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Research Article

A Modified Model of Failure Mode and Effects Analysis Based on Generalized Evidence Theory

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Due to the incomplete knowledge, how to handle the uncertain risk factors in failure mode and effects analysis (FMEA) is still an open issue. This paper proposes a new generalized evidential FMEA (GEFMEA) model to handle the uncertain risk factor, which may not be included in the conventional FMEA model. In GEFMEA, not only the conventional risk factors, the occurrence, severity, and detectability of the failure mode, but also the other incomplete risk factors are taken into consideration. In addition, the relative importance among all these risk factors is well addressed in the proposed method. GEFMEA is based on the generalized evidence theory, which is efficient in handling incomplete information in the open world. The efficiency and some merit of the proposed method are verified by the numerical example and a real case study on aircraft turbine rotor blades.

1. Introduction

Failure mode and effects analysis (FMEA) starts from aerospace industry in 1960s. It is a structural tool for analyzing the potential failure modes in a process, designing activity, service process, and so on. Based on the empirical knowledge from the FMEA team members, each failure mode will get a risk priority number to define its risk level, as well as some suggestions on how to control these failure modes to prevent them from having a bad effect on the customers. FMEA is an effective preventive approach to reduce the possibility of a failure. So far, FMEA has become a useful method in risk analysis being widely used in many real applications, like nuclear safety systems [1], software engineering [2], complex system analysis [3, 4], medical management [5–7], patient safety evaluation [8], shipping equipment [9, 10], automotive industry [11], food industry [12], and so on [13–17]. Currently, the study corresponding to FMEA mainly focuses on the following aspects.

(i) Applying FMEA approach to many more particular fields for risk analysis: except for those applications mentioned above, FMEA is also used as a risk assessment tool in other particular fields like agriculture and food domain [18], environment protection [19], and so on [20, 21].

(ii) Modifying the conventional risk priority number (RPN) model, which is the product of the three risks factors, occurrence (O), severity (S), and detection (D), to make it more rational for ranking the priority of failure modes: the modified RPN value is based on many theories like the fuzzy set theory [16, 22, 23], the grey theory [24], the Monte Carlo method [25], the evidence theory [23, 26, 27], and so on [28–30]. Some of the proposed methods are hybrid methods [14, 23, 31]. A more detailed literature review on this topic is studied by Liu et al. [32].

(iii) Addressing the subjective risk evaluation information of FMEA more flexibly: the evaluation information in FMEA method is effective and more flexible while it is combined with the approach of fuzzy theory [33–35], the grey theory [36, 37], the D-S evidence theory [23, 38, 39], the TOPSIS method [14, 40], the OWA operator [41], the D numbers [36], the AHP/ANP method [31, 42], the Bayesian reasoning method [28], and so on [5].

These studies mentioned above all make contribution to improve the conventional FMEA method or extend it to different particular fields as an efficient risk analysis tool. But little attention has been paid to the incomplete risk factor; in other words, the other uncertain risk factor except for O, S, and D should also be taken into consideration in real application. Other risk factors may be the period of development, the cost [30], the uncertain risk factors from many suppliers domestic and external, and so on. For example, from the commercial perspective, the cost can be the key factor in terms of a company's financial objectives [30, 43]. And these risk factors should be taken into consideration independently like O, S, and D. So, how to model these incomplete risk factors out of the conventional FMEA is addressed in this paper; then a modified model of FMEA is proposed based on the generalized evidence theory (GET) [44], namely, generalized evidential FMEA (GEFMEA). In addition, the relative importance among all these risk factors, not only the conventional one in RPN, but also the uncertain one, is taken into consideration in GEFMEA.

The generalized evidence theory [44], which is a more generalized situation of the Dempster-Shafer evidence theory (D-S evidence theory) [45, 46], is developed to handle the uncertain information in the open world. D-S evidence theory has been studied extensively during the past decades [47]; it is a useful mathematical theory for information fusion in real applications [26, 48–50]. Some key problems in D-S evidence theory are still worth further study, for example, the dependent evidence combination [51] and the determination of basic probability assignment [52]. The generalized evidence theory inherits the advantages of D-S evidence theory; what is more, if the frame of discernment is incomplete, the generalized basic probability assignment (GBPA) and generalized combination rule (GCR) in GET can handle the incomplete knowledge more efficiently [44]. In this paper, the incomplete risk factor which comes from the incomplete frame of discernment of FMEA in the open world is expressed by the empty set in the frame of generalized evidence theory. The GBPA is used to handle the relative importance of all these risk factors, including the incomplete one. In this way, the proposed GEFMEA model seemed as a more generalized model in the open world extended from conventional FMEA. GEFMEA can be degenerated to the conventional FMEA whenever it is necessary.

The rest of this paper is organized as follows. In Section 2, some preliminaries are briefly introduced. In Section 3, a new generalized evidential FMEA (GEFMEA) model is proposed. Two experiments based on GEFMEA are shown in Section 4. The conclusions are given in Section 5.

2. Preliminaries

In this section, some preliminaries are introduced, including the failure mode and effects analysis (FMEA) model [54], D-S evidence theory [45, 46], the generalized evidence theory (GET) [44], and the pignistic probability transformation (PPT) model [55].

Table 1: Suggested criteria of rating for occurrence of a failure in FME [11, 53].

