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*Research Article*

# Multiperiodic Procurement Problem with Option Contracts under Inflation

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This paper studies the problem of the multiperiod replenishment decisions for the retailer under inflation. In order to manage the risks of price and demand caused by inflation, the retailer has an opportunity to order products and purchase options from the supplier in each period. We formulate the multiperiod inventory model for the retailer with option contracts and then derive his myopic ordering policy in each period and his myopic expected total discounted profit over the entire time horizon. By taking the case without option contracts as a benchmark, we explore the effect of option contracts on the retailer's decisions and performance under inflation. We find that the application of option contracts might induce the retailer to reduce the firm order and increase the total order in each period under inflation. We also find that the application of option contracts might benefit the retailer under inflation.

## 1. Introduction

Since the global financial crisis of 2008, there exists a significant slowdown in the global economy growth and the inflationary pressure has been further emerging. Inflation rate in many countries has stayed above 2% target for an incredibly long time following the crisis. Over the past few years, the global food prices continue to rise and several categories, such as fruit, dairy, and beef, have experienced a dramatic surge. Not only that, but also the increase in the price level of many other commodities remains a strong momentum. Since the pace of wage growth has always lagged behind the pace of price growth, the real incomes of consumers decline and the purchasing power of consumers is eroded. More and more people switch to less-expensive substitutes or buy fewer commodities. Inflation exerts a direct impact on the market price and the consumer demand, which further influence the daily operations of the companies.

On the other hand, in order to protect against various risks derived from production, demand, and price, option contracts have been extensively used in many industries such as fashion apparel industry, food processing industry, and automobile industry [1]. It is worth noting that we limit our discussion to call option contracts in this paper. Comparing with other contracts, the advantage for option contracts is that it can provide the buyer with the right, not the obligation, to reorder the items at a prenegotiated price without sacrificing the interest of the seller. In recent years, option contracts are adopted by many famous firms such as Hewlett-Packard [2] and China Telecom [3]. For example, Hewlett-Packard use option contracts for 35% procurement value of inputs. China Telecom use option contracts for more than 100 billion RMB value of goods. Hence, we have good reasons to believe that option contracts are useful for protecting against the risks of price and demand caused by inflation.

Until now, inflation characterizes the new normal operating environment. Since the inflationary process takes a relatively long time span, the effect of inflation on the companies is also a long-term effect. Wan and Chen [4] study the problem of the ordering and production decisions for the retailer and the supplier under inflation. They prove that the application of option contracts benefits the supply chain members under inflation. However, they just discuss the single-period situation. Unlike the above paper, our work aims to discuss the multiperiod replenishment problem for the retailer in the presence of option contracts under inflation. We exhibit an important feature that distinguishes from the single-period problem: the retailer needs to adapt the successive orders to respond to the successively observed market demand. Several key questions are addressed in this paper:

(i) With and without option contracts, what is the structure of the optimal ordering policy for the retailer in each period under inflation?

(ii) With and without option contracts, what is the approximate value of the optimal expected total discounted profit for the retailer over the entire time horizon under inflation?

(iii) What effect do option contracts have on the retailer's decisions under inflation?

(iv) What effect do option contracts have on the retailer's performance under inflation?

The rest of this paper is structured as follows. The related literature review is presented in Section 2. Model description and assumptions are given in Section 3. A multiperiod inventory model with option contracts is proposed in Section 4. The effect of option contracts on the retailer's decisions and performance is discussed in Section 5. A numerical study is presented in Section 6. Conclusions and the possible future work are given in Section 7.

## 2. Literature Review

There exist three main research streams that are mostly related to our study. The first research stream is on enterprise operation management under inflation. Hsieh and Dye [5] analyze the optimal lot size and periodic pricing policies for a deterioration item inventory under inflation, in which partial backlogging is allowed and the demand is price and time dependent. Sarkar et al. [6] propose an EMQ model with a defective production process under inflation, in which the reliability parameter is considered and the demand is time dependent. Mirzazadeh [7] formulates an inventory model with partial backlogging, in which the time horizon is random and the demand is inflation dependent. Mousavi et al. [8] discuss the optimal replenishment solutions for a multi-item multiperiod inventory with constraints and discounts under inflation. Gilding [9] demonstrated that there exists a change on the optimal inventory control policy with time-dependent demand after considering the effect of inflation. Pal et al. [10] build up an EMQ model for deteriorating items under inflation in a fuzzy situation, in which time-dependent demand rate and Weibull distribution deterioration rate are considered. All of the related papers mainly focus on the inventory decision of one single company under inflation. They never consider option contracts.

The second research stream is on the use of option contracts. Xu [11] proves that the partners within a supply chain are better off in the presence of option contracts, when there are uncertainties in production, demand, and price. Zhao et al. [12] study the supply chain coordination problem based on the cooperation game method, when the retailer uses option contracts to purchase from the manufacturer. Xia et al. [13] demonstrate that the buyer tends to use firm order contracts under both low and high disruption probability whereas he tends to use option contracts under the moderate disruption probability. Moon and Kwon [14] consider a Nash bargaining game problem for an advertiser-publisher system, in which the advertiser uses option contracts to pay the advertising cost to the publisher. Chen and Shen [3] discuss how to set the parameters of portfolio contracts with options to attain the channel coordination in the presence of a service requirement. Lee et al. [15] study the procurement problem in the presence of option contracts and spot market. They also explore how the capacity constraints and fixed ordering costs influence the optimal solution. Chen et al. [16] discuss the optimal ordering policy for the loss-averse retailer and the optimal production policy for the risk-neutral supplier under option contracts. They also discuss how to design option contracts to achieve the supply chain coordination. Wang and Chen [1] study the joint ordering and pricing problem of a newsvendor with option contracts. All of the related papers mainly focus on the single-period problem under the stochastic demand condition. They never consider the effect of inflation.

The third research stream is on multiperiodic inventory/supply chain management. Chao et al. [17] study the joint ordering and pricing policies for a periodic-review inventory, in which random and dependent supply capacities for different periods are taken into account. Cheaitou et al. [18] use the dynamic programming approach to analyze a two-period production/inventory problem, in which slow and fast production modes as well as capacity constraints are considered. Pan et al. [19] discuss the problem of joint ordering and pricing decisions for a dominant retailer who faces a declining price environment in a two-period situation. Chiang [20] explores the optimal order expediting control policy for a continuous-review inventory, in which regular and fast replenishment modes are used. Linh and Hong [21] find that the supply chain coordination can be achieved in a two-period newsboy problem by properly setting the parameters of revenue-sharing contracts, in which the situations of one and two ordering opportunities are discussed, respectively. Chen and Xiao [22] investigate the effect of the midlife and end-of-life policies on the coordination issue of a two-period supply chain, in which the retailer has a single ordering opportunity over the whole selling cycle and the price declines in the middle of the selling cycle. Cheaitou et al. [23] propose a two-period inventory control model with dual supply sources and demand forecast updating. Wang et al. [24] investigate how to coordinate the two-period supply chain through properly setting revenue-sharing contracts in a declining price environment. All of the related papers never consider option contracts. They also never consider the effect of inflation.

The published papers that are most closely related to our work are those of Li et al. [25], Chen et al. [26], and Wan and Chen [4], respectively. Li et al. [25] deal with the problem of optimizing the multiperiod ordering and production decisions with a supplementary supply-order opportunity in each period. They obtain the optimal stationary solution for the Nash equilibrium over the planning horizon. Chen et al. [26] propose a multiperiod inventory model, in which two different types of orders, normal and expediting orders, are considered and the demand depends on the inventory level after ordering in that period. They find that the optimal replenishment strategy in each period is of base-stock type and of $(s, S)$ type without and with fixed order costs. However, the above two papers never consider option contracts and the effect of inflation. Wan and Chen [4] explore the effect of option contracts on the optimal ordering and production decisions for the retailer and the supplier under inflation. They prove that option contracts benefit two members and achieve an efficient channel under inflation. However, the above paper only discusses the single-period problem. Different from the above studies, the main intension of our study is to formulate a multiperiod inventory model in the presence of option contracts under inflation, which can characterize the feature of the long-lasting duration with respect to inflation.

