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Research Article

A Modified Combination Rule for *D* Numbers Theory

Ningkui Wang,[1] Fuyu Liu,[1] and Daijun Wei[1,2]

[1] *School of Science, Hubei University for Nationalities, Enshi, Hubei 445000, China*
[2] *School of Engineering, Vanderbilt University, Nashville, TN 37235, USA*

Correspondence should be addressed to Daijun Wei; prof.wei@hotmail.com

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D numbers theory is an appropriate method to deal with the information of uncertainty and incompleteness when making a reasonable decision. Previous *D* numbers theory provides a rule to combine multiple *D* numbers. However, the commutative law is not satisfied in the rule of combining multiple *D* numbers. In this paper, a modified method for multiple *D* numbers combination is proposed. The proposed method defines a new function for multiple *D* numbers combination which is mainly determined by the original value of *D* numbers. Then the proposed combination rule is applied to environmental impact assessment (EIA); our results show that the proposed method is efficient for multiple *D* numbers combination and it is useful when dealing with uncertainty and incompleteness.

1. Introduction

In the real world, it is difficult but necessary to make a comprehensive assessment to make a reasonable decision because much uncertainty and incompleteness are often involved in the assessments [1–4]. Several methods, such as probability theory [5, 6], fuzzy theory [7–13], rough set [14–17], uncertainty theory [18–20], and Dempster-Shafer theory of evidence (DST) [21–23], are widely used to deal with these problems. These methods have widely been used in kinds of fields, like supplier selection [24, 25], risk assessments [26–29], and so on [30, 31].

The DST needs weaker conditions than the Bayesian theory of probability, it is often regarded as an extension of the Bayesian theory [32, 33]. For the frame of discernment, which consists of mutually exclusive and collective elements, the basic probability assignment (BPA) can distribute confident degree to the power set of the frame of discernment. Furthermore, an overall assessment can be obtained by combining pairs of BPAs in the DST. Therefore, the DST has been widely applied to multiple criteria decision-making [34–44]. However, some strong hypotheses obviously exist in the DST because of the definitions of the frame of discernment and the BPA. Firstly, the elements in the frame of discernment require being mutually exclusive, but it is hard to be satisfied in the real life especially in linguistic assessments, such as the evaluation on the subjects; "good" and "very good" are two common linguistic evaluations, but they are not completely mutually exclusive so that the DST is unable to handle them. At the same time, the sum of all the BPAs must be equal to 1. However, lacking of some professional knowledge and inadequacy judgements may lead to incompleteness everywhere in the real word. These shortcomings have limited its usage in some fields [45, 46].

Regarded as the generalization of DST, *D* numbers theory is proposed by Deng [47, 48]. It removes these hypotheses reasonably; the elements in the framework of *D* numbers theory do not need to be mutually exclusive and incomplete assessments can also exist in *D* numbers theory. Because *D* numbers theory has the ability to deal with uncertainty and incompleteness, it has been used in EIA [48], failure mode and effects analysis [49], supplier selection [50], and curtain grouting efficiency assessment [51]. Nevertheless, associative property is not satisfied in the previous *D* numbers' combination rule. In [48], Deng et al. do some work for multiple *D* numbers combination in special circumstances. However, the associative property is not addressed in a general condition. In this paper, a modified method for multiple *D* numbers combination is proposed.

The remainder of this paper is organized as follows. In Section 2, preliminaries about DST and D numbers theory are described in detail. The problem of the previous D numbers combination rule and the proposed method is shown in Section 3. An illustrative numerical example is presented in Section 4. Conclusions are given in Section 5.

2. Problem Statement and Preliminaries

2.1. Dempster-Shafer Theory. DST is proposed by Dempster and Shafer; some basic concepts are introduced as follows [21, 22].

Definition 1. Establish that U is a set of mutually exclusive and collectively exhaustive elements which can be represented as follows:

$$U = (e_1, e_2, e_3, \ldots, e_n). \tag{1}$$

The power set of U is denoted as 2^U; any element belongs to the power set 2^U is said to be a proposition. For a frame of discernment U, a mass function is a mapping, which is denoted as follows:

$$m : 2^U \longrightarrow [0, 1] \tag{2}$$

in which the following conditions are satisfied:

$$\begin{gathered} m(\emptyset) = 0 \\ \sum_{A \subset U} m(A) = 1, \end{gathered} \tag{3}$$

where $\emptyset$ is an empty set and A is a subset of 2^U; the function $m(A)$ represents how strongly the evidence supports A.

Definition 2 (Dempster's rule of combination). Given two BPAs m_1 and m_2, Dempster's rule of combination donated as $m = m_1 \oplus m_2$ is defined as follows:

$$m(A) = \begin{cases} \dfrac{1}{1-K} \displaystyle\sum_{B \cap C = A} m_1(B) m_2(C), & A \neq \emptyset; \\ 0, & A = \emptyset; \end{cases} \tag{4}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C), \tag{5}$$

where A, B, and C are the elements of 2^U and K is a normalization constant which means the conflict coefficient of two BPAs.