Rating	Probability of occurrence	Possible failure rate
10	Extremely high: failure almost inevitable	≥1/2
9	Very high	1/3
8	Repeated failures	1/8
7	High	1/20
6	Moderately high	1/80
5	Moderate	1/400
4	Relatively low	1/2000
3	Low	1/15000
2	Remote	1/150000
1	Nearly impossible	≤1/1500000

2.1. Failure Mode and Effects Analysis. FMEA is one of the systematic techniques for risk analysis. Generally, FMEA model includes the following steps [54].

Step 1. Identifying the team: FMEA team members should be with the relevant experience and necessary authority.

Step 2. It includes defining the scope of the FMEA analysis and the customers of the FMEA process.

Step 3. It includes identifying the functions, requirements, and specifications relevant to the defined scope, as well as the potential failure modes, effects, causes, and controls.

Step 4. It includes identifying and assessing risk.

Step 5. It includes defining recommended actions and results.

Among all these five steps, Steps 1–3 are mainly based on empirical knowledge and qualitative analysis. In Step 4, the risk priority number (RPN) offers a useful way to assess the risk level of each failure mode. Step 5 is based on Step 4 and other more empirical knowledge. The risk evaluation in conventional FMEA is determined by the risk priorities of failure modes through the RPN value, which is defined as the product of three risk factors of a failure mode [29]:

$$\text{RPN} = O \times S \times D, \tag{1}$$

where O is the probability of occurrence of a failure mode, S is the severity of a failure effect, and D is the probability of a failure being detected. Each risk factor has a numerical rating from 1 to 10. The suggested criterion of rating for occurrence (O) is shown in Table 1. Similarly, the criteria of rating for severity and detection can be found in [11, 32, 54].

2.2. Dempster-Shafer Evidence Theory. The obtained information in real word is often vague and incomplete [56]. For example, in complex systems, the factors are influenced by each other with very complicated manners, which is hardly to represent with analytic methods [57, 58]. To address this issue, there are many math tools to handle uncertainty [59]. For example, fuzzy set theory is presented to deal with linguistic variables [60–62], logic problem [63, 64], and decision making [65]. On the other hand, as a generalization of the classic probability theory, the D-S theory has an ability to handle uncertainty of imprecision embedded in the evidence, and it has been increasingly applied in many fields, such as decision making [51, 66], fault diagnosis [36, 48], and data fusion [67]. Formally, the evidence theory concerns the following preliminary notations. Some basic concepts of the D-S evidence theory [45, 46] are introduced, including the frame of discernment, the basic probability assignment (BPA), and Dempster's rule of combination.

Definition 1 (frame of discernment). Assume a finite nonempty set of mutually exclusive events $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, the set of all subsets of Θ, which is power set $2^{|\Theta|}$, known as the frame of discernment, denoted as [45]

$$\Omega = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_n\}, \{\theta_1, \theta_2\}, \ldots, \{\theta_1, \theta_2 \ldots, \theta_n\}\}. \quad (2)$$

Definition 2 (basic probability assignment). The basic probability assignment function or mass function m is defined as a mapping from the power set of Θ to a number between 0 and 1, satisfying [45, 46]

$$\sum_{A \in \Theta} m(A) = 1, \quad m(\emptyset) = 0, \ 0 \le m(A) \le 1, \quad (3)$$

where $\emptyset$ is an empty set, A is any subsets of Θ, and the mass function $m(A)$ represents how strongly the evidence supports A. The mass $m(\Theta)$ represents the uncertainty of the evidence.

Definition 3 (Dempster's rule of combination). The rule of combination combines two BPAs in such a way that the new BPA represents a consensus of the contributing pieces of evidence. Dempster's rule of combination is the orthogonal sum of m_1 and m_2. Dempster's rule of combination is defined as [45]

$$m(A) = (m_1 \oplus m_2)(A) = \frac{1}{1-k} \sum_{B \cap C = A} m_1(B) \cdot m_2(C), \quad (4)$$

where k is a normalization constant worked as the conflict coefficient of two BPAs, defined as [45]

$$k = \sum_{B \cap C = \emptyset} m_1(B) \cdot m_2(C), \quad (5)$$

where A, B, and C are subsets of $2^{|\Theta|}$.

For more information about D-S evidence theory, one can refer to [45, 46].

2.3. Generalized Evidence Theory. The basic concepts of generalized evidence theory (GET) [44] are introduced in this section. In GET, the generalized basic probability assignment (GBPA) corresponds to basic probability assignment (BPA) in D-S evidence theory, which is used for data expression and modeling, the generalized combination rule (GCR) is provided for combining conflicting or inconsistent or uncertain body of evidence, and GCR is generalized from Dempster's rule of combination.

Definition 4. Suppose that U is a frame of discernment in the open world. Its power set, 2^U_G, is composed of 2^U propositions, $\forall A \subset U$; a mass function is a mapping $m_G : 2^U_G \to [0, 1]$, satisfying [44]

$$\sum_{A \in 2^U_G} m_G(A) = 1, \quad (6)$$

where m_G is the GBPA of the frame of discernment, U. The difference between GBPA and BPA is the restriction of $\emptyset$. In GET, $m_G(\emptyset) = 0$ is not necessary in GBPA. In other words, the empty set can also be a focal element. If $m_G(\emptyset) = 0$, the GBPA degenerates to a conventional BPA in D-S evidence theory.

