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Research Article

Multiservice Switching Networks with Overflow Links and Resource Reservation

Mariusz Głąbowski and Michał Dominik Stasiak

Faculty of Electronics and Telecommunications, Poznań University of Technology, Polanka 3, 60-965 Poznań, Poland

Correspondence should be addressed to Mariusz Głąbowski; mariusz.glabowski@put.poznan.pl

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This paper proposes a new analytical model of a multiservice switching network with overflow links in the first stage and resource reservation for selected call classes in output directions. The proposed model assumes that both the overflow mechanism and the resource reservation mechanism can be used in a number of selected, or all, classes of calls. A particular attention is given to the way the effective availability parameter for networks with overflow links in the point-to-point selection mode is determined. The proposed model makes it possible to determine dependencies between the internal blocking probability, capacity of overflow links, and the number of reserved resources for call classes to be analysed. Simulation experiments confirm high accuracy of the proposed method and potential applications of the model in engineering issues.

1. Introduction

Switching networks are the most important element of network nodes and significantly influence the effectiveness of telecommunication networks. Switching networks can be divided into nonblocking and blocking networks [1–7]. In nonblocking networks, the phenomenon of internal blocking, manifested by the lack of any possibility of setting up a connection between a free input and a free output, is nonexistent [8]. However, the use of nonblocking networks is not financially viable on account of their complex and extended structure that typically includes a large number of stages and switching elements, switches [9]. As a rule, a construction of a nonblocking network leads to a situation in which a potential increase in network effectiveness is not matched economically by relevant financial input necessitated by its construction [3]. In blocking networks, in turn, the occurrence of the phenomenon of internal blocking may lead to a rejection of a great number of offered calls. There are ways known in traffic engineering to efficiently counteract an increase in the internal blocking probability in multiservice switching networks. These methods include, for example, the application of appropriate control algorithms and the so-called repackaging and rearranging algorithms [3, 10, 11]. These algorithms do not impose any alterations to the structure of the switching network but are characterized by high computational complexity and, ultimately, their implementation may lead to a considerable increase in the load of control devices and significant delays in the process of setting up connections [12].

One of the most effective ways for decreasing the internal blocking probability in the switching network is the application of the so-called overflow links [13, 14]. This method is based on an introduction of additional links between inputs and outputs of neighboring (adjacent) switches of this stage of the network to which the overflow mechanism has been introduced. The introduction of overflow links leads then to only slight changes in the structure of the switching network. Overflow links were used for the first time in the Pentaconta switching system in the latter half of the twentieth century [13]. Their introduction led to at least twofold decrease in the level of the internal blocking probability in switching networks. Additionally, a possibility of executing the overflow mechanism in single-service digital switching networks has also been considered in traffic engineering, notably in [15–17]. It has been proved recently, for example,

in [14, 18, 19], that the application of overflow links is followed by a significant reduction of the internal blocking probability for selected or all classes of calls, including the case of switching networks servicing multiservice traffic [3, 20, 21], now dominant in modern computer and communications systems. Multiservice traffic is composed of a great number of call streams with different characteristics. Fairly significant here are traffic streams generated by real time services, such as VoIP (Voice over Internet Protocol) [22, 23] and DVoIP (Digital Video over Internet Protocol) [24]. References [14, 18, 19] prove (using simulation and analytical methods) that an introduction of overflow links between neighboring (adjacent) switches of the first stage in a multiservice switching network leads to the maximum decrease of the level of the internal blocking probability for selected or all call classes and consequently can lead to an increase in the number of serviced real time connections.

The traffic overflow mechanism can be used in networks created with the help of electronic technologies and in networks based on optical technologies alike [25–27]. The application of overflow links in optical networks seems to be very prospective, indeed. The present-day optical networks must provide reliable service for traffic streams with very differentiated demands, in terms of both bit rates and demanded quality of service parameters [28]. To effectively service traffic streams with varied demands within the range from kilobits per second to gigabits per second, the literature proposes the so-called hybrid optical networks, for example, Optical Burst Transport Network (OBTN) [29]. Networks of this type use in lower layers Dense Wavelength Division Multiplexing (DWDM) technology that provides in its most practical implementations bit rate of 10 Gb/s using a single wavelength to achieve this transmission. To execute channels with lower bit rates, the packet system Multiprotocol Label Switching (MPLS) is commonly used. MPLS makes a division of resources offered by a single wavelength possible. In addition, the traffic overflow mechanism is also considered as one of the essential and key mechanisms relative to the dimensioning stage for these networks (e.g., Optical Burst Switching Networks) [29]. This mechanism can be applied for the optimization of both traffic distribution between nodes and, in line with the considerations presented further in the paper, traffic within a single node (switching network).

The application of the traffic overflow mechanism in the switching network leads to a significant decrease in the internal blocking probability [14, 18, 19], though it has virtually no influence on the value of the external blocking probability in the switching network. In order to effectively manage the traffic distribution in network nodes, it is then necessary to introduce some additional traffic management mechanism in output links of the switching network. One of the most effective access management mechanisms is resource reservation [30–33] which, with appropriately done parametrization of the node, can lead to some privileged status of a number of selected traffic streams, for example, real time streams. This mechanism is based on controlling the admission of new calls to the system in relation to the current state of the load. For this purpose, the so-called reservation threshold is determined and established for each of traffic classes, that is, such a threshold occupancy state of the system below which call streams of a given class will be still admitted for service. All occupancy states "higher" than the reservation threshold belong to the so-called reservation space in which calls of a given class will be blocked. The operation and analysis of the reservation mechanism have been widely addressed in the literature of the subject, for example, in [31–36]. A possibility of the application of reservation mechanisms in shaping traffic service characteristics in switching networks and structuring mutual dependencies between the levels of blocking of individual call classes at output links of switching networks has been analysed in many works, for example, in [34, 37]. These works prove that an introduction of the resource reservation mechanism is followed by changes in the values of blocking probabilities for particular call classes, for example, for equalization of blocking probabilities for all call classes.

For the analytical modelling of switching networks, the methods based on Jacobaeus approach [38] and linear programming can be used [39]. One of the most universal methods for analytical modelling of multiservice switching networks is the so-called effective availability methods [21]. The concept of effective availability for two-stage single-service switching networks has been proposed in [40, 41]. In many other works, notably in [17, 21, 42, 43], this approach has been expanded into single-service switching networks with any number of stages. Subsequently, an appropriately modified concept of effective availability has been also proposed to model multiservice switching networks with various traffic streams [34, 44]. The concept of effective availability has also been used to model multiservice switching networks with overflow links and the point-to-group selection [14]. The models of switching networks with overflow traffic that have been hitherto developed assume the lack of any other additional traffic control mechanisms. This paper proposes the IPPD model (Iterative Point-to-Point blocking Direct method), a model of multiservice switching network with overflow links and the point-to-point selection, that will make it possible to take into consideration the influence of the reservation mechanism (placed in output links) on the blocking probability of a switching network. The basis for the model is an appropriately modified PPD method (Point-to-Point blocking Direct method) [34] in which the blocking probability in the multiservice switching network is determined iteratively. The proposed model will make analysis of switching networks in which any traffic classes can undergo reservation mechanism and be redirected onto overflow links possible.

The paper is structured in the following way. Section 2 provides a description of the structure and presents algorithms for setting up connections in multiservice three-stage switching network with overflow links and the point-to-point selection. Section 3 describes the proposed IPPD method. Section 4 discusses models for output groups, overflow, and interstage links used in the IPPD method. Section 5 presents methods for calculations of the effective availability parameter in switching networks with and without overflow links. In Section 6, the results of analytical modelling are compared with the results of simulation experiments for selected multiservice switching networks. Section 7 sums up the paper.

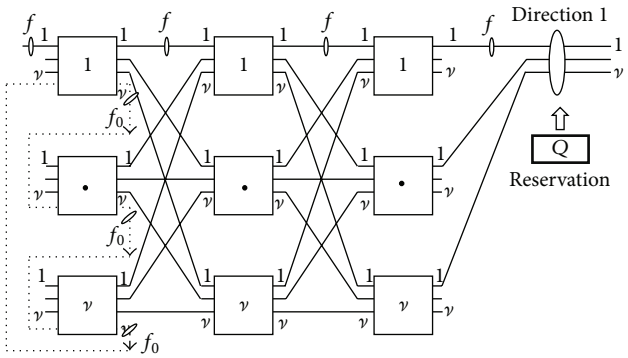

Figure 1: Multiservice three-stage switching network with Clos structure and overflow links and reservation.

2. Switching Networks with Overflow Links and Resource Reservation in Output Links

The present paper considers a network with Clos structure [2] composed of ν symmetrical switches with ν inputs and ν outputs. In this particular case of the analysed multiservice network, all links (input, output, and interstage) have identical capacity equal to f BBUs (Basic Bandwidth Units). In traffic engineering, BBU is calculated as the greatest common divisor of bit rates of all call classes offered to the system [30, 45]. All output links of the switching network are divided into directions. The model assumes that the first output links of each switch of the last stage create the first direction, the second links form the second direction, and so forth. Hence, in the considered switching network, we can define ν directions, each composed of ν output links.

Figure 1 shows a three-stage Clos network with a system of overflow links in the first stage of the network. Overflow links, marked with dotted line, connect an additional output of the first switch of the first stage with an additional input of the next switch of this stage. The additional output of the last switch is connected with the first switch of the first stage. The capacity of the overflow link is indicated in the figure with the symbol f_0 BBUs. The structure of the overflow links proposed in the figure, as well as its location in the first stage, leads to the maximum decrease in the internal blocking probability in the multiservice switching network [18].

