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Research Article

Adaptive Finite-Time Stabilization of High-Order Nonlinear Systems with Dynamic and Parametric Uncertainties

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Under the weaker assumption on nonlinear functions, the adaptive finite-time stabilization of more general high-order nonlinear systems with dynamic and parametric uncertainties is solved in this paper. To solve this problem, finite-time input-to-state stability (FTISS) is used to characterize the unmeasured dynamic uncertainty. By skillfully combining Lyapunov function, sign function, backstepping, and finite-time input-to-state stability approaches, an adaptive state feedback controller is designed to guarantee high-order nonlinear systems are globally finite-time stable.

1. Introduction

Since the concept of finite-time stability was introduced in [1], many efforts have been made to study the problem of finite-time stabilization because of faster convergence rates, higher accuracies, and better disturbance rejection properties. Based on the finite-time stability theorem in [2–4], some finite-time stabilization results have been achieved by combining finite-time stability with backstepping design method, for example, [5–9] and the references therein.

Recently, more attention of finite-time stability has been focused on a family of high-order nonlinear systems of the form

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}^{p_i}(t) + \phi_i\left(x_1(t), \ldots, x_i(t), d\right), \\ &\qquad i = 1, \ldots, n-1, \quad (1) \\ \dot{x}_n(t) &= u^{p_n}(t) + \phi_n\left(x_1(t), \ldots, x_n(t), d\right), \end{aligned}$$

where $u \in R$ is the control input, $x = (x_1, \ldots, x_n)^\top \in R^n$ is the measured state, and $d \in R^r$ denotes the unknown parameter vector. For $i = 1, \ldots, n$, $\phi_i : R^i \times R^{n_0} \times R^+ \to R$ is an unknown and Lipschitz continuous function. $p_i \in R_{\text{odd}}^{\geq 1} := \{p/q \in R^+ : p$ and q are odd integers, $p \geq q\}$. System (1) is called high-order system if there exists at least $p_i > 1$, $i \in \{1, \ldots, n\}$.

For system (1), when d is known, [10, 11] studied finite-time stability, where the order of state x_j $(j = 1, \ldots, i)$ in (10) and (11) can be taken value in $(0, 1/(p_j \ldots p_{i-1}))$ with $p_i p_{i-1} = 1$. The restrictive condition was relaxed by [12], in which all the states in the bounding condition were allowed to be of both low order and high order. When d is unknown, it is well known that adaptive technique is an effective way to deal with control problem of nonlinear systems with parametric uncertainty. Reference [13] developed a continuous adaptive finite-time controller with the bounding condition of ϕ_i being an order equal to 1. The latest paper [14] weakened the growth condition by allowing the order greater than 0. However, there is no dynamic uncertainty considered by these papers.

The analysis and control problem of nonlinear systems with dynamic uncertainty have been an active research topic because dynamic uncertainty often arises from many different control engineering applications; see [15–19] and the references therein. In view of the benefits of finite-time convergence, finite-time stabilization of nonlinear systems with dynamic uncertainty has been regarded as one of the important issues. By characterizing dynamic uncertainty with finite-time input-to-state stability (FTISS), [20] constructed a finite-time adaptive state feedback controller for one-order nonlinear systems with dynamic and parametric uncertainties. Reference [21] gave the explicit definition of FTISS and

developed a framework for the finite-time control analysis and synthesis based on FTISS. However, for more general high-order nonlinear systems, to the best of the authors' knowledge, no result on finite-time stabilization has been achieved until now.

Based on the above discussion, an interesting problem is put forward spontaneously: for more general high-order systems with dynamic and parametric uncertainties,

$$\begin{aligned} \dot{z}(t) &= \psi(z(t), x_1(t)), \\ \dot{x}_i(t) &= x_{i+1}^{p_i}(t) + \phi_i(x_1(t), \ldots, x_i(t), z(t), d), \\ &\qquad i = 1, \ldots, n-1, \\ \dot{x}_n(t) &= u^{p_n}(t) + \phi_n(x_1(t), \ldots, x_n(t), z(t), d), \end{aligned} \tag{2}$$

$z \in R^{n_0}$ is the unmeasured state, referred to as dynamic uncertainty, $d \in R^r$ is the unknown parameter vector, $\phi_i : R^i \times R^{n_0} \times R^+ \to R$ $(i = 1, \ldots, n)$ is an unknown and Lipschitz continuous function, $\psi : R^{n_0} \times R \to R$ is piecewise continuous with respect to z and Lipschitz continuous with respect to x_1, and $\psi(0,0) = 0$; under the weaker assumption than Assumption 1 in [14] (see Remark 5 for a detailed discussion), can a finite-time stabilized continuous controller be designed?

In this paper, an affirmative solution to this problem is given. To solve this problem, finite-time input-to-state stability (FTISS) is used to characterize the unmeasured dynamic uncertainty. By skillfully combining Lyapunov function, sign function, backstepping, and FTISS approaches and overcoming some obstacles emerging in design and analysis owing to the relaxed condition on nonlinear functions, an adaptive state feedback controller is designed to guarantee high-order nonlinear system (2) is globally finite-time stable. An example demonstrates the theoretical result.

This paper is organized as follows. Section 2 gives preliminaries. Sections 3 and 4 provide the design and analysis of adaptive finite-time state feedback controller, following a simulation example in Section 5. Section 6 concludes the paper. The appendix proves Proposition 6 of Section 3.

2. Mathematical Preliminaries

Some notations and lemmas are to be used throughout this paper.

R^+ stands for the set of all the nonnegative real numbers. For any vector $x = (x_1, \ldots, x_n)^\top \in R^n$, denote $\|x\| = (\sum_{i=1}^n x_i^2)^{1/2}$. For $i = 1, \ldots, n$, $\overline{x}_i \triangleq (x_1, \ldots, x_i)^\top \in R^i$. A function $f : R^n \to R$ is $\mathscr{C}$ if it is continuous and is $\mathscr{C}^1$ if it is continuously differential. $\mathscr{K}$ denotes the set of all functions: $R^+ \to R^+$ that are continuous, strictly increasing, and vanishing at zero, and $\mathscr{K}_\infty$ denotes the set of all functions that are of class $\mathscr{K}$ and unbounded. For simplicity, we sometimes denote a function $f(x(t))$ by $f(x)$ or f. Sign function $\operatorname{sgn}(x)$ is defined as $\operatorname{sgn}(x) = 1$ if $x > 0$, $\operatorname{sgn}(x) = 0$ if $x = 0$, and $\operatorname{sgn}(x) = -1$ if $x < 0$.

Definition 1 (see [13]). Consider a system

$$\dot{x} = f(x, t), \quad f(0, t) = 0, \ x \in U_0 \subset R^n, \tag{3}$$

where $f : U_0 \times R^+ \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is (local) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq U_0$ of the origin. By "finite-time convergence" one means the following: if, for any initial condition $x(t_0) = x_0 \in U$ at any given initial time t_0, there is a settling time $T > 0$, such that every solution $x(t; t_0, x_0)$ of system (3) is defined with $x(t; t_0, x_0) \in U/0$ for $t \in [t_0, T)$ and $\lim_{t \to T} x(t; t_0, x_0) = 0$, $x(t; t_0, x_0) = 0$ for any $t > T$. When $U = R^n$, the origin is a globally finite-time stable equilibrium.

Definition 2 (see [21]). Consider a system

$$\dot{x} = f(x, v), \quad f(0, 0) = 0, \ x \in R^n, \ v \in R^m, \tag{4}$$

where v is the input and f is continuous with respect to (x, v). A continuous function $V(x)$ is called FTISS-Lyapunov function for system (4) if there exist $\mathscr{K}_\infty$-functions ϕ_1, ϕ_2, and ϕ_3 and a positive constant c such that

$$\begin{aligned} &\phi_1(\|x\|) \le V(x) \le \phi_2(\|x\|), \quad \forall x \in R^n, \\ &\dot{V}(x(t)) \le -cV^\alpha(x(t)) + \phi_3(\|v(t)\|), \quad 0 < \alpha < 1. \end{aligned} \tag{5}$$

In the remainder of this section, we list several lemmas that serve as the basis for the design of state feedback controller for system (2). Lemma 3 is finite-time stability theorem. Lemmas 4–8 are used to enlarge inequalities. Lemmas 9 and 10 are used to deal with sign function.

