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Research Article

Pinning Cluster Synchronization in Linear Hybrid Coupled Delayed Dynamical Networks

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The problem on cluster synchronization will be investigated for a class of delayed dynamical networks based on pinning control strategy. Through utilizing the combined convex technique and Kronecker product, two sufficient conditions can be derived to ensure the desired synchronization when the designed feedback controller is employed to each cluster. Moreover, the inner coupling matrices are unnecessarily restricted to be diagonal and the controller design can be converted into solving a series of linear matrix inequalities (LMIs), which greatly improve the present methods. Finally, two numerical examples are provided to demonstrate the effectiveness and reduced conservatism.

1. Introduction

In past decade, the synchronization of various chaotic systems has received considerable attention since the pioneering works have appeared [1, 2]. Presently, it is widely known that many benefits of having synchronization can be existent. In particular, the synchronization in language emergence and development results can come up with the common vocabulary and agents' synchronization in organization management can improve their work efficiency. Thus recently, the synchronization has been widely studied owing to its great potential applications. Furthermore, since chaos synchronization in arrays of coupled dynamical networks was initially studied [3], various coupled networks have received the attention because they can exhibit some interesting phenomena [4, 5], and many elegant results have been reported [6–32]. In particular, in [6, 7, 33], time-delay is unavoidable and delayed neural networks (DNNs) are verified to exhibit some complex and unpredictable behaviors, such as periodic oscillations, bifurcation, and chaotic attractors; then, the impulsive and adaptive synchronization has also been studied [8–11], and some uneasy-to-test results have been presented. Most recently, through using Kronecker product, the global synchronization has been studied and elegant criteria have been obtained in terms of LMIs [12–20, 23]. Yet it is worth noting that, in the above works, some most developed techniques were not utilized and the addressed networks seemed to be of simple forms. Thus, researchers have used some effective tools to give less conservative results ensuring the synchronization for more general coupled DNNs [23].

In 1992, as the truth that the effective coupling among neurons varies temporally in a rather short time scale has been found [34], some researchers have mentioned that the degree of synchronization among pairs of neurons changed both temporally and by the choice of pairs. Therefore, the cluster synchronization has been imposed to various dynamical networks [21, 22, 24–31, 35]. However, due to the existence of embedding of invariant synchronization manifolds, it may occur that the system can reach different clustering patterns from the different initial conditions [24–29]. Thus, together with pinning control, some suitable methods have appeared but have been independent of initial states [30]. In [30], the pinning control strategy has been used to realize the cluster synchronization for stochastic coupled DNNs, in which the upper bound of delay variation was less than 1. Later, some effective techniques were used to overcome the shortcoming during tackling the delayed dynamical networks [31, 35]. Yet though these results above were elegant, there still exist some points waiting for the improvements. Firstly, most works above have not contained lower bound of delay variation and, in fact, its information can play an important role in reducing the conservatism. Secondly, in [30, 31], the inner coupling matrices had to be diagonal, which unavoidably limits the application areas. Thirdly, as for delay $\tau(t) \in [\tau_0, \tau_m]$, since the triple integral LKF terms such as $((\tau_m^2 - \tau_0^2)/2) \int_{-\tau_m}^{-\tau_0} \int_{\varrho}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Q \dot{x}(s) ds\, d\theta\, d\varrho$ were firstly put forward [35], it has been used and improved owing to the fact that it could help reduce the conservatism greatly [36]. Yet the authors noticed that some important terms have been ignored when estimating its derivative [35, 36], which also induces the conservatism. Therefore, the tighter estimation should be given. Overall, as for the pinning cluster synchronization of coupled networks, the mentioned points above have not been considered, which remains important and motivates this work.

Inspired by the above discussions, this paper aims to study the problem on cluster synchronization for a class of coupled time-delay networks with linear hybrid coupling by means of pinning control. Through choosing two augmented Lyapunov-Krasovskii functionals (LKFs) and using the combined convex technique, some novel sufficient conditions are presented via Kronecker product and LMIs, whose feasibility can be easily checked by resorting to Matlab LMI Toolbox. In particular, we will give the tighter upper bounds on time derivative of LKF terms. The efficiency and less conservatism can be verified on the basis of two numerical examples.

Notations. $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices; I_m represents the $m \times m$ identity matrix and $0_{m\cdot n}$ denotes the $m \times n$ zero matrix; $A \otimes B$ represents Kronecker product of matrices A and B.

2. Problem Formulations and Preliminaries

Firstly, suppose the nodes are coupled with states $x_i(t)$, $i \in \{1, \dots, N\}$; we consider the dynamical networks with each node being an n-dimensional DNN with linear hybrid coupling as

$$\begin{aligned}\dot{x}_i(t) = {} & -Cx_i(t) + A\overline{f}(x_i(t)) + B\overline{f}(x_i(t-\tau(t))) \\ & + I(t) + u_i(t) + \sum_{j=1}^{N} l_{ij}^1 G x_j(t) \\ & + \sum_{j=1}^{N} l_{ij}^2 H x_j(t-\tau(t)) \\ & + \sum_{j=1}^{N} l_{ij}^3 K \left(\int_{t-\tau(t)}^{t} x_j(s)\, ds \right),\end{aligned} \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{in}(t)]^T$ are the state vectors; here $C = \operatorname{diag}(c_1, c_2, \dots, c_n) > 0$, $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n}$, and $\overline{f}(x_i(\cdot)) = [\overline{f}_1(x_{i1}(\cdot)), \dots, \overline{f}_n(x_{in}(\cdot))]^T$ are the activation functions; also here we assume $G = [g_{ij}]_{n\times n}$, $H = [h_{ij}]_{n\times n}$, $K = [k_{ij}]_{n\times n}$ denote the inner coupling matrices, $u_i(t)$ is the control input, and $I(t) \in \mathbf{R}^n$ is the input vector.

Remark 1. In system (1), the hybrid coupling is utilized in model (1) and it should be emphasized that the inner coupling matrices G, H, and K are not necessarily restricted to be of diagonal form, which can represent more general cases than the ones in [30, 31].

Suppose that networks (1) will be controlled onto some desired inhomogeneous state as $\{x_1(t), \dots, x_{m_1}(t)\} \to s_1(t)$, $\{x_{m_1+1}(t), \dots, x_{m_2}(t)\} \to s_2(t), \dots, \{x_{m_{k-1}+1}(t), \dots, x_{m_k}(t)\} \to s_k(t)$; that is, $\mathscr{M} = \{\{s_1(t), \dots, s_1(t)\}, \{s_2(t), \dots, s_2(t)\}, \dots, \{s_k(t), \dots, s_k(t)\}\} \in \mathbf{R}^{n\times N}$ is the desired cluster synchronization pattern under the pinning control, where $x_i(t) \to s_l(t)$ means that $\lim_{t\to+\infty} \|x_i(t) - s_l(t)\| = 0$ for $i \in \{m_{l-1}, \dots, m_l\}$ with $m_0 = 0$ and $l \in \{1, \dots, k\}$. The function $s_l(t)$ is defined as

$$\begin{aligned}\dot{s}_l(t) = {} & -Cs_l(t) + A\overline{f}(s_l(t)) + B\overline{f}(s_l(t-\tau(t))) \\ & + I(t), \quad l = 1, \dots, k.\end{aligned} \tag{2}$$

For the dynamical networks described by (1), the following assumptions are utilized.

(**A1**) Here $\tau(t)$ denotes the interval time-varying delay satisfying

$$\begin{gathered} 0 \le \tau_0 \le \tau(t) \le \tau_m, \\ \mu_0 \le \dot{\tau}(t) \le \mu_m < +\infty. \end{gathered} \tag{3}$$

Moreover, we give the denotations as $\overline{\tau}_m = \tau_m - \tau_0$, $\overline{\mu}_m = \mu_m - \mu_0$, and $\delta_m = \tau_m^2 - \tau_0^2$.