Definition 5. Given a GBPA m, the generalized belief function is $\text{GBel} : 2^U \to [0, 1]$, which satisfies [44]

$$\begin{aligned} \text{GBel}(A) &= \sum_{B \subseteq A} m(B), \\ \text{GBel}(\emptyset) &= m(\emptyset). \end{aligned} \quad (7)$$

Definition 6. Given a GBPA m, the generalized belief function is $\text{GPl} : 2^U \to [0, 1]$, which satisfies [44]

$$\begin{aligned} \text{GPl}(A) &= \sum_{B \cap A \neq \emptyset} m(B), \\ \text{GPl}(\emptyset) &= m(\emptyset). \end{aligned} \quad (8)$$

Definition 7. In generalized evidence theory, $\emptyset_1 \cap \emptyset_2 = \emptyset$, which means that the intersection between two empty sets is still an empty set. Given two GBPAs (m_1 and m_2), the generalized combination rule (GCR) is defined as follows [44]:

$$\begin{aligned} m(A) &= \frac{(1 - m(\emptyset)) \sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1 - K}, \\ K &= \sum_{B \cap C = \emptyset} m_1(B) \cdot m_2(C), \\ m(\emptyset) &= m_1(\emptyset) \cdot m_2(\emptyset), \\ m(\emptyset) &= 1 \quad \text{iff } K = 1. \end{aligned} \quad (9)$$

2.4. Pignistic Probability Transformation. In the transferable belief model (TBM) [55], the pignistic probabilities are typically used for decision making.

Definition 8. Let m be a GBPA on the frame of discernment U. Its associated pignistic probability transformation (PPT), which represents a point estimate in a belief interval, BetPm : $U \rightarrow [0,1]$, is defined as [55]

$$\text{BetPm}(A_i) = \sum_{A \subset P(U), A_i \in A} \frac{m(A)}{|A|}, \quad (10)$$

where $|A|$ is the cardinality of subset A. The PPT process transforms GBPA to probability distribution.

3. Generalized Evidential FMEA

In real application, conventional FMEA only addresses the three risk factors, occurrence (O), severity (S), and detection (D); all the other potential risk factors are not handled in the conventional FMEA model. In this section, a novel failure mode and effects analysis model named generalized evidential FMEA (GEFMEA) is proposed. GEFMEA is a more generalized FMEA model; it is based on generalized evidence theory. The other risk factors in the open world can be modeled with the empty set in the frame of GET. In addition, with GBPA and PPT, the relative importance among all the risk factors, O, S, D, and the incomplete one, is addressed flexibly.

Each part of the proposed GEFMEA model is presented in this section, including the frame of discernment of GEFMEA, the GBPA of GEFMEA, the generalized evidential risk priority number (GERPN) in GEFMEA, and a flowchart of GEFMEA model.

3.1. Frame of Discernment in GEFMEA. As to the risk factors, the frame of discernment for the conventional FMEA, according to (1), includes the probability of the occurrence of a failure mode (O), the severity of a failure effect (S), and the probability of a failure to be detected (D). Only O, S, and D are in the frame of discernment of conventional FMEA. It only focuses on these three risk factors independently; this is incomplete, because it ignores all the other potential risk factors in real application, such as the period of developing a new safety component for an auto and the cost of the new project. In the open world, the frame of discernment may be incomplete; there are always some uncertain potential risk factors needed to be handled independently. So, the frame of discernment of the risk factors for GEFMEA is defined in this section.

Definition 9. The *incomplete* frame of discernment for GEFMEA includes the conventional three risk factors: occurrence (O), severity (S), and detection (D):

$$U_{\text{FMEA}} = \{O, S, D\}. \quad (11)$$

In Definition 9, the power set of U_{FMEA} (short for U), 2_G^U, is composed of 2^U propositions, including $\emptyset$. In GET, $\emptyset$ can be a focal element; it represents the union of focal elements that are out of the given frame of discernment in the open world. So, in GEFMEA, $\emptyset$ is used to model all the other uncertain risk factors in the open world due to the incomplete frame of discernment of conventional FMEA. If $m(\emptyset) = 0$, the GEFMEA degenerates to conventional FMEA; in this sense, the GEFMEA extends the classical FMEA; GEFMEA is a more generalized model of conventional FMEA.

3.2. GBPA in GEFMEA. For each failure mode, a team member in the FMEA team can assign the relative importance among all the risk factors, O, S, and D by GBPA. This is similar to assigning the risk level for each failure mode from level 1 to level 10 with regard to O, S, and D [26]. So, the relative importance among O, S, and D can be assigned subjectively by each member in the FMEA team according to their prior empirical knowledge. The GBPA for the uncertain risk factors, $m(\emptyset)$, can be got from two methods in Definition 11. Definition 10 defines the mass function of each proposition corresponding to risk factors.

Definition 10. For all the 2^U propositions in Definition 9, $\forall A \subset U_{\text{FMEA}}$, a mass function m is a mapping $m_G : 2_G^U \rightarrow [0,1]$ that satisfies

$$\sum_{A \in 2_G^U} m_G(A) = 1, \quad (12)$$

where all the possible propositions for A may be

$$2_G^U = (\{O\}, \{S\}, \{D\}, \{O,S\}, \{O,D\}, \{S,D\}, \{O,S,D\}, \{\emptyset\}), \quad (13)$$

and the proposition $\emptyset$ represents the uncertain risk factors.

