geqslant L_2(L_1 \leqslant L_2)$: this means matrix $(L_1 - L_2)$ is a semipositive (seminegative) definite matrix.

(iv) $L_1 > L_2(L_1 < L_2)$: this means matrix $(L_1 - L_2)$ is a positive (negative) definite matrix.

(v) $\lambda_{\max}(\Phi)$ and $\lambda_{\min}(\Phi)$ denote the largest and the smallest eigenvalue of matrix $\Phi$, respectively.

(vi) Denote $|L| = (|l_{ij}|)_{n\times n}$ for any matrix $L = (l_{ij})_{n\times n}$.

(vii) $|u| = (|u_1|, |u_2|, \ldots, |u_n|)^T$ for any vector $u = (u_1, u_2, \ldots, u_n)^T \in R^n$.

(viii) $u \leqslant (\geqslant) v$ implies that $u_i \leqslant (\geqslant) v_i$, $\forall i$, and $u < (>) v$ implies that $u_i < (>) v_i$, $\forall i$, for any vectors $u = (u_1, u_2, \ldots, u_n)^T \in R^n$ and $v = (v_1, v_2, \ldots, v_n)^T \in R^n$.

(ix) $I$ is identity matrix with compatible dimension.

*Remark 1.* Different from the methods of [28, 29], it will be the first time to utilize contraction mapping principle to infer directly the LMI-based stability criterion of BAM neural networks, convenient for computer programming. Recently, there have been a lot of good results and methods [21–29] enlightening our current work. In this paper, we shall propose the LMI-based criterion, novel against the existing results, published from 2013 to 2016 (see Remark 10 and Table 1).

## 2. Preliminaries

Consider the following delayed differential equations:

$$
\begin{aligned}
\frac{dx(t)}{dt} &= -Ax(t) + Cf(y(t-\tau(t))), \\
&\qquad t \in [0, +\infty),\ t \neq t_k,\ k = 1, 2, \ldots, \\
\frac{dy(t)}{dt} &= -By(t) + Dg(x(t-h(t))), \\
&\qquad t \in [0, +\infty),\ t \neq t_k,\ k = 1, 2, \ldots, \\
x(t_k^+) - x(t_k) &= \rho(x(t_k)), \\
y(t_k^+) - y(t_k) &= \rho(y(t_k)), \\
&\qquad k = 1, 2, \ldots, \\
x(s) &= \xi(s), \\
y(s) &= \eta(s), \\
&\qquad s \in [-\tau, 0],
\end{aligned} \tag{2}
$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in R^n$, and $\xi(s), \eta(s) \in \mathscr{C}([-\tau, 0], R^n)$. Here, $\mathscr{C}([-\tau, 0], R^n)$ represents the space of continuous functions from $[-\tau, 0]$ to $R^n$. Active functions $f(x(t-\tau(t))) = (f_1(x_1(t-\tau(t))), f_2(x_2(t-\tau(t))), \ldots, f_n(x_n(t-\tau(t))))^T$, $g(x(t-h(t))) = (g_1(x_1(t-h(t))), g_2(x_2(t-h(t))), \ldots, g_n(x_n(t-h(t))))^T \in R^n$, impulsive function $\rho(x(t)) = (\rho_1(x_1(t)), \rho_2(x_2(t)), \ldots, \rho_n(x_n(t)))^T \in R^n$, and time delays $0 \leqslant \tau(t), h(t) \leqslant \tau$. The fixed impulsive moments $t_k$ $(k = 1, 2, \ldots)$ satisfy $0 < t_1 < t_2 < \cdots$ with $\lim_{k\to\infty} t_k = \infty$. $x(t_k^+)$ and $x(t_k^-)$ stand for the right-hand and left-hand limit of $x(t)$ at time $t_k$, respectively. We always assume $x(t_k^-) = x(t_k)$, for all $k = 1, 2, \ldots$. Similar to [23], we assume in this paper that $f(0) = g(0) = \rho(0) = 0 \in R^n$. Constant matrices $A = \operatorname{diag}(a_1, a_2, \ldots, a_n)$, $B = \operatorname{diag}(b_1, b_2, \ldots, b_n)$ are positive definite diagonal matrices, and both $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are matrices with $n \times n$ dimension.

Throughout this paper, we assume that $F = \operatorname{diag}(F_1, F_2, \ldots, F_n)$, $G = \operatorname{diag}(G_1, G_2, \ldots, G_n)$, and $H = \operatorname{diag}(H_1, H_2, \ldots, H_n)$ are diagonal matrices, satisfying

(A1) $|f(x) - f(y)| \leqslant F|x - y|$, $x, y \in R^n$.