Lemma 3 (see [13]). *Suppose that, for system (3), there is a $\mathscr{C}^1$ positive-definite function $V(x, t)$ (defined on $\widehat{U} \times R^n$, where $\widehat{U} \subset U_0 \subset R^n$ is a neighborhood of the origin), and a real number $c > 0$ and $0 < \alpha < 1$, such that $\dot{V}(x, t) + cV^\alpha(x, t) \le 0$ (along the trajectory) on $\widehat{U}$. Then $V(x, t)$ is locally finite-time convergent or equivalently becomes 0 locally in finite time, with its settling time $T \le V^{1-\alpha}(x(t_0), t_0)/c(1-\alpha)$ for a given initial condition $x(t_0)$ in a neighborhood of the origin in $\widehat{U}$.*

Lemma 4 (Young's inequality). *Let real numbers $p \ge 1$ and $q \ge 1$ satisfy $1/p + 1/q = 1$; then for any $x, y \in R$ and any given positive number $\gamma > 0$, $xy \le \gamma|x|^p + (1/q)(p\gamma)^{-q/p}|y|^q$.*

Lemma 5 (Jensen's inequality). *If $0 < a_1 < a_2$, then $(\sum_{i=1}^n x_i^{a_2})^{1/a_2} \le (\sum_{i=1}^n x_i^{a_1})^{1/a_1}$ for any $x_i \ge 0$, $i = 1, \ldots, n$.*

Lemma 6 (see [22]). *If $p \ge 1$, then $|x + y|^{1/p} \le |x|^{1/p} + |y|^{1/p}$, $|x + y|^p \le 2^{p-1}|x^p + y^p|$ for any $x, y \in R$.*

Lemma 7 (see [22]). *If $p \in R_{odd}^{\ge 1}$, then $|x - y|^p \le 2^{p-1}|x^p - y^p|$, $|x^{1/p} - y^{1/p}| \le 2^{1-1/p}|x - y|^{1/p}$ for any $x, y \in R$.*

Lemma 8 (see [23]). *For a continuous function $f(x, y)$ with $x \in R^m$, $y \in R^n$, there exist smooth functions $a(x) \ge 0$, $b(y) \ge 0$, $c(x) \ge 1$, and $d(y) \ge 1$, such that $|f(x, y)| \le a(x) + b(y)$, $|f(x, y)| \le c(x)d(y)$.*

Lemma 9 (see [24]). *If $p = a/b \in R_{odd}^{\ge 1}$, $b \ge 1$, then $|x^p - y^p| \le 2^{1-1/b}|\operatorname{sgn}(x)|x|^a - \operatorname{sgn}(y)|y|^a|^{1/b}$ for any $x, y \in R$.*

Lemma 10 (see [24]). *$f(x) = \operatorname{sgn}(x)|x|^a$ is continuously differentiable, and $\dot{f}(x) = a|x|^{a-1}$, where $a \geq 1$, $x \in R$. Moreover, if $x = x(t)$, $t \geq 0$, then $df(x(t))/dt = a|x(t)|^{a-1}\dot{x}(t)$.*

3. Finite-Time Convergence Analysis

3.1. Problem Formulation and Assumptions. The purpose of this paper is to achieve a global finite-time control design for high-order nonlinear system (2) with dynamic and parametric uncertainties.

To achieve the purpose, we need the following assumptions.

Assumption 1. The z-subsystem has an FTISS-Lyapunov function $V_0(z)$ that satisfies

$$\dot{V}_0 \leq -c_0 V_0^{\alpha_0} + \gamma_0\left(|x_1|\right), \tag{6}$$

where $c_0 > 0$, $0 < \alpha_0 < 1$ are constants, and γ_0 is a $\mathscr{K}_\infty$ function. Moreover, π_1 and π_2 are $\mathscr{K}_\infty$ functions such that

$$\pi_1(\|z\|) \leq V_0(z) \leq \pi_2(\|z\|). \tag{7}$$

Assumption 2. For each $i = 1, \ldots, n$, there is a constant $\nu \in (0, 1/\sum_{j=1}^n p_0 \cdots p_{j-1})$ satisfying $(2-\nu)/(2-\nu-r_n) > 2\alpha_0$ and known $\mathscr{C}^1$ nonnegative function κ_1 and $\mathscr{C}$ nonnegative function κ_{i2} with $\kappa_1(0) = 0$ and $\kappa_{i2}(0, \ldots, 0) = 0$ such that

$$\begin{aligned} &\left|\phi_i(x_1, \ldots, x_i, z, d)\right| \\ &\quad \leq \kappa_1(\|z\|) + \gamma_i\left|x_{i+1}\right|^{p_i} \\ &\qquad + \theta\kappa_{i2}(x_1, \ldots, x_i)\sum_{j=1}^{i}\left|x_j\right|^{(r_i-\nu)/r_j+\mu_{ij}}, \end{aligned} \tag{8}$$

where $\theta > 0$ is an unknown constant, $\gamma_i \in [0, 1)$, $\mu_{ij} \in [0, +\infty)$, $p_0 = 1$, $x_{n+1}(t) = u(t)$, and

$$\begin{aligned} r_1 &= 1, \\ r_i &= \frac{r_{i-1} - \nu}{p_{i-1}}, \quad i = 2, \ldots, n. \end{aligned} \tag{9}$$

Assumption 3. We assume that $\limsup_{s\to 0+}(\gamma_0(s)/s^2) < +\infty$ and $\limsup_{s\to 0+}(\kappa_1^2(s)/\pi_1(s)) < +\infty$.

Remark 4. Assumption 1 implies that z-subsystem is characterized by finite-time input-to-state stability (FTISS). The inequalities in Assumption 3 are the small-gain conditions.

Remark 5. The following discussions in order demonstrate that Assumption 2 encompasses and generalizes the existing results.

(i) When $z = 0$ and θ is known, Assumption 2 includes the growth condition in [11]

$$\begin{aligned} &\left|\phi_i\right| \leq \gamma_i(x_1, \ldots, x_i)\sum_{j=1}^{i}\left|x_j\right|^{(r_i+\tau)/r_j}, \\ &\qquad \tau \in \left(-\frac{1}{\sum_{j=1}^n p_0 \cdots p_{j-1}}, 0\right) \end{aligned} \tag{10}$$

as its special case (i.e., $\mu_{ij} = 0$ in (8)), as well as the growth condition in [10]

$$\left|\phi_i\right| \leq c\sum_{j=1}^{i}\left|x_j\right|^{(r_i+\tau)/r_j}, \quad \tau \in \left(-\frac{1}{\sum_{j=1}^n p_0 \cdots p_{j-1}}, 0\right) \tag{11}$$

as a special case (i.e., $\mu_{ij} = 0$ and $\kappa_{i2} = c$ is a constant).

From $\nu \in (0, 1/\sum_{j=1}^n p_0 \cdots p_{j-1})$, $r_1 = 1$, $r_{i+1} = (r_i - \nu)/p_i$, it is easy to see that $0 < (r_i - \nu)/r_j < 1/(p_j \cdots p_{i-1})$, which implies that the power in condition (8) defined by $(r_i - \nu)/r_j + \mu_{ij}$ can take any value in an interval $(0, +\infty)$, while, for [10, 11], the powers only take values in $(0, 1/(p_j \cdots p_{i-1}))$.

(ii) When $z = 0$ and θ is unknown, (8) is reduced to Assumption 1 in [14]:

$$\begin{aligned} &\left|\phi_i\right| \leq \beta_i\left|x_{i+1}\right|^{p_i} + \theta\kappa_{i2}\sum_{j=1}^{i}\left|x_j\right|^{(r_i+\omega)/r_j+\mu_{ij}}, \\ &\omega \in \left(-\frac{1}{\sum_{j=1}^n p_0 \cdots p_{j-1}}, 0\right), \ \mu_{ij} \in [0, +\infty), \ \beta_i \in [0, 1). \end{aligned} \tag{12}$$

(iii) When $p_i = 1$ for all $i = 1, \ldots, n$, system (2) becomes

$$\begin{aligned} \dot{z} &= \psi(z, x_1), \\ \dot{x}_i &= x_{i+1} + \phi_i(x_1, \ldots, x_i, z, d), \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= u + \phi_n(x_1, \ldots, x_n, z, d), \end{aligned} \tag{13}$$

which is studied by [20, 21].

By the discussions, it is highlighted that this paper substantially extends the results of these papers; namely, for more general high-order nonlinear systems (2) with dynamic and parametric uncertainties, the finite-time control problem is to be solved under weaker condition (8).

3.2. Design of Adaptive Finite-Time Controller. In what follows, we denote $\sigma = \max\{\theta, \theta^{(2-\nu)/r_n}\}$, which is unknown because θ is unknown, $\widehat{\sigma}(t)$ is the estimate of σ, and $\widetilde{\sigma}(t) = \sigma - \widehat{\sigma}(t)$ is the estimation error. Denote $x = (x_1, \ldots, x_n)^\top$, $\overline{x}_i = (x_1, \ldots, x_i)^\top \in R^i$, $i = 1, \ldots, n$. For simplicity, denote $[x]^p = |x|^p\operatorname{sgn}(x)$, where $x \in R$, $\operatorname{sgn}(\cdot)$ is sign function whose definition is in the notations explained in Section 2, $[x]^p \cdot x = |x|^{p+1}$, and $[x]^p$ is obviously $\mathscr{C}^1$ if $p \geq 1$.