Note that Dempster's rule of combination is feasible only when $K < 1$ because $K = 1$ means that the two BPAs are one hundred percent conflicted. Associative property is well satisfied in Dempster's rule of combination.

2.2. D Numbers Theory. There are some strong hypotheses in DST which have limited its wide usage in some fields especially in linguistic assessments. D numbers theory is proposed in [47, 48] and it has overcome these hypotheses. The details about D numbers theory are introduced as follows.

Definition 3. Let Ω be a finite nonempty set; D numbers is a mapping:

$$D : \Omega \longrightarrow [0, 1], \tag{6}$$

where the following conditions are satisfied:

$$\begin{gathered} D(\emptyset) = 0 \\ \sum_{B \subset \Omega} D(B) \leq 1, \end{gathered} \tag{7}$$

where $\emptyset$ is an empty set and B is a subset of Ω. The elements in the set Ω of D numbers do not require mutual exclusiveness and the sum of the assessments can be less than 1 in D numbers theory.

Suppose that five linguistic assessments "extremely poor (EP)," "poor (P)," "average (A)," "good (G)," and "very good (VG)" are used for the evaluation of a car. The framework of DST must be mutually exclusive and D numbers theory providing the framework with nonexclusive hypotheses is more tallying with the actual situation. The differences of their framework of DST and D numbers are shown in Figure 1 [48]. In (7), D numbers theory is acceptable for incomplete information since $\sum_{B \subset U} D(B) \leq 1$ which is more close to the real situation.

Definition 4. For a discrete set $\Omega = (b_1, b_2, b_3, \ldots, b_n)$, where b_i belongs to R and $b_i \neq b_j$ if $i \neq j$, for any $v_i \geq 0$ and $\sum_{i=1}^{n} v_i \leq 1$, a special form of D numbers can be expressed by

$$\begin{gathered} D(b_1) = v_1 \\ D(b_2) = v_2 \\ D(b_3) = v_3 \\ \vdots \\ D(b_n) = v_n \end{gathered} \tag{8}$$

or be represented simply as

$$D = \{(b_1, v_1), (b_2, v_2), (b_3, v_3), \ldots, (b_n, v_n)\}. \tag{9}$$

Definition 5 (D numbers combination rule). Let D_1 and D_2 be two D numbers:

$$\begin{gathered} D_1 = \left\{ \left(b_1^1, v_1^1\right), \left(b_2^1, v_2^1\right), \left(b_3^1, v_3^1\right), \ldots, \left(b_n^1, v_n^1\right) \right\} \\ D_2 = \left\{ \left(b_1^2, v_1^2\right), \left(b_2^2, v_2^2\right), \left(b_3^2, v_3^2\right), \ldots, \left(b_n^2, v_n^2\right) \right\}. \end{gathered} \tag{10}$$

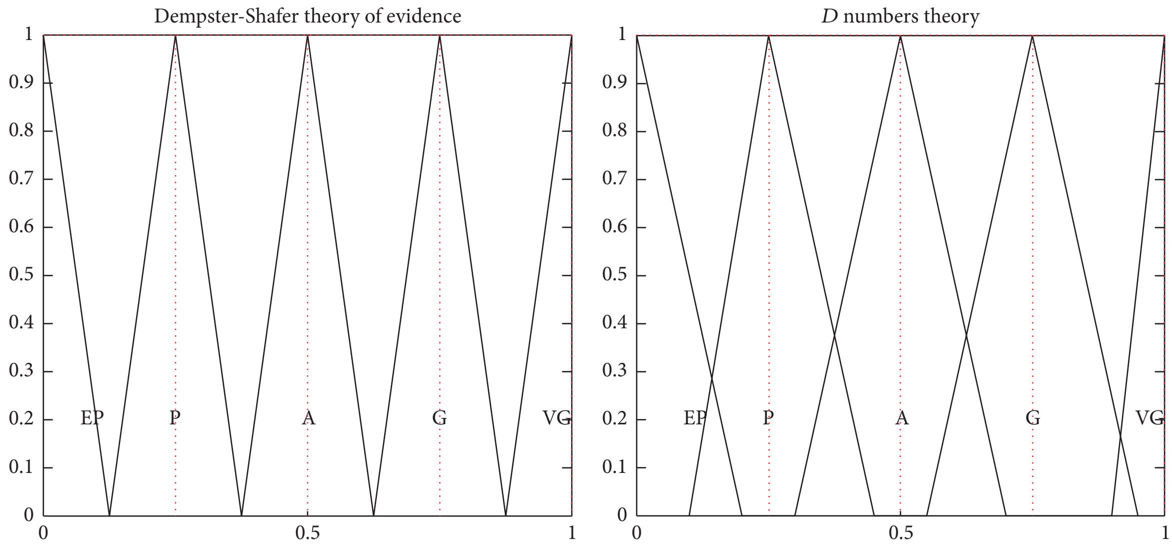

FIGURE 1: The framework of DST and D numbers theory.