The GBPA for each proposition A in Definition 10, for example, the $m(O)$, $m(S)$, and $m(D)$, is assigned by FMEA team member.

Definition 11. The GBPA for the uncertain risk factors, $m(\emptyset)$, may be defined according to one of the two methods. (1) The GBPA for incomplete risk factor, $\emptyset$, can be assigned directly by team members subjectively, which is similar to the rating of the three conventional risk factors. (2) The GBPA for incomplete risk factor: if it is not assigned directly by team member, it can be calculated by the following equation:

$$m(\emptyset) = 1 - \text{Gpl}(O, S, D), \quad (14)$$

subject to

$$\text{Gpl}(O, S, D) = \sum_{\{O,S,D\} \cap B \neq \emptyset} m(B), \quad (15)$$

where $B \subset U_{\text{FMEA}}$.

In this paper, the GBPA for the incomplete risk factors is derived from (14) and (15).

3.3. Generalized Evidential RPN in GEFMEA. In real application, in order to put the limited resources into the key process and reach the financial objectives [43], the risk priority number (RPN) is used to rank and select the failure modes that needed to be handled in advance. In GEFMEA, a generalized evidential RPN, namely, GERPN, is proposed as follows:

$$\text{GERPN} = O^{\text{BetPm}(O)} \times S^{\text{BetPm}(S)} \times D^{\text{BetPm}(D)} \times 10^{\text{BetPm}(\emptyset)}. \quad (16)$$

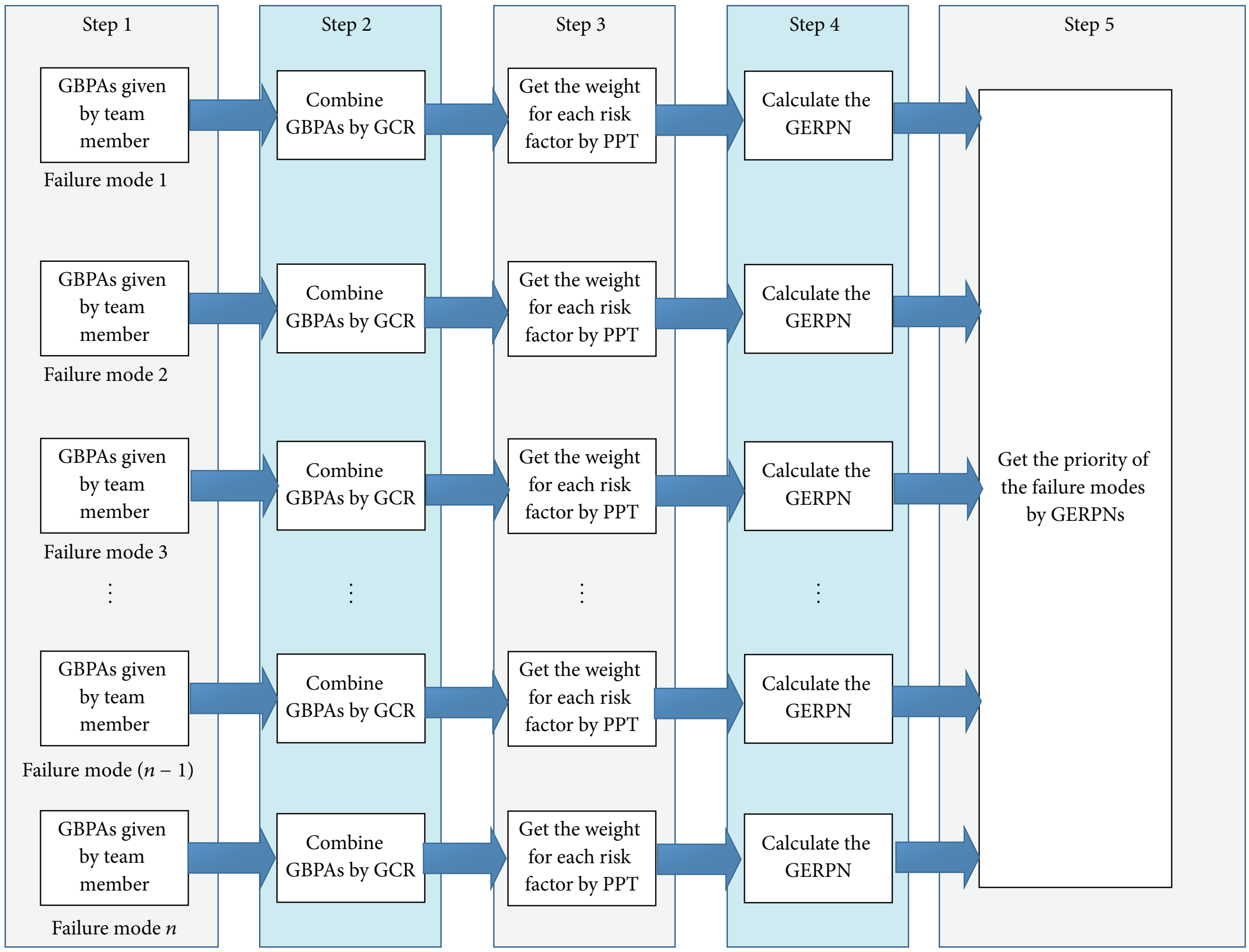

Figure 1: The flowchart in GEFMEA.