The assumption was that the switching network was offered multiservice traffic composed of M call classes with differentiated demands. A call of class i demands t_i BBUs to set up a connection; that is, it requires t_i free BBUs in the input link, interstage links creating a connection path, and the output link in a given direction. In the figure, the reservation mechanism according to which the reservation threshold was introduced is also indicated. The reservation mechanism is indicated by the symbol Q and expressed in the number of busy BBUs of the output direction in the switching network. The reservation threshold can be different for individual classes of calls or the same for all call classes and defines the maximum occupancy state for the output direction in which a call of class i that demands t_i BBU can be admitted for service.

Consider the algorithm for setting up a point-to-point connection in the switching network presented in Figure 1 for calls of class i, with the assumption that this call class can be directed to overflow links and can undergo reservation mechanism with the set threshold equal to Q_i. This algorithm, after identifying the demanded direction by a call of class i that appears at the input of the switch of the first stage, chooses randomly a switch of the last stage that has a free output link in the demanded direction. The notion "free output link" means that this link has t_i free BBUs and additionally, because the class i is to undergo the reservation mechanism, the total occupancy of the demanded direction (i.e., all links that belong to this direction) is not higher than the adopted reservation threshold Q_i. If the demanded direction does not have any free output links, then the call will be lost due to the phenomenon of external blocking. If the control algorithm finds a free output link in a given direction, then in its next step the algorithm will attempt to find a free connection path in the switching network between an input switch and an output switch that have been determined earlier. The existence of such a connection path ensures the connection to be executed. Otherwise, the considered connection is redirected, through an overflow link, to a neighboring switch of the first stage. Now, the control algorithm attempts to set up a connection between the neighboring switch of the first stage and an output switch that has been determined earlier. If this attempt failed, the call will be lost due to the internal blocking.

3. Model of Multiservice Point-to-Point Switching Network

Reference [46] proposes a method for modelling single-service switching networks, the so-called PPD method (Point-to-Point blocking Direct method). The method is based on an estimation of the average number of switches of the last stage that are available for a given call. Then, on the basis of the ratio of the average number of unavailable switches of the last stage to the total number of switches, the internal blocking probability in the switching network is determined. The notion "available switch of the last stage" means such a switch of the last stage to which a connection path from a given input of the switching network can be set up. In [34], the PPD method has been expanded to include multiservice switching networks and multiservice networks with a number of selected reservation mechanisms. In this section, we will propose such a modification of the PPD method, called the IPPD method (Iterative Point-to-Point blocking Direct method), that will iteratively make a determination of all important traffic characteristics of multiservice switching networks possible, including networks with introduced overflow links and the reservation mechanism in output links for all or a number of selected call classes.

3.1. The PPD Method. The internal blocking probability $\mathbf{E}_{\text{int}}$ in the multiservice switching network for calls of all traffic classes in the PPD method can be determined on the basis of the following formula:

$$\mathbf{E}_{\text{int}} = \left\{ E_{\text{int}}(i) = \frac{\nu - d(i)}{\nu} : 1 \le i \le M \right\}, \tag{1}$$

where ν is the capacity of the output direction, expressed in the number of links (equal to the number of last stage switches). The parameter $d(i)$ is the effective availability for calls of class i and defines the average number of switches that are available to a call of class i that appears at the input of the system. An available switch is considered to be such a switch of the last stage to which a connection path can be set up for a call of class i. Let us notice that availability of a given switch is not equal to the existence of a free output link for a call of class i in a demanded direction in the switch. The effective availability parameter for calls of class i in a switching network is the function of structure $\mathbf{S}$ of the switching network, offered traffic $\mathbf{A}$ and the capacity ν of the output direction. Effective availability parameters for all classes of traffic offered to the switching network will be written in the form of vector $\mathbf{d}$:

$$\mathbf{d} = \{d(i) = F(\mathbf{S}, \mathbf{A}, \nu) : 1 \le i \le M\}. \quad (2)$$

Vector $\mathbf{A}$ defines traffic offered to the switching network. Assuming that the number of offered traffic classes is M, we can then write

$$\mathbf{A} = \{A_i : 1 \le i \le M\}, \quad (3)$$

where A_i is the traffic intensity of class i traffic offered to the switching network. The model assumes that traffic of each class is Erlang traffic (the call stream is Poisson stream; service time is described by the exponential distribution). The assumption is that the number of demanded BBUs required to set up connections for individual classes is differentiated and a call of class i demands t_i BBUs to set up a connection. The way the value of effective availability $d(i)$ for calls of individual classes is determined will be presented in the following sections.

External blocking probability $E_{\text{ext}}(i)$ for calls of class i in the switching network results from the occupancy of all output links of a given direction. We can then express the external blocking probability in the form of the following dependence G:

$$\mathbf{E}_{\text{ext}} = \{E_{\text{ext}}(i) = G(\mathbf{A}, \nu, f) : 1 \le i \le M\}, \quad (4)$$

where f is the capacity of a single output link. In the PPD method, the probability $E_{\text{ext}}(i)$ is determined on the basis of the occupancy distribution in the model of limited-availability group (LAG) [34, 47] that approximates a given output direction of the switching network. The LAG model will be discussed in the following sections.

The total blocking probability in the switching network is defined as the sum of the external probability and the internal probability. Calculations of relevant probabilities in the PPD method do not take into consideration the sequential operation of the algorithm that controls setting up connections, which is manifested by the impossibility for the events of the internal and external blocking to occur simultaneously. The control algorithm is constructed in such a way as to first check if the phenomenon of the external blocking can occur (i.e., checking if there are free output links for a call of a given class in a given direction). If this phenomenon does not occur, then a possibility for an occurrence of the internal blocking is checked (i.e., the control algorithm checks a possibility of setting up a connection path in the switching network for a call of a given class). In order to include the sequential character of the operation of the control algorithm, the following formula is proposed for a determination of the total blocking probability $E_{\text{tot}}(i)$ for calls of class i in the PPD method:

$$\mathbf{E}_{\text{tot}} = \{E_{\text{tot}}(i) = E_{\text{ext}}(i) + E_{\text{int}}(i)[1 - E_{\text{ext}}(i)] : 1 \le i \le M\}. \quad (5)$$

3.2. The IPPD Method. The problem of the impossibility of simultaneous occurrence of internal and external blocking is solved in the proposed IPPD method following the underlying reasoning. The assumption is that output links in a given direction can be offered only this part of the total traffic which is not lost at interstage links. In a similar way, interstage links can only be offered this part of the total traffic which is not lost at output links. Such an approach means that while determining the internal blocking probability we take into consideration only this part of traffic that is not lost due to the external blocking. In calculations for the external blocking probability, in turn, we use this part of traffic that is not lost due to the internal blocking. With such an approach, the internal blocking probability can be determined on the basis of formula (1) in which the effective availability can be presented in the form of the following dependence F:

$$\mathbf{d} = \{d(i) = F(\mathbf{S}, \mathbf{A}_{\text{int}}, \nu) : 1 \le i \le M\}, \quad (6)$$

where $\mathbf{A}_{\text{int}}$ represents traffic that is not lost because of the phenomenon of external blocking:

$$\mathbf{A}_{\text{int}} = \{A_{\text{int}}(i) = A_i(1 - E_{\text{ext}}(i)) : 1 \le i \le M\}. \quad (7)$$

The external blocking probability is determined on the basis of the model of the limited-availability group LAG, with the assumption that this group is offered traffic $\mathbf{A}_{\text{ext}}$ that is not lost due to the internal blocking. With this assumption, the dependence G from formula (4) can be rewritten as follows:

$$\mathbf{E}_{\text{ext}} = \{E_{\text{ext}}(i) = G(\mathbf{A}_{\text{ext}}, \nu, f) : 1 \le i \le M\}, \quad (8)$$

where

$$\mathbf{A}_{\text{ext}} = \{A_{\text{ext}}(i) = A_i(1 - E_{\text{int}}(i)) : 1 \le i \le M\}. \quad (9)$$

Since the exclusion of the concurrency of events of internal and external blocking is considered in the proposed IPPD model at the level of offered traffic, then the total blocking probability can be written directly in the form of the sum of the external and internal blocking probabilities:

$$\mathbf{E}_{\text{tot}} = \{E_{\text{tot}}(i) = E_{\text{ext}}(i) + E_{\text{int}}(i) : 1 \le i \le M\}. \quad (10)$$

To determine the probabilities $E_{\text{int}}(i)$, $E_{\text{ext}}(i)$, and $E_{\text{tot}}(i)$ for calls of class i, it is necessary to construct iteration process

that can be written in its simplified form as the following algorithm, called the IPPD algorithm:

IPPD Algorithm

(1) Initiation of the iteration step $k = 0$.