The combination of D_1 and D_2 denoted by $D = D_1 \oplus D_2$ is defined as follows:

$$D(b) = v \tag{11}$$

$$b = \frac{\left(b_i^1 + b_j^2\right)}{2} \tag{12}$$

$$v = \frac{\left(v_i^1 + v_j^2\right)}{2 \times c} \tag{13}$$

$$C = \begin{cases} \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{v_i^1 + v_j^2}{2}\right) & \sum_{i=1}^{n} v_i^1 = 1,\ \sum_{i=1}^{m} v_j^1 = 1; \\ \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{v_i^1 + v_j^2}{2}\right) + \sum_{j=1}^{m}\left(\frac{v_c^1 + v_j^2}{2}\right) & \sum_{i=1}^{n} v_i^1 < 1,\ \sum_{i=1}^{m} v_j^1 = 1; \\ \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{v_i^1 + v_j^2}{2}\right) + \sum_{i=1}^{n}\left(\frac{v_i^1 + v_c^2}{2}\right) & \sum_{i=1}^{n} v_i^1 = 1,\ \sum_{i=1}^{m} v_j^1 < 1; \\ \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{v_i^1 + v_j^2}{2}\right) + \sum_{j=1}^{m}\left(\frac{v_c^1 + v_j^2}{2}\right) + \sum_{i=1}^{n}\left(\frac{v_i^1 + v_c^2}{2}\right) & \sum_{i=1}^{n} v_i^1 < 1,\ \sum_{i=1}^{m} v_j^1 < 1, \end{cases} \tag{14}$$

where $v_c^1 = 1 - \sum_{i=1}^{n} v_i^1$ and $v_c^2 = 1 - \sum_{j=1}^{m} v_j^2$, m and n are the assessment numbers in each D number, and the superscripts in above equations are not the exponent but the order of the D numbers.

Definition 6 (D numbers' integration). For given D numbers, the overall assessments can be calculated as follows:

$$I(D) = \sum_{i=1}^{n} b_i v_i. \tag{15}$$

3. Proposed Method

3.1. Problem of Existing D Numbers Combination Rule. It has to be pointed out that the associative property is not satisfied in the previous D numbers combination rule; that is to say that the sequence of multiple D numbers has great effects on the final results when they get combined. As can be seen in the previous D numbers combination rule, $b = (b_i^1 + b_j^2)/2$ and $v = (v_i^1 + v_j^2)/(2 \times c)$; when three D numbers D_1, D_2, and

D_3 get combined, the b and v of the combined results should be

$$b = \frac{(b_1 + b_2)/2 + b_3}{2}$$
$$v = \frac{(v_1 + v_2)/2c_1 + v_3}{2c_2} \qquad (16)$$

which means that the third D number D_3 has more effect on the final results. The associative property is not satisfied in the rule of combining multiple D numbers. Meanwhile, the calculated quantity may increase by multiplication with the evaluation grades increasing in D numbers theory.

Therefore, a method, with which to solve the EIA, is proposed [48]. In that method, an order variable for multiple D numbers combination is given. As each D number is given by a knowledgeable expert from different cultural or educational backgrounds, so all of them will be evaluated in different weights in the decision-making system. The higher the weight is, the more credible the expert should be. For example, three D numbers shown below, ω_1, ω_2, and ω_3, are the weights of the D numbers separately:

$$\begin{aligned} D_1 &= \{(0, 0.6), (1, 0.4)\}, \\ \omega_1 &= 0.3 \\ D_2 &= \{(0, 0.5), (1, 0.5)\}, \\ \omega_2 &= 0.5 \\ D_3 &= \{(0, 0.2), (1, 0.8)\}, \\ \omega_3 &= 0.2. \end{aligned} \qquad (17)$$

Since $\omega_3 < \omega_1 < \omega_2$, the combination sequence is $(D_3 \oplus D_1) \oplus D_2$. If experts' weights are set to be equal, all possible combination results need to be calculated and the highest value of D numbers integration is the best combination result. However, it is so hard to decide the weight of every decision-maker and deciding the weight will always involve human subjective judgements. What is more, when the weights are set to be equal, all possible combination results will have enormous computational complexity.

3.2. Unconfident-Confident Combination Rule of D Numbers. In this section, a new combination sequence for D numbers theory is proposed. The proposed combination rule includes two independent parts, which are "unconfident D numbers combination rule" and "confident D numbers combination rule," respectively. For given D number $D_i = \{(b_i, v_i)\}$ $(i = 1, 2, \ldots, n)$, b_i is the assessment grade the decision-makers made on the decision-making problems and v_i is the confident value to the assessment grade b_i. The value of v_i being more close to 1 means that decision-maker is more confident about the assessment grade. Therefore, the proposed method is given as follows.