In GERPN, the incomplete uncertain risk factors are assigned with rating 10, the highest rating. It seems that assigning rating 10 to the uncertain risk factor is a pessimistic strategy. However, it is reasonable since the uncertain risk factors mean quite a high risk level for the consumer. What is more, this is really important to improve the quality of the product.

3.4. GEFMEA Model. The flowchart of the key steps in GEFMEA is shown in Figure 1. According to Figure 1, there are 5 steps to get the priority of all the failure modes, which is described in detail as follows.

Step 1. For each failure mode, each team member gives the subjective relative importance among O, S, D, and the other unknown risk factors by GBPA.

Step 2. For each failure mode, the GBPAs are combined by GCR.

Step 3. For each failure mode, the final weight of each risk factor is derived from combined GBPA by PPT.

Step 4. For each failure mode, the GERPN is calculated, which takes into consideration the relative importance of each risk factor, including the incomplete risk factor.

Step 5. The priority among all the failure modes is got according to GERPNs.

4. Experiments

In this section, some numerical examples and a real case study on aircraft turbine rotor blades are used to illustrate the efficiency of the proposed method. The numerical examples are used to show how the proposed GEFMEA model may be used in industrial application, especially for those cases that incomplete risk factors should be taken into consideration independently. The case study in aircraft turbine rotor blades, which is also compared with the result of conventional D-S evidence theory in the literature [39], shows the compatibility of the proposed method.

4.1. The Illustrative Numerical Example. A few numerical examples of design FMEA (DFMEA) in automotive industry

Table 2: Five failure modes and their RPNs.

Failure mode	Rating of each risk factor	Conventional RPN	Auto parts
FM1	$O = 2, S = 8, D = 3$	48	Passive safety system
FM2	$O = 4, S = 7, D = 4$	112	Power supply system
FM3	$O = 3, S = 9, D = 4$	108	Active safety system
FM4	$O = 3, S = 4, D = 2$	24	Air conditioning system
FM5	$O = 2, S = 2, D = 3$	12	Entertainment system

Table 3: GBPA for relative importance among risk factors for FM1.

Team member	GBPA for O, S, and D	GBPA for uncertain risk factors
TM1	$m_1(S) = 0.8, m_1(O, D) = 0.1$	0.1
TM2	$m_2(S) = 0.7, m_2(O, D) = 0.2$	0.1
TM3	$m_3(S) = 0.6, m_3(O, D) = 0.3$	0.1

Table 4: GBPA for relative importance among risk factors for FM2.

Team member	GBPA for O, S, and D	GBPA for uncertain risk factors
TM1	$m_1(O, S) = 0.8, m_1(D) = 0.2$	0
TM2	$m_2(O, S, D) = 0.9$	0.1
TM3	$m_3(O) = 0.2, m_3(S) = 0.5, m_3(D) = 0.2$	0.1

Table 5: GBPA for relative importance among risk factors for FM3.

Team member	GBPA for O, S, and D	GBPA for uncertain risk factors
TM1	$m_1(O, S) = 0.8, m_1(D) = 0.1$	0.1
TM2	$m_2(O, D) = 0.2, m_2(S) = 0.7$	0.1
TM3	$m_3(O) = 0.2, m_3(S) = 0.5, m_3(D) = 0.2$	0.1

Table 6: GBPA for relative importance among risk factors for FM4.

Team member	GBPA for O, S, and D	GBPA for uncertain risk factors
TM1	$m_1(O, S, D) = 0.9$	0.1
TM2	$m_2(O, S, D) = 0.8$	0.2
TM3	$m_3(O, S) = 0.3, m_3(D) = 0.6$	0.1

Table 7: GBPA for relative importance among risk factors for FM5.

Team member	GBPA for O, S, and D	GBPA for uncertain risk factors
TM1	$m_1(O, D) = 0.6, m_1(S) = 0.2$	0.2
TM2	$m_2(O, D) = 0.7, m_2(S) = 0.2$	0.1
TM3	$m_3(O, D) = 0.6, m_3(S) = 0.1$	0.3

are presented in this section to illustrate the efficiency of the proposed method and its potential application in industrial environment.

4.1.1. Experimental Data. The experimental data to illustrate how the proposed GEFMEA model can be used in real application is empirical data which is proposed based on the process of DFMEA.

Assume that there are five failure modes chosen from the FMEA file for an advanced auto. Each failure mode comes from a different component or system in the auto; the relative importance among these risk factors should be different. What is more, some other risk factors should be taken into consideration, such as the cost and the period of development. The five failure modes (FM) and their conventional RPNs are shown in Table 2.