(2) Determination of initial approximations for external and internal blocking probabilities for all classes:

$$\mathbf{E}_{\text{int}}^{[0]} = \left\{E_{\text{int}}^{[0]}(i) = 0 : 1 \le i \le M\right\},$$
$$\mathbf{E}_{\text{ext}}^{[0]} = \left\{E_{\text{ext}}^{[0]}(i) = 0 : 1 \le i \le M\right\}. \tag{11}$$

(3) Increase in the iteration step:

$$k = k + 1. \tag{12}$$

(4) Determination of the value of offered traffic $\mathbf{A}_{\text{int}}^{[k]}$ and traffic $\mathbf{A}_{\text{ext}}^{[k]}$:

$$\mathbf{A}_{\text{int}}^{[k]} = \left\{A_{\text{int}}^{[k]}(i) = A_i\left[1 - E_{\text{ext}}^{[k-1]}(i)\right] : 1 \le i \le M\right\},$$
$$\mathbf{A}_{\text{ext}}^{[k]} = \left\{A_{\text{ext}}^{[k]}(i) = A_i\left[1 - E_{\text{int}}^{[k-1]}(i)\right] : 1 \le i \le M\right\}. \tag{13}$$

(5) Determination of the effective availability and the internal blocking probability of the switching network for individual classes of calls:

$$\mathbf{d}^{[k]} = \left\{d^{[k]}(i) = F\left(\mathbf{S}, \mathbf{A}_{\text{int}}^{[k]}, \nu\right) : 1 \le i \le M\right\}, \tag{14}$$

$$\mathbf{E}_{\text{int}}^{[k]} = \left\{E_{\text{int}}^{[k]}(i) = \frac{\nu - d^{[k]}(i)}{\nu} : 1 \le i \le M\right\}. \tag{15}$$

(6) Determination of the external blocking probability of the switching network for all call classes:

$$\mathbf{E}_{\text{ext}}^{[k]} = \left\{E_{\text{ext}}^{[k]}(i) = G\left(\mathbf{A}_{\text{ext}}^{[k]}, \nu, f\right) : 1 \le i \le M\right\}. \tag{16}$$

(7) Determination of the total blocking probability of the switching network for all call classes:

$$\mathbf{E}_{\text{tot}}^{[k]} = \left\{E_{\text{tot}}^{[k]}(i) = E_{\text{ext}}^{[k]}(i) + E_{\text{int}}^{[k]}(i) : 1 \le i \le M\right\}. \tag{17}$$

(8) Verification of the accuracy of calculations for each class of calls:

$$\left\{\left|\frac{E_{\text{tot}}^{[k]}(i) - E_{\text{tot}}^{[k-1]}(i)}{E_{\text{tot}}^{[k]}(i)}\right| \le \epsilon : 1 \le i \le M\right\}, \tag{18}$$

(a) if the condition is not satisfied, move to step (3),

(b) if the condition is satisfied,

$$\mathbf{E}_{\text{int}} = \mathbf{E}_{\text{int}}^{[k]},$$
$$\mathbf{E}_{\text{ext}} = \mathbf{E}_{\text{ext}}^{[k]}, \tag{19}$$
$$\mathbf{E}_{\text{tot}} = \mathbf{E}_{\text{tot}}^{[k]}.$$

(9) Termination of calculations.

In the algorithm presented above, the assumption is that $X^{[k]}$ denotes the value of the parameter X in the kth iteration. The ϵ parameter is the required relative error of the calculations that determines the accuracy of the iteration process. In the assumed construction of the iteration algorithm, during each iteration, values of internal, external, and the total blocking probabilities for individual call classes are determined on the basis of offered traffic $\mathbf{A}_{\text{int}}^{[k]}$ and traffic $\mathbf{A}_{\text{ext}}^{[k]}$. This traffic is, in turn, determined on the basis of the external and internal blocking probability determined in step (4) of IPPD algorithm.

The proposed IPPD algorithm is not complicated. It is virtually based on an execution of a number of calculations of normalized type, which results in the algorithm to be easily programmable. In the next sections, we will describe the method for a determination of functional dependencies F and G (steps (5) and (6) in the IPPD algorithm) for multiservice switching network with overflow links and reservation mechanism implemented in output directions of the switching network.

4. Models of Interstage, Overflow, and Output Links of the Switching Network with Resource Reservation

In this section, we will present models of interstage and overflow links as well as groups of output links of the switching network. These models will be then used to evaluate the value of effective availability of the switching network for particular call classes (function F in dependencies (2), (6), and (14)) and external blocking probabilities (function G in dependencies (4), (8), and (16)), internal blocking probabilities (formulas (1) and (15)), and total blocking probabilities (formulas (5), (10), and (17)).

4.1. Model of the Interstage Link. In the adopted model, the interstage link is approximated by a multiservice model of the full-availability group (FAG) [48, 49]. The occupancy distribution in FAG is determined on the basis of the following recurrence:

$$n\left[P_n\right]_f = \sum_{i=1}^{M} A_{\text{link}}(i)\, t_i \left[P_{n-t_i}\right]_f \quad \text{for } 0 \le n \le f,$$
$$\left[P_n\right]_f = 0 \quad \text{for } f < n < 0, \tag{20}$$
$$\sum_{n=0}^{f}\left[P_n\right]_f = 1,$$

where

$[P_n]_f$ is the occupancy probability of n BBUs in FAG (one interstage link) with capacity f BBUs,

$A_{\text{link}}(i)$ is traffic of class i offered to FAG (one interstage link),

t_i is the number of BBUs required for class i calls in FAG (one interstage link).

The blocking probability for calls of class i in FAG is defined by the lack of enough number of t_i BBUs to set up a connection. Therefore, the vector $\mathbf{E}_{\text{link}}$ of blocking probabilities in FAG, including all call classes, can be written as follows:

$$\mathbf{E}_{\text{link}} = \left\{ E_{\text{link}}(i) = \sum_{n=f-t_i+1}^{f} P[n]_f : 1 \le i \le M \right\}. \quad (21)$$

If the basis for the considerations is the switching network presented in Section 2, then the values of traffic intensities for individual classes offered to one interstage link $\mathbf{A}_{\text{link}}$,

$$\mathbf{A}_{\text{link}} = \{A_{\text{link}}(i) : 1 \le i \le M\}, \quad (22)$$

can be determined on the basis of the following reasoning: the number of interstage links between two neighboring stages of the switching network (Figure 1) is equal to ν^2. These links are offered traffic that is then directed to all ν directions of the switching network. The model assumes a complete symmetry of traffic distribution. This means that traffic $\mathbf{A}$ (formula (3)), offered in the switching network, is distributed symmetrically onto all interstage links between two consecutive stages of the network. Hence, traffic offered to a single interstage link $\mathbf{A}_{\text{link}}$ can be defined in the following way:

$$\mathbf{A}_{\text{link}} = \left\{ A_{\text{link}}(i) = \frac{A_i}{\nu^2} : 1 \le i \le M \right\}, \quad (23)$$

where A_i is the intensity of traffic of class i offered in the switching network (formula (3)).

4.2. Model of the Overflow Link. According to the algorithm for setting up point-to-point connections in the multiservice switching network with overflow links (Section 2), the overflow link will be offered traffic $\mathbf{R}_{\text{overflow}}$,

$$\mathbf{R}_{\text{overflow}} = \{R_{\text{overflow}}(i) : 1 \le i \le M\}, \quad (24)$$

which would be lost in a switching network without overflow links due to the internal point-to-point blocking. In formula (24), the parameter $R_{\text{overflow}}(i)$ denotes the average traffic of class i directed to one overflow link. A determination of the characteristics of overflow links necessitates then the knowledge of the internal blocking probability in the switching network without overflow links. Let us assume that the values of these probabilities are known. Traffic offered to one direction of the switching network $\mathbf{A}_{\text{dir}}$, according to the principle of full symmetry of the switching network adopted in the considerations, can be defined in the following way:

$$\mathbf{A}_{\text{dir}} = \left\{ A_{\text{dir}}(i) = \frac{A_i}{\nu} : 1 \le i \le M \right\}, \quad (25)$$

where $A_{\text{dir}}(i)$ is the average intensity of traffic of class i offered to a given direction of the switching network, whereas A_i is the intensity of traffic of class i offered in the switching network (formula (3)).

The average intensity value $R_{\text{overflow}}(i)$ of traffic of class i, offered to one overflow link in formula (24), can be evaluated on the basis of the following reasoning: if the internal blocking probability for calls of class i in the switching network without overflow links is $E_{\text{int},0}(i)$, then the average intensity of traffic that overflows to overflow links from one direction will be equal to $A_{\text{dir}}(i)E_{\text{int},0}(i)$. The average intensity of traffic overflowing from all ν directions (for the network presented in Figure 1), in line with the adopted symmetry principle, will be equal to $\nu A_{\text{dir}}(i)E_{\text{int},0}(i)$. This traffic, in turn, will be divided symmetrically between ν interstage links. Therefore, the average intensity of traffic that overflows to one-single overflow link will be $A_{\text{dir}}(i)E_{\text{int},0}(i)$. Taking into account formula (25), we are now in position to determine traffic $\mathbf{R}_{\text{overflow}}$ directed to single overflow traffic and rewrite (24) as follows:

$$\mathbf{R}_{\text{overflow}} = \left\{ R_{\text{overflow}}(i) = \frac{A_i}{\nu} E_{\text{int},0} : 1 \le i \le M \right\}, \quad (26)$$

where, as in the previous case, A_i is the intensity of traffic of class i offered in the switching network.