(A2) $|g(x) - g(y)| \leqslant G|x - y|$, $x, y \in R^n$.

(A3) $|\rho(x) - \rho(y)| \leqslant H|x - y|$, $x, y \in R^n$.

*Definition 2.* Dynamic equations (2) are said to be globally exponentially stable if, for any initial condition $\xi(s), \eta(s) \in \mathscr{C}([-\tau, 0], R^n)$, there exist a pair of positive constants $a$ and $b$ such that

$$
\left\| \begin{pmatrix} x(t; s, \xi, \eta) \\ y(t; s, \xi, \eta) \end{pmatrix} \right\| \leqslant b e^{-at}, \quad \forall t > 0, \tag{3}
$$

where the norm $\left\| \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \right\| = (\sum_{i=1}^n |x_i(t)|^2 + \sum_{i=1}^n |y_i(t)|^2)^{1/2}$, and $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n) \in R^n$.

*Definition 3.* For a diagonal constants matrix $B = \operatorname{diag}(b_1, b_2, \ldots, b_n)$, one denotes the matrix exponential function $e^{Bt} = \operatorname{diag}(e^{b_1 t}, e^{b_2 t}, \ldots, e^{b_n t})$ for all $t \in R$.

**Lemma 4** (see [31] contraction mapping theorem). *Let $P$ be a contraction operator on a complete metric space $\Theta$; then, there exists a unique point $\theta \in \Theta$ for which $P(\theta) = \theta$.*

**Lemma 5.** *Let $B$ be a diagonal constants matrix and $e^{Bt}$ be the matrix exponential function of $B$. Then, one has*

(1) $(d/dt)e^{Bt} = Be^{Bt}$, $t \in R$;

(2) $(d/dt)(e^{Bt}\alpha) = Be^{Bt}\alpha$, $t \in R$,

*where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T \in R^n$ and each $\alpha_i \in R$ $(i = 1, 2, \ldots, n)$ is a constant.*

## 3. Main Result

**Theorem 6.** *Assume that there exists a positive constant $\delta$ such that* $\inf_{k=1,2,\ldots}(t_{k+1}-t_k) \geqslant \delta$. *In addition, there exists a constant* $0 < \lambda < 1$ *such that*

$$\begin{aligned} &|C|\, F + \frac{1}{\delta}H + AH - \lambda A < 0, \\ &|D|\, G + \frac{1}{\delta}H + BH - \lambda B < 0, \end{aligned} \tag{4}$$

*and then the impulsive fuzzy dynamic equations (2) are globally exponentially stable, where* $\delta = \inf_{k=1,2,\ldots}(t_{k+1} - t_k) > 0$.

*Proof.* To apply the fixed point theory, we firstly define a complete metric space $\Omega = \Omega_1 \times \Omega_2$ as follows.

Let $\Omega_i$ $(i = 1, 2)$ be the space consisting of functions $q_i(t) : [-\tau, \infty) \to R^n$, satisfying the following:

(a) $q_i(t)$ is continuous on $t \in [0, +\infty) \setminus \{t_k\}_{k=1}^{\infty}$.
(b) $q_1(t) = \xi(t)$, $q_2(t) = \eta(t)$, for $t \in [-\tau, 0]$.
(c) $\lim_{t\to t_k^-} q_i(t) = q_i(t_k)$, and $\lim_{t\to t_k^+} q_i(t)$ exists, for all $k = 1, 2, \ldots$.
(d) $e^{\gamma t} q_i(t) \to 0 \in R^n$ as $t \to \infty$, where $\gamma > 0$ is a positive constant, satisfying $\gamma < \min\{\lambda_{\min} A, \lambda_{\min} B\}$.

It is not difficult to verify that the product space $\Omega$ is a complete metric space if it is equipped with the following metric:

$$\operatorname{dist}(\overline{q}, \widetilde{q}) = \max_{i=1,2,\ldots,2n-1,2n} \left( \sup_{t\geqslant-\tau} \left| \overline{q}^{(i)}(t) - \widetilde{q}^{(i)}(t) \right| \right), \tag{5}$$

where

$$\begin{aligned} \overline{q} = \overline{q}(t) &= \begin{pmatrix} \overline{q}_1(t) \\ \overline{q}_2(t) \end{pmatrix} = \left( \overline{q}^{(1)}(t), \overline{q}^{(2)}(t), \ldots, \overline{q}^{(2n)}(t) \right)^T \\ &\in \Omega, \\ \widetilde{q} = \widetilde{q}(t) &= \begin{pmatrix} \widetilde{q}_1(t) \\ \widetilde{q}_2(t) \end{pmatrix} = \left( \widetilde{q}^{(1)}(t), \ldots, \widetilde{q}^{(2n)}(t) \right)^T \in \Omega, \end{aligned} \tag{6}$$

and $\overline{q}_i \in \Omega_i$, $\widetilde{q}_i \in \Omega_i$, $i = 1, 2$.

