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Research Article

Multistage Warning Indicators of Concrete Dam under Influences of Random Factors

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Warning indicators are required for the real-time monitoring of the service conditions of dams to ensure safe and normal operations. Warnings are traditionally targeted at some "single point deformation" by deformation measuring points of concrete dam, and scientific warning theory on "overall deformation" measured is nonexistent. Furthermore, the influences of random factors are not considered. In this paper, the overall deformation of the dam was seen as a deformation system of single interactional observation points with different contribution degrees. The spatial deformation entropy, which describes the overall deformation, was established and the fuzziness indicator that measures the influence of complex random factors on monitoring values according to cloud theory was constructed. On this basis, multistage warning indicators of "spatial deformation" that consider fuzziness and randomness were determined. Analysis showed that the change law of information entropy of the dam' overall deformation is identical to the real change law of the dam; thus, it reflects the real deformation state of the dam. Moreover, the identified warning indicators improved the warning ability of concrete dams.

1. Introduction

Deformation is one of the major monitored items in dam safety. Concrete dams are exposed to influences of various nondeterministic settings such as the load effect of water level, uplift pressure, and wind waves caused by hydrologic and hydraulic uncertainties, as well as geological and material uncertainties such as shearing and compressive strength. Thus, a concrete dam is a complicated system of nondeterministic settings that are affected by various complex random factors [1, 2]. Considering a dam's long-term service, conducting timely and effective warning against emergencies through real-time monitoring is key to its safe operation [3].

The monitoring of dam safety is an important research subject in advanced mechanics and mathematics theories. In 1950, Tonini categorized the factors influencing the displacement of dam into water pressure, temperature, and effectiveness for a given period [4]. These factors were expressed in the polynomials of specific functions before a statistic model with regression analysis was established. Then, the deterministic model and mixed model were consulted for deformation of concrete dam and introduced finite element to monitor and evaluate the safety of dams [5]. Furthermore, many scholars brought new achievements in diagnosing dam safety from many aspects. In 2009, Gu and Wang established the catastrophe model of time-dependent component on the basis of catastrophe theory and proposed the method to determine the threshold value of the structure displacement of the dam [6]. On the basis of the POT model in extreme value theory, in 2012, Su et al. estimated the warning indicators by setting the threshold value and combining the probability of dam deformation with transfinite data sequence as the subject of modeling analysis [7]. Warning indicators are sure to have some fuzziness and randomness because of the influences of various complex random factors. On the basis of the fuzzy finite element, Chen (2006) realized the nondeterministic optimal control on roller compacted concrete dam [8]. Although the above-mentioned theories and methods complement and improve traditional methods in solving difficult problems in dam safety, such approaches only address the warning of "point" deformation and not the scientific "spatial and overall" deformation. Further studies on the deformation of concrete dams must consider the influences of complex random factors. Thus, the expression of a dam's overall deformation should be constructed and scientific and accurate warning indicators that consider randomness and fuzziness should be determined.

This paper began with an analysis of an indicator of fuzziness that affects the value of monitoring the dam by studying the influences of fuzziness and randomness from long-term service of the dam based on cloud theory [9–11]. Thereafter, the relationship between a dam's overall deformation and deformation of single observation points was analyzed through information entropy and synergetics. In the analysis, overall deformation refers to a system wherein single observation points with different contribution degrees (weights) influence one another. A criterion indicator measuring the overall deformation conditions was constructed. On this basis, multistage warning indicators for the overall deformation of concrete dam considering fuzziness and randomness were determined. Therefore, nondeterministic optimal control was achieved [12]. This paper concludes with a project case that verified the feasibility of the proposed theory.

2. Nondeterministic Optimal Control

Hydraulic engineering considers that some fuzziness and randomness in dams are inevitable because of the influences of various random factors such as the nondeterminacy of mechanical parameters, imposed load, and boundary conditions. Statistically, the smaller probability of dam displacement indicates that the dam is in a more dangerous state. $\mu + 3\sigma$ and $\mu - 3\sigma$ can be used as a warning indicator in one confidence coefficient of the dam if the displacement obeys the normal distribution of mean value and variance and is in the range of $\mu \pm 3\sigma$ in deterministic optimal control. In reality, given that dams are affected by random factors, warning indicators will have a range of variations. For example, in Figure 1, the warning value of upstream displacement has a maximum control value and minimum control value.