Engineering models of overflow systems are two-parametric models that make use of the average value of overflow traffic and its variance. In single-service models, variance is determined on the basis of Riordan formulas [50]. Then, the parameters of the so-called equivalent group are determined. The equivalent group is a certain fictitious group, such that traffic that overflows from the group is identical to traffic that overflows from real groups. The equivalent group serves then as basis for a determination of the blocking probability in an overflow group. The method presented in [51] represents another approach. After a determination of the average value and variance of overflow traffic, the peakedness overflow traffic coefficient is determined on the basis of both. Then, overflow traffic and the capacity of the overflow group are divided by the peakedness coefficient and as a result the parameters of the equivalent group with Erlang traffic that serves as a basis for a determination of the blocking probability in the overflow group are obtained. Overflow systems with multiservice traffic have been addressed in a great number of investigations, for example, in [52–56]. The basis for the model of the overflow link in the switching network adopted in this section is [54] that generalizes the model [51] into multiservice systems.

After determining the value of traffic directed to an overflow link (formula (26)), it is possible to determine approximately its variance [53], with the assumption that a given fictitious full-availability group services given fictitious traffic of class i, in such a way that the blocking probability in this group is equal to the internal blocking probability in the switching network without overflow links $E_{\text{int},0}(i)$. The adopted assumption allows us to use the Riordan formula to calculate the variance $\boldsymbol{\sigma}^2_{\text{overflow}}$ and peakedness coefficient $\mathbf{Z}_{\text{overflow}}$ for overflow traffic. Vector $\boldsymbol{\sigma}^2_{\text{overflow}}$ and vector

$\mathbf{Z}_{\text{overflow}}$ for each class i ($1 \le i \le M$) can be thus expressed in the following way:

$$\boldsymbol{\sigma}^2_{\text{overflow}} = \left\{\sigma^2_{\text{overflow}}(i) = R_{\text{overflow}}(i) \cdot \left[\frac{A_{\text{fictitious}}(i)}{V_{\text{fictitious}}(i) + 1 - A_{\text{fictitious}}(i) + R_{\text{overflow}}(i)} + 1 - R_{\text{overflow}}(i)\right] : 1 \le i \le M\right\},$$
$$\mathbf{Z}_{\text{overflow}} = \left\{Z_{\text{overflow}}(i) = \frac{\sigma^2_{\text{overflow}}(i)}{R_{\text{overflow}}(i)} : 1 \le i \le M\right\}, \tag{27}$$

where

$A_{\text{fictitious}}(i)$ is intensity of fictitious traffic of class i offered to a fictitious group of this class,

$V_{\text{fictitious}}(i)$ is capacity of the fictitious group, expressed in the number of calls (single-service model).

The capacity $V_{\text{fictitious}}(i)$ of the fictitious group can be determined on the basis of a single-service Erlang model [57] that assumes that the blocking probability in the fictitious group equals the internal blocking probability for calls of class i in the switching network without overflow links:

$$E^0_{\text{int}}(i) = E_{V_{\text{fictitious}}(i)}\left(A_{\text{fictitious}}(i)\right), \tag{28}$$

where $E_V(A)$ is Erlang function that defines the blocking probability for traffic A in a single-service FAG group with the capacity V. On the basis of formula (28), it is easy to determine, using, for example, Erlang tables, capacities of fictitious groups for all traffic classes directed to an overflow link.

Vectors $\mathbf{R}_{\text{overflow}}$, $\boldsymbol{\sigma}^2_{\text{overflow}}$, and $\mathbf{Z}_{\text{overflow}}$ describe traffic directed to one overflow link. Now, to approximate the occupancy distribution in the overflow link, the occupancy distribution in FAG, to which multiservice traffic is offered, can be used:

$$n\left[P_n\right]_{f_0/Z} = \sum_{i=1}^{M} \frac{R_{\text{overflow}}(i)}{Z_{\text{overflow}}(i)} t_i \left[P_{n-t_i}\right]_{f_0/Z} \quad \text{for } 0 \le n \le \frac{f_0}{Z},$$
$$\left[P_n\right]_{f_0/Z} = 0 \quad \text{for } \frac{f_0}{Z} < n < 0,$$
$$\sum_{n=0}^{f_0/Z} \left[P_n\right]_{f_0/Z} = 1, \tag{29}$$

where

f_0 is capacity of one overflow link, expressed in BBUs;

Z is aggregate peakedness coefficient; this parameter is determined on the basis of peakedness coefficients for individual classes of calls, assuming that the share of each of the coefficients $Z_{\text{overflow}}(i)$ in the aggregate coefficient Z is directly proportional to the average value of traffic of class i, expressed in BBUs, that is, traffic that is defined by the product $R_{\text{overflow}}(i)t_i$ and overflowing to the overflow link

$$Z = \sum_{i=1}^{M} Z_{\text{overflow}}(i) = \frac{R_{\text{overflow}}(i)\, t_i}{\sum_{j=1}^{M} R_{\text{overflow}}(j)\, t_j}; \tag{30}$$

$[P_n]_{f_0/Z}$ is occupancy probability of n BBUs in FAG with the capacity f_0/Z BBUs.

The vector of blocking probabilities in the overflow link is determined in the adopted model as follows:

$$\mathbf{E}_{\text{overflow}} = \left\{E_{\text{overflow}}(i) = \sum_{n=f_0/Z-t_i+1}^{f_0/Z} P[n]_{f_0/Z} : 1 \le i \le M\right\}. \tag{31}$$

The model of the overflow link adopted in the present considerations is based on a multiservice model of FAG to which a mixture of traffic with differentiated peakedness coefficient is offered [54]. This model (formulas (29)–(31)) is based on a modification of the occupancy distribution in the multiservice model of FAG to which a mixture of Erlang traffic is offered [48, 49]. The Hayward approach [51] was used in the modification. The approach is based on a division of both the intensity of offered traffic and the capacity of FAG by the peakedness coefficient.

Note that, in the case of redirecting traffic of only one traffic class to overflow links, the overflow group model can be basically directly reduced in its essence to the Fredericks-Hayward model [51]. This model is used in [19] to model multiservice switching networks with point-to-group selection in which only one, privileged, class of calls is directed to overflow links.

4.3. Model of the Output Direction of the Switching Network. An output group of the switching network, defined as the output direction (Figure 1), is composed of a number of ν output links. To model a direction in the IPPD model, a limited-availability group model with resource reservation was used. A limited-availability group without reservation, the so-called LAG, has been addressed scientifically in a number of works, for example, in [47, 58]. LAG is a system composed of ν independent links, each with the capacity f BBUs. LAG models with different capacities of independent links are also known [59]. Such a definition of call service in LAG results in the situation that there is no possibility of "dividing" t_i BBUs, demanded by a call of class i to set up a connection, between a number of links that belong to the same direction. The way calls are serviced in LAG is then consistent with the adopted call service algorithm for call service in the switching network.

In [30], the most universal LAG model with resource reservation is proposed. The model will provide the basis for the considerations in this section. The reservation mechanism introduced to LAG makes a new call of class i admitted

for service only when the total occupancy level is not higher than the adopted reservation threshold value of Q_i BBUs, regardless of the occupancy state in individual links that a group is composed of. The reservation threshold for calls of class i is thus defined as such an occupancy state in LAG in which a call of class i can be still admitted for service. All states higher than Q_i belong to the so-called reservation area R_i in which calls of class i are rejected. Reserved resources R_i can be used by other classes of calls that do not undergo the reservation mechanism. For those classes, the reservation threshold is equal to the capacity of LAG diminished by the number of t_i BBUs demanded by a call of class i to set up a connection:

$$Q_i = V - t_i = \nu f - t_i. \tag{32}$$

The limited-availability group under consideration is an example of a state-dependent system. In the case of such systems, in order to determine the occupancy distribution, it is necessary to determine first the so-called conditional transition probability [30]. This probability defines which part of the total traffic stream of class i can be transferred between neighboring occupancy states (for a given call class). The conditioning results from the factor that makes the system dependent on the occupancy state. If the system is affected by a number of independent factors that make the system state-dependent, then the conditional transition probability related to the relevant factors can be multiplied. LAG with resource reservation is influenced by two independent factors that make the system dependent on the occupancy state. The first is related to the resource allocation algorithm in LAG in setting up a new connection, while the other is with the introduced reservation mechanism. Taking into consideration conditional transition coefficients, the occupancy distribution in LAG with resource reservation can be approximated by the following recurrence equations [30]:

$$\begin{aligned} n\left[P_n\right]_V &= \sum_{i=1}^{M} A_{\text{dir}}(i)\, t_i \delta_i(n-t_i)\, \xi(n-t_i)\left[P_{n-t_i}\right]_V \\ &\qquad \text{for } 0 \le n \le V, \\ \left[P_n\right]_V &= 0 \quad \text{for } V < n < 0, \\ \sum_{n=0}^{V} &= 1, \end{aligned} \tag{33}$$

where

$[P_n]_V$ is occupancy probability of n BBUs in LAG with the capacity V BBUs,

$A_{\text{dir}}(i)$ is traffic intensity of class i offered to LAG (formula (25)),

$\delta_i(n)$ is conditional transition probability between states n and $n + t_i$ for calls of class i, conditioned by the method for resources allocation in LAG,

$\xi_i(n)$ is conditional transition probability between states n and $n + t_i$ for calls of class i, conditioned by the reservation mechanism in LAG.