Definition 7 (unconfident D numbers combination rule). For given D numbers, if they are different from each other, the maximum value of v_i should be calculated firstly. Suppose $D_1, D_2, \ldots, D_n$ are n D numbers:

$$\begin{aligned} D_1 &= \left\{\left(b_1^1, v_1^1\right), \left(b_2^1, v_2^1\right), \left(b_3^1, v_3^1\right), \ldots, \left(b_n^1, v_n^1\right)\right\} \\ D_2 &= \left\{\left(b_1^2, v_1^2\right), \left(b_2^2, v_2^2\right), \left(b_3^2, v_3^2\right), \ldots, \left(b_n^2, v_n^2\right)\right\} \\ &\vdots \\ D_n &= \left\{\left(b_1^n, v_1^n\right), \left(b_2^n, v_2^n\right), \left(b_3^n, v_3^n\right), \ldots, \left(b_n^n, v_n^n\right)\right\}, \end{aligned} \qquad (18)$$

where

$$\begin{aligned} V_{1\max} &= \max\left\{v_1^1, v_2^1, v_3^1, \ldots, v_n^1\right\} \\ V_{2\max} &= \max\left\{v_1^2, v_2^2, v_3^2, \ldots, v_n^2\right\} \\ &\vdots \\ V_{n\max} &= \max\left\{v_1^n, v_2^n, v_3^n, \ldots, v_n^n\right\}. \end{aligned} \qquad (19)$$

Then the combination operation of multiple D numbers is a mapping f_D, such that

$$\begin{aligned} &f_D(D_1, D_2, D_3, \ldots, D_n) \\ &\quad = \left[\cdots\left[\left[D_i \oplus D_j\right] \oplus D_k\right]\cdots\right], \end{aligned} \qquad (20)$$

where $V_{i\max} > V_{j\max} > V_{k\max}$ in unconfident D numbers combination rule and $V_{i\max}$, $V_{j\max}$, and $V_{k\max}$ are corresponding to D_i, D_j, and D_k.

In the unconfident D numbers combination rule, if some assessments are completely the same, then these assessments should be combined at the first step. Meanwhile, the combinatorial results should be the same to each of the D numbers since the same assessments indicate that all the experts have the same opinions on the object. For example, D_i $(i = 1, 2, 3, \ldots, n)$ are completely the same.

$$\begin{aligned} D_1 &= \left\{\left(b_1^1, v_1^1\right), \left(b_2^1, v_2^1\right), \ldots, \left(b_i^1, v_i^1\right), \ldots, \left(b_n^1, v_n^1\right)\right\}, \\ D_2 &= \left\{\left(b_1^2, v_1^2\right), \left(b_2^2, v_2^2\right), \ldots, \left(b_j^2, v_j^2\right), \ldots, \left(b_n^2, v_n^2\right)\right\}, \\ &\vdots \\ D_n &= \left\{\left(b_1^n, v_1^n\right), \left(b_2^n, v_2^n\right), \ldots, \left(b_j^n, v_j^n\right), \ldots, \left(b_n^n, v_n^n\right)\right\}, \end{aligned} \qquad (21)$$

where b_j^i $(i = 1, 2, \ldots, n)$ are of the same value and v_j^i $(i = 1, 2, \ldots, n)$ are the same confident value as their assessment correspondingly separately. When the n D numbers get combined, the final result D should be the same as each of them; that is to say,

$$D = D_1 \oplus D_2 \oplus \cdots \oplus D_n = D_1 = D_2 = \cdots = D_n. \qquad (22)$$

In (19), if the maximum V_i are of the same value, the better average assessment grades will be combined ahead of the lower average evaluated grades. That is to say, the order of combination is according to the value of average b_i from largest to smallest. The higher average assessment means evaluating it more positively and the lower average assessment means evaluating it more negatively.

In order to illustrate the law of combination of D numbers, for example, the assessment on one project is conducted. D_1, D_2, D_3, D_4, and D_5 are five D numbers given by five experts from different fields:

$$\begin{aligned} D_1 &= \{(0, 0.3), (1, 0.5)\}, \\ V_{1\max} &= 0.5 \\ D_2 &= \{(1, 0.5), (2, 0.4)\}, \\ V_{2\max} &= 0.5 \\ D_3 &= \{(0, 0.6), (1, 0.4)\}, \\ V_{3\max} &= 0.6 \\ D_4 &= \{(1, 0.8), (2, 0.2)\}, \\ V_{4\max} &= 0.8 \\ D_5 &= \{(1, 0.8), (2, 0.2)\}, \\ V_{5\max} &= 0.8. \end{aligned} \tag{23}$$

As D_4 and D_5 are completely the same assessments, we have $D_{45} = D_4 \oplus D_5 = D_4 = D_5$. Then the combined result will be combined with the left D numbers D_1, D_2, and D_3, as $V_{3\max}$ is the biggest value of the three D numbers. So D_{45} will combine with D_3 at the second step. As $V_{1\max}$ and $V_{2\max}$ are of the same value, the better average value of b will be chosen firstly. In D_1, the value of average b is 0.5. In D_2, the value of average b is 1.5. Therefore, D_2 is combined at the third step. The final combined result should be

$$D = D_4 \oplus D_5 \oplus D_3 \oplus D_2 \oplus D_1. \tag{24}$$

As the value of v shows the confident degree to the assessments, according to (13) and (14), the smaller the value of V_i is, the bigger the weight of the combination of V_i will be. The order of combination is from maximum value V to minimum value V. Thus, it is called "unconfident D numbers combination rule."