(**A2**) For $\nu = 1, 2, 3$ and the configuration matrices

$$L^\nu = \begin{bmatrix} L_{11}^\nu & L_{12}^\nu & \cdots & L_{1k}^\nu \\ L_{21}^\nu & L_{22}^\nu & \cdots & L_{2k}^\nu \\ \vdots & \vdots & \ddots & \vdots \\ L_{k1}^\nu & L_{k2}^\nu & \cdots & L_{kk}^\nu \end{bmatrix} \tag{4}$$

with $L_{ii}^\nu \in R^{(m_i - m_{i-1})\times(m_i - m_{i-1})}$ and $L_{ij}^\nu \in R^{(m_i - m_{i-1})\times(m_j - m_{j-1})}$, $i, j \in \{1, \dots, k\}$, assume that every matrix $L_{ii}^\nu = [l_{ii,gh}^\nu]$ for $i \in \{1, \dots, k\}$ satisfies $l_{ii,gg}^\nu \ge 0$, $l_{ii,gg}^\nu = -\sum_{h=1,h\ne g}^{m_i - m_{i-1}} l_{ii,gh}^\nu$, and the sums of all rows in every L_{ij}^ν $(i \ne j)$ are zeros.

(**A3**) There exist constants $\sigma_i^-, \sigma_i^+ \in \mathbf{R}$ such that the bounded functions $\overline{f}_i(\cdot)$ satisfy

$$\sigma_i^- \le \frac{\overline{f}_i(\alpha) - \overline{f}_i(\beta)}{\alpha - \beta} \le \sigma_i^+, \quad \forall \alpha, \beta \in \mathbf{R},\ i = 1, \dots, n. \tag{5}$$

Here we set $\Sigma^+ = \operatorname{diag}(\sigma_1^+, \dots, \sigma_n^+)$, $\Sigma^- = \operatorname{diag}(\sigma_1^-, \dots, \sigma_n^-)$, and

$$\begin{gathered} \Sigma_1 = \operatorname{diag}(\sigma_1^+ \sigma_1^-, \dots, \sigma_n^+ \sigma_n^-), \\ \Sigma_2 = \operatorname{diag}\left(\frac{\sigma_1^+ + \sigma_1^-}{2}, \dots, \frac{\sigma_n^+ + \sigma_n^-}{2} \right). \end{gathered} \tag{6}$$

In this paper, we consider one special case that the accurate information on time-delay is available. Without loss of generality, to achieve the goal of cluster synchronization in this work, we will apply the pinning control strategy on the nodes set $J = \{m_1, m_2, \ldots, m_k\}$ and adopt the following pinning controller as

$$u_i(t) = \begin{cases} -l_i G[x_i(t) - s_l(t)] - r_i H[x_i(t-\tau(t)) - s_l(t-\tau(t))], & i = m_l,\ l = 1, \ldots, k, \\ 0, & i \neq m_l. \end{cases} \tag{7}$$

Let $e_i(t) = x_i(t) - s_l(t)$; one can check that $\sum_{j=1}^{N} l_{ij}^1 G x_j(\cdot) = \sum_{j=1}^{N} l_{ij}^1 G e_j(\cdot)$, $\sum_{j=1}^{N} l_{ij}^2 H x_j(\cdot) = \sum_{j=1}^{N} l_{ij}^2 H e_j(\cdot)$, and $\sum_{j=1}^{N} l_{ij}^3 K x_j(\cdot) = \sum_{j=1}^{N} l_{ij}^3 K e_j(\cdot)$. Then, combining (1) and (2) with (7) yields

$$\begin{aligned} \dot{e}_i(t) = {} & -\left(C + \bar{l}_i G\right) e_i(t) + A f(e_i(t)) \\ & + B f(e_i(t-\tau(t))) - \bar{r}_i H e_i(t-\tau(t)) \\ & + \sum_{j=1}^{N} l_{ij}^1 G e_j(t) + \sum_{j=1}^{N} l_{ij}^2 H e_j(t-\tau(t)) \\ & + \sum_{j=1}^{N} l_{ij}^3 K \left(\int_{t-\tau(t)}^{t} e_j(s)\, ds \right), \end{aligned} \tag{8}$$

where $f(e_i(\cdot)) = \overline{f}((e_i(\cdot) + s_l(\cdot))) - \overline{f}(s_l(\cdot))$ and

$$\begin{aligned} \bar{l}_i &= \begin{cases} l_i, & i = m_l,\ l = 1, 2, \ldots, k, \\ 0, & i \neq m_l, \end{cases} \\ \bar{r}_i &= \begin{cases} r_i, & i = m_l,\ l = 1, 2, \ldots, k, \\ 0, & i \neq m_l. \end{cases} \end{aligned} \tag{9}$$

Then, we can easily check that the functions $f(\cdot)$ satisfy assumption (**A3**) and we set

$$\begin{aligned} \Xi &= \operatorname{diag}\left(\underline{0, \ldots, 0, l_{m_1}}, \ldots, \underline{0, \ldots, 0, l_{m_k}}\right), \\ \Theta &= \operatorname{diag}\left(\underline{0, \ldots, 0, r_{m_1}}, \ldots, \underline{0, \ldots, 0, r_{m_k}}\right). \end{aligned} \tag{10}$$

In what follows, some useful basic definition and denotations will be introduced.

Definition 2 (see [30]). The dynamical network (1) with N nodes is said to achieve the cluster synchronization, if the N nodes are split into k clusters $G_1, G_2, \ldots, G_k$ as $\{G_1 = (1, \ldots, m_1), G_2 = (m_1 + 1, \ldots, m_2), \ldots, G_k = (m_{k-1} + 1, \ldots, m_k)\}$ such that the nodes synchronize with each other in the same cluster; namely, for the states $x_i(t)$ and $x_j(t)$ of the arbitrary nodes i and j in the same cluster G_l $(l = 1, \ldots, k)$, $\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| = 0$ holds, in which $\|\cdot\|$ stands for the Euclidean norm.

Denotation 1. Denote the $3Nn \times 3Nn$ constant matrix $\mathbf{E}$ as

$$\begin{aligned} & \mathbf{E} \\ & = \left[\, \underline{\mathbf{E}_1\ \mathbf{E}_{N+1}\ \mathbf{E}_{2N+1}}\ \ \underline{\mathbf{E}_2\ \mathbf{E}_{N+2}\ \mathbf{E}_{2N+2}}\ \cdots\ \underline{\mathbf{E}_N\ \mathbf{E}_{2N}\ \mathbf{E}_{3N}} \,\right], \end{aligned} \tag{11}$$

in which the $3Nn \times n$ matrix $\mathbf{E}_i$ $(i = 1, \ldots, 3N)$ can be expressed as follows:

$$\mathbf{E}_i^T = \left[0_n\ \ 0_n\ \ \cdots\ \ I_n\ \ \cdots\ \ 0_n\ \ 0_n\right] \tag{12}$$

with the identity matrix I_n denoting the ith one in the matrix vector $\mathbf{E}_i$.

Denotation 2. Denote

$$\begin{aligned} e^T(\cdot) &= \left[e_1^T(\cdot)\ \ e_2^T(\cdot)\ \ \cdots\ \ e_N^T(\cdot)\right], \\ \dot{e}^T(t) &= \left[\dot{e}_1^T(t)\ \ \dot{e}_2^T(t)\ \ \cdots\ \ \dot{e}_N^T(t)\right]. \end{aligned} \tag{13}$$

3. Pinning Cluster Synchronization

Prior to addressing the main results, the following lemmas will be useful in the proof.