The conditional transition probability $\delta_i(n)$ is related to the resource allocation algorithm in LAG. According to this algorithm, a new call can be admitted for service only when at least one independent link that belongs to LAG has resources necessary for the execution of the service of this call. The probability $\delta_i(n)$ expresses then the probability of such a distribution of free BBUs in the occupancy state n in which an admittance of a new call of class i (demanding t_i BBUs) for service is possible:

$$\delta_i(n) = \begin{cases} \dfrac{\Psi(V-n,\nu,f,0) - \Psi(V-n,\nu,t_i-1)}{\Psi(V-n,\nu,f,0)} & \text{for } 0 \le n \le V - t_i, \\ 0 & \text{otherwise.} \end{cases} \tag{34}$$

In formula (22), the function $\Psi(x, \nu, f, t)$ defines the number of arrangements of x free BBUs in ν links, each with the capacity f BBUs. The assumption is that there is at least t free BBUs in each link. The function Ψ is determined in a combinatorial way on the basis of the following formula:

$$\Psi(x,\nu,f,t) = \sum_{r=0}^{\lfloor (x-\nu t)/(f-t+1) \rfloor} (-1)^r \cdot \binom{\nu}{r} \binom{x-\nu(t-1)-1-r(f-t+1)}{\nu-1}. \tag{35}$$

The other factor that makes LAG dependent on the occupancy state is conditioned by the introduced reservation mechanism. According to the principles of the operation of this mechanism, a given call of class i can be admitted for service when the total group occupancy level is not higher than the adopted reservation threshold value Q_i BBUs, independently of the occupancy state of the individual links that form the group. Therefore, the conditional transition probability $\xi_i(n)$, conditioned by the introduced reservation threshold, will be determined by the following formula:

$$\xi_i(n) = \begin{cases} 1 & \text{for } n \le Q_i, \\ 0 & \text{for } n > Q_i. \end{cases} \tag{36}$$

The blocking probability for calls of all classes in LAG with resource reservation can be determined on the basis of the following vector:

$$\mathbf{E}_{\text{LAG}} = \left\{ E_{\text{LAG}}(i) = \sum_{n=0}^{V} \left[1 - \delta_i(n)\,\xi_i(n)\right]\left[P_n\right]_V : 1 \le i \le M \right\}. \tag{37}$$

The difference in the square bracket in formula (37) determines the conditional blocking probability in LAG for the case of the operation of two mechanisms that make the system state-dependent. Formulas (33)–(37) create a model of the output group in the switching network. On the basis

of this model, the external blocking probability can be determined:

$$\mathbf{E}_{\text{ext}} = \mathbf{E}_{\text{LAG}} = \{E_{\text{ext}}(i) = E_{\text{LAG}}(i) : 1 \le i \le M\}. \quad (38)$$

The dependence G in the description of the PPD and IPPD methods (formulas (4), (8), and (16)) is then determined by model (33)–(37). Observe that if a given call class j does not undergo the reservation mechanism, then all that is needed in distribution (33) is to make an assumption that the reservation threshold for this class is equal to $V - t_j$ (formula (32)). Therefore,

$$\xi_j(n) = \begin{cases} 1 & \text{for } n \le V - t_j, \\ 0 & \text{for } n > V - t_j. \end{cases} \quad (39)$$

5. Effective Availability in the Switching Network

The phenomenon of internal blocking in the switching network causes the fact that not all output links in a given direction are available to a call that appears at the input of one of the switches of the first stage. This situation results from such an occupancy state of the switching network in which a free connecting path between a given switch of the first stage (at the input of which a new call has appeared) and a switch of the last stage that has at least one link in the demanded direction cannot be found. If, however, this connecting path can be set up, then we can say that a given switch of the last stage is available to the switch of the first stage. Note that in this definition of the available switch the problem of the occupancy of the output link in the demanded direction, that is, whether the output link of the switch of the last stage in the demanded direction is free or occupied, is not resolved. What is exclusively important is the fact of the existence of a free connecting path between selected switches of external stages [46].

The notion of effective availability is a key notion in effective availability methods. The assumption in these methods, developed for single-service switching networks, is that the blocking probability in a multistage switching network is equal to the blocking probability in the single-service system, the non-full-availability group (grading) [17, 40–43, 46]. The structure of the non-full-availability group is defined by two parameters: the total capacity and availability, that is, the number of output links that are available from one input of the group [60, 61]. If we now assume that the capacity of the non-full-availability group and the capacity of a given direction are identical, then the effective availability parameter can be defined on the basis of the average value of available output links in a given direction of the considered switching network. In the case of multiservice switching, the effective availability parameter is determined for each traffic class independently; that is, a fictitious single-service switching network, called the equivalent network in the literature of the subject, is constructed for each traffic class [46]. The equivalent network has identical structure as the multiservice network, whereas the capacity of input, interstage, and output links is equal to one. The assumption is that the load of one link of the equivalent network, called the fictitious load, is equal to the blocking probability in one link of the multiservice network. Effective availability for calls of class i in z-stage equivalent switching network is determined on the basis of the following formula [21]:

$$\begin{aligned} d(i) = {} & [1 - \pi_z(i)]\,\nu + \pi_z(i)\,\eta_1(i)\,Y_1(i) \\ & + \pi_z(i)\,[\nu - \eta_1(i)\,Y_1(i)]\,y(i)\,\omega_z(i), \end{aligned} \quad (40)$$

where

$d(i)$ is effective availability for calls of class i in the equivalent network;

$y(i)$ is fictitious load of an interstage link in the equivalent network;

$\pi_z(i)$ is probability of direct nonavailability for calls of class i in z-stage switching network; the method for a determination of this parameters will be presented further in the paper;

$Y_1(i)$ is the average fictitious traffic in the switch of the first stage in the equivalent network of class i;

$\eta_1(i)$ is the part of the average fictitious traffic of the first-stage switch in the equivalent network of class i that is directed to the considered direction;

$\omega_z(i)$ is the secondary availability coefficient in z-stage equivalent network of class i; the method for a determination of this parameter will be presented further in the paper.

Formula (40) is the sum of three components that correspond to the three (and just three) possibilities of the occupancy of links in non-full-availability groups. The first component element corresponds to a possibility of the occupancy of a free output from a given input (belonging to the so-called input group) in a non-full-availability group. The second component defines a possibility of an existence of a connection between a given input group and a given output of a non-full-availability group. The third component element determines, in turn, a possibility of the existence of a connection between a given output and another input group. Exact definitions of all component elements and the methods for the calculation of the effective availability parameter in multiservice switching networks are presented in [21, 46]. In the case of the Clos three-stage switching network shown in Figure 1, traffic directed to all of the directions of one switch will be, in line with the adopted principle of symmetry, identical. We can thus write

$$\eta_1(i) = \frac{1}{\nu}. \quad (41)$$

Fictitious traffic, serviced by one link of the switch of the first stage, is equal to $y(i)$. Since every switch of the switching network in Figure 1 has ν output links, then the average fictitious traffic, serviced in the switch of the first stage of the equivalent network of class i, is equal to

$$Y_1(i) = \nu y(i). \quad (42)$$

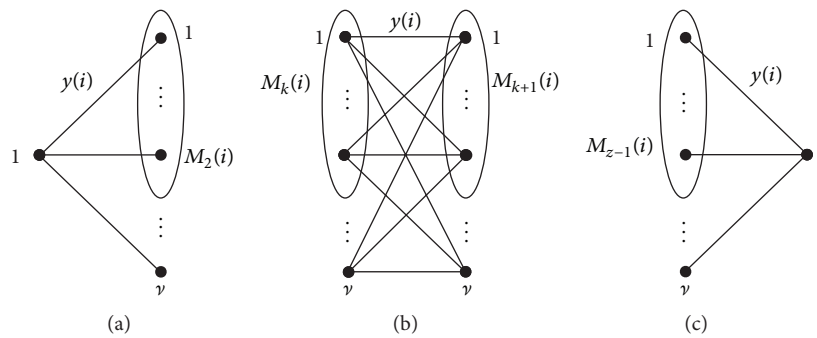

Figure 2: Typical connections in equivalent graphs of switching networks: (a) between stages 1 and 2, (b) between stages k and $k+1$, and (c) between stages $z-1$ and z in z-stage switching network.

Taking into consideration formulas (40), (41), and (42), vector **d** for the structure of the three-stage switching network shown in Figure 1 will be written as follows:

$$\mathbf{d} = \{d(i) = [1 - \pi_3(i)]\,\nu + \pi_3(i)\,y(i) + \pi_3(i)\,[\nu - y(i)]\,y(i)\,\omega_3(i) : 1 \le i \le M\}. \tag{43}$$

The value of the fictitious availability $y(i)$ for calls of a given class i is equal to the blocking probability for calls of this class in one interstage link of the multiservice network and can be determined on the basis of the model of the interstage link presented in Section 4.1. Therefore, on the basis of (20) and (21), for each particular traffic class i, we can write

$$\{y(i) = E_{\text{link}}(i) : 1 \le i \le M\}. \tag{44}$$

5.1. Graph Channel of the Equivalent Switching Network. The probability of direct nonavailability $\pi(i)$ is necessary to be known to be in position to determine effective availability on the basis of formula (40). This parameter is defined as the probability of an event that a free connection path between some selected switches of the external stages of an equivalent switching network cannot be set up. The probability $\pi(i)$ is typically determined on the basis of a graph channel for a given equivalent network. The graph shows all possible connection paths between two switches of external stages. Nodes of this graph correspond to the switches, while its edges correspond to the interstage links. Every edge of the graph is ascribed a certain value of the fictitious load of a corresponding interstage link. The probability of direct nonavailability can be interpreted as the blocking probability for all connection paths in the channel graph between the input node (switch of the first stage) and output node (switch of the last stage). The probability $\pi(i)$ can be determined on the basis of Lee's method [62] or, alternatively, on the basis of the DASSG (Direct Availabilities of Successive Stages of the Graph) method [46]. The assumption in both methods is that occupancies of all graph edges are independent of one another. Lee's method is a convenient method for switching networks in which graphs have serial and parallel character. In the case of switching networks in which graphs are of bridge nature, it is far more convenient to apply the DASSG method. In this method, it is possible to determine the direct availability for $k+1$ stage on the basis of the direct availability for stage k. A directly available node of stage k is then a node that can be connected with an input node via free edges. The average number of these nodes is just what we call the direct availability for stage k.