Next, we are to construct a contraction mapping $P : \Omega \to \Omega$, which may be divided into three steps.

*Step 1* (formulating the mapping). Let $(x^T(t), y^T(t))^T = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), \ldots, y_n(t))^T$ be a solution of system (2).

Then, for $t \geqslant 0$, $t \neq t_k$, we have

$$\begin{aligned} \frac{dx(t)}{dt}\left(e^{At}x(t)\right) &= Ae^{At}x(t) + e^{At}\frac{dx(t)}{dt} \\ &= e^{At}Cf\left(y(t-\tau(t))\right). \end{aligned} \tag{7}$$

Further, we get by the integral property

$$x(t) = e^{-At}\left[\int_0^t e^{As}Cf\left(y(s-\tau(s))\right)ds + \chi\right], \quad t \geqslant 0, \tag{8}$$

where $\chi \in R^n$ is the vector to be determined.

Next, we claim that $\chi = \xi(0) + \sum_{0<t_k<t} e^{At_k}\rho(x_{t_k})$, and

$$\begin{aligned} x(t) = e^{-At}\xi(0) + e^{-At}&\left[\int_0^t e^{As}Cf\left(y(s-\tau(s))\right)ds \right. \\ &\left. + \sum_{0<t_k<t} e^{At_k}\rho\left(x_{t_k}\right)\right], \quad t \geqslant 0. \end{aligned} \tag{9}$$

Indeed, on the one hand, we can conclude from (9) that

$$\begin{aligned} e^{At}x(t) = \xi(0) &+ \int_0^t e^{As}Cf\left(y(s-\tau(s))\right)ds \\ &+ \sum_{0<t_k<t} e^{At_k}\rho\left(x_{t_k}\right). \end{aligned} \tag{10}$$

For $t \geqslant 0$, $t \neq t_k$, taking the time derivative of both sides leads to

$$\begin{aligned} e^{At}\frac{dx(t)}{dt} + Ae^{At}x(t) &= \frac{d}{dt}\left(e^{At}x(t)\right) \\ &= e^{At}Cf\left(y(t-\tau(t))\right), \end{aligned} \tag{11}$$

or

$$\frac{dx(t)}{dt} + Ax(t) = Cf\left(y(t-\tau(t))\right), \tag{12}$$

which is the first equation of system (2).

Moreover, as $t \to t_j^-$, we can gain by (9)

$$\begin{aligned} x\left(t_j^-\right) &= \lim_{\varepsilon\to0^+} x\left(t_j - \varepsilon\right) = x\left(t_j\right), \quad j = 1, 2, \ldots, \\ x\left(t_j^+\right) &= \lim_{\varepsilon\to0^+} x\left(t_j + \varepsilon\right) = x\left(t_j\right) + \rho\left(x\left(t_j\right)\right), \\ &\qquad j = 1, 2, \ldots. \end{aligned} \tag{13}$$

On the other hand, multiplying both sides of the first equation of system (2) by $e^{At}$ yields

$$\begin{aligned} e^{At}\frac{dx(t)}{dt} + Ae^{At}x(t) = e^{At}Cf\left(y(t-\tau(t))\right), \\ t \geqslant 0, \ t \neq t_k. \end{aligned} \tag{14}$$

Moreover, integrating from $t_{k-1} + \varepsilon$ to $t \in (t_{k-1}, t_k)$ gives

$$\begin{aligned} e^{At}x(t) = e^{A(t_{k-1}+\varepsilon)}&x\left(t_{k-1}+\varepsilon\right) \\ &+ \int_{t_{k-1}+\varepsilon}^t e^{As}\left[Cf\left(y(s-\tau(s))\right)\right]ds, \end{aligned} \tag{15}$$

which yields, after letting $\varepsilon \to 0^+$,

$$\begin{aligned} e^{At}x(t) = e^{A(t_{k-1})}&x\left(t_{k-1}^+\right) \\ &+ \int_{t_{k-1}}^t e^{As}\left[Cf\left(y(s-\tau(s))\right)\right]ds, \\ &\qquad t \in \left(t_{k-1}, t_k\right). \end{aligned} \tag{16}$$