To give the design of controller, we first define the parameters $\beta_0, \beta_1, \ldots, \beta_n$ recursively as

$$\begin{aligned} \beta_0 &= 1, \\ (\beta_i p_i + 1)\, r_{i+1} &= (\beta_{i-1} p_{i-1} + 1)\, r_i > 0, \quad i = 1, \ldots, n. \end{aligned} \tag{14}$$

From (14), it follows that

$$\begin{aligned} &1 < r_i\beta_{i-1}p_{i-1} < r_{i+1}\beta_i p_i, \\ &1 < \beta_1 p_1 < \cdots < \beta_n p_n, \\ &\qquad \beta_i > 1, \ i = 1, \ldots, n. \end{aligned} \tag{15}$$

Besides, from (15), it leads to $r_i p_{i-1} + r_{i-1}\beta_{i-2}p_{i-2} = r_{i+1}p_i + r_i\beta_{i-1}p_{i-1}$; then

$$r_i p_{i-1} + r_{i-1}\beta_{i-2}p_{i-2} = r_2 p_1 + r_1\beta_0 p_0 = 2-\nu, \quad i = 1,\dots,n. \tag{16}$$

Secondly, we introduce the following coordinate transformation:

$$\begin{aligned} w_k(t) &= [x_k(t)]^{\beta_{k-1}p_{k-1}} - [v_{k-1}(\overline{x}_{k-1}(t), \widehat{\sigma}(t))]^{\beta_{k-1}p_{k-1}}, \quad k = 1,\dots,n, \\ u(t) &= v_n(x(t), \widehat{\sigma}(t)), \\ v_k^{p_k}(\overline{x}_k(t), \widehat{\sigma}(t)) &= -[w_k(t)]^{(r_k-\nu)/r_k\beta_{k-1}p_{k-1}}\Phi_k(\overline{x}_k(t), \widehat{\sigma}(t)), \quad k = 1,\dots,n, \end{aligned} \tag{17}$$

where Φ_k, $k = 1,\dots,n$, are $\mathscr{C}^1$ positive functions to be specified later. For the sake of consistency, we let $\Phi_0 = 0$ and $v_0 = 0$.

Finally, to obtain the detailed expression of u, we determine $\Phi_1,\dots,\Phi_n$ by induction.

Initial Step. Consider

$$\dot{x}_1 = x_2^{p_1} + \phi_1(x_1, z, d). \tag{18}$$

Take $V_1(x_1) = x_1^2/2$; then V_1 is $\mathscr{C}^1$ positive-definite and $\dot{V}_1(x_1) = x_1(x_2^{p_1} - v_1^{p_1}) + x_1 v_1^{p_1} + x_1\phi_1$. By (15), (17), Lemmas 4 and 6, Assumption 2, and $\sigma \geq \theta$,

$$\begin{aligned} x_1\phi_1 &\leq |x_1|\kappa_1(\|z\|) + x_1\cdot\gamma_1|x_2|^{p_1} \\ &\quad + |x_1|\kappa_{12}(x_1)|x_1|^{1-\nu+\mu_{11}}\theta \\ &\leq |w_1|^{2-\nu} + \kappa_1^{(2-\nu)/(1-\nu)} + \theta\overline{\kappa}_{12}(x_1)|x_1|^{2-\nu} \\ &\quad + \gamma_1|w_1||w_2|^{1/\beta_1} + \gamma_1|w_1|^{2-\nu}\Phi_1 \\ &\leq |w_1|^{2-\nu}(1+\sigma b_1(x_1)) + \kappa_1^{(2-\nu)/(1-\nu)} \\ &\quad + \gamma_1|w_1||w_2|^{1/\beta_1} + \gamma_1|w_1|^{2-\nu}\Phi_1, \end{aligned} \tag{19}$$

where $b_1(x_1) = \overline{\kappa}_{12}(x_1) = |x_1|^{\mu_{11}}\kappa_{12}(x_1)$ is a $\mathscr{C}$ nonnegative function and $b_1(0) = 0$. By Lemma 8, we choose a $\mathscr{C}^1$ positive function $\Phi_1(x_1, \widehat{\sigma})$ dominating the following function:

$$\Phi_1^0 = \frac{1}{1-\gamma_1}\left(1 + L_1 + \frac{\gamma_0(|x_1|)}{|x_1|^{2-\nu}}(\mu(x_1)+1) + \widehat{\sigma}b_1(x_1)\right), \quad L_1 > 0, \tag{20}$$

where $\mu(x_1)$ is a $\mathscr{C}^1$ function to be determined. Because of $\nu > 0$ and Assumption 3, Φ_1^0 is a continuous function. After some manipulations, we have

$$\begin{aligned} \dot{V}_1(x_1) &\leq -\gamma_0(|x_1|)(\mu(x_1)+1) - L_1|w_1|^{2-\nu} \\ &\quad + \kappa_1^{(2-\nu)/(1-\nu)} + |w_1||x_2^{p_1} - v_1^{p_1}| \\ &\quad + (\widetilde{\sigma} + \eta_1)\varphi_1 - \eta_1\dot{\widehat{\sigma}} + \gamma_1|w_1||w_2|^{1/\beta_1}, \end{aligned} \tag{21}$$

where $\eta_1 = 0$, $\varphi_1(x_1) = |x_1|^{2-\nu}b_1(x_1) \geq 0$, and $\dot{\widehat{\sigma}}$ will be determined later.

Inductive Step. We give this step by the following proposition.

Proposition 6. *Suppose at Step $i-1$, for system*

$$x_j = x_{j+1}^{p_j} + \phi_j(x_1,\dots,x_j,z,d), \quad j = 1,\dots,i-1, \tag{22}$$

there is a $\mathscr{C}^1$ function $V_{i-1}(\overline{x}_{i-1}, \widehat{\sigma})$ which is positive-definite with respect to $x_1,\dots,x_{i-1}$ and $\mathscr{C}^1$ positive function Φ_{i-1} such that

$$\begin{aligned} \dot{V}_{i-1}(\overline{x}_{i-1}, \widehat{\sigma}) &\leq -\gamma_0(|x_1|)(\mu(x_1)+1) - L_{i-1}Q_{i-1}^{2-\nu} \\ &\quad + |w_{i-1}||x_i^{p_{i-1}} - v_{i-1}^{p_{i-1}}| \\ &\quad + (\widetilde{\sigma} + \eta_{i-1})\varphi_{i-1} - \eta_{i-1}\dot{\widehat{\sigma}} \\ &\quad + \sum_{j=1}^{i-1}\kappa_1^{(2-\nu)/(2-\nu-r_j\beta_{j-1}p_{j-1})} \\ &\quad + \sum_{j=2}^{i-1}\kappa_1^{(2-\nu)/(2-\nu-r_j)} \\ &\quad + \gamma_{i-1}|w_{i-1}||w_i|^{1/\beta_{i-1}}, \quad L_{i-1} > 0, \end{aligned} \tag{23}$$

where

$$Q_{i-1}(\overline{x}_{i-1}, \widehat{\sigma}) = \left(|w_1|^{2-\nu} + |w_2|^{(2-\nu)/r_2\beta_1 p_1} + \cdots + |w_{i-1}|^{(2-\nu)/r_{i-1}\beta_{i-2}p_{i-2}}\right)^{1/(2-\nu)}, \tag{24}$$

$$\eta_{i-1}(\overline{x}_{i-1}, \widehat{\sigma}) = -\sum_{l=1}^{i-1}\frac{\partial W_l}{\partial\widehat{\sigma}}, \tag{25}$$

$\varphi_{i-1}(\overline{x}_{i-1}, \widehat{\sigma}) \geq 0$ is $\mathscr{C}^1$. Then, for system

$$x_j = x_{j+1}^{p_j} + \phi_j(x_1,\dots,x_j,z,d), \quad j = 1,\dots,i, \tag{26}$$

the ith function $V_i(\overline{x}_i,\widehat{\sigma}) = V_{i-1}(\overline{x}_{i-1},\widehat{\sigma}) + W_i(\overline{x}_i,\widehat{\sigma})$ is $\mathscr{C}^1$ and positive-definite with respect to $x_1,\ldots,x_i$, and one can find a $\mathscr{C}^1$ positive function Φ_i (see (A.14) in the Appendix) such that

$$\begin{aligned}\dot{V}_i(\overline{x}_i,\widehat{\sigma}) \le{}& -\gamma_0(|x_1|)(\mu(x_1)+1) - L_iQ_i^{2-\nu} \\ &+ |w_i|\left|x_{i+1}^{p_i} - v_i^{p_i}\right| + (\widetilde{\sigma}+\eta_i)\varphi_i - \eta_i\dot{\widehat{\sigma}} \\ &+ \sum_{j=1}^{i}\kappa_1^{(2-\nu)/(2-\nu-r_j\beta_{j-1}p_{j-1})} \\ &+ \sum_{j=2}^{i}\kappa_1^{(2-\nu)/(2-\nu-r_j)} + \gamma_i|w_i|\left|w_{i+1}\right|^{1/\beta_i},\end{aligned} \tag{27}$$

where $L_i = ((n-1)/n)L_{i-1} > 0$, $\varphi_i(\overline{x}_i,\widehat{\sigma}) = \varphi_{i-1}(\overline{x}_{i-1},\widehat{\sigma}) + |w_i|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}b_i(\overline{x}_i,\widehat{\sigma})$ (see (A.12) in the Appendix) is continuous, and

$$W_i(\overline{x}_i,\widehat{\sigma}) = \int_{v_{i-1}}^{x_i}\left([s]^{\beta_{i-1}p_{i-1}} - [v_{i-1}]^{\beta_{i-1}p_{i-1}}\right)\mathrm{d}s, \quad 1\le i\le n. \tag{28}$$