Lemma 3 (see [35]). *For any constant matrix $X \in \mathbf{R}^{n\times n}$, $X = X^T \geq 0$, two scalars $h_2 \geq h_1 \geq 0$, such that the following integrations are well defined; then*

$$\begin{aligned} & -(h_2 - h_1) \int_{t-h_2}^{t-h_1} x^T(s) X x(s)\, ds \\ & \quad \leq -\left(\int_{t-h_2}^{t-h_1} x(s)\, ds\right)^T X \left(\int_{-h_2}^{-h_1} x(s)\, ds\right); \\ & -\frac{h_2^2 - h_1^2}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x^T(s) X x(s)\, ds\, d\theta \\ & \quad \leq -\left(\int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x(s)\, ds\, d\theta\right)^T \\ & \quad \cdot X \left(\int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x(s)\, ds\, d\theta\right). \end{aligned} \tag{14}$$

Lemma 4 (see [37]). *For any vectors ζ_1 and ζ_2, constant matrices R and S, and real scalars $\alpha \geq 0$ and $\beta \geq 0$ satisfying that $\left[\begin{smallmatrix} R & S \\ * & R \end{smallmatrix}\right] \geq 0$ and $\alpha + \beta = 1$, the inequality holds:*

$$-\frac{1}{\alpha} \zeta_1^T R \zeta_1 - \frac{1}{\beta} \zeta_2^T R \zeta_2 \leq -\begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ * & R \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}. \tag{15}$$

Lemma 5 (see [23]). *Suppose that Ω, Ξ_1, and Ξ_2 are the constant matrices of appropriate dimensions, $\alpha \in [0, 1]$; then $\Omega + \alpha \Xi_1 + (1 - \alpha)\Xi_2 < 0$ holds, if and only if $\Omega + \Xi_1 < 0$, $\Omega + \Xi_2 < 0$.*

Lemma 6 (see [37]). *For the symmetric appropriately dimensional matrices $R > 0, \Xi$, and matrix Γ, the two following statements are equivalent: (i) $\Xi - \Gamma^T R \Gamma < 0$; (ii) there exists a matrix of appropriate dimension Π such that*

$$\begin{bmatrix} \Xi + \Gamma^T \Pi^T + \Pi\Gamma & \Pi R^{-1} \\ * & -R^{-1} \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} \Xi + \Gamma^T \Pi^T + \Pi\Gamma & \Pi \\ * & -R \end{bmatrix} < 0. \tag{16}$$

Now, together with the pinning control strategy, two less conservative criteria will be presented for the cluster synchronization based on Kronecker product and LMI approach.

Theorem 7. *Suppose that assumptions* (**A1**)–(**A3**) *are true; then, the controlled dynamical networks (1) can achieve the desired cluster synchronization, if there exist two $6Nn \times Nn$ matrices Π_i $(i = 1, 2)$ making $\Pi = [\Pi_1 \ \ \Pi_2]$, $n \times n$ constant matrices $P > 0$, $P_i > 0$ $(i = 1, 2, 3, 4)$, $Q_j > 0$, $R_j > 0$, S_j $(j = 1, 2, 3)$, and $M_l, N_l > 0$ $(l = 1, 2)$ guaranteeing $\begin{bmatrix} R_j & S_j \\ * & R_j \end{bmatrix} \geq 0$ and $n \times n$ diagonal matrices $E > 0$, $F > 0$, $U > 0$, and $V > 0$ such that the LMIs in (17)-(18) hold:*

$$\begin{bmatrix} \Phi + [\mathbf{IE}(I_N \otimes \Upsilon_i)]\Pi^T + \Pi[\mathbf{IE}(I_N \otimes \Upsilon_i)]^T & \Pi_1 & \Pi_2 \\ * & -I_N \otimes R_1 & -I_N \otimes S_1 \\ * & * & -I_N \otimes R_1 \end{bmatrix} < 0, \quad i = 1, 2, \tag{17}$$

$$\begin{gathered} \Omega + \overline{\mu}_m \Gamma_1 \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix} \Gamma_1^T < 0, \\ \Omega + \overline{\mu}_m \Gamma_2 P_4 \Gamma_2^T < 0, \end{gathered} \tag{18}$$

where **E** *is expressed in Denotation 1 and*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & 0 & \Phi_{16} \\ * & \Phi_{22} & 0 & 0 & 0 & \Phi_{26} \\ * & * & \Phi_{33} & 0 & 0 & \Phi_{36} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \Omega_{11} & Q_3 & M_1 B & 0 & \Omega_{15} & M_1 B & \Omega_{17} \\ * & \Omega_{22} & \Omega_{23} & S_3 & 0 & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & V\Sigma_2 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & \Omega_{57} \\ * & * & * & * & * & \Omega_{66} & B^T M_2^T \\ * & * & * & * & * & * & \Omega_{77} \end{bmatrix},$$

$$\mathbf{I}^T = \begin{bmatrix} I_{Nn} & 0_{Nn\cdot 3Nn} & 0_{Nn\cdot 2Nn} \\ 0_{Nn\cdot 3Nn} & I_{Nn} & 0_{Nn\cdot 2Nn} \\ 0_{Nn\cdot 4Nn} & I_{Nn} & 0_{Nn\cdot Nn} \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} \overline{\tau}_m I_n & 0_n \\ -I_n & 0_n \\ 0_n & -I_n \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} 0_n & \overline{\tau}_m I_n \\ -I_n & 0_n \\ 0_n & -I_n \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0_{5n\cdot n} & 0_{3n\cdot n} \\ I_n & I_n \\ 0_n & 0_{3n\cdot n} \end{bmatrix}, \tag{19}$$

and $\Gamma_2 = [0_{n\cdot 3n} \ \ I_n \ \ 0_{n\cdot 3n}]^T$ *with*

$$\Phi_{11} = L^1 \otimes (M_2 G) + (L^1 \otimes M_2 G)^T - 2\Xi \otimes (M_1 G) - I_N \otimes (\tau_0^2 Q_1),$$

$$\Phi_{12} = [(L^2)^T - \Theta] \otimes (M_1 H),$$

$$\Phi_{13} = I_N \otimes (\tau_0 Q_1) + L^3 \otimes (M_1 K),$$

$$\Phi_{14} = L^3 \otimes (M_1 K),$$

$$\Phi_{16} = [(L^1)^T - \Xi] \otimes (M_2 G)^T,$$

$$\Phi_{22} = -I_N \otimes N_2,$$

$$\Phi_{26} = [(L^2)^T - \Theta] \otimes (M_2 H)^T,$$

$$\Phi_{33} = -I_N \otimes (Q_1 + Q_2),$$

$$\Phi_{36} = (L^3 \otimes M_2 K)^T,$$

$$\Phi_{44} = -I_N \otimes R_2,$$
$$\Phi_{45} = -I_N \otimes S_2,$$
$$\Phi_{46} = \left(L^3 \otimes M_2 K\right)^T,$$
$$\Phi_{55} = -I_N \otimes R_2,$$
$$\Phi_{66} = -I_N \otimes N_1,$$
$$\Omega_{11} = P_2 + \tau_0^2 Q_2 + \overline{\tau}_m^2 R_2 - Q_3 - U\Sigma_1 - M_1 C - CM_1^T,$$
$$\Omega_{15} = M_1 A + U\Sigma_2,$$
$$\Omega_{17} = P - CM_2^T - M_1 + F\Sigma^+ - E\Sigma^-,$$
$$\Omega_{22} = -P_2 - Q_3 - R_3 + P_3,$$
$$\Omega_{23} = R_3 - S_3,$$
$$\Omega_{33} = (\mu_0 - 1) P_3 + (1 - \mu_m) P_4 - 2R_3 + S_3 + S_3^T - V\Sigma_1 + N_2,$$
$$\Omega_{34} = R_3 - S_3,$$
$$\Omega_{44} = -P_4 - R_3,$$
$$\Omega_{55} = -U + P_1,$$
$$\Omega_{57} = E^T - F^T + A^T M_2^T,$$
$$\Omega_{66} = -V + (\mu_0 - 1) P_1,$$
$$\Omega_{77} = -M_2 - M_2^T + N_1 + \tau_0^2 Q_3 + \overline{\tau}_m^2 R_3 + \frac{\tau_0^4}{4} Q_1 + \frac{\delta_m^2}{4} R_1. \tag{20}$$

In what follows, based on Theorem 7, we can consider the pinning cluster synchronization for the dynamical networks composed of N time-delay Lur'e systems [32]:

$$\dot{x}_i(t) = -Cx_i(t) + A\overline{f}\left(W^T x_i(t)\right) + \sum_{j=1}^{N} l_{ij}^1 G x_j(t) + \sum_{j=1}^{N} l_{ij}^2 H x_j(t - \tau(t)) + u_i(t), \tag{21}$$

where $C = [a_{ij}]_{n\times n}$, $A = [c_{ij}]_{n\times n_1}$, and $W = [w_{ij}]_{n\times n_1} = [W_1, W_2, \ldots, W_{n_1}]$ are constant matrices; here $\overline{f}(W^T x(t))$ denotes the nonlinear function satisfying (**A3**) and $\overline{f}(0) = 0$. Then, by using the pinning controller (7), we can derive the following theorem.