Throughout this paper, we assume that $\varepsilon$ is a sufficient small positive number. Now, taking $t = t_k - \varepsilon$ in the above equation yields

$$e^{At_k-\varepsilon} x(t_k - \varepsilon) = e^{At_{k-1}} x(t_{k-1}^+) + \int_{t_{k-1}}^{t_k-\varepsilon} e^{As} [Cf(y(s-\tau(s)))] \, ds, \tag{17}$$

which yields by letting $\varepsilon \to 0^+$

$$e^{At_k} x(t_k) = e^{At_{k-1}} x(t_{k-1}^+) + \int_{t_{k-1}}^{t_k} e^{As} [Cf(y(s-\tau(s)))] \, ds. \tag{18}$$

So, we have actually got

$$\begin{aligned} e^{At} x(t) &= e^{At_{k-1}} x(t_{k-1}^+) + \int_{t_{k-1}}^{t} e^{As} [Cf(y(s-\tau(s)))] \, ds \\ &= e^{At_{k-1}} x(t_{k-1}) + \int_{t_{k-1}}^{t} e^{As} [Cf(y(s-\tau(s)))] \, ds + e^{At_{k-1}} \rho(x(t_{k-1})), \end{aligned} \tag{19}$$

for all $t \in (t_{k-1}, t_k]$, $k = 1, 2, \ldots$. Furthermore, (19) generates

$$\begin{aligned} e^{At_{k-1}} x(t_{k-1}) &= e^{At_{k-2}} x(t_{k-2}) + \int_{t_{k-2}}^{t_{k-1}} e^{As} [Cf(y(s-\tau(s)))] \, ds + e^{At_{k-2}} \rho(x(t_{k-2})), \\ &\ \vdots \\ e^{At_2} x(t_2) &= e^{At_1} x(t_1) + \int_{t_1}^{t_2} e^{As} [Cf(y(s-\tau(s)))] \, ds + e^{At_1} \rho(x(t_1)), \\ e^{At_1} x(t_1) &= \xi(0) + \int_{0}^{t_1} e^{As} [Cf(y(s-\tau(s)))] \, ds. \end{aligned} \tag{20}$$

Making a synthesis of the above equations results in (9).

Similarly, we can obtain

$$y(t) = e^{-Bt} \eta(0) + e^{-Bt} \left[ \int_0^t e^{Bs} Dg(x(s-h(s))) \, ds + \sum_{0<t_k<t} e^{Bt_k} \rho(y_{t_k}) \right], \quad t \geqslant 0. \tag{21}$$

So, we may define the mapping $P$ on the space $\Omega$ as follows:

$$P\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} e^{-At} \xi(0) + e^{-At} \left[ \int_0^t e^{As} Cf(y(s-\tau(s))) \, ds + \sum_{0<t_k<t} e^{At_k} \rho(x_{t_k}) \right] \\ e^{-Bt} \eta(0) + e^{-Bt} \left[ \int_0^t e^{Bs} Dg(x(s-h(s))) \, ds + \sum_{0<t_k<t} e^{Bt_k} \rho(y_{t_k}) \right] \end{pmatrix}, \quad \text{for } t \in [0, +\infty), \tag{22}$$

$$P\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \xi(t) \\ \eta(t) \end{pmatrix}, \quad \text{for } t \in [-\tau, 0]. \tag{23}$$

*Step 2.* We claim that $P\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in \Omega$ for any $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in \Omega$. In other words, we need to prove that $P\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ must satisfy conditions (a)–(d) of $\Omega$.

Indeed, $P(\cdot)$ satisfies condition (b) due to (23). Besides, let $\varepsilon$ be a real number; we have

$$P\begin{pmatrix} x(t+\varepsilon) \\ y(t+\varepsilon) \end{pmatrix} = \begin{pmatrix} e^{-A(t+\varepsilon)} \xi(0) + e^{-A(t+\varepsilon)} \left[ \int_0^{t+\varepsilon} e^{As} Cf(y(s-\tau(s))) \, ds + \sum_{0<t_k<t+\varepsilon} e^{At_k} \rho(x_{t_k}) \right] \\ e^{-B(t+\varepsilon)} \eta(0) + e^{-B(t+\varepsilon)} \left[ \int_0^{t+\varepsilon} e^{Bs} Dg(x(s-h(s))) \, ds + \sum_{0<t_k<t+\varepsilon} e^{Bt_k} \rho(y_{t_k}) \right] \end{pmatrix}. \tag{24}$$

In (24), letting $\varepsilon \to 0$ brings about

$$\lim_{\varepsilon\to 0} P\begin{pmatrix} x(t+\varepsilon) \\ y(t+\varepsilon) \end{pmatrix} = P\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \forall t \neq t_k,\ t \geqslant 0, \tag{25}$$

which implies that $P\binom{x(t)}{y(t)}$ is continuous on $t \geqslant 0$, $t \neq t_k$. Here, the convergences are under the metric defined on the space $\Omega$. Below, all the convergences of vector functions are in this sense. Thus, condition (a) is satisfied.