Figure 2 shows the most typical interstage connections in channel graphs for equivalent switching networks. The symbol $M_k(i)$ denotes direct availability for stage k in the channel graph of the equivalent network for calls of class i. The direct availability for the interstage connections in Figure 2, determined on the basis of the DASSG method, is expressed by the following formulas [46]:

$$M_2(i) = \nu_2\,[1 - y(i)], \tag{45}$$

$$M_k(i) = \nu_k\left\{1 - [y(i)]^{M_{k-1}(i)}\right\}, \tag{46}$$

$$\pi_z(i) = [y(i)]^{M_{z-1}(i)}, \tag{47}$$

where ν_k is the number of nodes in stage k of the channel graph. In the symmetrical switching networks considered in the paper, we always have

$$\{\nu_k = \nu : 1 \le k \le z\}. \tag{48}$$

Formula (45) determines the direct availability of the second stage of the graph (Figure 2(a)), that is, the average number of switches of the second stage in the equivalent switching network that is accessible via free interstage links for one switch of the first stage where a call of class i has appeared. Formula (46) determines in turn the direct availability $M_k(i)$ of stage k in the graph (Figure 2(b)), determined with the assumption that the direct availability $M_{k-1}(i)$ of the preceding stage is known. After a determination of the direct availability of the penultimate stage $M_{z-1}(i)$ in a z-stage graph of the equivalent switching network (Figure 2(c)), it is possible to evaluate, on the basis of formula (47), the probability of direct nonavailability for calls of class i.

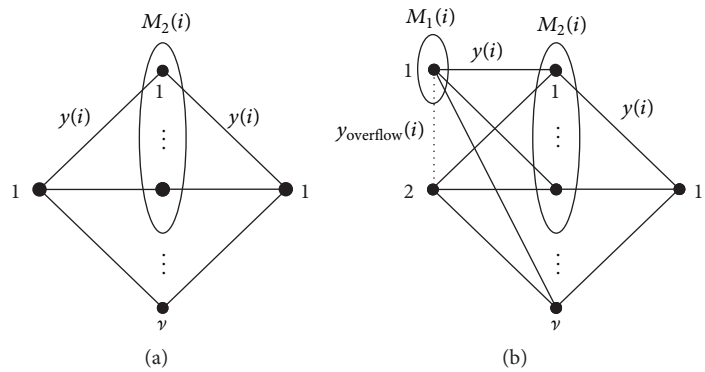

Figure 3: Channel graph of a Clos equivalent three-stage switching network: (a) network without overflow links, (b) network with overflow links.

Let us note that, in order to determine the direct availability k $(1 \le k \le z)$ in a z-stage channel graph for the switching network, one of the possibilities is to use the probability of direct nonavailability $\pi_k(j)$ for a k-stage subgraph [21]. Such a subgraph corresponds to a k-stage switching network, that is, a network that is composed of k first stages. Such areasoning leads us then to the following dependence:

$$M_k(i) = \nu_k \{1 - \pi_k(j)\}. \tag{49}$$

In symmetrical networks, the parameter ν_k always takes on identical value (formula (48)).

5.2. Probability $\pi(i)$ in the Network with and without Overflow Links. Figure 3 shows a channel graph of a three-stage Clos switching network (Figure 1). Figure 3(a) corresponds to a switching network without overflow links. Direct availability of the first stage of the graph is equal to one because it is only on the input of one switch of the first stage that a new call of a given traffic class can appear.

It is possible to determine, on the basis of formula (45), the direct availability for the second stage of the graph and then, on the basis of formula (47) in which $z = 3$, we can proceed to determine the direct nonavailability probability. Therefore, for a three-stage Clos network without overflow links, we can write

$$\pi_3(i) = [y(i)]^{\nu[1-y(i)]}. \tag{50}$$

Figure 3(b) presents a graph of a three-stage equivalent Clos switching network with overflow links. The edge of the graph corresponding to the overflow link between two neighboring switches of the first stage is marked with dotted line (Figures 1 and 3(b)). This edge is ascribed some fictitious load $y_{\text{overflow}}(i)$, equal to the blocking probability for calls of class i in an overflow link of the multiservice switching network. The model of the overflow link is presented in Section 4.2. Therefore, on the basis of (27)–(31), for each traffic class of traffic i, we can write

$$\{y_{\text{overflow}}(i) = E_{\text{overflow}}(i) : 1 \le i \le M\}. \tag{51}$$

Note that, because of the existence of the overflow link, direct availability of the first stage of the graph is not equal to one. To a given call, with the probability equal to one, one node (one that corresponds to a switch of the first stage where a new call appears at the input) is available, and, with the probability $1 - y_{\text{overflow}}(i)$, the second node is available (the one that corresponds to a switch of the first stage, i.e., connected, via an overflow link, with this switch where a new call appears at the input). Hence, in this particular case, the availability of the first stage of the graph will be equal to

$$\begin{aligned} M_1(i) &= 1 \cdot 1 + 1 \cdot [1 - y_{\text{overflow}}(i)] \\ &= 2 - y_{\text{overflow}}(i). \end{aligned} \tag{52}$$

By substituting $M_1(i)$ into formula (46), and adopting that $k = 2$, we get

$$M_2(i) = \nu \left\{1 - [y(i)]^{2-y_{\text{overflow}}(i)}\right\}. \tag{53}$$

Eventually, the obtained result (53) can be substituted to formula (47) in which $z = 3$, thus obtaining the probability of direct nonavailability for the three-stage Clos switching network with overflow links:

$$\pi_3(i) = [y(i)]^{\nu\{1-[y(i)]^{2-y_{\text{overflow}}(i)}\}}. \tag{54}$$

5.3. The Coefficient $\omega(i)$ in the Network with and without Overflow Links. The secondary availability coefficient $\omega_z(i)$ can be interpreted as the probability of an event that a given connection passes through directly available switches of at least one interstage (20). Therefore, for a z-stage switching network, we can write

$$\omega_z(i) = 1 - \prod_{k=2}^{z-1} [1 - \phi_k(i)]. \tag{55}$$

The parameter $\phi_k(i)$ in formula (55) is the probability of an event that the connection path of a given connection of class i goes through directly available switches of at least one of the interstages of the z-stage switching network:

$$\phi_k(i) = \frac{M_k(i)}{\nu_k}. \tag{56}$$

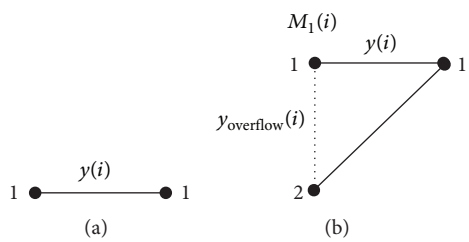

Figure 4: Channel graph of the equivalent, two-stage switching network: (a) network without overflow links, (b) network with overflow links.

In a three-stage switching network, the interstage is the second stage. Therefore, by including formulas (48) and (49) in (56) and substituting the obtained result to (55), we eventually get

$$\omega_3(i) = 1 - \pi_2(i). \tag{57}$$

Formula (57) shows that, in order to determine the secondary availability parameter in a three-stage switching network, it is necessary to determine the direct nonavailability probability for calls of class i in the 2-stage switching network. This probability can be evaluated on the basis of the 2-stage subgraph of the 3-stage graph presented in Figure 3. The relevant subgraphs for 2-stage equivalent switching networks without overflow links and with overflow links are presented in Figure 4.

In a two-stage equivalent switching network without overflow links, created by the elimination of the last stage from the network structure presented in Figure 1, there is only one connection path between a given switch of the first and the second stage. Hence, the relevant channel graph (Figure 4(a)) is composed of one edge. Since the direct availability of the first stage is equal to one ($M_1(i) = 1$), then by substituting $z = 2$ to formula (47) we get the value of the direct nonavailability probability $\pi_2(i)$ for calls of class i in the z-stage switching network. Now, on the basis of (55), we can determine the value of the secondary availability coefficient in the three-stage switching network without overflow links. After a number of transformations, we get

$$\omega_3(i) = 1 - y(i). \tag{58}$$

Figure 4(b) shows a channel graph of a 2-stage equivalent switching network with overflow links. Direct availability of the first stage of this graph is defined by formula (52). By substituting this result into formula (47), we get the value of the parameter $\pi_2(i)$ that is necessary to determine the secondary availability coefficient (formula (55)) in the three-stage switching network with overflow links. After all necessary transformations, we get

$$\omega_3(i) = 1 - \left[y(i)\right]^{2-y_{\text{overflow}}(i)}. \tag{59}$$

5.4. Commentary. The blocking probability of calls of individual classes in the multiservice switching network with overflow links and reservation mechanism at output links can be determined by a double reference to the IPPD method presented in Section 3.2. First, the IPPD method is used to determine the internal blocking probability in the switching network with reservation mechanism, but without overflow links. In this case, we use formula (43) to model the effective availability. In this formula, for each class i ($1 \le i \le M$), the direct nonavailability probability $\pi_3(i)$ and the secondary availability coefficient $\omega_3(i)$ are determined by formulas (50) and (58), respectively. The internal blocking probability in the network without overflow links forms the basis then for a determination of the traffic parameters of traffic overflowing to an overflow link and the fictitious load of this link, which is necessary to determine the effective availability of the switching network with overflow links. Then, we use the IPPD method again to determine the internal and external blocking probability and the total blocking probability in the multiservice switching network with reservation mechanism and overflow links. In this case, to model the effective availability, we use formula (43), in which for each class of calls the parameters $\pi_3(i)$ and $\omega_3(i)$ are determined on the basis of formulas (54) and (59), respectively.