## 3. Model Description and Assumptions

Consider one retailer that manages the periodic-review inventory system over a finite planning horizon with $n$ period under inflation. Let $t_i$ denote the length of period $i$ ($i = 1, 2, \ldots, n$). As we know, in the classic periodic-review inventory model, the ordering decisions are made at fixed time points and so the length of each period is deterministic. However, this assumption is not realistic in a stochastic environment owing to several reasons such as the variation of costs and changes in technology. In reality, the period length is always stochastic and sometimes exogenous. In this paper, the length of period $i$ is assumed to be an exogenous random variable with probability density $g_i(\cdot)$ and cumulative distribution $G_i(\cdot)$ within the interval $(0, T_i)$. Thereby, it is concerned with the size of the order in that period.

Owing to the effect of inflation, both the retail price and the market demand will change with time during each period. Similar to Jaggi and Khanna [27], we assume that the retail price in period $i$, denoted by $P_i(t_i)$, is represented as $P_i(t_i) = p_i e^{rt_i}$. Here, $p_i$ represents the initial retail price in period $i$ and $r$ represents the inflation rate over the entire time horizon. Moreover, similar to Xiao et al. [28], we assume that the market demand in period $i$, denoted by $d_i(t_i, \epsilon_i)$, is represented as $d_i(t_i, \epsilon_i) = x_i(t_i) + \epsilon_i$. Here, $x_i(t_i)$ represents a decreasing function of the period length $t_i$ and $\epsilon_i$ represents a stochastic error with probability density $f_i(\cdot)$ and cumulative distribution $F_i(\cdot)$ within the interval $(0, +\infty)$. $F(0) = 0$, $E(x) = \mu$, and $\overline{F}(x) = 1 - F(x)$ denotes the tail distribution.

We describe the sequence of the event as follows: before the beginning of period $i$, the retailer has an opportunity to order $Q_i^1$ products at unit wholesale price $w_i$ and purchase $q_i^1$ options at unit purchase price $o_i$ according to the initial inventory level before ordering $y_i^1$ and the market demand in that period. If the two types of orders are placed, the retailer can obtain the products through the firm order and instantly replenish the inventory level up to $z_i^1$ at the beginning of period $i$. During period $i$, the retailer can exercise option contracts to obtain some additional items at unit exercise price $e_i$ when the realized market demand in that period exceeds the inventory level $z_i^1$. If only the options order is placed, the retailer can first use the initial inventory level, namely, the leftover inventory level in the previous period, to meet the market demand in that period. During period $i$, the retailer can exercise option contracts when the realized market demand in that period exceeds the inventory level $y_i^1$. If the two types of orders are not placed, the retailer can only use the initial inventory level to meet the market demand in that period. At the end of period $i$, the unsatisfied market demand becomes lost sale and the unsold products are held in the inventory at unit holding cost $h_i$. The retailer carries the unsold products to meet the market demand in the next period. At the end of the last period, each unsold product is salvaged at unit salvage value $v$.

We list the notations throughout this paper in the Notations.

The retailer is assumed to be in a competitive marketplace and all the market price is exogenous. The lead time of either the firm or options order is assumed to be zero. The retailer has no opportunity to place a supplementary order during each period. Moreover, we make the following assumptions. (1) The first assumption is as follows: $h_i < \alpha h_{i+1}$, $e_i < \alpha e_{i+1}$, $o_i < \alpha o_{i+1}$, $w_i < \alpha w_{i+1}$, and $p_i < \alpha p_{i+1}$. These conditions ensure that unit holding cost, unit exercise and purchase price of option, unit wholesale price, and unit initial retail price in period $i+1$ are higher than those in period $i$ under inflation. They indicate that there exists an increase in the market prices in different periods due to the effect of inflation. (2) The second assumption is as follows: $p_i > o_i + e_i > w_i > o_i$. This condition ensures the profit for the retailer in the presence of option contracts in each period. They indicate that the retailer can realize the incomes by exercising option contracts in each period. (3) The third assumption is as follows: $h_i + w_i > \alpha w_{i+1}$. This condition ensures that the retailer never orders more products than necessary in period $i$ in order to meet the market demand in period $i + 1$. (4) The fourth assumption is as follows: $h_n + w_n > \alpha v$. This condition ensures the retailer never orders infinite products more than necessary in the last period in order to obtain the terminal value.

## 4. Model with Option Contracts

In this section, we plan to propose a multiperiod ordering model with option contracts under inflation. With option contracts, the retailer's expected profit in period $i$, denoted by $\pi_{ri}^1(z_i^1, q_i^1)$, is

$$\begin{aligned}\pi_{ri}^1\left(z_i^1, q_i^1\right) = {} & E\left\{p_i e^{rt_i} \min\left[z_i^1 + q_i^1, d_i\left(t_i, \epsilon_i\right)\right]\right\} \\ & - w_i\left(z_i^1 - y_i^1\right) - o_i q_i^1 \\ & - e_i E \min\left[q_i^1, \left(d_i\left(t_i, \epsilon_i\right) - z_i^1\right)^+\right] \\ & - h_i E\left[z_i^1 - d_i\left(t_i, \epsilon_i\right)\right]^+ .\end{aligned} \tag{1}$$

The first term is the revenue realized by the sales of products in period $i$. The last four terms are the costs of placing the firm order, placing the options order, exercising the options order, and holding the unsold products in period $i$, respectively. Let

$l_i^1 = z_i^1 + q_i^1$. Note that determining $(z_i^1, q_i^1)$ is equivalent to determining $(z_i^1, l_i^1)$. Then,

$$\begin{aligned}\pi_{ri}^1\left(z_i^1, l_i^1\right) &= E\left\{p_i e^{rt_i}\min\left[l_i^1, d_i\left(t_i, \epsilon_i\right)\right]\right\} \\ &\quad - w_i\left(z_i^1 - y_i^1\right) - o_i\left(l_i^1 - z_i^1\right) \\ &\quad - e_i E\min\left[\left(l_i^1 - z_i^1\right), \left(d_i\left(t_i, \epsilon_i\right) - z_i^1\right)^+\right] \\ &\quad - h_i E\left[z_i^1 - d_i\left(t_i, \epsilon_i\right)\right]^+ .\end{aligned} \tag{2}$$

With option contracts, the retailer's expected total discounted profit over the entire time horizon, denoted by $\Pi_r^1(z_1^1, \ldots, z_n^1; l_1^1, \ldots, l_n^1)$, is

$$\begin{aligned}\Pi_r^1\left(z_1^1, \ldots, z_n^1; l_1^1, \ldots, l_n^1\right) \\ = \sum_{i=1}^{n}\alpha^{i-1}\pi_{ri}^1\left(z_i^1, l_i^1\right) + \alpha^n v y_{n+1}^1.\end{aligned} \tag{3}$$

Thereby, with option contracts, the optimization problem for the retailer can be described as

$$\max_{l_i^1 \geq z_i^1 \geq y_i^1, i=1,2,\ldots,n} \Pi_r^1\left(z_1^1, \ldots, z_n^1; l_1^1, \ldots, l_n^1\right). \tag{4}$$