Three team members (TM1, TM2, and TM3) in a FMEA team assess the relative importance of the risk factors by GBPAs, as shown in Tables 3, 4, 5, 6, and 7, respectively. FM1 is assumed to be a failure mode in a passive safety system, such as a supplemental restraint system (SRS); three members from a FMEA team judge the relative importance of these three risk factors O, S, and D, by GBPAs, as is shown in Table 3. Note that because it is a failure mode for a safety part of an auto, all the GBPAs give a high relative importance to the risk factor S. This is reasonable, because it is a matter of lives. What is more, there are some potential factors needed to be taken into consideration, such as the number of loops of the SRS for an auto, the cost, and the period of development of a SRS supplier. Here, the GBPA for uncertain risk factor is not zero. FM2 is assumed to be a failure mode in a power supply system; the GBPAs are shown in Table 4. FM3 is assumed to be a failure mode in an active safety system, such as the head light assembly, which is a key function part at night; the GBPAs are shown in Table 5. FM4 is assumed to be a failure mode in a heating ventilation air conditioning system (HVAC), which should be worked in quite different kinds of environment or even with no need in some area, so the GBPAs sometimes can be very flexible, as is shown in Table 6. FM5 is assumed to be a failure mode in an audio-video entertainment system, which is only for fun, but the cost can range from very cheap to very expensive; its GBPAs are shown in Table 7.

4.1.2. Experiment with GEFMEA Model. According to the proposed method in Section 3, in order to get the ranking of all the failure modes, the GERPN for each failure mode is calculated. The following is the detail of each step to get the GERPN for FM1.

Step 1. The relative importance among O, S, and D in the form of GBPAs is given in Table 3. The GBPA for the uncertain risk factors is calculated by (14) and (15) in Definition 11. For TM1, $m(\emptyset)$ is calculated as follows:

$$\begin{aligned} m(\emptyset) &= 1 - \mathrm{Gpl}(O,S,D) = 1 - \sum_{\{O,S,D\}\cap B\neq\emptyset} m(B) \\ &= 1 - (0.8 + 0.1) = 0.1. \end{aligned} \quad (17)$$

Similarly, $m(\emptyset)$ for TM2 and TM3 can be calculated. It should be pointed out that all the team members give a high relative importance to the risk factor S in the FMEA process since it is a matter of lives for the final consumers.

Step 2. The GBPAs in Table 3 are combined by GCR.

(1) Combine GBPAs from TM1 and TM2 by GCR in Definition 7 with (9):

$$\begin{aligned} K_{\mathrm{TM1,TM2}} &= \sum_{B\cap C=\emptyset} m_1(B)\cdot m_2(C) \\ &= m_1(S)\times[m_2(O,D)+m_2(\emptyset)] \\ &\quad + m_1(O,D)\times[m_2(S)+m_2(\emptyset)] \\ &\quad + m_1(\emptyset) \\ &\quad \times[m_2(S)+m_2(O,D)+m_2(\emptyset)] \\ &= 0.8\times(0.2+0.1)+0.1\times(0.7+0.1) \\ &\quad + 0.1\times(0.7+0.2+0.1) = 0.42, \\ m_{\mathrm{TM1,TM2}}(\emptyset) &= m_1(\emptyset)\cdot m_2(\emptyset) = 0.1\times 0.1 = 0.01. \end{aligned} \quad (18)$$

With the result of (18), then the combined result from TM1 and TM2 is as follows:

$$\begin{aligned} m_{\mathrm{TM1,TM2}}(S) &= \frac{(1-m(\emptyset))\sum_{B\cap C=A} m_1(B)\cdot m_2(C)}{1-K} \\ &= \frac{(1-m_{\mathrm{TM1,TM2}}(\emptyset))\sum_{B\cap C=A} m_1(S)\cdot m_2(S)}{1-K_{\mathrm{TM1,TM2}}} \\ &= \frac{(1-0.01)\times(0.8\times 0.7)}{1-0.42} = 0.9559, \\ m_{\mathrm{TM1,TM2}}(O,D) & \\ &= \frac{(1-m(\emptyset))\sum_{B\cap C=A} m_1(B)\cdot m_2(C)}{1-K} \\ &= \frac{(1-m_{\mathrm{TM1,TM2}}(\emptyset))\sum_{B\cap C=A} m_1(O,D)\cdot m_2(O,D)}{1-K_{\mathrm{TM1,TM2}}} \\ &= \frac{(1-0.01)\times(0.1\times 0.2)}{1-0.42} = 0.0341. \end{aligned} \quad (19)$$

(2) Again, with (9) in Definition 7, we can calculate the result of the combined GBPAs from TM1, TM2, and TM3; the results are as follows:

$$\begin{aligned} K_{(\mathrm{TM1,TM2}),\mathrm{TM3}} &= 0.4162, \\ m_{(\mathrm{TM1,TM2}),\mathrm{TM3}}(\emptyset) &= 0.001, \\ m_{(\mathrm{TM1,TM2}),\mathrm{TM3}}(S) &= 0.9814, \\ m_{(\mathrm{TM1,TM2}),\mathrm{TM3}}(O,D) &= 0.0175. \end{aligned} \quad (20)$$

(3) Finally, the GBPAs of FM1 are as follows:

$$\begin{aligned} m(\emptyset) &= 0.001, \\ m(S) &= 0.9814, \\ m(O,D) &= 0.0175. \end{aligned} \quad (21)$$

Step 3. The final weight of each risk factor is derived from combined GBPA by PPT.

With (10) in Definition 8, the weight for each risk factor is as follows:

$$\begin{aligned} \mathrm{BetPm}(O) = \mathrm{BetPm}(D) &= \sum_{A\subset P(\Omega),A_i\in A} \frac{m(A)}{|A|} \\ &= \frac{0.0175}{2} = 0.0088, \\ \mathrm{BetPm}(S) &= 0.9814, \\ \mathrm{BetPm}(\emptyset) &= 0.001. \end{aligned} \quad (22)$$

Step 4. The GERPN is calculated, which takes into consideration the relative importance of each risk factor, including the other uncertain risk factors.