As the foundation of cloud theory, the cloud model is precisely both controlled and uncontrolled in microscope scale. U is the time series of monitoring the dam deformation, and C is the qualitative judgment on dam safety. If the quantitative value $x \in U$, $u(x)$ fall in $[0,1]$ and follow the probability distribution law:

$$\mu : U \longrightarrow [0,1], \quad \forall x \in U, \; x \longrightarrow \mu(x). \tag{1}$$

The distribution of x in U is called cloud and (x, u) is the cloud droplet.

In Figure 2, for observation point i, if the range wherein the cloud droplet falls is given, the upper bound and lower bound of the cloud droplet in the cloud model y_i^u and y_i^l can be expressed as follows:

$$\begin{aligned} y_i^u\left(x_i, Ex_i, En_i, He_i\right) &= e^{-(x_i - Ex_i)^2/2(En_i + 3He_i)^2}, \\ y_i^l\left(x_i, Ex_i, En_i, He_i\right) &= e^{-(x_i - Ex_i)^2/2(En_i - 3He_i)^2}. \end{aligned} \tag{2}$$

x_{ij} is the value j of observation point i, and the fuzziness Δ_{ij} can be calculated by

$$\Delta_{ij} = y_{ij}^u - y_{ij}^l. \tag{3}$$

In this equation, y_{ij}^u is the upper limit value of x_{ij} in the range and y_{ij}^l is the lower limit value of x_{ij} in the range.

Given the influence of random factors, when $x_{ij} \geq 0$, x_{ij} changes in the range of $[x_{ij} - x_{ij}\Delta_{ij}, x_{ij} + x_{ij}\Delta_{ij}]$; when $x_{ij} < 0$, it is in the range of $[x_{ij} + x_{ij}\Delta_{ij}, x_{ij} - x_{ij}\Delta_{ij}]$. When drawing up the warning indicator for downstream deformation, the maximum and minimum control values of the indicator can be determined when the significance level is α; thus, indicating that the nondeterministic optimal control has been achieved.

3. Methods of Characterizing Contributions of Single Observation Point

The overall deformation condition of the concrete dam is usually exposed to the influences of water pressure and temperature and is related to many factors including the physical and mechanical properties of dam materials, body structure, geology, and hydrology and it could be referred to in Figure 3. The principles of synergetics posit that a concrete dam is a synthesis of feature points with different contributions (weights) that influence one another. The contribution of a single observation point needs to be studied to construct a reasonable expression of overall deformation.

3.1. Construction of the Indicator Set of a Single Observation Point Weight.

Entropy [13–15], a basic concept in thermodynamics, refers to a state function in a system. The concept of information entropy is a measurement of the system's disorder and nondeterminacy [15]. The measured value j on observation point i is x_{ij} and its corresponding entropy is S_{ij}. According to entropy theory, when an observation point is in a more dangerous state, the system is in greater disorder and its entropy value is smaller. Thus, the following is obtained:

$$S_{ij} = -\left[\mu_{ij} \ln \mu_{ij} + \left(1 - \mu_{ij}\right) \ln\left(1 - \mu_{ij}\right)\right], \tag{4}$$

$$\mu_{ij} = \begin{cases} \displaystyle\int_{-\infty}^{x_{ij}} f(\varsigma)\, d\varsigma, & x_{ij} \geq 0 \\ \displaystyle\int_{x_{ij}}^{+\infty} f(\varsigma)\, d\varsigma, & x_{ij} < 0. \end{cases} \tag{5}$$

Formula (4) defines the information entropy of the measurement value. No matter what distribution $\{x_{ij}\}$ obeys, if the probability density function of the measured value is