Define $V_i^1(y^1)$ as the retailer's maximum expected total discounted profit with option contracts over periods $i, \ldots, n$, providing $y_i^1 = y^1$ in period $i$. The terminal value is $V_{n+1}^1(y^1) = vy^1$. Since the recursion $y_{i+1}^1 = z_i^1 - d_i(t_i, \epsilon_i)$ holds for the entire time horizon, we can develop the dynamic formulation as follows:

$$V_i^1\left(y^1\right) = w_i y^1 + \max_{l^1 \geq z^1 \geq y^1}\left\{J_i^1\left(z^1, l^1\right)\right\}, \tag{5}$$

where $V_{n+1}^1(y^1) = vy^1$ $(y^1 \geq 0)$ and

$$\begin{aligned}J_i^1\left(z^1, l^1\right) &= H_i^1\left(z^1, l^1\right) \\ &\quad + \alpha E\left\{V_{i+1}^1\left[z^1 - d_i\left(t_i, \epsilon_i\right)\right]^+\right\}, \\ H_i^1\left(z^1, l^1\right) &= E\left\{p_i e^{rt_i}\min\left[l^1, d_i\left(t_i, \epsilon_i\right)\right]\right\} - w_i z^1 \\ &\quad - o_i\left(l^1 - z^1\right) \\ &\quad - e_i E\min\left[\left(l^1 - z^1\right), \left(d_i\left(t_i, \epsilon_i\right) - z^1\right)^+\right] \\ &\quad - h_i E\left[z^1 - d_i\left(t_i, \epsilon_i\right)\right]^+ .\end{aligned} \tag{6}$$

Here, $J_i^1(z^1, l^1)$ represents the retailer's expected total discounted profit with option contracts over periods $i, \ldots, n$, when the inventory level is replenished up to $z^1$ after receiving the firm order and the inventory level is replenished up to $l^1$ after receiving the firm order and exercising the options order in period $i$. $H_i^1(z^1, l^1)$ represents the retailer's expected profit with option contracts in period $i$, when there is no initial inventory and there is no salvage value.

By using the same method, Bellman's principle of optimality, as Li et al. [25] and Chen et al. [26], we characterize the optimal ordering policy for the retailer with option contracts in each period. Define $w_{n+1} = v$ and $V_i^{1+}(y^1) = V_i^1(y^1) - w_i y^1$. Then, the following recursion is equivalent to (5)-(6):

$$V_i^{1+}\left(y^1\right) = \max_{l^1 \geq z^1 \geq y^1}\left\{J_i^1\left(z^1, l^1\right)\right\}, \tag{7}$$

where $V_{n+1}^{1+}(y^1) = 0$ $(y^1 \geq 0)$ and

$$\begin{aligned}J_i^1&\left(z^1, l^1\right) \\ &= H_i^{1+}\left(z^1, l^1\right) + \alpha E\left\{V_{i+1}^{1+}\left[z^1 - d_i\left(t_i, \epsilon_i\right)\right]^+\right\},\end{aligned} \tag{8}$$

$$\begin{aligned}H_i^{1+}&\left(z^1, l^1\right) \\ &= E\left\{p_i e^{rt_i}\min\left[l^1, d_i\left(t_i, \epsilon_i\right)\right]\right\} - w_i z^1 - o_i\left(l^1 - z^1\right) \\ &\quad - e_i E\min\left[\left(l^1 - z^1\right), \left(d_i\left(t_i, \epsilon_i\right) - z^1\right)^+\right] \\ &\quad - \left(h_i - \alpha w_{i+1}\right)E\left[z^1 - d_i\left(t_i, \epsilon_i\right)\right]^+ .\end{aligned} \tag{9}$$

Here, $H_i^{1+}(z^1, l^1)$ represents the retailer's expected profit with option contracts in period $i$, when there is no initial inventory.

**Lemma 1.** *With option contracts, there exist the optimal ordering policies for the retailer over the entire time horizon under inflation.*

*Proof.* See the Appendix. □

From the above lemma, we find that with option contracts the optimal solution for $z_i^1$ in period $i$, denoted by $z_i^{1*}$, is characterized by

$$z_i^{1*} = \begin{cases} s_i^{1*} & \text{if } y_i^1 \leq s_i^{1*}, \\ y_i^1 & \text{if } y_i^1 > s_i^{1*} \end{cases} \tag{10}$$

and the optimal solution for $l_i^1$ in period $i$, denoted by $l_i^{1*}$, is characterized by

$$l_i^{1*} = \begin{cases} S_i^{1*} & \text{if } z_i^1 \leq S_i^{1*}, \\ z_i^{1*} & \text{if } z_i^1 > S_i^{1*}. \end{cases} \tag{11}$$

Here, $s_i^{1*}$ represents the minimum value of $z^1$ maximizing $J_i^1(z^1, l^1)$ and $S_i^{1*}$ represents the minimum value of $l^1$ maximizing $J_i^1(z^1, l^1)$. At this moment, $s_i^{1*}$ is an optimal policy for the retailer's firm order with option contracts in period $i$ and $S_i^{1*}$ is an optimal policy for the retailer's total order with option contracts in period $i$. That is to say, with option contracts, the optimal firm order policy for the retailer in period $i$ is up to the optimal base-stock level $s_i^{1*}$ and the optimal total order policy for the retailer in period $i$ is up to the optimal base-stock level $S_i^{1*}$. From (9), we derive that $\partial^2 J_i^1(z^1, l^1)/\partial z^1 \partial w_i < 0$, $\partial^2 J_i^1(z^1, l^1)/\partial z^1 \partial o_i > 0$, and $\partial^2 J_i^1(z^1, l^1)/\partial z^1 \partial e_i > 0$. Then, we conclude that $J_i^1(z^1, l^1)$ is submodular with respect to $w_i$ and $z^1$, supermodular with respect to $o_i$ and $z^1$, and supermodular with respect to $e_i$ and $z^1$. Thereby, $s_i^{1*}$ is decreasing in $w_i$ and increasing in

$(o_i, e_i)$. Similarly, we derive that $\partial^2 J_i^1(z^1, l^1)/\partial l^1 \partial o_i < 0$ and $\partial^2 J_i^1(z^1, l^1)/\partial l^1 \partial e_i < 0$. Then, we conclude that $J_i^1(z^1, l^1)$ is submodular with respect to $o_i$ and $l^1$ and submodular with respect to $e_i$ and $l^1$. Thereby, $S_i^{1*}$ is decreasing in $(o_i, e_i)$. This proceeding demonstrates that when unit wholesale price in period $i$ tends to increase, the retailer might order more options in that period. When unit purchase and exercise price of option in period $i$ tend to increase, the retailer might order fewer options in that period.

As the previous analysis, we find that with option contracts the optimal ordering policy for the retailer in period $i$ is characterized by two optimal base-stock levels $s_i^{1*}$ and $S_i^{1*}$. It is not easy to compute the exact value of $s_i^{1*}$ and $S_i^{1*}$. However, we can calculate the approximate value of them. Define $s_i^{1+}$ as the minimum value of $z^1$ maximizing $H_i^{1+}(z^1, l^1)$ and $S_i^{1+}$ as the minimum value of $l^1$ maximizing $H_i^{1+}(z^1, l^1)$. The corresponding base-stock policy maximizes the current profit while ignoring the future, so we call it the myopic policy. Hence, up-to-level $s_i^{1+}$ is regarded as a myopic policy for the retailer's firm order with option contracts in period $i$ and up-to-level $S_i^{1+}$ is regarded as a myopic policy for the retailer's total order with option contracts in period $i$. Note that $s_i^{1*} \le s_i^{1+}$ and $S_i^{1*} \le S_i^{1+}$. With some algorithm, we get the following proposition.