Theorem 8. *Suppose that assumptions* (**A1**)–(**A3**) *are true; then, the controlled dynamical networks (21) can achieve the desired cluster synchronization, if there exist two* $6Nn \times Nn$ *matrices* Π_i $(i = 1, 2)$ *making* $\Pi = [\Pi_1 \ \ \Pi_2]$, $n \times n$ *constant matrices* $P > 0$, $P_i > 0$, $Q_i > 0$, $R_i > 0$, S_i $(i = 1, 2, 3)$, *and* $M_l, N_l > 0$ $(l = 1, 2)$ *guaranteeing* $\begin{bmatrix} R_i & S_i \\ * & R_i \end{bmatrix} \geq 0$, *and* $n_1 \times n_1$ *diagonal matrices* $E > 0$, $F > 0$, *and* $U > 0$ *such that the LMIs in (22) hold:*

$$\begin{bmatrix} \Phi + [\mathbf{IE}(I_N \otimes \Upsilon_i)]\Pi^T + \Pi[\mathbf{IE}(I_N \otimes \Upsilon_i)]^T & \Pi_1 & \Pi_2 \\ * & -I_N \otimes R_1 & -I_N \otimes S_1 \\ * & * & -I_N \otimes R_1 \end{bmatrix} < 0, \quad i = 1, 2,$$
$$\Omega + \overline{\mu}_m \Gamma P_2 \Gamma^T < 0,$$
$$\Omega + \overline{\mu}_m \Gamma P_3 \Gamma^T < 0, \tag{22}$$

where $\mathbf{E}$ *is expressed in Denotation 1,* $\mathbf{I}$ *is expressed in Theorem 7, and*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & 0 & \Phi_{16} \\ * & \Phi_{22} & 0 & 0 & 0 & \Phi_{26} \\ * & * & \Phi_{33} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \Omega_{11} & Q_3 & 0 & 0 & \Omega_{15} & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & S_3 & 0 & 0 \\ * & * & \Omega_{33} & R_3 - S_3 & 0 & 0 \\ * & * & * & -P_3 - R_3 & 0 & 0 \\ * & * & * & * & -U & \Omega_{56} \\ * & * & * & * & * & \Omega_{66} \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} \overline{\tau}_m I_n & 0_n \\ -I_n & 0_n \\ 0_n & -I_n \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} 0_n & \overline{\tau}_m I_n \\ -I_n & 0_n \\ 0_n & -I_n \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 0_{3n \cdot n} \\ I_n \\ 0_{2n \cdot n} \end{bmatrix} \tag{23}$$

with

$$\begin{aligned}
\Phi_{11} &= L^1 \otimes (M_2 G) + (L^1 \otimes M_2 G)^T - 2\Xi \otimes (M_1 G) \\
&\quad - I_N \otimes (\tau_0^2 Q_1), \\
\Phi_{12} &= \left[(L^2)^T - \Theta\right] \otimes (M_1 H), \\
\Phi_{13} &= I_N \otimes (\tau_0 Q_1), \\
\Phi_{16} &= \left[(L^1)^T - \Xi\right] \otimes (M_2 G)^T, \\
\Phi_{22} &= -I_N \otimes N_2, \\
\Phi_{26} &= \left[(L^2)^T - \Theta\right] \otimes (M_2 H)^T, \\
\Phi_{33} &= -I_N \otimes (Q_1 + Q_2), \\
\Phi_{44} &= -I_N \otimes R_2, \\
\Phi_{45} &= -I_N \otimes S_2, \\
\Phi_{55} &= -I_N \otimes R_2, \\
\Phi_{66} &= -I_N \otimes N_1, \\
\Omega_{11} &= P_1 + \tau_0^2 Q_2 + \overline{\tau}_m^2 R_2 - Q_3 - WU\Sigma_1 W^T - M_1 C \\
&\quad - CM_1^T, \\
\Omega_{15} &= M_1 A + WU\Sigma_2, \\
\Omega_{16} &= -M_1 + P - CM_2^T + F\Sigma^+ W^T - E\Sigma^- W^T, \\
\Omega_{22} &= -P_1 - Q_3 - R_3 + P_2, \\
\Omega_{23} &= R_3 - S_3, \\
\Omega_{33} &= (\mu_0 - 1) P_2 + (1 - \mu_m) P_3 - 2R_3 + S_3 + S_3^T \\
&\quad - V\Sigma_1 + N_2, \\
\Omega_{56} &= E^T - F^T + A^T M_2^T, \\
\Omega_{66} &= N_1 + \tau_0^2 Q_3 + \overline{\tau}_m^2 R_3 + \frac{\tau_0^4}{4} Q_1 + \frac{\delta_m^2}{4} R_1 - M_2 \\
&\quad - M_2^T.
\end{aligned} \tag{24}$$

4. Numerical Examples

Two numerical examples will be provided to illustrate the derived results with some typical cases.

Example 1. Consider one 2-dimensional delayed dynamical network (1) described by

$$\begin{aligned}
\dot{x}_i(t) &= -Cx_i(t) + A\overline{f}(x_i(t)) + B\overline{f}(x_i(t - \tau(t))) \\
&\quad + u_i(t) + \sum_{j=1}^{4} l_{ij}^1 G x_j(t) + \sum_{j=1}^{4} l_{ij}^1 H x_j(t - \tau(t)) \\
&\quad + \sum_{j=1}^{4} l_{ij}^1 K \left(\int_{t-\tau(t)}^{t} x_j(s)\, ds \right), \quad i = 1, 2, 3, 4,
\end{aligned} \tag{25}$$

with the following parameters:

$$\begin{aligned}
C &= \begin{bmatrix} 1.25 & 0 \\ 0 & 1.25 \end{bmatrix}, \\
A &= \begin{bmatrix} 1.8 & -0.15 \\ -5.2 & -3.5 \end{bmatrix}, \\
B &= \begin{bmatrix} -1.7 & -0.12 \\ -0.26 & -2.5 \end{bmatrix}, \\
G &= \begin{bmatrix} 2.5 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}, \\
H &= \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix}, \\
K &= \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix}, \\
\overline{f}(x_i) &= \begin{bmatrix} \tanh(x_{i1}) \\ \tanh(x_{i2}) \end{bmatrix}.
\end{aligned} \tag{26}$$

Then, through adopting the pinning controller in (7),

$$u_i(t) = \begin{cases} -1.5G\left[x_i(t) - s_l(t)\right] - 0.25H\left[x_i(t - \tau(t)) - s_l(t - \tau(t))\right], & i = m_l,\ l = 1, 2, \\ 0, & i \neq m_l, \end{cases} \tag{27}$$

with $\Xi = \mathrm{diag}(0, 1.5, 0, 1.5)$ and $\Theta = \mathrm{diag}(0, 0.25, 0, 0.25)$, we assume that the desired cluster synchronization states of DNNs (25) are $s_1(t)$ and $s_2(t)$, which can satisfy

$$\dot{s}_i(t) = -Cs_i(t) + A\overline{f}(s_i(t)) + B\overline{f}(s_i(t - \tau(t))), \quad i = 1, 2, \tag{28}$$

TABLE 1: The calculated MAUBs τ_{max} by setting $\tau_0 = 0$, $\mu_0 = -0.2, -0.1$, and unavailable μ_0.

Methods	μ_m	0.4	0.6	0.8	1.2
Li and Cao [30]		1.112	1.035	1.004	—
Wang et al. [31]	−0.2	1.154	1.074	1.048	0.987
Theorem 7		1.243	1.126	1.100	1.022
Li and Cao [30]		1.112	1.035	1.004	—
Wang et al. [31]	−0.1	1.165	1.084	1.077	0.996
Theorem 7		1.263	1.132	1.127	1.065
Li and Cao [30]		1.112	1.035	1.004	—
Wang et al. [31]	—	1.127	1.046	1.035	0.945
Theorem 8		1.195	1.094	1.067	0.996

with different initial conditions. In order to reduce the number of controllers and realize the cluster synchronization, we can use the controlled networks sets $\{m_1, m_2\} = \{2, 4\}$ and, respectively, choose the configuration matrices as $L^1 = \begin{bmatrix} L^1_{11} & L^1_{12} \\ L^1_{21} & L^1_{22} \end{bmatrix}$ with

$$L^1_{11} = \begin{bmatrix} -1.5 & 1.5 \\ 1.5 & -1.5 \end{bmatrix}, \quad L^1_{12} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \quad L^1_{21} = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix}, \quad L^1_{22} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}. \tag{29}$$

In what follows, two cases will be given to illustrate the efficiency and reduced conservatism of our results.