Furthermore, letting $t = t_j$, $j = 1, 2, \ldots$, in (24), we have

$$\begin{gathered} \lim_{\varepsilon\to 0^-} P\begin{pmatrix} x(t_j+\varepsilon) \\ y(t_j+\varepsilon) \end{pmatrix} = P\begin{pmatrix} x(t_j) \\ y(t_j) \end{pmatrix}, \\ \lim_{\varepsilon\to 0^+} P\begin{pmatrix} x(t_j+\varepsilon) \\ y(t_j+\varepsilon) \end{pmatrix} = P\begin{pmatrix} x(t_j) \\ y(t_j) \end{pmatrix} + \begin{pmatrix} \rho(x(t_j)) \\ \rho(y(t_j)) \end{pmatrix}, \end{gathered} \tag{26}$$

which implies that condition (c) is also satisfied.

Next, condition (d) is satisfied if only

$$e^{\gamma t}\begin{pmatrix} e^{-At}\xi(0) + e^{-At}\left[\int_0^t e^{As} Cf(y(s-\tau(s)))\,ds + \sum_{0<t_k<t} e^{At_k}\rho(x_{t_k})\right] \\ e^{-Bt}\eta(0) + e^{-Bt}\left[\int_0^t e^{Bs} Dg(x(s-h(s)))\,ds + \sum_{0<t_k<t} e^{Bt_k}\rho(y_{t_k})\right] \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} \in R^{2n}, \quad t \longrightarrow +\infty. \tag{27}$$

Indeed, obviously, $e^{\gamma t}e^{-At}\xi(0) \to 0 \in R^n$ and $e^{\gamma t}e^{-Bt}\eta(0) \to 0 \in R^n$ as $t \to \infty$.

Below, we may firstly prove

$$e^{\gamma t}e^{-At}\int_0^t e^{As} Cf(y(s-\tau(s)))\,ds \longrightarrow 0 \in R^n, \quad t \longrightarrow \infty. \tag{28}$$

In fact, it follows from $e^{\gamma t}x(t) \to 0$ that, for any given $\varepsilon > 0$, there exists a corresponding constant $t^* > \tau$ such that

$$\left|e^{\gamma t}x(t)\right| + \left|e^{\gamma t}y(t)\right| < \varepsilon\mu, \quad \forall t \geqslant t^*, \text{ where } \mu = (1, 1, \ldots, 1)^T \in R^n. \tag{29}$$

Next, we get by (A1)

$$\begin{aligned} &\left|e^{\gamma t}e^{-At}\int_0^t e^{As} Cf(y(s-\tau(s)))\,ds\right| \\ &\quad \leqslant e^{-(A-\gamma I)t}\int_0^{t^*} e^{As}|C|F\left|y(s-\tau(s))\right|ds \\ &\quad + e^{-(A-\gamma I)t}\int_{t^*}^t e^{As}|C|F\left|y(s-\tau(s))\right|ds. \end{aligned} \tag{30}$$

On the one hand,

$$\begin{aligned} &e^{-(A-\gamma I)t}\int_0^{t^*} e^{As}|C|F\left|y(s-\tau(s))\right|ds \\ &\quad \leqslant t^* e^{-(A-\gamma I)t}e^{At^*}|C|F\left[\max_i\left(\sup_{s\in[-\tau,t^*]}\left|y_i(s)\right|\right)\right]\mu \\ &\quad \longrightarrow 0 \in R^n, \quad t \longrightarrow \infty. \end{aligned} \tag{31}$$