**Proposition 2.** *With option contracts, the myopic firm order quantity for the retailer in period i, denoted by $Q_i^{1+}$, is*

$$Q_i^{1+} = \begin{cases} s_i^{1+} - y_i^1 & \text{if } y_i^1 \le s_i^{1+}, \\ 0 & \text{if } y_i^1 > s_i^{1+}. \end{cases} \tag{12}$$

*And the myopic options quantity for the retailer in period i, denoted by $q_i^{1+}$, is*

$$q_i^{1+} = \begin{cases} S_i^{1+} - s_i^{1+} & \text{if } y_i^1 \le s_i^{1+}, \\ S_i^{1+} - y_i^1 & \text{if } s_i^{1+} < y_i^1 \le S_i^{1+}, \\ 0 & \text{if } y_i^1 > S_i^{1+}, \end{cases} \tag{13}$$

*where $\int_0^{T_i} F_i[s_i^{1+} - x_i(t_i)] g_i(t_i) dt_i = (o_i + e_i - w_i)/(e_i + h_i - \alpha w_{i+1})$ and $\int_0^{T_i} (p_i e^{r t_i} - e_i) \overline{F}_i [S_i^{1+} - x_i(t_i)] g_i(t_i) dt_i = o_i$.*

*Proof.* See the Appendix. □

Note that $S_i^{1+} > s_i^{1+}$ is equivalent to $o_i < (w_i + h_i - \alpha w_{i+1})(\int_0^{T_i} (p_i e^{r t_i} - e_i) g_i(t_i) dt_i / \int_0^{T_i} (p_i e^{rt} + h_i - \alpha w_{i+1}) g_i(t_i) dt_i)$. This inequality implies that if unit purchase price of option in each period is too high, the retailer will never purchase any number of options.

From the above proposition, we find that with option contracts the myopic ordering policy for the retailer in each period depends on the initial inventory level before ordering as well as the two myopic base-stock levels $s_i^{1+}$ and $S_i^{1+}$ in that period. When $y_i^1 \le s_i^{1+}$, there exist the myopic solutions with $Q_i^{1+} = s_i^{1+} - y_i^1$ and $q_i^{1+} = S_i^{1+} - s_i^{1+}$. This means that it is necessary for the retailer to place the two types of orders before the beginning of period $i$. In this case, the retailer can instantly replenish the inventory level up to $s_i^{1+}$ by receiving the firm order at the beginning of period $i$. The retailer can obtain the extra items by exercising the options order when the market demand in that period surpasses $s_i^{1+}$. At this moment, the myopic firm order quantity in period $i$ is a variable, dependent on the initial inventory level in that period, whereas the myopic options order quantity in period $i$ is a fixed value, independent of the initial inventory level in that period. When $s_i^{1+} < y_i^1 \le S_i^{1+}$, there exist the myopic solutions with $Q_i^{1+} = 0$ and $q_i^{1+} = S_i^{1+} - y_i^1$. This means that it is necessary for the retailer to place the options order, not the firm order, before the beginning of period $i$. In this case, the retailer first uses the initial inventory level to meet the market demand in that period and then obtain the extra items by exercising the options order when the market demand in that period exceeds $y_i^1$. At this moment, the myopic options order quantity in period $i$ is a variable, dependent on the initial inventory level in that period. When $y_i^1 > S_i^{1+}$, there exist the myopic solutions with $Q_i^{1+} = 0$ and $q_i^{1+} = 0$. This means that it is not necessary for the retailer to place the two types of orders before the beginning of period $i$. In this case, the retailer can only use the initial inventory level to meet the market demand in that period.

Similarly, it is not easy to compute the exact value of $\Pi_r^1(z_1^{1*}, \dots, z_n^{1*}; l_1^{1*}, \dots, l_n^{1*})$. However, the above myopic approach provides a good approximation. The approximate value is precise especially when $s_i^{1+}$ and $S_i^{1+}$ are close to $s_i^{1*}$ and $S_i^{1*}$. With option contracts, the retailer's myopic expected total discounted profit over the entire time horizon, denoted by $\Pi_r^1(s_1^{1+}, \dots, s_n^{1+}; S_1^{1+}, \dots, S_n^{1+})$, is

$$\Pi_r^1 \left( s_1^{1+}, \dots, s_n^{1+}; S_1^{1+}, \dots, S_n^{1+} \right) = \sum_{i=1}^{n} \alpha^{i-1} H_i^{1+} \left( s_i^{1+}, S_i^{1+} \right). \tag{14}$$

Note that $\Pi_r^1(z_1^{1*}, \dots, z_n^{1*}; l_1^{1*}, \dots, l_n^{1*}) \ge \Pi_r^1(s_1^{1+}, \dots, s_n^{1+}; S_1^{1+}, \dots, S_n^{1+})$. Then,

$$\Pi_r^1 \left( s_1^{1+}, \dots, s_n^{1+}; S_1^{1+}, \dots, S_n^{1+} \right) = \sum_{i=1}^{n} \alpha^{i-1} \left\{ \begin{array}{l} S_i^{1+} \int_0^{T_i} \left( p_i e^{r t_i} - o_i - e_i \right) g_i(t_i) \, dt_i - \int_0^{T_i} \int_0^{S_i^{1+} - x_i(t_i)} \left( p_i e^{r t_i} - e_i \right) F_i(\epsilon_i) \, g_i(t_i) \, d\epsilon_i \, dt_i \\ s_i^{1+} \left( o_i + e_i - w_i \right) - \int_0^{T_i} \int_0^{s_i^{1+} - x_i(t_i)} \left( e_i + h_i - \alpha w_{i+1} \right) F_i(\epsilon_i) \, g_i(t_i) \, d\epsilon_i \, dt_i \end{array} \right\}. \tag{15}$$

## 5. Discussion

*5.1. Model without Option Contracts.* To begin with, we plan to propose a multiperiod ordering model without option contracts under inflation. This model can be used as a benchmark to compare with the case of option contracts. We describe the sequence of the event as follows: before the beginning of period $i$, the retailer has an opportunity to order $Q_i^0$ products according to the initial inventory level before ordering $y_i^0$ and the market demand in that period. If the firm order is placed, the retailer can obtain the products through the firm order and instantly replenish the inventory level up to $z_i^0$ at the beginning of period $i$. If the firm order is not placed, the retailer can only use the initial inventory level to meet the market demand in that period.

Without option contracts, the retailer's expected profit in period $i$, denoted by $\pi_{ri}^0(z_i^0)$, is

$$\pi_{ri}^0\left(z_i^0\right) = E\left\{p_i e^{rt_i} \min\left[z_i^0, d_i\left(t_i, \epsilon_i\right)\right]\right\} - w_i\left(z_i^0 - y_i^0\right) - h_i E\left[z_i^0 - d_i\left(t_i, \epsilon_i\right)\right]^+ . \quad (16)$$

The first term is the revenue realized by the sales of products in period $i$. The last two terms are the costs of placing the firm order and holding the unsold products in period $i$, respectively. Without option contracts, the retailer's expected total discounted profit over the entire time horizon, denoted by $\Pi_r^0(z_1^0, \dots, z_n^0)$, is

$$\Pi_r^0\left(z_1^0, \dots, z_n^0\right) = \sum_{i=1}^{n} \alpha^{i-1} \pi_{ri}^0\left(z_i^0\right) + \alpha^n v y_{n+1}^0. \quad (17)$$

Thereby, without option contracts, the optimization problem for the retailer can be described as

$$\max_{z_i^0 \geq y_i^0, i=1,2,\dots,n} \Pi_r^0\left(z_1^0, \dots, z_n^0\right). \quad (18)$$