Proof. See the Appendix. □

At Step n, (27) holds with $w_{n+1} = 0$. Hence, by choosing $V_n(x,\widehat{\sigma}) = V_{n-1}(\overline{x}_{n-1},\widehat{\sigma}) + W_n(x,\widehat{\sigma})$, we can get

$$u^{p_n} = v_n^{p_n} = -[w_n]^{(r_n-\nu)/r_n\beta_{n-1}p_{n-1}}\Phi_n(x,\widehat{\sigma}) \tag{29}$$

with an appropriate function Φ_n and

$$\dot{\widehat{\sigma}} = -\dot{\widetilde{\sigma}} = \varphi_n, \tag{30}$$

such that

$$\begin{aligned}\dot{V}_n(x,\widehat{\sigma}) \le{}& -\gamma_0(|x_1|)(\mu(x_1)+1) - L_nQ_n^{2-\nu} + \widetilde{\sigma}\varphi_n \\ &+ \sum_{j=1}^{n}\kappa_1^{(2-\nu)/(2-\nu-r_j\beta_{j-1}p_{j-1})} \\ &+ \sum_{j=2}^{n}\kappa_1^{(2-\nu)/(2-\nu-r_j)}, \quad L_n > 0.\end{aligned} \tag{31}$$

4. Finite-Time Convergence Analysis

We state the main result in this paper.

Theorem 7. *The solutions of the closed-loop system (2), (29), and (30) are bounded, and the trajectory $(z(t),x(t))$ is finite-time convergent to the origin $0\in R^{n+n_0}$.*

Proof. Define

$$\begin{aligned}\overline{\kappa}_1(s) ={}& \sum_{i=1}^{n}\kappa_1(s)^{(2-\nu)/(2-\nu-r_i\beta_{i-1}p_{i-1})} \\ &+ \sum_{i=2}^{n}\kappa_1(s)^{(2-\nu)/(2-\nu-r_i)}.\end{aligned} \tag{32}$$

From Assumption 3, $\kappa_1(s)$ is $\mathscr{C}^1$, $(2-\nu)/(2-\nu-\beta_0) = 1+1/(1-\nu) > 2$, and, for $i = 2,\ldots,n$, $(2-\nu)/(2-\nu-r_i\beta_{i-1}p_{i-1}) > 2$, $(2-\nu)/(2-\nu-r_i) > 2\alpha_0$, it follows that

$$\limsup_{s\to 0+}\frac{\overline{\kappa}_1(s)}{\pi_1^{\alpha_0}(s)} < +\infty. \tag{33}$$

The proof is divided into two parts.

Step 1. We show that the trajectories $(z(t),x(t),\widehat{\sigma}(t))$ of the closed-loop system (2), (29), and (30) are bounded.

Take a Lyapunov function as

$$\begin{aligned}V_c(x,z,\widehat{\sigma}) &= V_n(x,\widehat{\sigma}) + V_*(z) + \frac{1}{2}\widetilde{\sigma}^2, \\ V_*(z) &= \int_0^{V_0(z)}\rho(s)\,\mathrm{d}s,\end{aligned} \tag{34}$$

where $\rho : R^+ \to R^+$ is a continuous nondecreasing function with $\rho(0) > 0$. It follows from (7) that when $\|z\| > (\gamma_0(\|x_1\|)/c_0\epsilon)^{1/\alpha_0}$, $-c_0\epsilon\rho(V_0(z))V_0^{\alpha_0}(z) + \rho(V_0(z))\gamma_0(|x_1|) < 0$; when $\|z\| \le (\gamma_0(\|x_1\|)/c_0\epsilon)^{1/\alpha_0}$, $\rho(V_0(z))\gamma_0(|x_1|) \le \rho(\pi_2(\pi_1^{-1}((\gamma_0(|x_1|)/c_0\epsilon)^{1/\alpha_0})))\gamma_0(|x_1|)$. Then we observe from (30), (31), and (34) that

$$\begin{aligned}&\dot{V}_c \\ &\le -\gamma_0(|x_1|)(\mu(x_1)+1) - L_nQ_n^{2-\nu} + \overline{\kappa}_1(\|z\|) \\ &\quad - c_0\rho(V_0(z))V_0^{\alpha_0}(z) + \rho(V_0(z))\gamma_0(|x_1|) \\ &\le -\gamma_0(|x_1|)(\mu(x_1)+1) - L_nQ_n^{2-\nu} + \overline{\kappa}_1(\|z\|) \\ &\quad - c_0(1-\epsilon)\rho(V_0(z))V_0^{\alpha_0} \\ &\quad + \rho\left(\pi_2\left(\pi_1^{-1}\left(\left(\frac{\gamma_0(|x_1|)}{c_0\epsilon}\right)^{1/\alpha_0}\right)\right)\right)\gamma_0(|x_1|), \\ &\qquad 0<\epsilon<1.\end{aligned} \tag{35}$$

According to (33) and Lemma 8, we can find a desired function ρ and a $\mathscr{C}^1$ function μ such that

$$\begin{aligned}&c_0(1-\epsilon)\rho(\pi_1(\|z\|))\pi_1^{\alpha_0}(\|z\|) \ge 2\overline{\kappa}_1(\|z\|), \\ &\mu(x_1)+1 \ge \rho\left(\pi_2\left(\pi_1^{-1}\left(\left(\frac{\gamma_0(|x_1|)}{c_0\epsilon}\right)^{1/\alpha_0}\right)\right)\right).\end{aligned} \tag{36}$$

Substituting (36) into (35) leads to

$$\dot{V}_c \le -L_nQ_n^{2-\nu} - \frac{c_0(1-\epsilon)}{2}\rho(V_0(z))V_0^{\alpha_0}(z) \le 0, \tag{37}$$

which implies the boundedness of $z(t)$, $x(t)$, and $\widehat{\sigma}(t)$.

Step 2. We show that the trajectories $(z(t),x(t))$ are finite-time convergent.

Consider $\overline{V}_c(x,z,\widehat{\sigma}) = V_n(x,\widehat{\sigma}) + V_*(z)$, which is positive-definite with respect to (z,x). Then

$$\dot{\overline{V}}_c \le -L_nQ_n^{2-\nu} - \frac{c_0(1-\epsilon)}{2}\rho(V_0(z))V_0^{\alpha_0}(z) + \widetilde{\sigma}\varphi_n. \tag{38}$$

At first, we consider local finite-time convergence in a small neighborhood around $(z,x)=(0,0)$. It is easy to see that, locally around $z=0$, $\lim_{\|z\|\to 0}(V_0^{\alpha_0}(z)/\rho(V_0(z))V_0^{\alpha_0}(z))=1/\rho(0)<\infty$, $\lim_{\|z\|\to 0}(V_*(z)/V_0(z))=\rho(0)<\infty$, and then $\lim_{\|z\|\to 0}(V_*^{\alpha_0}(z)/\rho(V_0(z))V_0^{\alpha_0}(z))=1$, which implies that when $\|z\|$ is small enough, there is a constant c_1 such that $V_*(z)^{\alpha_0}\le (c_0(1-\epsilon)/c_1)\rho(V_0(z))V_0^{\alpha_0}(z)$. Thus, locally around $(z,x)=(0,0)$, by (38), one has

$$\dot{\overline{V}}_c=-L_nQ_n^{2-\nu}-\frac{c_1}{2}V_*^{\alpha_0}+\tilde{\sigma}\varphi_n. \tag{39}$$