Meanwhile, another D numbers combination rule called "confident D numbers combination rule" is used accompanying "unconfident D numbers combination rule." In confident D numbers combination rule, the first step is the same as the unconfident method and all the same assessments should be combined with the same results as each of the D numbers.

Definition 8 (confident D numbers combination rule). In (19), the lower value of "V" will be chosen firstly; that is to say, in confident combination rule,

$$\begin{aligned} & f_D(D_1, D_2, D_3, \ldots, D_n) \\ & \quad = \left[\cdots\left[\left[D_i \oplus D_j\right] \oplus D_k\right]\cdots\right], \end{aligned} \tag{25}$$

where $V_{i\max} < V_{j\max} < V_{k\max}$ and $V_{i\max}$ and $V_{j\max}$ and $V_{k\max}$ are corresponding to D_i, D_j, and D_k.

The confident D numbers combination rule is contrary to the unconfident combination rule. Then when minimum values are of the same value, the lower average assessment grade will be combined ahead of the better average evaluated grade.

4. Examples and Applications

In this section, the proposed method is adopted to EIA. EIA usually contains four steps. Firstly the hierarchical structure model for assessment needs to be established, the second step is the assessment for each environmental impact factor, the third step is the calculation of all the evaluated factors, and the last step is to rank the entire projects. In an EIA example, the assessment on the impact of four projects for the conservation of the area of Rupa Tal is taken as follows [52, 53].

Project 1. Keep it the way it is and do not make changes. The lake is disappearing and a small gorge is formed to control the streams because the present sedimentation is still continuing.

Project 2. A high retaining dam is created to raise the overall water level along the southern edge and the in-lake areas created by sedimentation over the last few decades would be overflowed because of the build of retaining dam.

Project 3. Between two precipices, a smaller high dam is built. This dam is smaller than that built in project 2 but has similar upstream effects.

Project 4. A single large sedimentation reservoir is in the upstream area, or a series of smaller retaining walls which would be used to form a sedimentation cascade. The water area may remain intact by this project.

In order to assess these four projects, each factor has some primary subfactors which is shown in Table 1 in detail; every subfactor has different influences on the assessment of the projects.

Second the calculation of the assessment should be done. Nevertheless most of the assessments are represented by linguistic grades like "good" and "poor" and "A," "B," and "C," and so on. First of all, translating such a kind of assessment into numerical grade is necessary. In the existing world, a seven-point scale and five grades are presented [54]. In this method, 3, 2, 1, 0, −1, −2, and −3 represent "very good" to "moderate" to "very bad". The original grades are represented by the letters "A," "B," "C," and so on [52]. In [48], the grade is translated into numerical and shown in Table 2.

From Table 2, the assessment E means major positive impacts and the numerical number is 5. The assessment N means no impact; we translate it into 0. Then the D numbers are obtained from the assessment of experts. For example, when ten experts give the assessments for the conservation of Rupa Tal, six experts believe it is major positive impacts and other four evaluate it to be moderately positive impact; then D numbers should be $\{(5, 0.6), (3, 0.4)\}$. If five experts assess it to be positive impact while four experts evaluate it to be no impact, the remaining expert does not give any evaluation

TABLE 1: The meanings of factors and subfactors in EIA in literature [52].

Factor	Subfactor
Physical/chemical (P/C)	
P/C_1	The impacts of lake water volume
P/C_2	The impacts of the lake sedimentation
P/C_3	The impacts of crop and grazing areas
Biological/ecological (B/E)	
B/E_1	The impacts of lake fisheries
B/E_2	The impacts of biodiversity
B/E_3	The impacts of primary production
B/E_4	The impacts of aquatic macrophytes
B/E_5	The impacts of disease vector populations
Sociological/cultural (S/C)	
S/C_1	The loss of housing
S/C_2	The loss of shops/public buildings
S/C_3	The impacts of accessing routes
S/C_4	The impacts induced by changes of tourism patterns
S/C_5	The impacts of water supplies
S/C_6	The impacts of diet/nutrition
S/C_7	The impacts of aesthetic landscapes
S/C_8	The impacts of water/vector borne disease
S/C_9	The impacts of upstream quality of life
S/C_{10}	The impacts of downstream quality of life
Economic/operational (E/O)	
E/O_1	The impacts of crop-generated incomes
E/O_2	The impacts of fishery generated incomes
E/O_3	The convenience of operation and maintenance of option
E/O_4	The cost of operation and maintenance of option
E/O_5	The cost of resettlement/compensation for land loss
E/O_6	The cost of rehabilitation and restoration of shops
E/O_7	The cost of restoration of accessing routes
E/O_8	The impacts of tourism-generated incomes

TABLE 2: An assessment standard for EIA.