According to (16), the GERPN is calculated as follows:

$$\begin{aligned} \mathrm{GRPN}_1 &= O^{\mathrm{BetPm}(O)}\times S^{\mathrm{BetPm}(S)}\times D^{\mathrm{BetPm}(D)} \\ &\quad \times 10^{\mathrm{BetPm}(\emptyset)} \\ &= 2^{0.0088}\times 8^{0.9814}\times 3^{0.0088}\times 10^{0.001} \\ &= 7.8368. \end{aligned} \quad (23)$$

Repeat the four steps above, the GERPNs for FM2, FM3, FM4, and FM5 can be calculated, and the results are shown as (24), (25), (26), and (27), respectively:

$$\mathrm{GRPN}_2 = 4^{0.2667}\times 7^{0.6667}\times 4^{0.0667}\times 10^{0} = 5.8097, \quad (24)$$

$$\mathrm{GRPN}_3 = 3^{0}\times 9^{0.6281}\times 4^{0.009}\times 10^{0.001} = 7.9713, \quad (25)$$

$$\begin{aligned} \mathrm{GRPN}_4 &= 3^{0.1664}\times 4^{0.1664}\times 2^{0.6653}\times 10^{0.002} \\ &= 2.4091, \end{aligned} \quad (26)$$

$$\begin{aligned} \mathrm{GRPN}_5 &= 2^{0.4901}\times 2^{0.0078}\times 3^{0.4901}\times 10^{0.012} \\ &= 2.4873. \end{aligned} \quad (27)$$

Table 8: Comparing GERPN with RPN.

Failure mode	GERPN	Priority by GERPN	Conventional RPN	Priority by RPN
FM1	7.8368	2	48	3
FM2	5.8097	3	112	1
FM3	7.9713	1	108	2
FM4	2.4091	5	24	4
FM5	2.4627	4	12	5

4.1.3. Results and Discussions. According to conventional RPN, as is shown in Table 2, the priority among all the five risk factors should be FM2 ≻ FM3 ≻ FM1 ≻ FM4 ≻ FM5, where "≻" means "a higher priority than," while according to the results of GERPNs, the risk priority among these five risk factors is FM3 ≻ FM1 ≻ FM2 ≻ FM5 ≻ FM4, as is shown in Table 8.

The result of GERPN is compatible with the conventional RPN value in general, which assures that the proposed model is reasonable and effective. The difference between the GERPNs and the RPN hints the influence of the consideration of the relative importance among each risk factor. More importantly, the uncertain risk factors out of the incomplete frame of discernment (O, S, and D), which may have a great effect on the failure mode analysis, are modeled in GEFMEA. This is distinguished, reflected in the first and third failure modes (FM1 and FM3). Both FM1 and FM3 correspond to safety. As a result, FM1 and FM3 get the higher priority with GERPN. There are two main reasons. One is that other incomplete and uncertain risk factors are taken into consideration by assigning a nonzero mass function to the empty set. As is mentioned above, the incomplete risk factors may be the cost required by different suppliers, the qualified suppliers period of development, and so on. The other reason is that a high relative importance is assigned to the risk factor S. The incomplete risk factor and the relative importance of each risk also have an effect on the risk priority of FM4 and FM5.

In order to investigate the influence of the generalized evidential theory, assume that the proposed method is based on classical D-S evidence theory; now (9) of GCR in the Step 2 of GEFMEA approach will be replaced by (4) and (5) of Dempster's rule of combination. In this case, the other uncertain risk factor, which is represented by empty set in GEFMEA, can no longer be taken into consideration anymore, and the mass of empty set will be redistributed among the three conventional risk factors O, S, and D according to the normalization process with (4) and (5). The value of the RPNs based on classical D-S evidence theory, denoted as ERPN, is presented in Table 9. The risk priority is the same, that is, FM3 ≻ FM1 ≻ FM2 ≻ FM5 ≻ FM4. The value of ERPN is close to GERPN, sometimes the same as each other if one assesses that there is no other uncertain risk factor in a failure mode, such as in the case FM2. The comparative result shows that if the other risk factor is not taken into consideration even if the FMEA team supports that there exists another risk factor except for O, S, and D, GEFMEA is compatible with the method based on classical D-S evidence theory. In other words, the GEFMEA approach can be degenerated to the approach based on classical D-S evidence theory proposed by Yang et al. [38] once the relative importance among the risk factors is also taken into consideration in [38].

Table 9: Comparing GERPN with ERPN.

Failure mode	FM1	FM2	FM3	FM4	FM5
GERPN	7.8368	5.8097	7.9713	2.4091	2.4627
ERPN	7.8351	5.8097	7.9720	2.4015	2.4418

Table 10: The combined results of the 17 failure modes in literature [38].