Define $V_i^0(y^0)$ as the retailer's maximum expected total discounted profit without option contracts over periods $i, \dots, n$, providing $y_i^0 = y^0$ in period $i$. The terminal value is $V_{n+1}^0(y^0) = vy^0$ and the recursion is $y_{i+1}^0 = z_i^0 - d_i(t_i, \epsilon_i)$. We develop the dynamic formulation as follows:

$$V_i^0\left(y^0\right) = w_i y^0 + \max_{z^0 \geq y^0}\left\{J_i^0\left(z^0\right)\right\}, \quad (19)$$

where $V_{n+1}^0(y^0) = vy^0$ $(y^0 \geq 0)$ and

$$J_i^0\left(z^0\right) = H_i^0\left(z^0\right) + \alpha E\left\{V_{i+1}^0\left[z^0 - d_i\left(t_i, \epsilon_i\right)\right]^+\right\},$$

$$H_i^0\left(z^0\right) = E\left\{p_i e^{rt_i} \min\left[z^0, d_i\left(t_i, \epsilon_i\right)\right]\right\} - w_i z^0 - h_i E\left[z^0 - d_i\left(t_i, \epsilon_i\right)\right]^+ . \quad (20)$$

Here, $J_i^0(z^0)$ represents the retailer's expected total discounted profit without option contracts over periods $i, \dots, n$, when the inventory level is replenished up to $z^0$ after receiving the firm order in period $i$. $H_i^0(z^0)$ represents the retailer's expected profit without option contracts in period $i$, when there is no initial inventory and there is no salvage value.

Similarly, define $w_{n+1} = v$ and $V_i^{0+}(y^0) = V_i^0(y^0) - w_i y^0$. Then, the following recursion is equivalent to (19)-(20):

$$V_i^{0+}\left(y^0\right) = \max_{z^0 \geq y^0}\left\{J_i^0\left(z^0\right)\right\}, \quad (21)$$

where $V_{n+1}^{0+}(y^0) = 0$ $(y^0 \geq 0)$ and

$$J_i^0\left(z^0\right) = H_i^{0+}\left(z^0\right) + \alpha E\left\{V_{i+1}^{0+}\left[z^0 - d_i\left(t_i, \epsilon_i\right)\right]^+\right\}, \quad (22)$$

$$H_i^{0+}\left(z^0\right) = E\left\{p_i e^{rt_i} \min\left[z^0, d_i\left(t_i, \epsilon_i\right)\right]\right\} - w_i z^0 - \left(h_i - \alpha w_{i+1}\right) E\left[z^0 - d_i\left(t_i, \epsilon_i\right)\right]^+ . \quad (23)$$

Here, $H_i^{0+}(z^0)$ represents the retailer's expected profit without option contracts in period $i$, when there is no initial inventory.

**Lemma 3.** *Without option contracts, there exist the optimal ordering policies for the retailer over the entire time horizon under inflation.*

*Proof.* See the Appendix. □

From the above lemma, we find that without option contracts the optimal solution for $z_i^0$ in period $i$, denoted by $z_i^{0*}$, is characterized by

$$z_i^{0*} = \begin{cases} s_i^{0*} & \text{if } y_i^0 \leq s_i^{0*}, \\ y_i^0 & \text{if } y_i^0 > s_i^{0*}. \end{cases} \quad (24)$$

Here, $s_i^{0*}$ represents the minimum value of $z^0$ maximizing $J_i^0(z^0)$. At this moment, $s_i^{0*}$ is an optimal policy for the retailer's firm order without option contracts in period $i$. That is to say, without option contracts, the optimal firm order policy for the retailer in period $i$ is up to the optimal base-stock level $s_i^{0*}$. From (23), we derive that $\partial^2 J_i^0(z^0)/\partial z^0 \partial w_i < 0$. Then, we conclude that $J_i^0(z^0)$ is submodular with respect to $w_i$ and $z^0$. Thereby, $s_i^{0*}$ is decreasing in $w_i$. This proceeding demonstrates that when unit wholesale price in period $i$ tends to increase, the retailer might order fewer products in that period. When unit wholesale price in period $i$ tends to decrease, the retailer might order more products in that period.

Similarly, define $s_i^{0+}$ as the minimum value of $z^0$ maximizing $H_i^{0+}(z^0)$. Hence, up-to-level $s_i^{0+}$ is regarded as a myopic policy for the firm order without option contracts in period $i$. Note that $s_i^{0*} \leq s_i^{0+}$. With some algorithm, we get the following proposition.

**Proposition 4.** *Without option contracts, the myopic firm order quantity for the retailer in period i, denoted by* $Q_i^{0+}$, *is*

$$Q_i^{0+} = \begin{cases} s_i^{0+} - y_i^0 & \textit{if } y_i^0 \leq s_i^{0+}, \\ 0 & \textit{if } y_i^0 > s_i^{0+}, \end{cases} \quad (25)$$

*where* $\int_0^{T_i}(p_i e^{rt_i} + h_i - \alpha w_{i+1})\overline{F}_i[s_i^{0+} - x_i(t_i)]g_i(t_i)dt_i = w_i + h_i - \alpha w_{i+1}$.

*Proof.* See the Appendix. □

From the above proposition, we find that without option contracts the myopic ordering policy for the retailer in each period depends on the initial inventory level before ordering as well as the myopic base-stock level $s_i^{0+}$ in that period. When $y_i^0 \leq s_i^{0+}$, there exists the myopic solution with $Q_i^{0+} = s_i^{0+} - y_i^0$. This means that it is necessary for the retailer to place the firm order before the beginning of period $i$. In this case, the retailer can instantly replenish the inventory level up to $s_i^{0+}$ by receiving the firm order at the beginning of period $i$. At this moment, the myopic firm order quantity in period $i$ is a variable, dependent on the initial inventory level in that period. When $y_i^0 > s_i^{0+}$, there exists the myopic solution with $Q_i^{0+} = 0$. This means that it is not necessary for the retailer to place the firm order before the beginning of period $i$. In this case, the retailer can only use the initial inventory level to meet the market demand in that period.

Without option contracts, the retailer's myopic expected total discounted profit over the entire time horizon, denoted by $\Pi_r^0(s_1^{0+}, \ldots, s_n^{0+})$, is

$$\Pi_r^0\left(s_1^{0+}, \ldots, s_n^{0+}\right) = \sum_{i=1}^{n} \alpha^{i-1} H_i^{0+}\left(s_i^{0+}\right). \tag{26}$$

Note that $\Pi_r^0(z_1^{0*}, \ldots, z_n^{0*}) \geq \Pi_r^0(s_1^{0+}, \ldots, s_n^{0+})$. Then,

$$\begin{aligned} \Pi_r^0\left(s_1^{0+}, \ldots, s_n^{0+}\right) = \sum_{i=1}^{n} \alpha^{i-1} \Bigg\{ & s_i^{0+} \int_0^{T_i} \left(p_i e^{rt_i} - w_i\right) g_i\left(t_i\right) dt_i \\ & - \int_0^{T_i} \int_0^{s_i^{0+} - x_i(t_i)} \left(p_i e^{rt_i} + h_i - \alpha w_{i+1}\right) F_i\left(\epsilon_i\right) g_i\left(t_i\right) d\epsilon_i \, dt_i \Bigg\}. \end{aligned} \tag{27}$$

*5.2. The Effect of Option Contracts.* Now, we analyze the effect of option contracts on the retailer's decisions under inflation and get the following proposition.