Note that

$$\begin{aligned}\varphi_n(x,\hat{\sigma})&=\sum_{j=1}^{n}\left|w_j\right|^{(2-\nu)/r_j\beta_{j-1}p_{j-1}}b_j\left(\overline{x}_j,\hat{\sigma}\right)\\&\le Q_n^{2-\nu}\varphi_0(x,\hat{\sigma}),\end{aligned} \tag{40}$$

where $\varphi_0=\sum_{j=1}^{n}b_j$ is continuous with $\varphi_0(0,\hat{\sigma})=0$ because $b_j(0,\hat{\sigma})=0$. Substituting (40) into (39), it is easy to obtain

$$\begin{aligned}\dot{\overline{V}}_c=&-\frac{L_n}{2}Q_n^{2-\nu}-\frac{c_1}{2}V_*^{\alpha_0}\\&-L_nQ_n^{2-\nu}\left(\frac{1}{2}-\frac{1}{L_n}(\sigma+\bar{c})\varphi_0\right),\end{aligned} \tag{41}$$

where $\bar{c}$ is a positive constant. By (15) and Lemma 9, then $V_n=\sum_{j=1}^{n}\int_{v_{j-1}}^{x_j}([s]^{\beta_{j-1}p_{j-1}}-[v_{j-1}]^{\beta_{j-1}p_{j-1}})ds\le\sum_{j=1}^{n}2|w_j|^{2/r_j\beta_{j-1}p_{j-1}}$, which together with (24) and Lemma 5 mean that $V_n^{(2-\nu)/2}\le 2Q_n^{2-\nu}$. Hence, (41) becomes

$$\dot{\overline{V}}_c\le-\frac{L_n}{4}V_n^{(2-\nu)/2}-\frac{c_1}{2}V_*^{\alpha_0}-L_nQ_n^{2-\nu}\left(\frac{1}{2}-\widetilde{V}\right), \tag{42}$$

where $\widetilde{V}(x,\hat{\sigma})=(1/L_n)(\sigma+\bar{c})\varphi_0(x,\hat{\sigma})$ is continuous with $\widetilde{V}(0,\hat{\sigma})=0$. Because $\widetilde{V}$ is continuous and $\widetilde{V}(0,\hat{\sigma})=0$, it is obvious that there is a constant $0<\varrho\le 1$ such that $\widetilde{V}\le 1/2$ if $\overline{V}_c\le\varrho$. By Lemma 5 and $\overline{V}_c\le\varrho\le 1$, choosing $c_*=\min\{L_n/4,c_1/2\}$, $\alpha=\max\{(2-\nu)/2,\alpha_0\}$, we have $(L_n/4)V_n^{(2-\nu)/2}+(c_1/2)V_*^{\alpha_0}\ge c_*(V_n^{\alpha}+V_*^{\alpha})\ge c_*(V_n+V_*)^{\alpha}$. Thus, in a neighborhood $\Omega=\{(z,x,\hat{\sigma}):\overline{V}_c\le\varrho\}$, (42) becomes

$$\dot{\overline{V}}_c\le-c_*\overline{V}_c^{\alpha}, \tag{43}$$

which implies the local finite-time convergence in Ω by Lemma 3.

We consider the global finite-time convergence. The finite-time convergence in Ω has been achieved. We now study the situation outside Ω. When the initial condition is outside Ω, one has $\overline{V}_c\ge\varrho$. It is easy to see that there is a constant c_2 such that $(L_n/2)V_n^{(2-\nu)/2}+(c_1/2)V_*^{\alpha_0}\ge c_2\varrho^{\alpha}$. Let T_1 be the first time that $(z(t),x(t),\hat{\sigma}(t))$ intersects the boundary of Ω. Using (37) and the above-mentioned argument, for any $\tau\in[0,T_1)$, one has

$$\begin{aligned}&V_c(z(0),x(0),\hat{\sigma}(0))\\&\quad\ge V_c(z(0),x(0),\hat{\sigma}(0))-V_c(z(\tau),x(\tau),\hat{\sigma}(\tau))\\&\quad=\int_0^{\tau}-\dot{V}_c(z(s),x(s),\hat{\sigma}(s))\,ds\\&\quad=\int_0^{\tau}\left(-\dot{\overline{V}}_c+\tilde{\sigma}\varphi_n\right)ds\\&\quad\ge\int_0^{\tau}\left(\frac{L_n}{2}V_n^{(2-\nu)/2}+\frac{c_1}{2}V_*^{\alpha_0}\right)ds\ge c_2\varrho^{\alpha}\tau.\end{aligned} \tag{44}$$

If $(z(t),x(t),\hat{\sigma}(t))$ do not enter Ω in finite time, then $T_1=\infty$. When $\tau\to T_1$, $c_2\varrho^{\alpha}\tau\to\infty$, while $V_c(z(0),x(0),\hat{\sigma}(0))$ is a finite constant, then (44) leads to a contradiction. So $(z(t),x(t),\hat{\sigma}(t))$ will reach Ω in finite time. □

Remark 8. We estimate the settling time. In practice, although we do not know the real value of σ (or θ), we always have its range. Without loss of generality, we assume that $0<\sigma<\Sigma$, where $\Sigma>0$ is a constant. We consider two cases.

Case 1 ($(z(0),x(0),\hat{\sigma}(0))\in\Omega$). By (43), we have

$$T\le\frac{\overline{V}_c(z(0),x(0),\hat{\sigma}(0))^{1-\max\{(2-\nu)/2,\alpha_0\}}}{c_*\left(1-\max\left\{(2-\nu)/2,\alpha_0\right\}\right)}. \tag{45}$$

Case 2 ($(z(0),x(0),\hat{\sigma}(0))\notin\Omega$). Before the state reaches Ω, from (34) and (37), it follows that $\tilde{\sigma}^2\le 2V_c(z(0),x(0),\hat{\sigma}(0))\le 2\overline{V}_c(z(0),x(0),\hat{\sigma}(0))+(|\hat{\sigma}(0)|+\Sigma)^2$; then $V_c(z(0),x(0),\hat{\sigma}(0))\le\overline{V}_c(z(0),x(0),\hat{\sigma}(0))+(1/2)\tilde{\sigma}(0)^2\le 2\overline{V}_c(z(0),x(0),\hat{\sigma}(0))+(1/2)(|\hat{\sigma}(0)|+\Sigma)^2$. Therefore, by (44), $(x,\hat{\sigma})$ will reach Ω within T_1:

$$T_1\le\frac{2\overline{V}_c(z(0),x(0),\hat{\sigma}(0))+(|\hat{\sigma}(0)|+\Sigma)^2}{c_2\varrho^{\max\{(2-\nu)/2,\alpha_0\}}}. \tag{46}$$

After T_1, it will stay in Ω, and it will take

$$T_2\le\frac{\varrho^{1-\max\{(2-\nu)/2,\alpha_0\}}}{c_*\left(1-\max\left\{(2-\nu)/2,\alpha_0\right\}\right)} \tag{47}$$

to reach the origin. Therefore, in this case, the settling time can be estimated as T_1+T_2.

Remark 9. Compared with [14], due to the appearance of dynamic and parametric uncertainties, and the weaker condition on nonlinear functions, the main difficulty in this paper is how to skillfully combine Lyapunov function, sign function, backstepping, and FTISS approaches to give the design and rigorous analysis of finite-time controller.