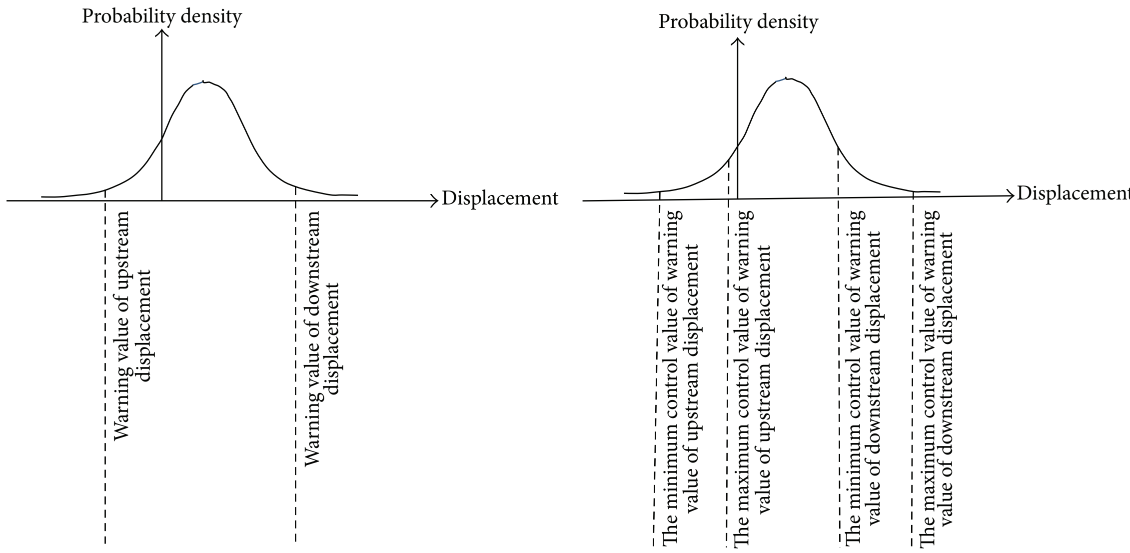

Figure 1: Diagram of deterministic optimal control and nondeterministic optimal control.

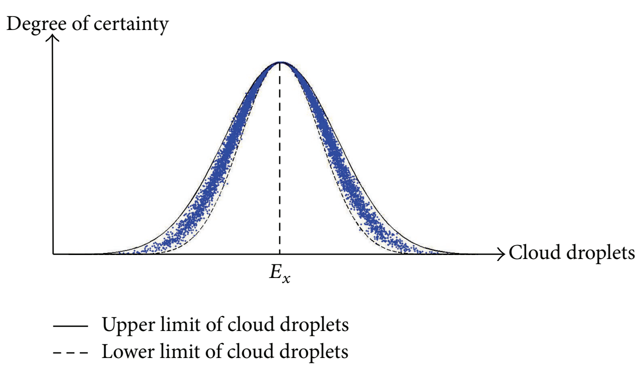

Figure 2: Diagram of the range where the cloud droplet falls.

known, the corresponding information entropy sequence can be calculated. Deformation of dam can be divided into three parts: water pressure component, temperature component, and aging component. Aging component comprehensively reflects the creep and plastic deformation of dam concrete and rock foundation and compression deformation of geological structure of rock foundation. At the same time, it also includes the irreversible displacement caused by the dam crack and the autogenous volume deformation. It changes dramatically in the early stage and gradually tends to be stable in the later stage. The project selected in this paper is a dam which has worked for many years, and its aging components tend to be stable, and the deformation value is stable in annual period rule, which obeys normal distribution.

The indicator measuring how much information is contained in x_{ij} is the inverse entropy $D_{ij} : D_{ij} = 1 - S_{ij}$. When one measured value reflects more information, the entropy value S_{ij} will be smaller and its inverse entropy D_{ij} will be greater. Thus, D_{ij} can be used to measure how much information is reflected by a single measured value.

Suppose the weight distribution of all observation points is $\{\omega_i \mid i = 1, 2, \ldots n\}$, where n is the number of points observed and ω_i will meet the following requirement: $\omega_i \geq 0$ and $\sum \omega_i = 1$. The entropy S_{ij} of the object matrix of the deformation measured value $\{x_{ij} \mid i = 1 \sim n,\ j = 1 \sim m\}$ can be computed through (4) and (5). The inverse entropy matrix is expressed as follows:

$$D_{ij} = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1m} \\ D_{21} & D_{22} & \cdots & D_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ D_{n1} & D_{n2} & \cdots & D_{nm} \end{bmatrix}. \tag{6}$$

In this matrix, D_{ij} is the inverse entropy of x_{ij}. The weight of the feature points was traced by the projection pursuit method.