Case 1. Given $\tau_0 = 0$, choose three inner coupling matrices of diagonal form as

$$G = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad H = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}, \quad K = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}. \tag{30}$$

Then, through, respectively, setting $\mu_0 = -0.2, -0.1$, and unavailable μ_0, we can compute the corresponding maximum allowable upper bounds (MAUBs) in Table 1 based on Theorem 7 and Remark A.2 by resorting to Matlab LMI Toolbox.

In Table 1, the term "—" means that the corresponding value is unavailable. Based on the MAUBs in Table 1, one can verify that our results can be less conservative than some existent ones. In particular, as the inner coupling matrices G, H, and K are not diagonal, our theorems still can be applicable while [30, 31] fail.

Case 2. Choosing $\tau(t) = 0.2 + 0.8\sin^2(2t)$ and the inner coupling matrices G, H, and K as Case 1, one can derive $\tau_0 = 0.2$, $\tau_m = 1.0$, $\mu_0 = -1.6$, and $\mu_m = 1.6$. Owing to the fact that $\mu_m > 1$ and G, H, and K are not diagonal, the methods in [30, 31] fail to verify the synchronization. Yet, by resorting to Matlab LMI Toolbox, Theorem 7 can guarantee the pinning cluster synchronization and the feasible solution to LMIs in (17)-(18) can be obtained as follows:

$$P_1 = \begin{bmatrix} 0.562 & 0.121 \\ 0.121 & 0.532 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.343 & 0.022 \\ 0.022 & 0.346 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 1.206 & 0.202 \\ 0.202 & 0.987 \end{bmatrix}, \quad P_4 = \begin{bmatrix} 1.633 & 0.193 \\ 0.193 & 0.857 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.002 & 0.001 \\ 0.001 & 0.001 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.005 & 0.001 \\ 0.001 & 0.012 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 1.214 & 0.314 \\ 0.314 & 0.685 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 1.073 & 0.511 \\ 0.511 & 1.034 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1.428 & 0.544 \\ 0.544 & 1.428 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 1.224 & 0.124 \\ 0.124 & 0.985 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.335 & 0.050 \\ 0.132 & 0.325 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.544 & 0.224 \\ 0.327 & 0.675 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 0.220 & 0.091 \\ 0.068 & 0.202 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 1.304 & -0.101 \\ -0.114 & 0.132 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1.142 & 1.102 \\ 0.786 & 0.132 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.562 & 0.121 \\ 0.121 & 0.867 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.445 & 0.342 \\ 0.342 & 0.766 \end{bmatrix},$$
$$P = \begin{bmatrix} 0.154 & 0.105 \\ 0.105 & 0.445 \end{bmatrix},$$
$$E = 0.202 I_2,$$
$$F = 0.175 I_2,$$
$$U = 0.006 I_2,$$
$$V = 0.010 I_2. \quad (31)$$

Example 2. In this example, we consider the well-known Chua's circuit to illustrate our synchronization results, which can be expressed as

$$\begin{aligned} \dot{x}_1 &= \alpha \left[x_2 - m_1 x_1 + h(x_1) \right], \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \quad (32)$$

with $h(x_1) = (1/2)(m_1 - m_0)(|x_1 + c| - |x_1 - c|)$ and the parameters $m_0 = -1/7$, $m_1 = 2/7$, $\alpha = 9$, $\beta = 14.286$, and $c = 1$. Then, the circuit model can be represented as the Lur'e system:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -\frac{18}{7} & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -14.286 & 0 \end{bmatrix} x(t) \\ &\quad + \begin{bmatrix} \frac{27}{7} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \overline{f}\left(W^T x(t) \right) \end{aligned} \quad (33)$$

with $W = [1 \ \ 0 \ \ 0]^T$ and $\overline{f}_1(\xi) = (1/2)(|\xi + 1| - |\xi - 1|)$ belonging to the sector $[0, 1]$. Now we consider the cluster synchronization of the dynamical networks with each node being a 2-dimensional system (33) with linear coupling as

$$\begin{aligned} \dot{x}_i(t) &= -C x_i(t) + A \overline{f}\left(W^T x_i(t) \right) + \sum_{j=1}^{4} l_{ij}^1 G x_j(t) \\ &\quad + \sum_{j=1}^{4} l_{ij}^2 H x_j(t - \tau(t)) + u_i(t). \end{aligned} \quad (34)$$

In order to reduce the number of controllers and realize the cluster synchronization, we adopt the pinning controller (7) as

$$u_i(t) = \begin{cases} -1.5 G \left[x_i(t) - s_l(t) \right], & i = m_l, \ l = 1, 2, \\ 0, & i \neq m_l \end{cases} \quad (35)$$

and the desired cluster synchronization states of (A.15) satisfy

$$\dot{s}_i(t) = -C s_i(t) + A \overline{f}\left(W^T s_i(t) \right), \quad i = 1, 2 \quad (36)$$

with the initial conditions $s_1(0) = \left[\begin{smallmatrix} 0.4 \\ -0.5 \end{smallmatrix}\right]$ and $s_2(0) = \left[\begin{smallmatrix} -0.2 \\ 0.3 \end{smallmatrix}\right]$. Now through setting $\Xi = \operatorname{diag}(0, 1.5, 0, 1.5)$, $G = \left[\begin{smallmatrix} 1.5 & 0.2 \\ 0.2 & 1.5 \end{smallmatrix}\right]$, and the configuration matrices as $L^1 = \left[\begin{smallmatrix} L_{11}^1 & L_{12}^1 \\ L_{21}^1 & L_{22}^1 \end{smallmatrix}\right]$ with

$$L_{11}^1 = \begin{bmatrix} -1.5 & 1.5 \\ 1.5 & -1.5 \end{bmatrix},$$
$$L_{12}^1 = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix},$$
$$L_{21}^1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$L_{22}^1 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}, \quad (37)$$

together with Theorem 8 and LMI in Matlab Toolbox, we can easily verify that network (34) can achieve the desired cluster synchronization, which can be further supported by the synchronization error states in Figure 1.

5. Conclusions

This paper has investigated the problem on pinning cluster synchronization for delayed dynamical networks with linearly hybrid coupling. Two novel conditions have been derived by employing the Lyapunov-Krasovskii stability theory. It is worth pointing out that some most recently developed techniques such as combined convex technique and triple integral LKF terms have been employed, which can help extend the application areas. The synchronization criteria are presented in the forms of LMIs, which can be checked easily by referring to Matlab LMI Toolbox. Finally, two numerical examples can illustrate the less conservatism of our theorems based on some comparing results.