**Proposition 5.** *For all $i$ ($i = 1, 2, \ldots, n$), $s_i^{1+} < s_i^{0+} < S_i^{1+}$.*

*Proof.* See the Appendix. □

From the above proposition, we find that the application of option contracts has a significant effect on the myopic base-stock levels under inflation. Without option contracts, the retailer needs to guarantee that the inventory level after receiving the firm order in each period is not less than $s_i^{0+}$. With option contracts, the retailer needs to guarantee that the inventory level after receiving the firm order in each period is not less than $s_i^{1+}$. In addition, the retailer also guarantees that the inventory level after receiving the firm order and excising the option order is not less than $S_i^{1+}$. Thereby, we conclude that the application of option contracts might prompt the retailer to increase the total order and reduce the firm order in each period under inflation.

Now, we analyze the effect of option contracts on the retailer's performance under inflation and get the following proposition.

Table 1: The effect of $\gamma$ on the retailer's decisions.

| $\gamma$ | $s_1^{1+}$ | $S_1^{1+}$ | $s_2^{1+}$ | $S_2^{1+}$ | $s_1^{0+}$ | $s_2^{0+}$ |
|---|---|---|---|---|---|---|
| 0.0030 | 254.209 | 266.339 | 165.653 | 179.503 | 264.618 | 176.240 |
| 0.0035 | 254.209 | 266.347 | 165.653 | 179.535 | 264.630 | 176.272 |
| 0.0040 | 254.209 | 266.354 | 165.653 | 179.567 | 264.642 | 176.304 |
| 0.0045 | 254.209 | 266.362 | 165.653 | 179.598 | 264.654 | 176.336 |
| 0.0050 | 254.209 | 266.370 | 165.653 | 179.630 | 264.666 | 176.368 |

**Proposition 6.** *For all $i$ ($i = 1, 2, \ldots, n$), $\Pi_r^1(s_1^{1+}, \ldots, s_n^{1+}; S_1^{1+}, \ldots, S_n^{1+}) > \Pi_r^0(s_1^{0+}, \ldots, s_n^{0+})$.*

*Proof.* See the Appendix. □

From the above proposition, we find that the application of option contracts has a significant effect on the retailer's myopic expected total discounted profit over the entire time horizon under inflation. We observe that the myopic expected total discounted profit of the retailer over the entire time horizon is greater with option contracts than without under inflation. Thereby, we conclude that the application of option contracts might help the retailer improve the performance under inflation.

## 6. Numerical Example

In this section, we plan to use several groups of numerical examples to show the effect of inflation and the effect of option contracts on the retailer's decisions and performance in a multiperiod situation. Similar to Wan and Chen [4], we assume that the function of the nonstochastic part $x_i(t_i)$ declines exponentially; that is, $x_i(t_i) = \lambda_i e^{-\delta_i t_i}$. Here, $\lambda_i$ is the initial market demand in period $i$ and $\delta_i$ is the demand contraction factor in period $i$. We study the two-period case and the corresponding parameters are as follows: $\lambda_1 = 250$, $\lambda_2 = 150$, $t_1 \sim \mathrm{U}(0, 10)$, $t_2 \sim \mathrm{U}(0, 10)$, $\epsilon_1 \sim \mathrm{U}(0, 20)$, $\epsilon_2 \sim \mathrm{U}(0, 40)$, $p_1 = 10$, $p_2 = 15$, $w_1 = 4$, $w_2 = 5.5$, $o_1 = 0.8$, $o_2 = 2.3$, $e_1 = 3.5$, $e_2 = 5$, $h_1 = 2$, $h_2 = 2.8$, $v = 4.5$, and $\alpha = 0.8$.

We show the effect of $\gamma$ on the retailer's decisions in Table 1.

From Table 1, we find that when there exists an obvious increase in $\gamma$, the myopic base-stock levels $S_1^{1+}$ and $S_2^{1+}$ tend to increase while the myopic base-stock levels $s_1^{1+}$ and $s_2^{1+}$ tend to remain unchanged. This means that with option contracts the increase in $\gamma$ might cause the retailer to raise the total order and maintain the firm order in the first and second period under inflation. When there exists an obvious increase in $\gamma$, the myopic base-stock levels $s_1^{0+}$ and $s_2^{0+}$ also tend to increase. This means that without option contracts the increase in $\gamma$ might cause the retailer to raise the total order, namely, the firm order, in the first and second period under inflation.

We show the effect of $\gamma$ on the retailer's performance in Table 2.

From Table 2, we also find that when there exists an obvious increase in $\gamma$, the myopic expected total discounted profits $\Pi_r^1(s_1^{1+}, s_2^{1+}; S_1^{1+}, S_2^{1+})$ and $\Pi_r^0(s_1^{0+}, s_2^{0+})$ tend to increase. This means that with and without option contracts the increase

Table 2: The effect of $\gamma$ on the retailer's performance.

| $\gamma$ | $\Pi_r^1(s_1^{1+}, s_2^{1+}; S_1^{1+}, S_2^{1+})$ | $\Pi_r^0(s_1^{0+}, s_2^{0+})$ |
|---|---|---|
| 0.0030 | 3165.048 | 3158.180 |
| 0.0035 | 3178.048 | 3171.140 |
| 0.0040 | 3191.098 | 3184.150 |
| 0.0045 | 3204.178 | 3197.200 |
| 0.0050 | 3217.318 | 3210.300 |

Table 3: The effect of $\delta_1$ on the retailer's decisions.

| $\delta_1$ | $s_1^{1+}$ | $S_1^{1+}$ | $s_2^{1+}$ | $S_2^{1+}$ | $s_1^{0+}$ | $s_2^{0+}$ |
|---|---|---|---|---|---|---|
| 0.0010 | 254.209 | 266.339 | 165.653 | 179.503 | 264.618 | 176.240 |
| 0.0015 | 253.589 | 265.715 | 165.653 | 179.503 | 263.994 | 176.240 |
| 0.0020 | 252.971 | 265.092 | 165.653 | 179.503 | 263.372 | 176.240 |
| 0.0025 | 252.355 | 264.472 | 165.653 | 179.503 | 262.752 | 176.240 |
| 0.0030 | 251.742 | 263.854 | 165.653 | 179.503 | 262.135 | 176.240 |

Table 4: The effect of $\delta_1$ on the retailer's performance.

| $\delta_1$ | $\Pi_r^1(s_1^{1+}, s_2^{1+}; S_1^{1+}, S_2^{1+})$ | $\Pi_r^0(s_1^{0+}, s_2^{0+})$ |
|---|---|---|
| 0.0010 | 3165.048 | 3158.180 |
| 0.0015 | 3161.288 | 3154.220 |
| 0.0020 | 3157.498 | 3150.220 |
| 0.0025 | 3153.678 | 3146.190 |
| 0.0030 | 3149.838 | 3142.130 |

in $\gamma$ might cause the retailer to obtain more profit under inflation.

We show the effect of $\delta_1$ on the retailer's decisions in Table 3.

From Table 3, we find that when there exists an obvious increase in $\delta_1$, the myopic base-stock levels $S_1^{1+}$ and $s_1^{1+}$ tend to decrease. This means that with option contracts the increase in $\delta_1$ might cause the retailer to reduce the total order and the firm order simultaneously in the first period under inflation. When there exists an obvious increase in $\delta_1$, the myopic base-stock level $s_1^{0+}$ tends to decrease. This means that without option contracts the increase in $\delta_1$ might cause the retailer to reduce the total order, namely, the firm order, in the first period under inflation.

We show the effect of $\delta_1$ on the retailer's performance in Table 4.

From Table 4, we also find that when there exists an obvious increase in $\delta_1$, the myopic expected total discounted profits $\Pi_r^1(s_1^{1+}, s_2^{1+}; S_1^{1+}, S_2^{1+})$ and $\Pi_r^0(s_1^{0+}, s_2^{0+})$ tend to decrease. This means that with and without option contracts the increase in $\delta_1$ might cause the retailer to obtain less profit under inflation.

We show the effect of $\delta_2$ on the retailer's decisions in Table 5.