If not all but only a number of selected call classes are directed to overflow links, then the effective availability for those traffic classes that overflow to overflow links is determined by formulas (43), (54), and (59), whereas the effective availability of the remaining classes that are not directed to overflow links is determined on the basis of formulas (43), (50), and (58).

6. Numerical Examples

The analytical IPPD method proposed to model multiservice switching network with overflow links is an approximate method. Therefore, the results of the analytical calculations were compared with the results of the simulation experiments for three-stage Clos switching networks (Figure 1). For the simulation experiments, a switching network with the following structural parameters was used:

(i) size of each of the switches of the first stage with overflow links—5×5,

(ii) size of each of the switches of the second and third stage—4×4,

(iii) capacity of the input, output, and interstage links $f = 30$ BBUs,

(iv) capacity of the overflow link—$f_0 = (10, 20, 30, 40, 50, 60)$ BBUs.

The following parameters for traffic offered to the switching network were adopted in the experiments:

(i) the number of traffic classes $M = 3$,

(ii) demanded number of BBUs for calls of particular classes: $t_1 = 10$, $t_2 = 5$, and $t_3 = 2$ BBUs,

(iii) the proportions of offered traffic: $A_1t_1 : A_2t_2 : A_3t_3 = 1 : 1 : 1$.

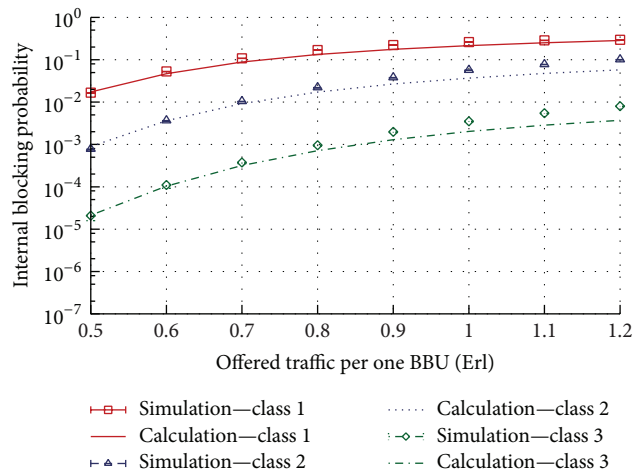

FIGURE 5: The internal blocking probability in the switching network without reservation mechanism and overflow links.

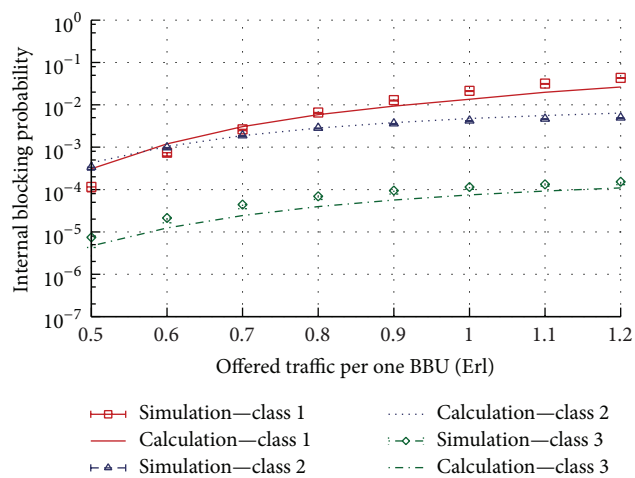

FIGURE 6: The internal blocking probability in the switching network with introduced reservation mechanism and overflow links.

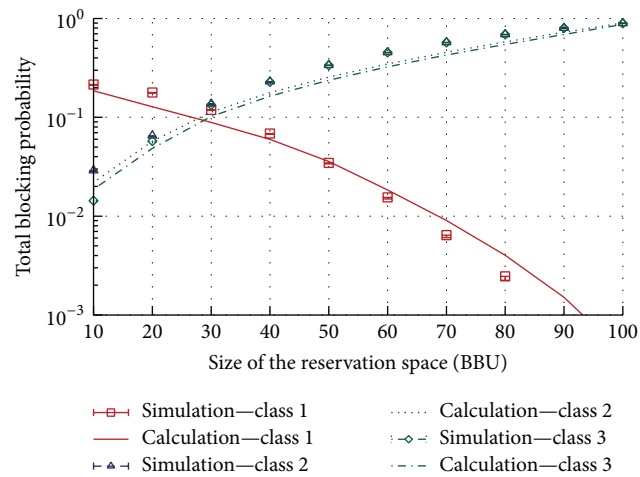

FIGURE 7: Dependence of the total blocking probability on the size of reservation space in the switching network without a system of overflow links.

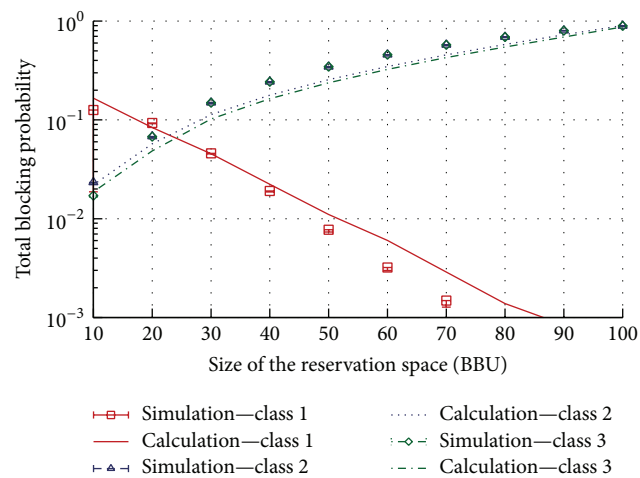

FIGURE 8: Dependence of the total blocking probability on the size of the reservation space in the switching network with a system of overflow links.

Figures 5–9 show the results of the calculations and the simulation of the blocking probability in the switching network under consideration. The results of the simulation experiments are presented in the figures in the form of dots with 95% confidence interval, determined on the basis of the t-Student distribution, for 10 series, each with at least 100000 calls of each class in each of the series. All the results are presented in relation to the value of traffic a offered to one BBU in the output link of the switching network:

$$a = \sum_{i=1}^{M} \frac{A_i t_i}{\nu^2 f}. \tag{60}$$

Figure 5 shows the dependence of the internal blocking probability on traffic offered to one BBU in a switching network without reservation mechanism and without the system of overflow links. Figure 6 presents the relevant results for a switching network in which both the reservation mechanism and overflow links were introduced. The purpose of these mechanisms is to maximize a decrease in the internal blocking probability of the first class that demands 10 BBUs to set up a connection. The assumption in the experiment was that an overflow link had the capacity equal to the capacity of an interstage link ($f_0 = f = 30$ BBUs). The use of overflow links was limited to servicing calls of the first class, whereas the reservation area was introduced for the second and the third class: $R_2 = R_3 = 40$ BBUs (reservation threshold for the second and the third class was equal to $Q_1 = Q_2 = 80$ BBUs). This means that in occupancy states higher than 80 BBUs the system can admit for service only calls of the first class. The obtained results show a considerable decrease in internal blocking probabilities for all classes once a reservation mechanism and overflow links have been introduced. The blocking probability for calls of the first class

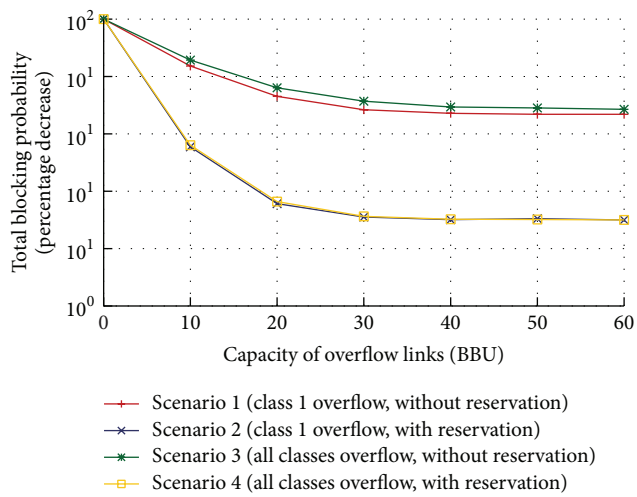

FIGURE 9: Percentage change in the total blocking probability of class 1 ($t_1 = 10$ BBUs) in relation to the capacity of the overflow link for different scenarios.

was decreased to the highest degree, nearly levelling with the blocking probability for calls of the second class, that is, the one that demanded a twofold lower number of BBUs to set up a connection. What is characteristic is that the effect of the privileged treatment of traffic of the first class (redirecting to overflow links, no possibility for calls of other classes to be admitted for service in the reservation area) is not followed by any increase in the internal blocking probability for other classes of calls.