Failure mode	Rating of each risk factor	MVRPN
FM1	$O = 3.04$, $S = 7$, $D = 2$	42.56
FM2	$O = 2$, $S = 8$, $D = 4$	64
FM3	$O = 1$, $S = 10$, $D = 3$	30
FM4	$O = 1$, $S = 6$, $D = 3$	18
FM5	$O = 1$, $S = 3$, $D = 1.39$	4.17
FM6	$O = 2$, $S = 6$, $D = 5$	60
FM7	$O = 1$, $S = 7$, $D = 3$	21
FM8	$O = 3$, $S = 5$, $D = 1$	15
FM9	$O = 1.99$, $S = 9.91$, $D = 4$	78.92
FM10	$O = 1$, $S = 10$, $D = 6$	60
FM11	$O = 1$, $S = 10$, $D = 5$	50
FM12	$O = 1$, $S = 10$, $D = 5$	50
FM13	$O = 1$, $S = 10$, $D = 5$	50
FM14	$O = 1$, $S = 10$, $D = 6$	60
FM15	$O = 2$, $S = 7$, $D = 3$	42
FM16	$O = 1.99$, $S = 4$, $D = 3$	23.88
FM17	$O = 2$, $S = 5.01$, $D = 3$	30.05

4.2. A Real Case Study on Aircraft Turbine Rotor Blades. A real case study on aircraft turbine rotor blades with the proposed method is studied in this section. Aircraft system is very complicated [68], where the FEMA plays an important role. The experiment result with GEFMEA model is compared with the result of the modified FMEA method based on D-S evidence theory in the literature [38].

4.2.1. Experimental Data. For the 17 failure modes of the aircraft turbine rotor blades in the literature [27, 38, 39], Yang et al. [38], Su et al. [39], and Jiang et al. [27] pay no attention to the relative importance of the three risk factors, O, S, and D, not to say taking into consideration the potential incomplete

Table 11: The GERPNs of the 17 failure modes.

Failure mode	GERPN	MVRPN	Priority by GERPN	Priority by MVRPN
FM1	3.4910	42.56	9	9
FM2	3.9994	64	2	2
FM3	3.1069	30	12	12
FM4	2.6205	18	15	15
FM5	1.6095	4.17	17	17
FM6	3.9143	60	3	3
FM7	2.7586	21	14	14
FM8	2.4660	15	16	16
FM9	4.2881	78.92	1	1
FM10	3.9143	60	3	3
FM11	3.6836	50	6	6
FM12	3.6836	50	6	6
FM13	3.6836	50	6	6
FM14	3.9143	60	3	3
FM15	3.4756	42	10	10
FM16	2.8794	23.88	13	13
FM17	3.1089	30.05	11	11

risk factor. In this paper, in order to use the data in these literatures to show the compatibility of the proposed method, it is assumed that (1) there is no other incomplete risk factor among these 17 failure modes, except for O, S, and D; (2) the relative importance among the three risk factors is the same, which means 33.33% for O, 33.33% for S, and 33.33% for D.

In the literature [38], according to the assessments from three experts, the combined result of each risk factor is shown in Table 10, as well as the modified RPN noted as MVRPN [38]. (Note that the MVRPN values for FM13 and FM17 are wrong in literature [38]; they are corrected in Table 10 in this paper.)

4.2.2. Experiment with GEFMEA Model. In Section 4.2.1, it is assumed that the relative importance among each risk factor is the same, and there is no other risk factor, which means $m(\emptyset) = 0$. So, the GBPAs for each risk factor of each failure mode in Table 10 are the same, shown as follows:

$$m(O) = m(S) = m(D) = 0.3333. \quad (28)$$

With (10) in Definition 8, the weight for each risk factor is as follows:

$$\begin{aligned} &\text{BetPm}(O) = \text{BetPm}(S) = \text{BetPm}(D) = 0.3333, \\ &\text{BetPm}(\emptyset) = 0. \end{aligned} \quad (29)$$

So, the GERPN for the first failure mode in Table 10, according to (16) and (28), is calculated as follows:

$$\begin{aligned} \text{GERPN}_{\text{FM1}} &= 3.04^{\text{BetPm}(O)} \times 7^{\text{BetPm}(S)} \times 2^{\text{BetPm}(D)} \\ &\quad \times 10^{\text{BetPm}(\emptyset)} = 3.4910. \end{aligned} \quad (30)$$

In this way, the GERPNs of all the 17 failure modes in Table 10 are calculated; the result is in Table 11, as well as the ranking by GERPN and MVRPN.

4.2.3. Results and Discussions. The results in Table 11 show that the priority of each failure mode calculated by GERPN method is the same as it is from MVRPN. This is because the relative importance among each risk factor is not taken into consideration in fact, and there is really no information about other risk factors in the experimental data of the literature [27, 38, 39]. This experiment shows that the GEFMEA is compatible with classical FMEA method. The risk priority based on GERPN and MVRPN [38] will be the same if the FMEA approach is applying in a closed world where there is no incomplete risk factor, and the FMEA team considers that relative importance among each risk factor is the same.

5. Conclusions

In this paper, a generalized evidential FMEA model is proposed based on the generalized evidence theory; the incomplete risk factors apart from O, S, and D are modeled by the empty set in the frame of the generalized evidence theory. The proposed GEFMEA model is a more generalized condition of the classical FMEA model; it can be degenerated to the conventional one if needed. In addition, the relative importance of each risk factor, including the other incomplete risk factor, is addressed in the proposed method. Compared with the other modified FMEA model, the proposed model is simple, which is also a merit in real application. For further study, how to generate the GBPA for GEFMEA intelligently and more flexibly is a challenging and promising problem, and the authors are beginning to work on this topic.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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