From Table 5, we find that when there exists an obvious increase in $\delta_2$, the myopic base-stock levels $S_2^{1+}$ and $s_2^{1+}$ tend to decrease. This means that with option contracts the increase in $\delta_2$ might cause the retailer to reduce the total order and the firm order simultaneously in the second period under inflation. When there exists an obvious increase in $\delta_2$, the myopic base-stock level $s_2^{0+}$ tends to decrease. This means that without option contracts the increase in $\delta_2$ might cause the retailer to reduce the total order, namely, the firm order, in the second period under inflation.

Table 5: The effect of $\delta_2$ on the retailer's decisions.

| $\delta_2$ | $s_1^{1+}$ | $S_1^{1+}$ | $s_2^{1+}$ | $S_2^{1+}$ | $s_1^{0+}$ | $s_2^{0+}$ |
|---|---|---|---|---|---|---|
| 0.0020 | 254.209 | 266.339 | 165.653 | 179.503 | 264.618 | 176.240 |
| 0.0025 | 254.209 | 266.339 | 165.283 | 179.131 | 264.618 | 175.869 |
| 0.0030 | 254.209 | 266.339 | 164.915 | 178.760 | 264.618 | 175.498 |
| 0.0035 | 254.209 | 266.339 | 164.548 | 178.391 | 264.618 | 175.130 |
| 0.0040 | 254.209 | 266.339 | 164.182 | 178.022 | 264.618 | 174.762 |

Table 6: The effect of $\delta_2$ on the retailer's performance.

| $\delta_2$ | $\Pi_r^1(s_1^{1+}, s_2^{1+}; S_1^{1+}, S_2^{1+})$ | $\Pi_r^0(s_1^{0+}, s_2^{0+})$ |
|---|---|---|
| 0.0020 | 3165.048 | 3158.180 |
| 0.0025 | 3162.038 | 3154.490 |
| 0.0030 | 3159.038 | 3150.810 |
| 0.0035 | 3156.038 | 3147.110 |
| 0.0040 | 3153.028 | 3143.420 |

We show the effect of $\delta_2$ on the retailer's performance in Table 6.

From Table 6, we also find that when there exists an obvious increase in $\delta_2$, the myopic expected total discounted profits $\Pi_r^1(s_1^{1+}, s_2^{1+}; S_1^{1+}, S_2^{1+})$ and $\Pi_r^0(s_1^{0+}, s_2^{0+})$ tend to decrease. This means that with and without option contracts the increase in $\delta_2$ might cause the retailer to obtain less profit under inflation.

From all the tables, we find that $s_1^{0+}$ is always higher than $s_1^{1+}$ but lower than $S_1^{1+}$, except that $s_2^{0+}$ is always higher than $s_2^{1+}$ but lower than $S_2^{1+}$. This means that the application of option contracts might prompt the retailer to reduce the firm order and increase the total order under inflation. We also find that $\Pi_r^1(s_1^{1+}, \dots, s_n^{1+}; S_1^{1+}, \dots, S_n^{1+})$ is always greater than $\Pi_r^0(s_1^{0+}, \dots, s_n^{0+})$. This means that the application of option contracts might help the retailer obtain more profit under inflation.

## 7. Conclusions and Suggestions for Further Research

This paper studies the multiperiod inventory problem for one retailer, who operates the inventory under a periodic review over a finite planning horizon, in the presence of option contracts under inflation. Owing to the effect of inflation, the retailer has to face the rising price and the shrinking demand. To manage the risks of price and demand caused by inflation, the retailer has an opportunity to order products and purchase options from the supplier in each period. To the best of our knowledge, our study is among the first efforts to integrate option contracts and the effect of inflation into a multiperiod modeling framework. This work provides several interesting observations.

*Observation 1.* We demonstrate that there exist the optimal ordering policies for the retailer with and without option contracts under inflation. By characterizing the relationship between the initial inventory level and the optimal base-stock levels in each period, we also exhibit the expressions of the optimal ordering policy in each period with and without option contracts under inflation.

*Observation 2.* We derive the myopic ordering policy for the retailer in each period with and without option contracts under inflation. We also derive the myopic expected total discounted profit of the retailer over the entire time horizon with and without option contracts under inflation. Our analysis provides scientific suggestion for companies on how to make appropriate replenishment decisions to maximize the profit under inflation.

*Observation 3.* Compared with the case without option contracts, we prove that the application of option contracts might cause the retailer to reduce the firm order and increase the total order in each period under inflation. We also prove that the application of option contracts might help the retailer improve the performance under inflation. Our analysis provides scientific suggestion for the companies on how to select the appropriate contract type to enhance the profit under inflation.

In the near future, we plan to extend this study in the following directions: (1) we plan to consider the case of price-dependent demand and analyze the joint optimal ordering and pricing policies in a multiperiod setting. (2) We plan to formulate this problem as a multi-inventory game with different market power, such as supplier Stackelberg, retailer Stackelberg, and vertical Nash [29, 30]. (3) We plan to consider the risk preference of the retailer, such as the case of loss aversion.

## Appendix

*Proof of Lemma 1.* (1) Consider the case $i = n$. Note that $V_{n+1}^{1+}(y^1) = 0$. Then, $J_n^1(z^1, l^1) = H_n^{1+}(z^1, l^1)$ is a jointly concave function of $z^1$ and $l^1$. Thereby, with option contracts, there exists an optimal ordering policy for the retailer in period $n$ under inflation.

(2) Consider the case $i \neq n$. Given that $[z^1 - d_i(t_i, \epsilon_i)]^+$ and $V_{i+1}^{1+}(\cdot)$ are concave, we see that $V_{i+1}^{1+}\{[z^1 - d_i(t_i, \epsilon_i)]^+\}$ is a jointly concave function of $z^1$ and $l^1$. Since $H_i^{1+}(z^1, l^1)$ is also a jointly concave function of $z^1$ and $l^1$, $J_i^1(z^1, l^1)$ is the sum of two concave functions. Thereby, with option contracts, there exists an optimal ordering policy for the retailer in period $i$ under inflation. This completes the proof. □

*Proof of Proposition 2.* From (9), we derive that $\partial H_i^{1+}(z^1, l^1)/\partial z^1 = (o_i + e_i - w_i) - \int_0^{T_i}(e_i + h_i - \alpha w_{i+1})F_i[z^1 - x_i(t_i)]g_i(t_i)dt_i$, $\partial^2 H_i^{1+}(z^1, l^1)/\partial (z^1)^2 = -\int_0^{T_i}(e_i + h_i - \alpha w_{i+1})f_i[z^1 - x_i(t_i)]g_i(t_i)dt_i < 0$, $\partial H_i^{1+}(z^1, l^1)/\partial l^1 = \int_0^{T_i}(p_i e^{rt_i} - o_i - e_i)g_i(t_i)dt_i - \int_0^{\tau}(p_i e^{rt_i} - e_i)F_i[l^1 - x_i(t_i)]g_i(t_i)dt_i$, and $\partial^2 H_i^{1+}(z^1, l^1)/\partial (l^1)^2 = -\int_0^{T_i}(p_i e^{rt_i} - e_i)f_i[l^1 - x_i(t_i)]g_i(t_i)dt_i < 0$. Since $\partial^2 H_i^{1+}(z^1, l^1)/\partial z^1 \partial l^1 = \partial^2 H_i^{1+}(z^1, l^1)/\partial l^1 \partial z^1 = 0$, we have derived that

$$D = \begin{vmatrix} \dfrac{\partial^2 H_i^{1+}\left(z^1, l^1\right)}{\partial \left(z^1\right)^2} & \dfrac{\partial^2 H_i^{1+}\left(z^1, l^1\right)}{\partial z^1 \partial l^1} \\ \dfrac{\partial^2 H_i^{1+}\left(z^1, l^1\right)}{\partial l^1 \partial z^1} & \dfrac{\partial^2 H_i^{1+}\left(z^1, l^1\right)}{\partial \left(l^1\right)^2} \end{vmatrix} > 0. \tag{A.1}$$