Figure 7 shows the dependence of the total blocking on the size of the reservation space introduced to the second and third class in the considered three-stage Clos network without overflow links. This means that within the reservation space calls of only the first class ($t_1 = 10$ BBUs) are admitted for service. The assumption in the study was that traffic offered to one BBU was equal to 0.7 Erl. When the size of the reservation area exceeds 30 BBUs, then the total blocking probability for the first class is virtually equal to the total blocking probabilities of the remaining classes of calls. Any further increase in the size of the reservation space (exceeding 30 BBUs) leads to a decrease in the blocking probability of the first class of calls and to an increase in the equalized total blocking probability for the remaining two call classes. This effect results from the adopted scenario for traffic service in which only calls of the first class are admitted for service in the reservation space. The bigger the reservation space, the better the conditions for the service of this traffic. An increase in the reservation space is followed, in turn, by a reduction of the resources for the remaining call classes and, in consequence, leads to an increase in the total blocking probability for the second and third class.

Figure 8 presents an identical scenario for traffic service in which the additional assumption is that the network has a system of overflow links reserved for calls of the first class. After an introduction of overflow links with the capacity equal to the capacity of the remaining links in the network ($f_0 = f = 30$ BBUs), the total blocking probability for the first class will be significantly decreased, whereas the total blocking probabilities for the remaining classes of calls will not be essentially changed. The graphs in Figures 7 and 8 show the new possibilities offered by the introduced traffic management mechanisms. With an appropriate choice of reservation thresholds, it is possible, just as given requirements of the operator are, to change the relations between the quality of service for individual call classes. For example, it is possible to equalize the blocking probability for all call classes or make some of the classes privileged in such a way that their blocking probability, despite a higher number of demanded BBUs to set up a connection, will be lower than the blocking probability for remaining call classes serviced in the switching network.

Figure 9 shows a percentage decrease in the value of the total blocking probability for calls of the first class ($t_1 = 10$ BBUs) in the switching network with overflow links depending on the capacity of overflow links. The presented results are the results of the analytical calculations. The assumption in the study was that traffic offered to one BBU was equal to 0.7 Erl. The two uppermost curves correspond to the switching network without the reservation mechanism, while the upmost curve relates to this scenario for traffic service in which all classes can use overflow links. The other curve then relates to the scenario for traffic service in which only the first class is directed to overflow links. The two lowermost overlapping curves correspond to the switching network to which the reservation mechanisms for the second and third call classes have been introduced ($R_2 = R_3 = 40$ BBUs). In this particular case, the overflow traffic service scenario is virtually of no importance (the curves overlap).

The results of the analytical calculations presented in Figure 9 show a similar nature of changes in the total blocking probability in relation to changes in the capacity of an overflow link, regardless of the adopted traffic distribution scenario directing traffic to overflow links and the introduced reservation mechanism. It is observable that with the capacities of an overflow link higher than 30 BBUs, the total point-to-point blocking probability is virtually stable and does not change. This means that, in the case of the point-to-point selection, the most effective is an application of overflow links with capacities that are equal to capacities of interstage links ($f_0 = f$). This result confirms the conclusions resulting from earlier simulation and analytical studies for switching networks with point-to-group selection [14, 18, 19] that prove that an appropriate increase in the capacity of an overflow link leads to a virtual elimination of the phenomenon of the internal blocking in the switching network that becomes a quasi-nonblocking network.

7. Conclusions

An application of overflow links in the first stage of multiservice switching networks as well as application of reservation mechanisms in output links is followed by a significant improvement in traffic effectiveness and efficiency of network nodes. The introduction of overflow links leads to a significant reduction in the phenomenon of the internal blocking, whereas the reservation mechanism leads to an introduction of differentiated service conditions for different

classes of calls. As a consequence, the application of those two mechanisms makes it possible to shape traffic characteristics of the network according to any given requirements (e.g., those that have been set up by the operator). This paper addresses the problem of modelling of multiservice switching networks with point-to-point selection, overflow links, and reservation mechanism implemented in output directions. The proposed model makes it possible to determine all important characteristics of a switching network for any number of traffic classes (selected from among all traffic classes that are offered to the network) that are directed to overflow links and any number of traffic classes that undergo reservation mechanism. The study proves that a connection with an additional overflow link, with the capacity equal to the capacity of the interstage link, of each of neighboring switches of the first stage leads to a significant reduction in the value of the internal blocking probability. The reservation mechanism, in turn, increases the external blocking probability (and in this way the total blocking probability) of a number of selected classes of calls and decreases the blocking probability for the remaining classes. The conducted simulation experiments have confirmed high accuracy of the proposed model. The model can be used in engineering practice providing methods of solving designing optimization problems in multiservice network nodes. The provided results of the conducted research study also indicate that further studies on switching networks with overflow links are clearly needed and that emerging problems should be addressed scientifically. These studies should be then aimed at finding the most effective structures for overflow systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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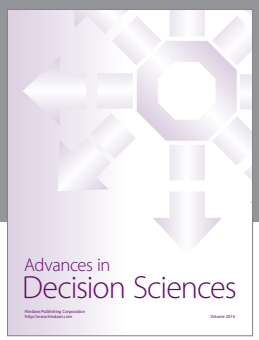

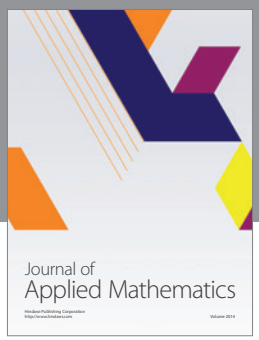

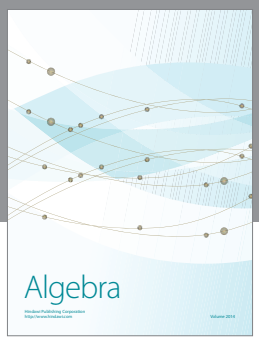

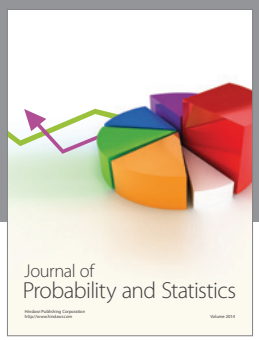

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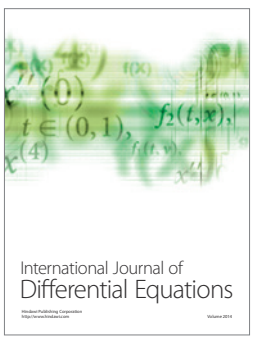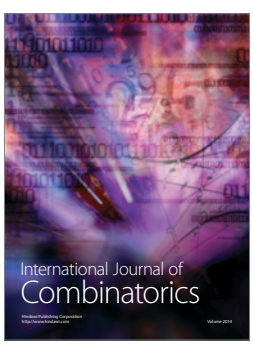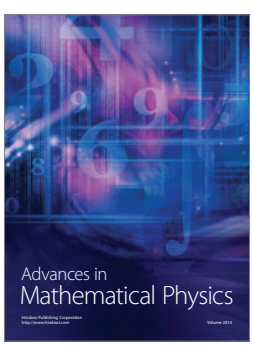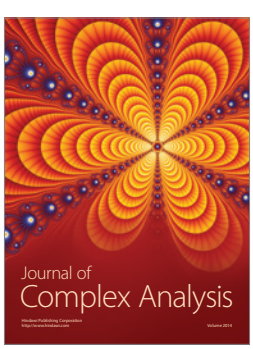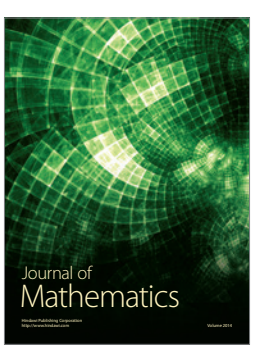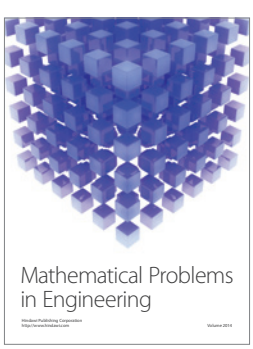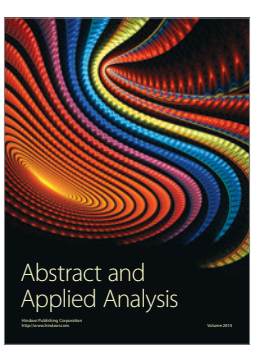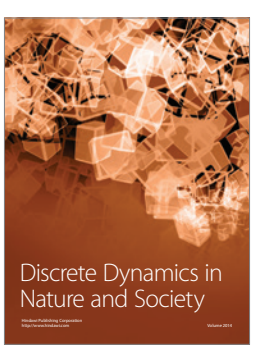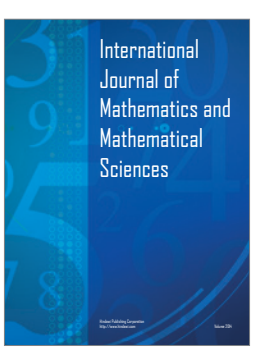
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