Appendix

Proof of Theorem 7. Based on assumptions (**A1**)–(**A3**), we can choose the Lyapunov-Krasovskii functional as

$$V(e(t)) = V_1(e(t)) + V_2(e(t)) + V_3(e(t)), \quad (A.1)$$

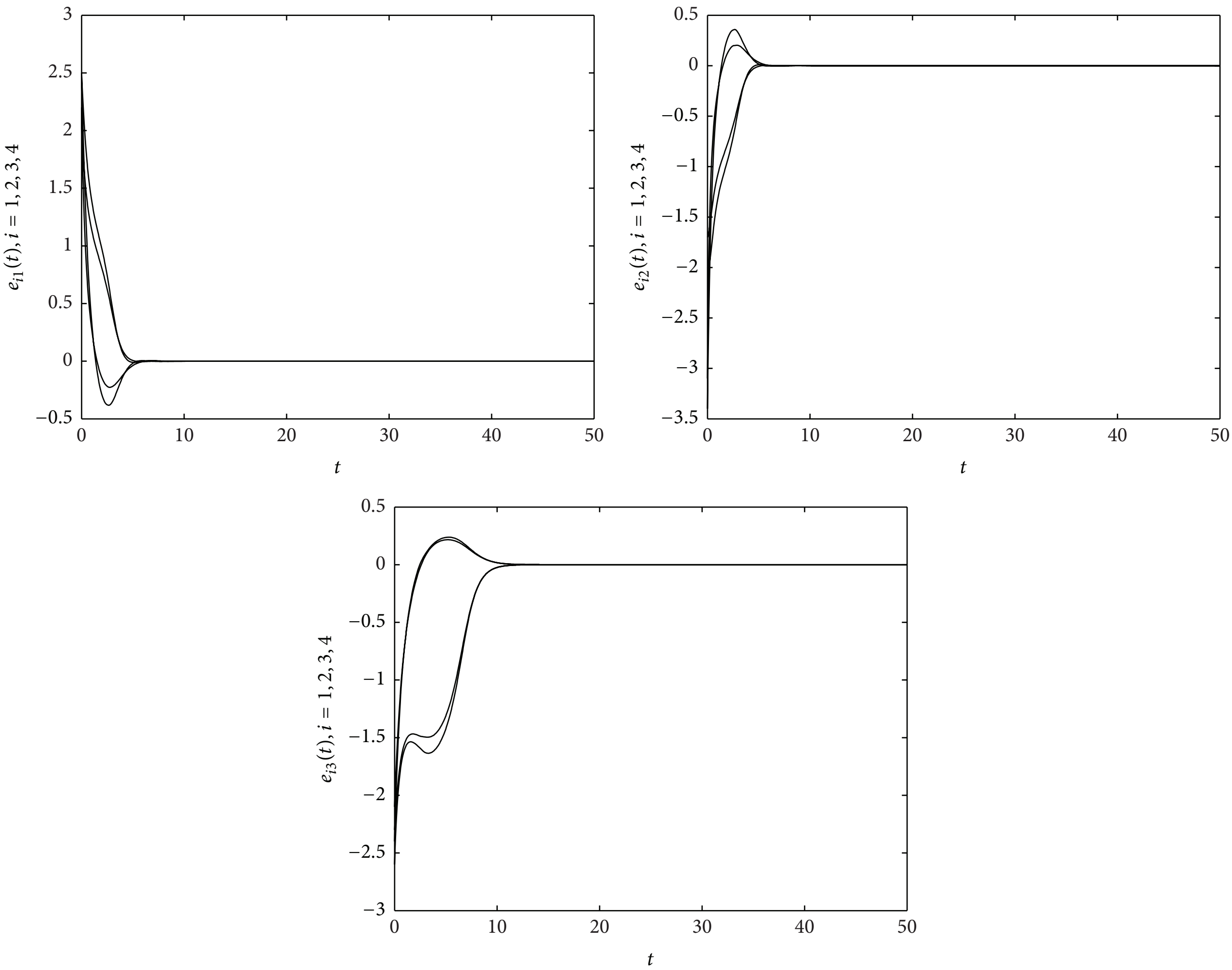

Figure 1: Phase and state trajectories of the error states.

where

$$\begin{aligned} V_1(e(t)) = \sum_{i=1}^{N} \Bigg[& e_i^T(t) P e_i(t) \\ & + \int_{t-\tau(t)}^{t} f^T(e_i(s)) P_1 f(e_i(s))\, ds \\ & + \int_{t-\tau_0}^{t} e_i^T(s) P_2 e_i(s)\, ds + \int_{t-\tau(t)}^{t-\tau_0} e_i^T(s) P_3 e_i(s)\, ds \\ & + \int_{t-\tau_m}^{t-\tau(t)} e_i^T(s) P_4 e_i(s)\, ds \\ & + 2\sum_{j=1}^{n} e_j \int_0^{e_{ij}(t)} \left[f_j(s) - \sigma_j^- s \right] ds \\ & + 2\sum_{j=1}^{n} f_j \int_0^{e_{ij}(t)} \left[\sigma_j^+ s - f_j(s) \right] ds \Bigg], \end{aligned}$$

$$\begin{aligned} V_2(e(t)) = \sum_{i=1}^{N} \Bigg[& \frac{\tau_0^2}{2} \int_{-\tau_0}^{0} \int_{\varrho}^{0} \int_{t+\theta}^{t} \dot{e}_i^T(s) Q_1 \dot{e}_i(s)\, ds\, d\theta\, d\varrho \\ & + \frac{\delta_m}{2} \int_{-\tau_m}^{-\tau_0} \int_{\varrho}^{0} \int_{t+\theta}^{t} \dot{e}_i^T(s) R_1 \dot{e}_i(s)\, ds\, d\theta\, d\varrho \Bigg], \\ V_3(e(t)) & \\ = \sum_{i=1}^{N} \tau_0 & \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \left[e_i^T(s) Q_2 e_i(s) + \dot{e}_i^T(s) Q_3 \dot{e}_i(s) \right] ds\, d\theta \\ + \sum_{i=1}^{N} \overline{\tau}_m & \int_{-\tau_m}^{-\tau_0} \int_{t+\theta}^{t} \left[e_i^T(s) R_2 e_i(s) + \dot{e}_i^T(s) R_3 \dot{e}_i(s) \right] ds\, d\theta \end{aligned} \tag{A.2}$$

with $n \times n$ constant matrices $P > 0$, $P_i > 0$ $(i = 1, 2, 3, 4)$, $Q_j > 0$, and $R_j > 0$ $(j = 1, 2, 3)$ and $n \times n$ diagonal matrices $E = \mathrm{diag}(e_1, \ldots, e_n) > 0$ and $F = \mathrm{diag}(f_1, \ldots, f_n) > 0$ waiting to be determined.

Then, the time derivative of $V_i(e(t))$ $(i = 1,2)$ along system (8) can be directly computed as

$$\begin{aligned}\dot{V}_1(e(t)) = \sum_{i=1}^{N} \Big[& 2e_i^T(t) P\dot{e}_i(t) \\ & + f^T(e_i(t)) P_1 f(e_i(t)) \\ & - [1-\dot{\tau}(t)] f^T(e_i(t-\tau(t))) P_1 f(e_i(t-\tau(t))) \\ & + e_i^T(t) P_2 e_i(t) - e_i^T(t-\tau_0)(P_2-P_3) e_i(t-\tau_0) \\ & - [1-\dot{\tau}(t)] e_i^T(t-\tau(t))(P_3-P_4) e_i(t-\tau(t)) \\ & - e_i^T(t-\tau_m) P_4 e_i(t-\tau_m) \\ & + 2f^T(e_i(t))(E-F)\dot{e}_i(t) \\ & + 2e_i^T(t)(F\Sigma^+ - E\Sigma^-)\dot{e}_i(t)\Big],\end{aligned} \tag{A.3}$$

$$\begin{aligned}\dot{V}_2(e(t)) = \sum_{i=1}^{N} \Big[& \dot{e}_i^T(t)\left(\frac{\tau_0^4}{4}Q_1 + \frac{\delta_m^2}{4}R_1\right)\dot{e}_i(t) \\ & - \frac{\tau_0^2}{2}\int_{-\tau_0}^{0}\int_{t+\theta}^{t} \dot{e}_i^T(s) Q_1 \dot{e}_i(s)\,ds\,d\theta \\ & - \frac{\delta_m}{2}\int_{-\tau_m}^{-\tau_0}\int_{t+\theta}^{t} \dot{e}_i^T(s) R_1 \dot{e}_i(s)\,ds\,d\theta\Big].\end{aligned} \tag{A.4}$$