Assessment grade	Numerical rating	Description
E	5	Major positive impact
D	4	Signification positive impact
C	3	Moderately positive impact
B	2	Positive impact
A	1	Slight impact
N	0	No impact
$-A$	−1	Slightly negative impact
$-B$	−2	Negative impact
$-C$	−3	Moderately negative impact
$-D$	−4	Significant negative impact
$-E$	−5	Major negative impact

because of lacking information; the D numbers can be $\{(3, 0.5), (0, 0.4)\}$; this kind of information is incomplete. The assessment matrix for project 1 and project 2 and project 3 and project 4 are shown in Tables 3 and 4, respectively.

The overall assessment for different projects is calculated via unconfident and confident D numbers combination rule, respectively. For example, in unconfident D numbers combination rule, for the evaluation of project 4, the environmental factors are biological and ecological. To the subfactors B/E_1, B/E_2, B/E_3, and B/E_4, all the assessments are $\{(0, 1)\}$; to subfactor B/E_5, the assessment is $\{(-1, 0.4), (0, 0.5)\}$. Firstly, the same assessment should be combined:

$$\begin{aligned}&\{(0,1)\} \oplus \{(0,1)\} \oplus \{(0,1)\} \oplus \{(0,1)\} \oplus \{(0,1)\}\\&\quad = \{(0,1)\}.\end{aligned} \tag{26}$$

Secondly, the combined result $\{(0, 1)\}$ needs to be fused with B/E_5:

$$\begin{aligned}&\{(-1,0.4),(0,0.5)\} \oplus \{(0,1)\}\\&\quad = \left\{\left(0.5, \frac{1.4}{4}\right), \left(0, \frac{1.5}{4}\right)\right\}.\end{aligned} \tag{27}$$

Then, all assessments are combined by same process.

Table 3: Assessment matrix of environment impact factors for projects 1 and 2 [52].

Environmental factors	Project 1	Project 2
Physical/chemical		
P/C_1	$\{(-4, 0.3), (-3, 0.7)\}$	$\{(1, 0.1), (2, 0.9)\}$
P/C_2	$\{(-2, 0.8), (-1, 0.2)\}$	$\{(1, 0.1), (2, 0.85)\}$
P/C_3	$\{2, 0.45), (3, 0.35)\}$	$\{(-3, 0.2), (-2, 0.8)\}$
Biological/ecological		
B/E_1	$\{(-3, 0.5), (-2, 0.4)\}$	
B/E_2	$\{(-2, 0.5), (-1, 0.5)\}$	$\{(-1, 1.0)\}$
B/E_3	$\{(-2, 1.0)\}$	$\{(-2, 1.0)\}$
B/E_4	$\{(-2, 1.0)\}$	$\{(-2, 1.0)\}$
B/E_5		$\{(1, 1.0)\}$
Sociological/culture S/C		
S/C_1	$\{(0, 1.0)\}$	$\{(-1, 1.0)\}$
S/C_2	$\{(0, 1.0)\}$	$\{(-1, 0.65), (0, 0.03)\}$
S/C_3	$\{(1, 0.5), (2, 0.5)\}$	$\{(-1, 1.0)\}$
S/C_4	$\{(-2, 0.2), (1, 0.8)\}$	$\{(2, 0.8), (3, 0.2)\}$
S/C_5	$\{(-2, 0.3), (1, 0.7)\}$	$\{(3, 1.0)\}$
S/C_6	$\{(0, 1.0)\}$	$\{(1, 0.8), (2, 0.2)\}$
S/C_7	$\{(-2, 1.0)\}$	$\{(2, 1.0)\}$
S/C_8	$\{(1, 0.5), (2, 0.3)\}$	$\{(-1, 1.0)\}$
S/C_9	$\{(0, 1.0)\}$	$\{(1, 1.0)\}$
S/C_{10}	$\{(-1, 1.0)\}$	$\{(2, 1.0)\}$
Economical/operational		
E/O_1	$\{(2, 0.8)\}$	$\{(-1, 1.0)\}$
E/O_2	$\{(-2, 1.0)\}$	$\{(2, 1.0)\}$
E/O_3	$\{(0, 1.0)\}$	$\{(-1, 1.0)\}$
E/O_4	$\{(0, 1.0)\}$	$\{(-1, 1.0)\}$
E/O_5	$\{(0, 1.0)\}$	$\{(-1, 1.0)\}$
E/O_6	$\{(0, 1.0)\}$	$\{(-1, 1.0)\}$
E/O_7	$\{(0, 1.0)\}$	$\{(-1, 1.0)\}$
E/O_8	$\{(-1, 1.0)\}$	$\{(3, 0.7)\}$

Lastly, by (15), the last score can be calculated and the example above is taken into consideration:

$$I(D) = 0.5 \times \frac{1.4}{4} + 0 \times \frac{1.5}{4} = 0.175. \tag{28}$$

The final results and ranking are obtained and shown in Table 5 by unconfident and confident D numbers combination rule.