3.2. Process of Calculating the Weight of a Single Observation Point. By using the projection pursuit method [16, 17], the high-dimensional data can be projected to low dimension space, and projection that reflects the structure or features of high-dimensional data is pursued to analyze high-dimensional data. This method is advantageous because it is highly objective, robust, resistant to interference, and accurate. The steps are as follows.

Step 1. The extreme value of the inverse entropy matrix was normalized through the following equation:

$$D_{ij}^{*} = \frac{D_{ij} - [D_j]_{\min}}{[D_j]_{\max} - [D_j]_{\min}}, \tag{7}$$

where $[D_j]_{\max}$ and $[D_j]_{\min}$ are the maximum and minimum values of line j in the matrix, respectively.

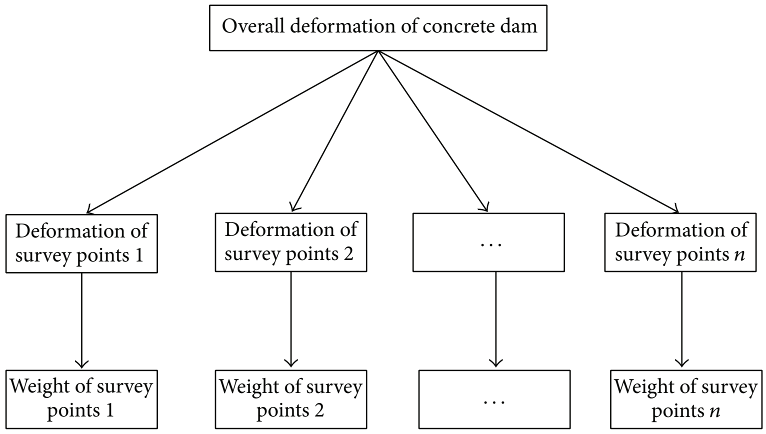

Figure 3: Diagram of overall deformation system of concrete dam.

Step 2. The normalized value D_{ij}^* was projected to unit direction P: $P = (p_1, p_2, \ldots, p_j)$ and $p_1^2 + p_2^2 + \cdots + p_j^2 = 1$. The indicator function of the projection $G(i)$ was constructed:

$$G(i) = \sum_{j=1}^{m} p_j D_{ij}^*, \quad (i = 1, 2, \ldots, n). \tag{8}$$

Step 3. The objective function of the projection was constructed. The best direction for the projection was estimated by solving the maximization problem of the objective function in the constraint condition:

$$\begin{gathered} \text{Objective function: Max} : H(p) = S_G \cdot Q_G, \\ \text{Constraint condition: } \sum_{j=1}^{m} p_j^2 = 1. \end{gathered} \tag{9}$$

In this equation, S_G is the divergent degree of the projection. Q_G is the local density of 1D data points along P and is expressed as follows:

$$\begin{gathered} S_G = \left[\frac{\sum_{i=1}^{n} (G(i) - \overline{g}(i))^2}{n-1} \right]^{0.5} \\ Q_G = \sum_{i=1}^{n} \sum_{j=1}^{n} (R - r_{ij}) \cdot f(R - r_{ij}), \end{gathered} \tag{10}$$

where $\overline{g}(i)$ is the mean value of this sequence, R is the window radius of the local density and $R = 0.1 S_G$ in this paper, r_{ij} is the distance between two projection values, and $f(t)$ is the unit step function. $f(t)$ is equal to 1 as t is greater than 0. Otherwise, $f(t)$ is equal to 0.