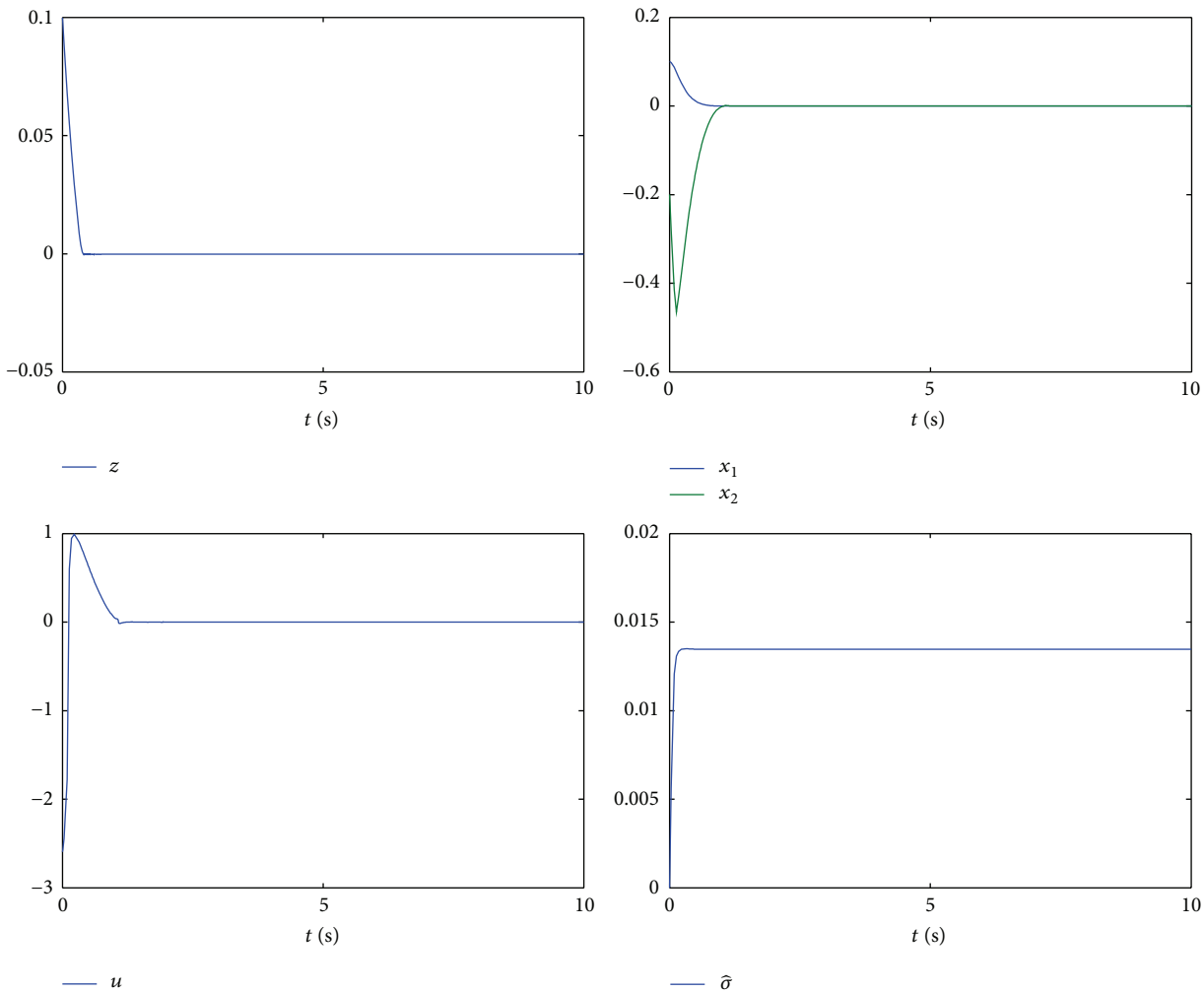

Figure 1: Trajectories of z, x_1, x_2, u, $\widehat{\sigma}$.

5. A Simulation Example

Consider a simple system

$$\begin{aligned} \dot{z} &= -z^{1/3} + x_1, \\ \dot{x}_1 &= x_2^{9/7} + \theta \sin x_1 \cos z, \\ \dot{x}_2 &= u - z^2 \cos x_2, \end{aligned} \tag{48}$$

where θ is an unknown parameter and z is the unmeasurable dynamic uncertainty.

Choose $\nu = 2/11$; then $r_1 = 1$, $r_2 = 7/11$. By (48), we have $\gamma_1 = \gamma_2 = 0$, $|\phi_1| = |\theta \sin x_1 \cos z| \le \theta|\sin x_1| \le \theta|x_1|^{9/11}$, $|\phi_2| = |z^2 \cos x_2| \le \|z\|^2$, $\kappa_1(\|z\|) = \|z\|^2$, $\kappa_{12}(x_1) = 1$, $\kappa_{22}(x_1, x_2) = 0$, and $\mu_{11} = \mu_{21} = \mu_{22} = 0$. Choose $V_0(z) = z^2$; then $\dot{V}_0 \le -(1/2)z^{4/3} + (1/2)x_1^4 \le -(1/2)V_0^{2/3} + (1/2)x_1^4$; z-subsystem is obviously finite-time input-to-state stable with $\gamma_0(x_1) = (1/2)x_1^4$. Hence, Assumptions 1–3 hold.

Take $\beta_0 = 1$; then $r_2 p_1 + r_1 \beta_0 p_0 = 2 - \nu = 20/11$. Following the design procedure in Section 3, one can obtain the adaptive finite-time controller

$$\begin{aligned} u &= -0.2\left(x_2^{15/7} + 2x_1^{15/11}\Phi_1^{5/3}\right)^{1/3} \\ &\quad \cdot \left(4 + 2\Phi_1 + 3.2\Phi_1^2 + 2\widehat{\sigma}x_1^{4/9}\right), \quad \Phi_1 = 4\widehat{\sigma}^2 x_1^4 + 2, \\ \dot{\widehat{\sigma}} &= x_1^{20/11} + 2x_1^{2/9}\left(x_2^{15/7} + 2x_1^{15/11}\Phi_1^{5/3}\right)^{4/3}. \end{aligned} \tag{49}$$

Choosing the initial conditions $z(0) = 0.1$, $x_1(0) = 0.1$, $x_2(0) = -0.2$, and $\widehat{\sigma}(0) = 0$, we estimate the settling time. Since $\Omega = \{(z, x, \widehat{\sigma}) : \overline{V}_c \le 0.2\}$, $\rho(s) = 8(s^{14/9} + s^3 + s^{81/78})$, one has $\overline{V}_c(z(0), x(0), \widehat{\sigma}(0)) = 0.0338$, which is in Ω. According to Case 1 in Remark 8, by $\max\{(2-\nu)/2, \alpha_0\} = 10/11$, $c_* = \min\{0.25, 0.1813\} = 0.1813$, one has $T \le 44.5923$.

With $\theta = 1$, Figure 1 shows the trajectories of z, x_1, x_2, u, $\widehat{\sigma}$, and one can see that z, x_1, x_2 reach the origin within T.

6. Conclusions

By characterizing the dynamic uncertainty with FTISS, under the weaker assumption on nonlinear functions, the problem of adaptive finite-time stabilization for more general high-order nonlinear systems with dynamic and parametric uncertainties is solved.

Some interesting problems are still remaining: (1) For system (2) with possibly nonvanishing disturbances and more general dynamic uncertainty, can a finite-time convergent controller be given? (2) How can we construct output feedback to stabilize system (2) in finite time? (3) In recent years, some results on stochastic nonlinear systems with SISS/SiISS dynamic uncertainty have been obtained, for example, [25–36] and the references therein, but these papers only consider the global asymptotic stabilization. An important problem is whether finite-time stabilization can be obtained for stochastic nonlinear systems with dynamic and parametric uncertainties.

Appendix

Proof of Proposition 6. We first prove that V_i is $\mathscr{C}^1$. From (15) and $\Phi_{i-1}(\overline{x}_{i-1},\widehat{\sigma})$ being positive and $\mathscr{C}^1$ and $[v_{i-1}]^{\beta_{i-1}p_{i-1}}$ being $\mathscr{C}^1$, then W_i is $\mathscr{C}^1$, so is V_i.

Secondly, we prove that V_i is positive-definite with respect to $x_1,\dots,x_i$. When $x_i > v_{i-1}$, by (15), (28), and Lemmas 7 and 9, one has

$$\begin{aligned} W_i &\geq 2^{1-\beta_{i-1}p_{i-1}}\int_{v_{i-1}}^{x_i}(s-v_{i-1})^{\beta_{i-1}p_{i-1}}\,ds \\ &\geq \frac{2^{1-\beta_{i-1}p_{i-1}}}{\beta_{i-1}p_{i-1}+1}\left|x_i-v_{i-1}\right|^{\beta_{i-1}p_{i-1}+1}\geq 0. \end{aligned} \tag{A.1}$$

When $x_i \leq v_{i-1}$, it can be shown that (A.1) also holds in a similar way.

From (A.1), the definition of V_i, and the positive definiteness of V_{i-1} with respect to $x_1,\dots,x_{i-1}$, it is easy to see that $V_i \geq 0$ and, for fixed $\widehat{\sigma}$, $V_i(\overline{x}_i,\widehat{\sigma}) = 0$ if and only if $\overline{x}_i = 0$, which implies that V_i is positive-definite with respect to $x_1,\dots,x_i$.

Finally, we prove (27). From the definition of V_i and (23), it follows that

$$\begin{aligned} &\dot{V}_i(\overline{x}_i,\widehat{\sigma}) \\ &\quad\leq -\gamma_0(|x_1|)(\mu(x_1)+1)-L_{i-1}Q_{i-1}^{2-\nu} \\ &\qquad+\sum_{j=1}^{i-1}\kappa_1^{(2-\nu)/(2-\nu-r_j\beta_{j-1}p_{j-1})}+\sum_{j=2}^{i-1}\kappa_1^{(2-\nu)/(2-\nu-r_j)} \\ &\qquad+\left(|w_{i-1}|\left|x_i^{p_{i-1}}-v_{i-1}^{p_{i-1}}\right|+\gamma_{i-1}|w_{i-1}||w_i|^{1/\beta_{i-1}}\right) \\ &\qquad+\sum_{j=1}^{i}\frac{\partial W_i}{\partial x_j}\left(x_{i+1}^{p_i}+\phi_i\right) \\ &\qquad+\left((\widetilde{\sigma}+\eta_{i-1})\varphi_{i-1}-\eta_{i-1}\dot{\widehat{\sigma}}+\frac{\partial W_i}{\partial\widehat{\sigma}}\dot{\widehat{\sigma}}\right). \end{aligned} \tag{A.2}$$

We estimate the last three terms on the right-hand side of (A.2).