From Table 5, the final ranking is project 2 > project 3 > project 4 > project 1 by using unconfident D numbers combination rule. According to confident D numbers combination rule, the ranking is project 3 > project 2 > project 4 > project 1. The results by the evidential reasoning approach (shortly ER approach) [52] and previous D numbers combination rule (shortly previous D method) [48] are shown in Table 6. From Tables 5 and 6, our results of unconfident D numbers combination rule are the same as risk-taking method [52]. The results of confident D numbers combination rule are the same as decision-optimistic method in [48]. Meanwhile, project 1 is always the worst choice for all methods. In ER approach, the best choice is project 2 or project 4. In previous D method, the best choice is project 3 or project 4. In our method, the best choice is project 2 and project 3; there is the same option for these researches. Furthermore, the unconfident-confident combination rule of D numbers is only determined by the original data of D numbers, any other information about D numbers is no longer needed.

5. Conclusions

How to deal with uncertain and incomplete information to make decisions is an open issue. D numbers theory, which is an extension of DST, has the ability to combine multiple evidence and is wildly used to deal with uncertain and incomplete information problems. However, the associative property for multiple D numbers combination is not satisfied. In this paper, A modified method for multiple D numbers combination denoted as unconfident-confident combination rule is proposed. In our method, the combination rule only depends on the values of D numbers themselves. The proposed method is applied to EIA and the numerical results indicate the effectiveness of the proposed method.

Table 4: Assessment matrix of environmental impact factors for projects 3 and 4 [52].

Environmental factors	Project 3	Project 4
Physical/chemical		
P/C_1	$\{(2, 0.8), (3, 0.2)\}$	$\{(0, 1)\}$
P/C_2	$\{(-1, 0.85), (3, 0.15)\}$	$\{(1, 0.3), (2, 0.7)\}$
P/C_3	$\{(0, 0.5), (1, 0.5)\}$	$\{(0, 1)\}$
Biological/ecological		
B/E_1	$\{(0, 0.2), (1, 0.8)\}$	$\{(0, 1)\}$
B/E_2	$\{(-2, 0.8), (-1, 0.1)\}$	$\{(0, 1)\}$
B/E_3	$\{(-1, 1.0)\}$	$\{(0, 1)\}$
B/E_4	$\{(-1, 1.0)\}$	$\{(0, 1)\}$
B/E_5	$\{(1, 1.0)\}$	$\{(-1, 0.4), (0, 0.5)\}$
Sociological/culture S/C		
S/C_1	$\{(-1, 1.0)\}$	$\{(0, 1.0)\}$
S/C_2	$\{(-1, 1.0)\}$	$\{(0, 1.0)\}$
S/C_3		$\{(0, 1.0)\}$
S/C_4	$\{(1, 1.0)\}$	$\{(0, 1.0)\}$
S/C_5	$\{(1, 1.0)\}$	$\{(1, 0.8)\}$
S/C_6	$\{(1, 0.5), (2, 0.5)\}$	$\{(0, 1.0)\}$
S/C_7	$\{(-1, 0.85), (3, 0.15)\}$	$\{(0, 1.0)\}$
S/C_8	$\{(1, 1.0)\}$	$\{(-1, 1.0)\}$
S/C_9	$\{(0, 0.2), (1, 0.7)\}$	$\{(-1, 1.0)\}$
S/C_{10}	$\{(-2, 0.8), (-1, 0.2)\}$	$\{(0, 1.0)\}$
Economical/operational		
E/O_1	$\{(-2, 0.9)\}$	$\{(0, 1.0)\}$
E/O_2	$\{(0, 1.0)\}$	$\{(0, 1.0)\}$
E/O_3	$\{(-1, 1.0)\}$	$\{(-2, 1.0)\}$
E/O_4	$\{(-1, 1.0)\}$	$\{(-2, 1.0)\}$
E/O_5	$\{(-1, 1.0)\}$	$\{(0, 1.0)\}$
E/O_6	$\{(-1, 1.0)\}$	$\{(0, 1.0)\}$
E/O_7	$\{(-1, 1.0)\}$	$\{(0, 1.0)\}$
E/O_8	$\{(1, 1.0)\}$	$\{(0, 1.0)\}$

Table 5: Overall environmental impacts and ranking of each project.