Step 4. The projection value of one sample point was computed by substituting the best direction P^* into (8); ω_i can be calculated by normalizing the projection value:

$$\omega_i = \frac{G^*(i)}{\sum_{j=1}^{n} G^*(j)}, \quad i = 1, 2, \ldots, n. \tag{11}$$

4. Study on Equivalent Model of Dam's Overall Deformation

The overall deformation of a dam can be considered a deformation system of feature points with different contributions that influence one another, as well as observation points of the deformation as feature points. The deformation condition was analyzed systemically, and the overall deformation was expressed by the evolution of equation of all feature points. The deformation condition was described qualitatively by the tectonic type of information entropy. The absolute value of the information entropy measures the danger level of the deformation value. Smaller absolute value means higher danger level. The positive and negative values indicate the direction of the deformation. A positive value corresponds to downstream deformation, whereas a negative value corresponds to upstream deformation. The influences of random factors were considered and the fuzzy information entropy was constructed.

4.1. Constructing Fuzzy Information Entropy of Single Measured Value. The downstream deformation is positive, whereas the upstream deformation is negative.

When the observation point moves downstream, make $\mu_{ij} = \int_{-\infty}^{x_{ij}} f(\varsigma) d\varsigma$ and according to the definition of information entropy, the information entropy of x_{ij} is expressed as (4).

Considering the influence of Δ_{ij}, μ_{ij} will float in $[\mu_{ij}^0, \mu_{ij}^1]$; the following is then obtained:

$$\begin{gathered} \mu_{ij}^0 = \int_{-\infty}^{x_{ij} - x_{ij}\Delta_{ij}} f(\varsigma)\, d\varsigma, \\ \mu_{ij}^1 = \int_{-\infty}^{x_{ij} + x_{ij}\Delta_{ij}} f(\varsigma)\, d\varsigma. \end{gathered} \tag{12}$$

When the observation point moves upstream, make $\mu_{ij} = \int_{x_{ij}}^{+\infty} f(\varsigma) d\varsigma$; the information entropy of x_{ij} is defined as follows:

$$S_{ij} = \mu_{ij} \ln \mu_{ij} + (1 - \mu_{ij}) \ln (1 - \mu_{ij}). \tag{13}$$

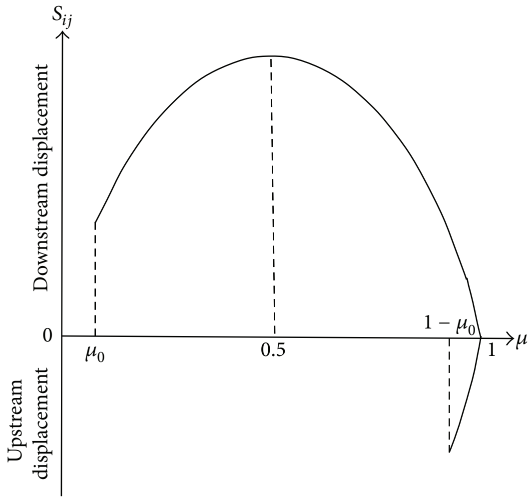

Figure 4: Diagram of S_{ij} changes with the change of μ_{ij} if $E_x \geq 0$.

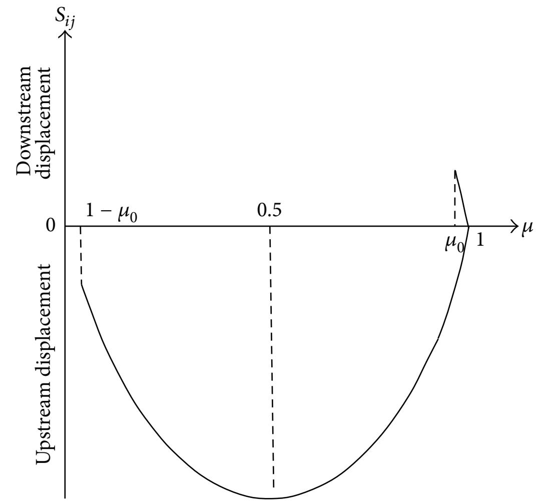

Figure 5: Diagram of S_{ij} changes with the change of μ_{ij} if $E_x < 0$.