On the other hand, obviously, there exists a positive number $a_0$ such that $|C|F\mu \leqslant a_0\mu$. So, we have

$$\begin{aligned} &e^{-(A-\gamma I)t}\int_{t^*}^t e^{As}|C|F\left|y(s-\tau(s))\right|ds \\ &\quad \leqslant \varepsilon e^{\gamma\tau}e^{-(A-\gamma I)t}\int_{t^*}^t e^{(A-\gamma I)s}|C|F\mu ds \\ &\quad \leqslant \varepsilon a_0 e^{\gamma\tau}e^{-(A-\gamma I)t}\begin{pmatrix} \frac{e^{(a_1-\gamma)t}}{a_1-\gamma} & 0 & \cdots & 0 & 0 \\ 0 & \frac{e^{(a_2-\gamma)t}}{a_2-\gamma} & 0 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & \cdots & 0 & \frac{e^{(a_n-\gamma)t}}{a_n-\gamma} \end{pmatrix}\mu \\ &\quad = \varepsilon a_0 e^{\gamma\tau}\left(\frac{1}{a_1-\gamma}, \frac{1}{a_2-\gamma}, \ldots, \frac{1}{a_n-\gamma}\right)^T. \end{aligned} \tag{32}$$

Combining (30)–(32) yields (28). Similarly, we can prove

$$e^{\gamma t}e^{-Bt}\int_0^t e^{Bs} Dg(x(s-h(s)))\,ds \longrightarrow 0 \in R^n, \quad t \longrightarrow \infty. \tag{33}$$

Next, we need to prove that if $t \to +\infty$,

$$\begin{aligned} &e^{\gamma t}\begin{pmatrix} e^{-At}\sum_{0<t_k<t} e^{At_k}\rho(x_{t_k}) \\ e^{-Bt}\sum_{0<t_k<t} e^{Bt_k}\rho(y_{t_k}) \end{pmatrix} \\ &\quad = e^{\gamma t}\begin{pmatrix} e^{-At}\sum_{0<t_k<t^*} e^{At_k}\rho(x_{t_k}) \\ e^{-Bt}\sum_{0<t_k<t^*} e^{Bt_k}\rho(y_{t_k}) \end{pmatrix} \\ &\quad + e^{\gamma t}\begin{pmatrix} e^{-At}\sum_{t^*<t_k<t} e^{At_k}\rho(x_{t_k}) \\ e^{-Bt}\sum_{t^*<t_k<t} e^{Bt_k}\rho(y_{t_k}) \end{pmatrix} \longrightarrow 0 \in R^{2n}. \end{aligned} \tag{34}$$

Indeed, it is obvious that

$$e^{\gamma t}\begin{pmatrix} e^{-At}\sum\limits_{0<t_k<t^*} e^{At_k}\rho\left(x_{t_k}\right) \\ e^{-Bt}\sum\limits_{0<t_k<t^*} e^{Bt_k}\rho\left(y_{t_k}\right)\end{pmatrix} = \begin{pmatrix} e^{(\gamma I-A)t}\sum\limits_{0<t_k<t^*} e^{At_k}\rho\left(x_{t_k}\right) \\ e^{(\gamma I-B)t}\sum\limits_{0<t_k<t^*} e^{Bt_k}\rho\left(y_{t_k}\right)\end{pmatrix} \longrightarrow 0 \in R^{2n}, \quad t \longrightarrow +\infty. \tag{35}$$

Below, we shall prove

$$e^{\gamma t}\begin{pmatrix} e^{-At}\sum\limits_{t^*<t_k<t} e^{At_k}\rho\left(x_{t_k}\right) \\ e^{-Bt}\sum\limits_{t^*<t_k<t} e^{Bt_k}\rho\left(y_{t_k}\right)\end{pmatrix} \longrightarrow 0 \in R^{2n}, \quad t \longrightarrow +\infty. \tag{36}$$

Firstly, we may assume that $t_{m-1} < t^* \leqslant t_m$ and $t_j < t \leqslant t_{j+1}$ for any given $t > t^*$. Hence,

$$\begin{aligned} &\left|e^{\gamma t}e^{-At}\sum_{t^*<t_k<t} e^{At_k}\rho\left(x_{t_k}\right)\right| \\ &\leqslant e^{\gamma t}e^{-At}\sum_{t^*<t_k<t} e^{At_k-\gamma I t_k}He^{\gamma t_k}\left|x_{t_k}\right| \leqslant \frac{\varepsilon}{\delta} \\ &\cdot\begin{pmatrix} e^{-(\gamma-a_1)t}\left(\delta e^{(a_1-\gamma)t_{j+1}} + \int_{t^*}^{t} e^{(a_1-\gamma)s}ds\right)H_1 \\ \vdots \\ e^{-(\gamma-a_n)t}\left(\delta e^{(a_n-\gamma)t_{j+1}} + \int_{t^*}^{t} e^{(a_n-\gamma)s}ds\right)H_n \end{pmatrix} \\ &\leqslant \frac{\varepsilon}{\delta} \\ &\cdot\begin{pmatrix} e^{-(\gamma-a_1)t}\left(\delta e^{(a_1-\gamma)t_{j+1}} + \dfrac{1}{a_1-\gamma}e^{(a_1-\gamma)t}\right)H_1 \\ \vdots \\ e^{-(\gamma-a_n)t}\left(\delta e^{(a_n-\gamma)t_{j+1}} + \dfrac{1}{a_n-\gamma}e^{(a_1-\gamma)t}\right)H_n \end{pmatrix}, \end{aligned} \tag{37}$$