Assessment grade	Impact rating	Ranking
Unconfident method		
Project 1	−0.18720	4
Project 2	0.09550	1
Project 3	0.04909	2
Project 4	−0.03760	3
Confident method		
Project 1	−0.3453	4
Project 2	−0.0444	2
Project 3	0.2984	1
Project 4	−0.29727	3

Table 6: Overall environmental impacts and ranking of each project.

Method	Ranking
ER approach	
Risk-neutral	Project 2 > project 4 > project 3 > project 1
Risk-taking	Project 2 > project 3 > project 4 > project 1
Risk-average	Project 4 > project 2 > project 3 > project 1
D method	
Decision-optimistic	Project 3 > project 2 > project 4 > project 1
Decision-pessimistic	Project 4 > project 2 > project 3 > project 1

In the future, more work should be done for multiple D numbers combination. D numbers theory is regarded as the generation of DST; many mathematical theorems including the associative property are satisfied in DST. It is reasonable for us to believe that the associative property should be satisfied in D numbers theory. More attempts will be made to find out the solution in which many mathematical theorems are satisfied in the multiple D numbers combination rule. Meanwhile, D numbers theory should be put into applications in more fields to deal with uncertainty and incompleteness, like risk evaluation and so on.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this article.

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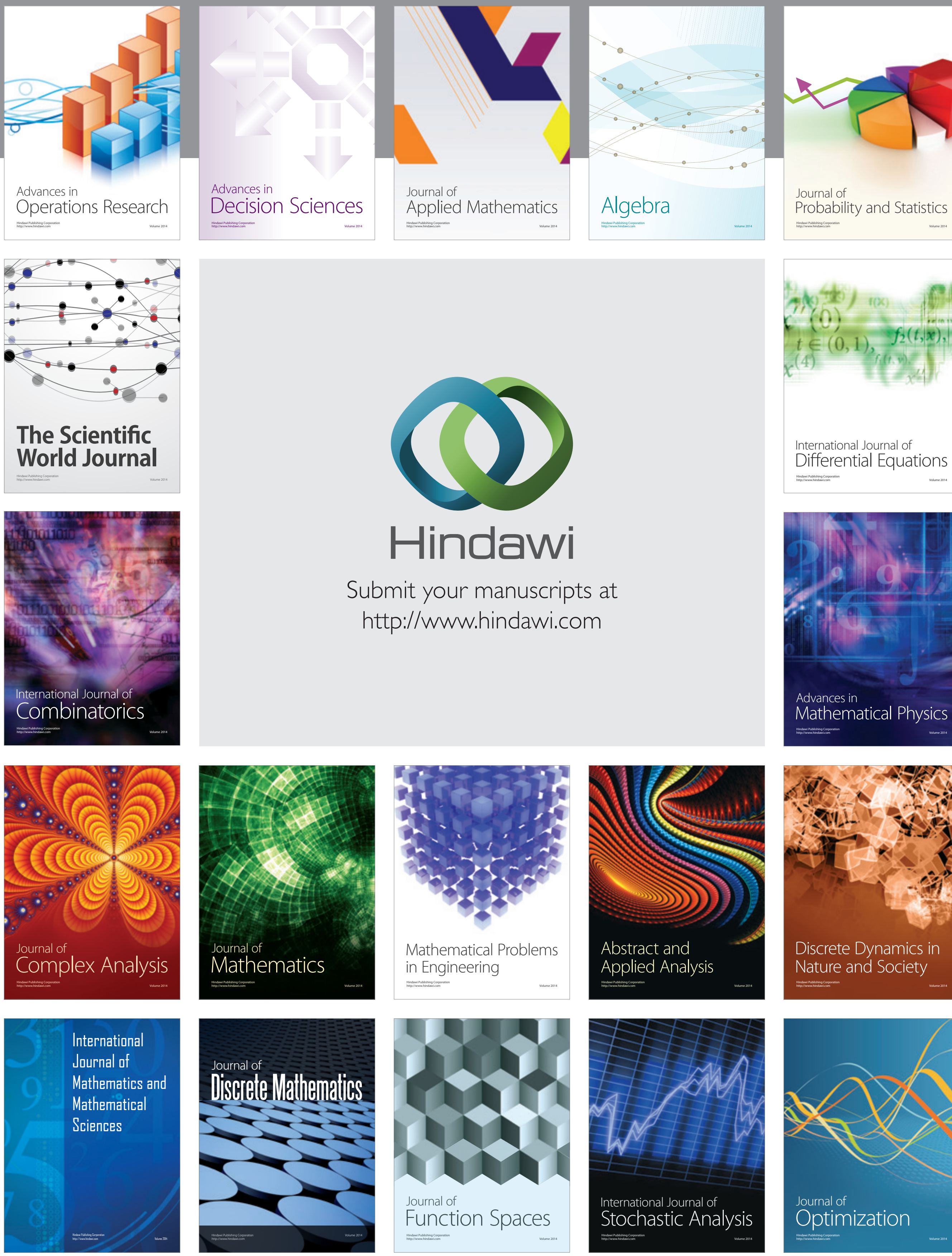
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