Through employing Lemmas 3 and 4 and $\begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} > 0$, we can estimate two terms in (A.4) as

$$\begin{aligned}& -\frac{\tau_0^2}{2}\int_{-\tau_0}^{0}\int_{t+\theta}^{t} \dot{e}_i^T(s) Q_1 \dot{e}_i(s)\,ds\,d\theta \\ & \leq \left[\int_{-\tau_0}^{t}\int_{t+\theta}^{t} \dot{e}_i(s)\,ds\,d\theta\right]^T R_1 \left[\int_{-\tau_0}^{t}\int_{t+\theta}^{t} \dot{e}_i(s)\,ds\,d\theta\right] \\ & = -\left[\tau_0 e_i(t) - \int_{t-\tau_0}^{t} e_i(s)\,ds\right]^T Q_1 \left[\tau_0 e_i(t) - \int_{t-\tau_0}^{t} e_i(s)\,ds\right], \\ & -\frac{\tau_m^2-\tau_0^2}{2}\int_{-\tau_m}^{-\tau_0}\int_{t+\theta}^{t} \dot{e}_i^T(s) R_1 \dot{e}_i(s)\,ds\,d\theta \\ & = -\frac{\tau_m^2-\tau_0^2}{2}\left[\frac{\tau^2(t)-\tau_0^2}{\tau^2(t)-\tau_0^2}\int_{-\tau(t)}^{-\tau_0}\int_{t+\theta}^{t} \dot{e}_i^T(s) R_1 \dot{e}_i(s)\,ds\,d\theta \right. \\ & \left. + \frac{\tau_m^2-\tau^2(t)}{\tau_m^2-\tau^2(t)}\int_{-\tau_m}^{-\tau(t)}\int_{t+\theta}^{t} \dot{e}_i^T(s) R_1 \dot{e}_i(s)\,ds\,d\theta\right] \\ & \leq -\left[[\tau(t)-\tau_0] e_i(t) - \int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds\right]^T \\ & \cdot R_1 \left[[\tau(t)-\tau_0] e_i(t) - \int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds\right] \\ & - \left[[\tau(t)-\tau_0] e_i(t) - \int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds\right]^T (2S_1) \\ & \cdot \left[[\tau_m-\tau(t)] e_i(t) - \int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds\right] \\ & - \left[[\tau_m-\tau(t)] e_i(t) - \int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds\right]^T \\ & \cdot R_1 \left[[\tau_m-\tau(t)] e_i(t) - \int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds\right] \\ & = -\begin{bmatrix} e_i(t) \\ \phi_i(t) \\ \psi_i(t)\end{bmatrix}^T \\ & \cdot \begin{bmatrix} \alpha(t) I_n & \beta(t) I_n \\ -I_n & 0 \\ 0 & -I_n \end{bmatrix} \begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \begin{bmatrix} \alpha(t) I_n & \beta(t) I_n \\ -I_n & 0 \\ 0 & -I_n \end{bmatrix}^T \\ & \cdot \begin{bmatrix} e_i(t) \\ \phi_i(t) \\ \psi_i(t)\end{bmatrix}\end{aligned} \tag{A.5}$$

with denoting

$$\begin{aligned}\alpha(t) &= \tau(t) - \tau_0, \\ \beta(t) &= \tau_m - \tau(t), \\ \phi_i(t) &= \int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds, \\ \psi_i(t) &= \int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds.\end{aligned} \tag{A.6}$$

Furthermore, together with Lemmas 3 and 4 and $\begin{bmatrix} R_i & S_i \\ * & R_i \end{bmatrix} > 0$ $(i = 2,3)$, we can compute

$$\begin{aligned}\dot{V}_3(e(t)) \leq \sum_{i=1}^{N} \Big\{ & e_i^T(t)\left(\tau_0^2 Q_2 + \overline{\tau}_m^2 R_2\right) e_i(t) + \dot{e}_i^T(t) \\ & \cdot \left(\tau_0^2 Q_3 + \overline{\tau}_m^2 R_3\right)\dot{e}_i(t) - \left(\int_{t-\tau_0}^{t} e_i(s)\,ds\right)^T \\ & \cdot Q_2 \left(\int_{t-\tau_0}^{t} e_i(s)\,ds\right) - \left(\int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds\right)^T \\ & \cdot R_2 \left(\int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds\right) - 2\left(\int_{t-\tau(t)}^{t-\tau_0} e_i(s)\,ds\right)^T \\ & \cdot S_2 \left(\int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds\right) - \left(\int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds\right)^T \\ & \cdot R_2 \left(\int_{t-\tau_m}^{t-\tau(t)} e_i(s)\,ds\right) - [e_i(t) - e_i(t-\tau_0)]^T\end{aligned}$$

$$\cdot Q_3 \left[e_i(t) - e_i(t-\tau_0)\right] - \left[e_i(t-\tau_0) - e_i(t-\tau(t))\right]^T \cdot R_3 \left[e_i(t-\tau_0) - e_i(t-\tau(t))\right] - 2\left[e_i(t-\tau_0) - e_i(t-\tau(t))\right]^T \cdot S_3 \left[e_i(t-\tau(t)) - e_i(t-\tau_m)\right] - \left[e_i(t-\tau(t)) - e_i(t-\tau_m)\right]^T \cdot R_3 \left[e_i(t-\tau(t)) - e_i(t-\tau_m)\right]\Bigg\}. \tag{A.7}$$

For any $n \times n$ constant matrices M_i $(i = 1, 2)$ and $N_j > 0$ $(j = 1, 2)$, it follows from (8) and Denotation 2 that

$$\begin{aligned} 0 = {} & 2\sum_{i=1}^{N} \left[e_i^T(t) M_1 + \dot{e}_i^T(t) M_2\right]\left[-\dot{e}_i(t) - Ce_i(t) + Af(e_i(t)) + Bf(e_i(t-\tau(t)))\right] - 2e^T(t) \\ & \cdot \left[(\Xi \otimes M_1 G) e(t) + (\Theta \otimes M_1 H) e(t-\tau(t))\right] - 2\dot{e}^T(t)\left[(\Xi \otimes M_2 G) e(t) + (\Theta \otimes M_2 H) e(t-\tau(t))\right] + 2e^T(t) \\ & \cdot \left[\left(L^1 \otimes M_1 G\right) e(t) + \left(L^2 \otimes M_1 H\right) e(t-\tau(t)) + \left(L^3 \otimes M_1 K\right)\left(\int_{t-\tau(t)}^{t} e(s)\,ds\right)\right] + 2\dot{e}^T(t) \\ & \cdot \left[\left(L^1 \otimes M_2 G\right) e(t) + \left(L^2 \otimes M_2 H\right) e(t-\tau(t)) + \left(L^3 \otimes M_2 K\right)\left(\int_{t-\tau(t)}^{t} e(s)\,ds\right)\right], \end{aligned}$$

$$\begin{aligned} 0 = {} & \sum_{i=1}^{N} \begin{bmatrix} \dot{e}_i(t) \\ e_i(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} \dot{e}_i(t) \\ e_i(t-\tau(t)) \end{bmatrix} - \begin{bmatrix} \dot{e}(t) \\ e(t-\tau(t)) \end{bmatrix}^T \\ & \cdot \begin{bmatrix} I_N \otimes N_1 & 0 \\ 0 & I_N \otimes N_2 \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ e(t-\tau(t)) \end{bmatrix}. \end{aligned} \tag{A.8}$$

Together with (**A3**) and any $n \times n$ diagonal matrices $U > 0$ and $V > 0$, one can easily derive

$$\begin{aligned} 0 \le {} & -\sum_{i=1}^{N} \Big\{\left[e_i^T(t) U \Sigma_1 e_i(t) - 2e_i^T(t) U \Sigma_2 f(e_i(t)) + f^T(e_i(t)) U f(e_i(t))\right] \\ & + \left[e_i^T(t-\tau(t)) V \Sigma_1 e_i(t-\tau(t)) - 2e_i^T(t-\tau(t)) V \Sigma_2 f(e_i(t-\tau(t))) + f^T(e_i(t-\tau(t))) V f(e_i(t-\tau(t)))\right]\Big\}. \end{aligned} \tag{A.9}$$