(i) By (15), (16), (17), (24), and Lemmas 4 and 9, one has

$$\begin{aligned} &|w_{i-1}|\left|x_i^{p_{i-1}}-v_{i-1}^{p_{i-1}}\right|+\gamma_{i-1}|w_{i-1}||w_i|^{1/\beta_{i-1}} \\ &\quad\leq(2+\gamma_{i-1})|w_{i-1}||w_i|^{1/\beta_{i-1}} \\ &\quad\leq\frac{L_{i-1}}{4n}Q_{i-1}^{2-\nu}+l_i|w_i|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}, \end{aligned} \tag{A.3}$$

where $l_i > 0$ is a constant dependent on ν, L_{i-1}, γ_{i-1}, $p_1,\dots,p_{i-1}$.

(ii) For $j \leq i-1$, from (15), (17), (24), and Lemmas 4, 8, and 10, the following holds:

$$\begin{aligned} &\left|\frac{\partial[v_{i-1}]^{\beta_{i-1}p_{i-1}}}{\partial x_j}\right|\leq|w_{i-1}|^{r_ip_{i-1}\beta_{i-1}/r_{i-1}\beta_{i-2}p_{i-2}}g(\overline{x}_{i-1},\widehat{\sigma}) \\ &\quad+\sum_{k=j+1}^{i-2}\left(\prod_{l=k+1}^{i-1}|w_l|^{(r_{l+1}\beta_lp_l-r_l\beta_{l-1}p_{l-1})/r_l\beta_{l-1}p_{l-1}}\right) \\ &\quad\cdot|w_k|^{r_{k+1}\beta_kp_k/r_k\beta_{k-1}p_{k-1}}\cdot g(\overline{x}_{i-1},\widehat{\sigma}) \\ &\quad+\prod_{l=j+1}^{i-1}|w_l|^{(r_{l+1}\beta_lp_l-r_l\beta_{l-1}p_{l-1})/r_l\beta_{l-1}p_{l-1}}\left|\frac{\partial[v_j]^{\beta_jp_j}}{\partial x_j}\right| \\ &\quad\cdot g(\overline{x}_{i-1},\widehat{\sigma})\leq|w_{i-1}|^{r_i\beta_{i-1}p_{i-1}/r_{i-1}\beta_{i-2}p_{i-2}}g(\overline{x}_{i-1},\widehat{\sigma}) \\ &\quad\cdot\sum_{k=j}^{i-2}\left(\prod_{l=k+1}^{i-1}|w_l|^{(r_{l+1}p_l\beta_l-r_l\beta_{l-1}p_{l-1})/r_l\beta_{l-1}p_{l-1}}\right) \\ &\quad\cdot|w_k|^{r_{k+1}\beta_kp_k/r_k\beta_{k-1}p_{k-1}}g(\overline{x}_{i-1},\widehat{\sigma}) \\ &\quad+\prod_{l=j}^{i-1}|w_l|^{(r_{l+1}\beta_lp_l-r_l\beta_{l-1}p_{l-1})/r_l\beta_{l-1}p_{l-1}} \\ &\quad\cdot\left(|w_j|^{(r_j\beta_{j-1}p_{j-1}-r_j)/r_j\beta_{j-1}p_{j-1}}\right. \\ &\quad\left.+|w_{j-1}|^{(r_j\beta_{j-1}p_{j-1}-r_j)/r_{j-1}\beta_{j-2}p_{j-2}}\right)g(\overline{x}_{i-1},\widehat{\sigma}) \\ &\quad\leq\left(\sum_{l=1}^{i-1}|w_l|^{(r_i\beta_{i-1}p_{i-1}-r_j)/r_l\beta_{l-1}p_{l-1}}\right)g(\overline{x}_{i-1},\widehat{\sigma}) \\ &\quad\leq Q_{i-1}^{r_i\beta_{i-1}p_{i-1}-r_j}h_{j,i-1}(\overline{x}_{i-1},\widehat{\sigma}), \end{aligned} \tag{A.4}$$

where $g(\overline{x}_{i-1},\widehat{\sigma})$ is a general $\mathscr{C}^1$ nonnegative function that may be implicitly changed in various places and $h_{j,i-1}(\overline{x}_{i-1},\widehat{\sigma})$ is a $\mathscr{C}^1$ nonnegative function. From (15), (17), (28), (A.4), and Lemma 9, it follows that

$$\begin{aligned} \frac{\partial W_i}{\partial x_j} &\leq|x_i-v_{i-1}|\left|\frac{\partial[v_{i-1}]^{\beta_{i-1}p_{i-1}}}{\partial x_j}\right| \\ &\leq 2|w_i|^{1/\beta_{i-1}p_{i-1}}Q_{i-1}^{r_i\beta_{i-1}p_{i-1}-r_j}h_{j,i-1},\quad j\leq i-1. \end{aligned} \tag{A.5}$$

For $i = 1, \ldots, n$, by Assumption 2, one has

$$\begin{aligned} &\left|\phi_i\left(x_1, \ldots, x_i, z, d\right)\right| \\ &\quad \leq \kappa_1\left(\|z\|\right) + \gamma_i\left|x_{i+1}\right|^{p_i} \\ &\qquad + \theta\overline{\kappa}_{i2}\left(x_1, \ldots, x_i\right)\sum_{l=1}^{i}\left|x_l\right|^{(r_i-\nu)/r_l}, \end{aligned} \tag{A.6}$$

where $\overline{\kappa}_{i2} = \sum_{l=1}^{i}|x_l|^{\mu_{ij}}\kappa_{i2}$ is a $\mathscr{C}$ nonnegative function vanishing at the origin. For $j < i$, by (15), (17), (24), (A.6), and Lemmas 6 and 9,

$$\begin{aligned} x_{j+1}^{p_j} + \phi_j &\leq \left|x_{j+1}^{p_j} - v_j^{p_j}\right| + \left|v_j^{p_j}\right| + \left|\phi_j\right| \leq 2\left|w_{j+1}\right|^{1/\beta_j} \\ &\quad + \left|w_j\right|^{(r_j-\nu)/r_j\beta_{j-1}p_{j-1}}\Phi_j + \left|\phi_j\right| \leq \left(2 + \Phi_j\right)Q_{j+1}^{r_j-\nu} \\ &\quad + \kappa_1 + \gamma_j\left|x_{j+1}\right|^{p_j} + \theta\overline{\kappa}_{j2}\left(x_1, \ldots, x_j\right)\sum_{l=1}^{j}\left|x_l\right|^{(r_j-\nu)/r_l} \\ &\leq \left(2 + \Phi_j\right)Q_{j+1}^{r_j-\nu} + \kappa_1 + \gamma_j\left(\left|w_{j+1}\right|^{1/\beta_j}\right. \\ &\quad \left. + \left|w_j\right|^{(r_j-\nu)/r_j\beta_{j-1}p_{j-1}}\Phi_j\right) + \theta\overline{\kappa}_{j2}\left(x_1, \ldots, x_j\right) \\ &\quad \cdot \sum_{l=1}^{j}\left(\left|w_l\right|^{(r_j-\nu)/r_l\beta_{l-1}p_{l-1}}\right. \\ &\quad \left. + \left|w_{l-1}\right|^{(r_j-\nu)/r_{l-1}\beta_{l-2}p_{l-2}}\Phi_{l-1}^{(r_j-\nu)/r_lp_{l-1}}\right). \end{aligned} \tag{A.7}$$