which together with the arbitrariness of the positive number $\varepsilon$ implies that

$$e^{-At}\sum_{t^*<t_k<t} e^{At_k}\rho\left(x_{t_k}\right) \longrightarrow 0 \in R^{n}, \quad t \longrightarrow +\infty. \tag{38}$$

Similarly, we can also get

$$e^{-Bt}\sum_{t^*<t_k<t} e^{Bt_k}\rho\left(y_{t_k}\right) \longrightarrow 0 \in R^{n}, \quad t \longrightarrow +\infty. \tag{39}$$

Hence, we have proved (36) and (34). And so we can conclude (27) from (28), (33), and (34). This means that condition (d) is satisfied, too.

Therefore, $P\binom{x(t)}{y(t)} \in \Omega$ for any $\binom{x(t)}{y(t)} \in \Omega$.

*Step 3.* Below, we only need to prove that $P$ is a contraction mapping.

Indeed, for any $\binom{x(t)}{y(t)}, \binom{\overline{x}(t)}{\overline{y}(t)} \in \Omega$, we have

$$\begin{aligned} &\left|P\begin{pmatrix} x(t) \\ y(t)\end{pmatrix} - P\begin{pmatrix} \overline{x}(t) \\ \overline{y}(t)\end{pmatrix}\right| \\ &\leqslant \begin{pmatrix} e^{-At}\int_0^t e^{As}\left|C\right|\left|f\left(y\left(s-\tau(s)\right)\right) - f\left(\overline{y}\left(s-\tau(s)\right)\right)\right|ds \\ e^{-Bt}\int_0^t e^{Bs}\left|D\right|\left|g\left(x\left(s-h(s)\right)\right) - g\left(\overline{x}\left(s-h(s)\right)\right)\right|ds \end{pmatrix} \\ &+ \begin{pmatrix} e^{-At}\sum\limits_{0<t_k<t} e^{At_k}\left|\rho\left(x_{t_k}\right) - \rho\left(\overline{x}_{t_k}\right)\right| \\ e^{-Bt}\sum\limits_{0<t_k<t} e^{Bt_k}\left|\rho\left(y_{t_k}\right) - \rho\left(\overline{y}_{t_k}\right)\right| \end{pmatrix} \leqslant \left[\begin{pmatrix} A^{-1}\left|C\right|F\mu \\ B^{-1}\left|D\right|G\mu \end{pmatrix}\right. \\ &+ \frac{1}{\delta} \\ &\cdot\left.\begin{pmatrix} e^{-At}\left(\int_0^t e^{As}ds + \delta e^{At}\right)H\mu \\ e^{-Bt}\left(\int_0^t e^{Bs}ds + \delta e^{Bt}\right)H\mu \end{pmatrix}\right] \operatorname{dist}\left(\begin{pmatrix} x(t) \\ y(t)\end{pmatrix}, \begin{pmatrix} \overline{x}(t) \\ \overline{y}(t)\end{pmatrix}\right) \\ &\leqslant \begin{pmatrix} \left(A^{-1}\left|C\right|F + \dfrac{1}{\delta}A^{-1}H + H\right)\mu \\ \left(B^{-1}\left|D\right|G + \dfrac{1}{\delta}B^{-1}H + H\right)\mu \end{pmatrix} \operatorname{dist}\left(\begin{pmatrix} x(t) \\ y(t)\end{pmatrix},\right. \\ &\left.\begin{pmatrix} \overline{x}(t) \\ \overline{y}(t)\end{pmatrix}\right) < \lambda\begin{pmatrix} \mu \\ \mu \end{pmatrix}\operatorname{dist}\left(\begin{pmatrix} x(t) \\ y(t)\end{pmatrix}, \begin{pmatrix} \overline{x}(t) \\ \overline{y}(t)\end{pmatrix}\right), \end{aligned} \tag{40}$$