Then, through employing Denotations 1 and 2 and the right terms in (A.3), (A.4), (A.5), (A.7), (A.8), and (A.9), the term $\dot{V}(e(t))$ can be estimated as

$$\begin{aligned} \dot{V}(e(t)) \le {} & \sum_{i=1}^{N} \zeta_i^T(t)\left[\Omega + [\dot{\tau}(t) - \mu_0]\Gamma_1 \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix} \Gamma_1^T + [\mu_m - \dot{\tau}(t)]\Gamma_2 P_4 \Gamma_2^T\right]\zeta_i(t) + \xi^T(t)\Big[\Phi \\ & - \mathbf{IE}\left(I_N \otimes \Upsilon(t) \begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \Upsilon^T(t)\right)(\mathbf{IE})^T\Big]\xi(t), \end{aligned} \tag{A.10}$$

where $\Upsilon^T(t) = \left[\begin{smallmatrix} \alpha(t)I_n & -I_n & 0_n \\ \beta(t)I_n & 0_n & -I_n \end{smallmatrix}\right]$, the denotations Φ, Ω, $\mathbf{I}$, Γ_1, and Γ_2 are expressed in (17)-(18), and

$$\begin{aligned} \xi^T(t) &= \left[e^T(t) \;\; e^T(t-\tau(t)) \;\; \left(\int_{t-\tau_0}^{t} e(s)\,ds\right)^T \;\; \left(\int_{t-\tau(t)}^{t-\tau_0} e(s)\,ds\right)^T \;\; \left(\int_{t-\tau_m}^{t-\tau(t)} e(s)\,ds\right)^T \;\; \dot{e}^T(t)\right], \\ \zeta_i^T(t) &= \left[e_i^T(t) \;\; e_i^T(t-\tau_0) \;\; e_i^T(t-\tau(t)) \;\; e_i^T(t-\tau_m) \;\; f^T(e_i(t)) \;\; f^T(e_i(t-\tau(t))) \;\; \dot{e}_i^T(t)\right]. \end{aligned} \tag{A.11}$$

Now as for the terms in (A.10), if the two following inequalities

$$\Omega + (\dot{\tau}(t) - \mu_0)\Gamma_1 \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix} \Gamma_1^T + (\mu_m - \dot{\tau}(t))\Gamma_2 P_4 \Gamma_2^T < 0,$$

$$\Phi - \mathbf{IE}\left(I_N \otimes \Upsilon(t) \begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \Upsilon^T(t)\right)(\mathbf{IE})^T < 0 \tag{A.12}$$

can be true simultaneously, then we can obtain $\dot{V}(e(t)) < 0$ for any $e(t) \neq 0$.

Together with Lemma 5, the LMIs in (18) can make $\Omega + [\dot{\tau}(t) - \mu_0]\Gamma_1 \left[\begin{smallmatrix} P_1 & 0 \\ 0 & P_3 \end{smallmatrix}\right]\Gamma_1^T + [\mu_m - \dot{\tau}(t)]\Gamma_2 P_4 \Gamma_2^T < 0$ hold.

Meanwhile, with the denotations Υ_1 and Υ_2 in (17), the LMI results in (17) mean the following inequalities

$$\begin{aligned}&\Phi + \mathbf{IE}\left(I_N \otimes \Upsilon_1\right)\Pi^T + \Pi\left(I_N \otimes \Upsilon_1^T\right)(\mathbf{IE})^T < 0,\\ &\Phi + \mathbf{IE}\left(I_N \otimes \Upsilon_2\right)\Pi^T + \Pi\left(I_N \otimes \Upsilon_2^T\right)(\mathbf{IE})^T < 0\end{aligned} \quad (A.13)$$

to be true. Then, based on Lemma 5, the terms $\Phi + \mathbf{IE}(I_N \otimes \Upsilon_i)\Pi^T + \Pi(I_N \otimes \Upsilon_i^T)(\mathbf{IE})^T < 0$ $(i = 1, 2)$ in (A.13) can guarantee

$$\Phi + \left[\mathbf{IE}\left(I_N \otimes \Upsilon(t)\right)\right]\Pi^T + \Pi\left[\mathbf{IE}\left(I_N \otimes \Upsilon(t)\right)\right]^T < 0. \quad (A.14)$$

Furthermore, one can easily check

$$\begin{bmatrix}\Phi + \left[\mathbf{IE}\left(I_N \otimes \Upsilon(t)\right)\right]\Pi^T + \Pi\left[\mathbf{IE}\left(I_N \otimes \Upsilon(t)\right)\right]^T & \Pi_1 & \Pi_2\\ * & -I_N \otimes R_1 & -I_N \otimes S_1\\ * & * & -I_N \otimes R_1\end{bmatrix} < 0. \quad (A.15)$$

Then, based on Lemma 6, the matrix inequality in (A.15) can make the following term true:

$$\Phi - \left[\mathbf{IE}\left(I_N \otimes \Upsilon(t)\right)\right] \cdot \begin{bmatrix} I_N \otimes R_1 & I_N \otimes S_1\\ * & I_N \otimes R_1\end{bmatrix}\left[\mathbf{IE}\left(I_N \otimes \Upsilon(t)\right)\right]^T < 0. \quad (A.16)$$

Therefore, the LMIs in (17)-(18) can make $\dot{V}(e(t)) < 0$ for any $e(t) \neq 0$. Then, based on Lyapunov stability theory and Definition 2, the error system (8) is asymptotically stable; that is, the controlled networks (1) can achieve the desired cluster synchronization. □

Remark A.1. During the proof procedure in Theorem 7, one can easily check that, firstly, two triple integral Lyapunov terms in $V_2(e(t))$ could play an essential role in reducing the conservatism, and some tighter upper bounds on the derivatives were presented; secondly, we combined reciprocal convex technique with normal convex one to tackle time-delay issue; thirdly, through employing the Kronecker product in (A.10), (A.13), and (A.15), the forms of the inner coupling matrices G, H, and K can be described more generally than the ones in [30, 31].

Remark A.2. As for time-delay issue, most present works on delayed dynamical networks have only considered the case $\dot{\tau}(t) \leq \mu_m$. Thus, through replacing $\int_{t-\tau_m}^{t-\tau(t)} e_i^T(s)P_4e_i(s)ds$ by $\int_{t-\tau_m}^{t-\tau_0} e_i^T(s)P_4e_i(s)ds$ in (A.1) and using the similar proving procedure, we also can derive some relevant results.

Proof of Theorem 8. Based on assumptions (**A1**)–(**A3**), we can choose the Lyapunov-Krasovskii functional as

$$V(e(t)) = V_1(e(t)) + V_2(e(t)) + V_3(e(t)), \quad (A.17)$$

where

$$\begin{aligned}V_1(e(t)) = \sum_{i=1}^{N}\Bigg[& e_i^T(t)Pe_i(t) + \int_{t-\tau_0}^{t} e_i^T(s)P_1e_i(s)ds\\ &+ \int_{t-\tau(t)}^{t-\tau_0} e_i^T(s)P_2e_i(s)ds\\ &+ \int_{t-\tau_m}^{t-\tau(t)} e_i^T(s)P_3e_i(s)ds\\ &+ 2\sum_{j=1}^{n_1} e_j\int_0^{W_i^Te_j(t)}\left[f_j(s) - \sigma_j^- s\right]ds\\ &+ 2\sum_{j=1}^{n_1} f_j\int_0^{W_i^Te_j(t)}\left[\sigma_j^+ s - f_j(s)\right]ds\Bigg],\end{aligned} \quad (A.18)$$

with setting $n\times n$ diagonal matrices $E = \mathrm{diag}(e_1, \ldots, e_{n_1}) > 0$, $F = \mathrm{diag}(f_1, \ldots, f_{n_1}) > 0$, and $V_i(e(t))$ $(i = 2, 3)$ identical to the ones in Theorem 7. □

Remark A.3. Theorems 7 and 8 provide two less conservative criteria that ensure the cluster synchronization of network models (1) and (28) via pinning control, which can be easily checked by resorting to LMI in the Matlab Toolbox and do not require the inner coupling matrices to be of diagonal form. Moreover, as for $h = 1$, the derived theorems can be reduced to guarantee the pinning global synchronization.

Remark A.4. Presently, the reciprocal convex technique in [37] has been widely put forward to tackle time-delay systems, owing to the fact that it could reduce the conservatism more efficiently than some previous ones [23, 31]. Yet it comes to our attention that the reciprocal technique has not been utilized to tackle the pinning cluster synchronization on delayed complex networks and it has been fully considered in this work.

Competing Interests

The authors declare that they have no competing interests.

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