From (16), (A.5), (A.7), Lemma 4, and $(2-\nu)/r_i \leq (2-\nu)/r_n$,

$$\begin{aligned} &\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial x_j}\left(x_{j+1}^{p_j} + \phi_j\right) \leq 2\left|w_i\right|^{1/\beta_{i-1}p_{i-1}}\left(Q_i^{r_i\beta_{i-1}p_{i-1}-\nu}\sum_{j=1}^{i-1}h_{j,i-1}\left(2\right.\right. \\ &\quad \left. + \Phi_j\right) + \sum_{j=1}^{i-1}Q_i^{r_i\beta_{i-1}p_{i-1}-r_j}h_{j,i-1}\left(\kappa_1 + \gamma_j\left(\left|w_{j+1}\right|^{1/\beta_j}\right.\right. \\ &\quad \left. + \left|w_j\right|^{(r_j-\nu)/r_j\beta_{j-1}p_{j-1}}\Phi_j\right) + \theta\overline{\kappa}_{j2}\left(x_1, \ldots, x_j\right) \\ &\quad \left.\left. \cdot \sum_{l=1}^{j}\left(\left|w_l\right|^{(r_j-\nu)/r_l\beta_{l-1}p_{l-1}} + \left|w_{l-1}\right|^{(r_j-\nu)/r_{l-1}\beta_{l-2}p_{l-2}}\Phi_{l-1}^{(r_j-\nu)/r_lp_{l-1}}\right)\right)\right) \\ &\quad \leq \frac{L_{i-1}}{4n}Q_i^{2-\nu} + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\left(\psi_{i,1} + \sigma b_{i,1}\right) + \kappa_1^{(2-\nu)/(2-\nu-r_i)}, \end{aligned} \tag{A.8}$$

where $\psi_{i,1}(\overline{x}_{i-1}, \widehat{\sigma})$ and $b_{i,1}(\overline{x}_{i-1}, \widehat{\sigma})$ are $\mathscr{C}$ nonnegative functions with $b_{i,1}(0, \widehat{\sigma}) = 0$. It follows from (15), (16), (17), (24), (28), (A.6), and Lemmas 4 and 6 that

$$\begin{aligned} &\frac{\partial W_i}{\partial x_i}\left(x_{i+1}^{p_i} + \phi_i\right) \leq \left|w_i\right|\left|x_{i+1}^{p_i} - v_i^{p_i}\right| + w_iv_i^{p_i} + \left|w_i\right|\left(\kappa_1\right. \\ &\quad + \gamma_i\left(\left|w_{i+1}\right|^{1/\beta_i} + \left|w_i\right|^{(r_i-\nu)/r_i\beta_{i-1}p_{i-1}}\Phi_i\right) + \theta\overline{\kappa}_{i2}\left(x_1,\right. \\ &\quad \left.\ldots, x_i\right)\sum_{l=1}^{i}\left(\left|w_l\right|^{(r_i-\nu)/r_l\beta_{l-1}p_{l-1}}\right. \\ &\quad \left.\left. + \left|w_{l-1}\right|^{(r_j-\nu)/r_{l-1}\beta_{l-2}p_{l-2}}\Phi_{l-1}^{(r_j-\nu)/r_lp_{l-1}}\right)\right) \leq \frac{L_{i-1}}{4n}Q_i^{2-\nu} \\ &\quad + \left|w_i\right|\left|x_{i+1}^{p_i} - v_i^{p_i}\right| + w_iv_i^{p_i} + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\left(\psi_{i,2}\right. \\ &\quad \left. + \sigma b_{i,2}\right) + \kappa_1^{(2-\nu)/(2-\nu-r_i\beta_{i-1}p_{i-1})} + \gamma_i\left|w_i\right|\left|w_{i+1}\right|^{1/\beta_i} \\ &\quad + \gamma_i\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\Phi_i, \end{aligned} \tag{A.9}$$

where $\psi_{i,2}(\overline{x}_i, \widehat{\sigma})$ and $b_{i,2}(\overline{x}_i, \widehat{\sigma})$ are $\mathscr{C}$ nonnegative functions with $b_{i,2}(0, \widehat{\sigma}) = 0$. Combining (A.8) with (A.9) leads to

$$\begin{aligned} \sum_{j=1}^{i}\frac{\partial W_i}{\partial x_j}\left(x_{j+1}^{p_j} + \phi_j\right) &\leq \left|w_i\right|\left|x_{i+1}^{p_i} - v_i^{p_i}\right| + w_iv_i^{p_i} \\ &\quad + \frac{L_{i-1}}{2n}Q_i^{2-\nu} \\ &\quad + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\left(\psi_i + \sigma b_i\right) \\ &\quad + \kappa_1^{(2-\nu)/(2-\nu-r_i)} \\ &\quad + \kappa_1^{(2-\nu)/(2-\nu-r_i\beta_{i-1}p_{i-1})} \\ &\quad + \gamma_i\left|w_i\right|\left|w_{i+1}\right|^{1/\beta_i} \\ &\quad + \gamma_i\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\Phi_i, \end{aligned} \tag{A.10}$$

where $\psi_i = \psi_{i,1} + \psi_{i,2}$, $b_i = b_{i,1} + b_{i,2}$.

(iii) For the last term of (A.2), from (17), (28), and Lemma 8, there is a $\mathscr{C}^1$ nonnegative function $\delta_j(\overline{x}_j, \widehat{\sigma})$ such that $|\partial[v_j]^{\beta_jp_j}/\partial\widehat{\sigma}| \leq \delta_j(\overline{x}_j, \widehat{\sigma})$ $(j \leq i-1)$. By (24), (25), and Lemma 4,

$$\begin{aligned} &\left(\widetilde{\sigma} + \eta_{i-1}\right)\varphi_{i-1} - \eta_{i-1}\dot{\widehat{\sigma}} + \frac{\partial W_i}{\partial\widehat{\sigma}}\dot{\widehat{\sigma}} \\ &\quad = \left(\widetilde{\sigma} + \eta_i + \frac{\partial W_i}{\partial\widehat{\sigma}}\right)\left(\varphi_i - \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}b_i\right) \\ &\qquad - \eta_i\dot{\widehat{\sigma}} \end{aligned}$$

$$
\begin{aligned}
&= (\tilde{\sigma} + \eta_i)\varphi_i - (\tilde{\sigma} + \eta_i)\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}} b_i \\
&\quad + \frac{\partial W_i}{\partial \hat{\sigma}}\varphi_i - \frac{\partial W_i}{\partial \hat{\sigma}}\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}} b_i - \eta_i\dot{\hat{\sigma}} \\
&\le (\tilde{\sigma} + \eta_i)\varphi_i \\
&\quad - \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}} b_i\left(\tilde{\sigma} + \eta_i + \delta_{i-1}\left|w_i\right|^{1/\beta_{i-1}p_{i-1}}\right) \\
&\quad - \eta_i\dot{\hat{\sigma}} + \left|w_i\right|^{1/\beta_{i-1}p_{i-1}}\delta_{i-1}\sum_{j=1}^{i}\left|w_j\right|^{(2-\nu)/r_j\beta_{j-1}p_{j-1}} b_j \\
&\le \frac{L_{i-1}}{4n}Q_i^{2-\nu} + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\left(\phi_i^0 - \tilde{\sigma}b_i\right) \\
&\quad + (\tilde{\sigma} + \eta_i)\varphi_i - \eta_i\dot{\hat{\sigma}},
\end{aligned}
\tag{A.11}
$$

where $\phi_i^0(\bar{x}_i, \hat{\sigma})$ is $\mathscr{C}$ nonnegative function;

$$
\begin{aligned}
\varphi_i(\bar{x}_i, \hat{\sigma}) &= \varphi_{i-1}(\bar{x}_{i-1}, \hat{\sigma}) \\
&\quad + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}} b_i(\bar{x}_i, \hat{\sigma}).
\end{aligned}
\tag{A.12}
$$

Substituting (A.3), (A.10), and (A.11) into (A.2), one has

$$
\begin{aligned}
\dot{V}_i(\bar{x}_i, \hat{\sigma}) &\le -\gamma_0(|x_1|)(\mu(x_1) + 1) \\
&\quad - \frac{n-1}{n}L_{i-1}Q_{i-1}^{2-\nu} + |w_i|\left|x_{i+1}^{p_i} - v_i^{p_i}\right| \\
&\quad + (\tilde{\sigma} + \eta_i)\varphi_i - \eta_i\dot{\hat{\sigma}} \\
&\quad + \sum_{j=1}^{i}\kappa_1^{(2-\nu)/(2-\nu-r_j\beta_{j-1}p_{j-1})} \\
&\quad + \sum_{j=2}^{i}\kappa_1^{(2-\nu)/(2-\nu-r_j)} + \gamma_i|w_i|\left|w_{i+1}\right|^{1/\beta_i} \\
&\quad + w_i v_i^{p_i} + \gamma_i\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\Phi_i \\
&\quad + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\left(l_i + \psi_i + \phi_i^0 + \hat{\sigma}b_i\right).
\end{aligned}
\tag{A.13}
$$

Using Lemma 8, one can choose a $\mathscr{C}^1$ positive function

$$
\begin{aligned}
&\Phi_i \\
&\quad \ge \frac{1}{1-\gamma_i}\left(\frac{n-1}{n}L_{i-1} + l_i + \psi_i + \phi_i^0 + \left(1 + \hat{\sigma}^2\right)b_i\right),
\end{aligned}
\tag{A.14}
$$

such that

$$
\begin{aligned}
&w_i v_i^{p_i} + \left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\left(l_i + \psi_i + \phi_i^0 + \hat{\sigma}b_i\right) \\
&\quad + \gamma_i\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}}\Phi_i \\
&\quad \le -L_i\left|w_i\right|^{(2-\nu)/r_i\beta_{i-1}p_{i-1}},
\end{aligned}
\tag{A.15}
$$

where $L_i = ((n-1)/n)L_{i-1}$. Substituting (A.15) into (A.13) leads to (27). □

Competing Interests

The authors declare that they have no competing interests.

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