Thereby, $H_i^{1+}(z^1, l^1)$ is a jointly concave function of $z^1$ and $l^1$. Let $\partial H_i^{1+}(z^1, l^1)/\partial z^1 = 0$ and $\partial H_i^{1+}(z^1, l^1)/\partial l^1 = 0$, which yield $\int_0^{T_i} F_i[s_i^{1+} - x_i(t_i)]g_i(t_i)dt_i = (o_i + e_i - w_i)/(e_i + h_i - \alpha w_{i+1})$ and $\int_0^{T_i}(p_i e^{rt_i} - e_i)\overline{F}_i[S_i^{1+} - x_i(t_i)]g_i(t_i)dt_i = o_i$. This completes the proof. □

*Proof of Lemma 3.* (1) Consider the case $i = n$. Note that $V_{n+1}^{0+}(y^0) = 0$. Then, $J_n^0(z^0) = H_n^{0+}(z^0)$ is a concave function of $z^0$. Thereby, without option contracts, there exists an optimal ordering policy for the retailer in period $n$ under inflation.

(2) Consider the case $i \neq n$. Given that $[z^0 - d_i(t_i, \epsilon_i)]^+$ and $V_{i+1}^{0+}(\cdot)$ are concave, we see that $V_{i+1}^{0+}\{[z^0 - d_i(t_i, \epsilon_i)]^+\}$ is a concave function of $z^0$. Since $H_i^{0+}(z^0)$ is also a concave function of $z^0$, $J_i^0(z^0)$ is the sum of two concave functions. Thereby, without option contracts, there exists an optimal ordering policy for the retailer in period $i$ under inflation. This completes the proof. □

*Proof of Proposition 4.* From (23), we derive that $dH_i^{0+}(z^0)/dz^0 = \int_0^{T_i}(p_i e^{rt_i} - w_i)g_i(t_i)dt_i - \int_0^{T_i}(p_i e^{rt} + h_i - \alpha w_{i+1})F_i[z^0 - x_i(t_i)]g_i(t_i)dt_i$ and $d^2 H_i^{0+}(z^0)/d(z^0)^2 = -\int_0^{T_i}(p_i e^{rt_i} + h_i - \alpha w_{i+1})f_i[z^0 - x_i(t_i)]g_i(t_i)dt_i < 0$. Hence, $H_i^{0+}(z^0)$ is a concave function of $z^0$. Let $\partial H_i^{0+}(z^0)/\partial z^0 = 0$, which yields $\int_0^{T_i}(p_i e^{rt_i} + h_i - \alpha w_{i+1})\overline{F}_i[s_i^{0+} - x_i(t_i)]g_i(t_i)dt_i = w_i + h_i - \alpha w_{i+1}$. This completes the proof. □

*Proof of Proposition 5.* From Propositions 2 and 4, we derive that $\partial H_i^{1+}(z^1, l^1)/\partial l^1|_{l^1 = s_i^{0+}} = \int_0^{T_i}(p_i e^{rt_i} - o_i - e_i)g_i(t_i)dt_i - \int_0^{T_i}(p_i e^{rt_i} - e_i)F_i[s_i^{0+} - x_i(t_i)]g_i(t_i)dt_i > \int_0^{T_i}((p_i e^{rt_i} - e_i)/(p_i e^{rt_i} + h_i - \alpha w_{i+1}))\{(p_i e^{rt_i} - w_i) - (p_i e^{rt_i} + h_i - \alpha w_{i+1})F_i[s_i^{0+} - x_i(t_i)]\}g_i(t_i)dt_i = 0$. Thereby $S_i^{1+} > s_i^{0+}$. Moreover, we derive that $dH_i^{0+}(z^0)/dz^0|_{z^0 = s_i^{1+}} = \int_0^{T_i}(p_i e^{rt_i} - w_i)g_i(t_i)dt_i - \int_0^{T_i}(p_i e^{rt_i} + h_i - \alpha w_{i+1})F_i[s_i^{1+} - x_i(t_i)]g_i(t_i)dt_i > \int_0^{T_i}((p_i e^{rt_i} + h_i - \alpha w_{i+1})/(e_i + h_i - \alpha w_{i+1}))\{(o_i + e_i - w_i) - (e_i + h_i - \alpha w_{i+1})F_i[s_i^{1+} - x_i(t_i)]\}g_i(t_i)dt_i = 0$. Thereby, $s_i^{0+} > s_i^{1+}$. This completes the proof. □

*Proof of Proposition 6.* Let $\Delta(s_i^{0+}) = H_i^{1+}(s_i^{0+}, S_i^{1+}) - H_i^{0+}(s_i^{0+})$. Then, $\Delta(s_i^{0+}) = (S_i^{1+} - s_i^{0+})\int_0^{T_i}(p_i e^{rt_i} - o_i - e_i)g_i(t_i)dt_i - \int_0^{T_i}\int_{s_i^{0+} - x_i(t_i)}^{S_i^{1+} - x_i(t_i)}(p_i e^{rt_i} - e_i)F_i(\epsilon_i)g_i(t_i)d\epsilon_i\, dt_i$. We derive that $\Delta(S_i^{1+}) = 0$ and $d\Delta(s_i^{0+})/ds_i^{0+}|_{s_i^{0+} = S_i^{1+}} = -\int_0^{T_i}(p_i e^{rt_i} - o_i - e_i)g_i(t_i)dt_i + \int_0^{T_i}(p_i e^{rt_i} - e_i)F_i[S_i^{1+} - x_i(t_i)]g_i(t_i)dt_i = 0$. Hence,

$H_i^{1+}(s_i^{0+}, S_i^{1+}) > H_i^{0+}(s_i^{0+})$. Equivalently, $H_i^{1+}(s_i^{1+}, S_i^{1+}) > H_i^{0+}(s_i^{0+})$. It follows that $\Pi_r^1(s_1^{1+}, \ldots, s_n^{1+}; S_1^{1+}, \ldots, S_n^{1+}) > \Pi_r^0(s_1^{0+}, \ldots, s_n^{0+})$. This completes the proof. □

## Notations

$t_i$: The length of period $i$ with probability density $g_i(\cdot)$ and cumulative distribution $G_i(\cdot)$ within the interval $(0, T_i)$

$d_i(t_i, \epsilon_i)$: The stochastic market demand in period $i$

$\epsilon_i$: The stochastic error with probability density $f_i(\cdot)$ and cumulative distribution $F_i(\cdot)$ within the interval $(0, +\infty)$

$y_i^1$: The initial inventory level before ordering in period $i$

$z_i^1$: The inventory level after receiving the firm order in period $i$

$l_i^1$: The inventory level after receiving the firm order and exercising the options order in period $i$

$Q_i^1$: The firm order quantity in period $i$. Note: $Q_i^1 = z_i^1 - y_i^1$

$q_i^1$: The options order quantity in period $i$. Note: $q_i^1 = l_i^1 - z_i^1$

$p_i$: Unit initial retail price in period $i$

$w_i$: Unit wholesale price in period $i$

$o_i$: Unit purchase price of option in period $i$

$e_i$: Unit exercise price of option in period $i$

$h_i$: Unit holding cost in period $i$

$v$: Unit salvage value in the last period

$\gamma$: Inflation rate. Note: $\gamma > 0$

$\alpha$: Discount factor. Note: $0 < \alpha \leq 1$.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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