and hence

$$\begin{aligned} &\operatorname{dist}\left(P\begin{pmatrix} x(t) \\ y(t)\end{pmatrix}, P\begin{pmatrix} \overline{x}(t) \\ \overline{y}(t)\end{pmatrix}\right) \\ &\qquad \leqslant \lambda\operatorname{dist}\left(\begin{pmatrix} x(t) \\ y(t)\end{pmatrix}, \begin{pmatrix} \overline{x}(t) \\ \overline{y}(t)\end{pmatrix}\right), \end{aligned} \tag{41}$$

where $A^{-1}$ and $B^{-1}$ are the inverse matrices of $A$ and $B$, respectively.

Therefore, $P : \Omega \rightarrow \Omega$ is a contraction mapping such that there exists the fixed point $\binom{x(t)}{y(t)}$ of $P$ in $\Omega$, which implies that $\binom{x(t)}{y(t)}$ is a solution of the impulsive fuzzy dynamic equations (2), satisfying $e^{\gamma t}\left\|\binom{x(t)}{y(t)}\right\| \rightarrow 0$ as $t \rightarrow +\infty$. And the proof is completed. □

*Remark 7.* Impulsive BAM neural networks model brings some mathematical difficulties to contraction mapping technique. However, in this paper, we set up the contraction mapping on the complete product space to overcome obstacles.

## 4. Numerical Example

*Example 8.* We equip the impulsive system with the following parameters:

$$\begin{aligned} A &= \begin{pmatrix} 1.8 & 0 \\ 0 & 2.1 \end{pmatrix}, \\ B &= \begin{pmatrix} 2 & 0 \\ 0 & 1.9 \end{pmatrix}, \end{aligned}$$

Table 1: Comparing Theorem 6 with other existing results.

| | Types of equations | Fixed point methods | LMI-based? |
|---|---|---|---|
| Theorem 6 | Impulsive BAM neural networks | Contraction mapping principle | Yes |
| [28, Th. 2–4] | BAM neural networks | Brouwer's fixed theorem | No |
| [29, Th. 2.1, Th. 3.1] | BAM neural networks | Contraction mapping principle | No |
| [22, Th. 1.2] | Neural networks | Contraction mapping principle | No |
| [23, Th. 3.1] | Cellular neural networks | Krasnoselskii fixed point theorem | No |
| [21, Th. 2.1] | Neutral differential equations | Contraction mapping principle | No |
| [30, Th. 1.1-1.2] | Neural networks | Schauder fixed point theorem | No |

$$C = \begin{pmatrix} -0.2 & 0.01 \\ 0 & 0.3 \end{pmatrix},$$
$$D = \begin{pmatrix} 0.3 & 0.02 \\ 0 & -0.1 \end{pmatrix},$$
$$F = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix},$$
$$G = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix},$$
$$H = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.2 \end{pmatrix}. \tag{42}$$

Let $\delta = 1.5$. Then, we can use Matlab LMI toolbox to solve LMI conditions (4), obtaining the datum feasible as follows:

$$\lambda = 0.9913. \tag{43}$$

Obviously, $0 < \lambda < 1$. Thereby, we can conclude from Theorem 6 that the impulsive equations (2) are globally exponentially stable.

*Remark 9.* Example 8 illustrates the effectiveness of LMI-based criterion (Theorem 6). Table 1 presents a comparable result among the related literature, mainly published from 2013 to 2016.

*Remark 10.* From Table 1, we know that there are many existing literatures involving fixed point technique and stability analysis, and a lot of interesting conclusions are derived [21–23, 28–30]. Motivated by some methods of those literatures, we utilized Banach contraction mapping theorem to obtain the LMI-based stability criterion applicable to computer Matlab LMI toolbox. It is well known that computer software can solve large-scale computations in actual engineering, which demonstrates the superiority of the proposed method in this paper to a certain extent.

## 5. Conclusion

Recently, fixed point technique and methods are employed to the stability analysis to BAM neural networks, and some stability criteria are derived (see, e.g., [28, 29]). Some good methods and results of related literature [21–23, 28–30] inspire our current work. Different from existing papers, we utilized Banach fixed point theorem deriving immediately the exponential stability criterion applicable to computer Matlab LMI toolbox. Computer programming is suitable for large-scale computation in practical engineering.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

Ruofeng Rao wrote the original manuscript, Zhilin Pu and Shouming Zhong checked it, and Xinggui Li and Ruofeng Rao were in charge of correspondence. All authors typed, read, and approved the final manuscript.

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