Considering the influence of Δ_{ij}, μ_{ij} floats in $[\mu_{ij}^0, \mu_{ij}^1]$. The following is then obtained:

$$\mu_{ij}^0 = \int_{x_{ij}-x_{ij}\Delta_{ij}}^{+\infty} f(\varsigma)\, d\varsigma$$
$$\mu_{ij}^1 = \int_{x_{ij}+x_{ij}\Delta_{ij}}^{+\infty} f(\varsigma)\, d\varsigma. \tag{14}$$

Considering the influences of random factors, the information entropy of $x_{ij}-S_{ij}$ floats in $[S_{ij}^0, S_{ij}^1]$, which was defined as the fuzzy information entropy of x_{ij}.

Take the downstream as an example. Influenced by determined and random factors, the fuzzy entropy of S_{ij} changes into the range of S_{ij} in the range of $[\mu_{ij}^0, \mu_{ij}^1]$.

4.2. Methods for Determining the Range of the Information Entropy of Single Measured Value. For Figure 4, the expectation of one deformation monitoring sequence sample at one observation point ($E_x \geq 0$, S_{ij}) changes with the change of μ_{ij}, as shown in Figure 4, where $\mu_0 = \int_{-\infty}^{0} f(\varsigma)d\varsigma$.

When the dam deforms downstream, the change law of S_{ij} is as follows: S_{ij} will increase with increasing μ_{ij} when μ_{ij} is in the range of $(\mu_0, 0.5)$; S_{ij} will decrease with decreasing μ_{ij} when μ_{ij} is in the range of $(0.5, 1)$; when $\mu_{ij} = 0.5$, S_{ij} will reach the maximum value. If 0.5 is in the range of $[\mu_{ij}^0, \mu_{ij}^1]$, the maximum of S_{ij} is S_{ij}^1 when $\mu_{ij} = 0.5$ and its minimum value is S_{ij}^0 at the endpoint. If 0.5 is not in the range of $[\mu_{ij}^0, \mu_{ij}^1]$, S_{ij} will have its maximum value S_{ij}^1 and minimum value S_{ij}^0 at endpoints. When the dam deforms upstream, S_{ij} will rise with the rise of μ_{ij} and it will have its maximum value S_{ij}^1 and minimum value S_{ij}^0 at endpoints.

The expectation of one deformation monitoring sequence sample at one observation point ($E_x < 0$, S_{ij}) changes with the change of μ_{ij} as shown in Figure 5, where $\mu_0 = \int_{-\infty}^{0} f(\varsigma)d\varsigma$ and Figure 5 is presented for S_{ij} changes with the change of μ_{ij}.

When the dam deforms downstream, the change law of S_{ij} is as follows: S_{ij} will increase with decreasing μ_{ij} and will have its maximum value S_{ij}^1 and minimum value S_{ij}^0 at endpoints. When the dam deforms upstream, the figure shows that when μ_{ij} is in the range of $(1-\mu_0, 0.5)$, S_{ij} will increase with the drop of μ_{ij}; when μ_{ij} is in the range of $(0.5, 1)$, S_{ij} will decrease with the increase of μ_{ij}; when $\mu_{ij} = 0.5$, S_{ij} will reach the minimum value. If 0.5 is contained in the range of $[\mu_{ij}^0, \mu_{ij}^1]$, S_{ij} will have its minimum value S_{ij}^0 at $\mu_{ij} = 0.5$ and have its maximum value S_{ij}^1 at endpoint; if 0.5 is not contained in the range, S_{ij} will have its maximum value S_{ij}^1 and minimum value S_{ij}^0 at endpoints.

4.3. Construction of the Fuzzy Information Entropy of Overall Deformation. On the basis of the above results, the expression of information entropy of overall deformation can be deduced. The contribution of the order degree of observation point i is $\omega_i\mu_{ij}$; make $\mu_{ij}^1 = \mu_{ij}$ and $\mu_{ij}^2 = 1-\mu_{ij}$. According to the broad definition of information entropy, when the dam move deforms downstream, the expression of information entropy of overall deformation is expressed as follows:

$$\begin{aligned} S_j &= -\sum_{i=1}^{m}\sum_{k=1}^{2}\omega_i\mu_{ij}^k \ln\left(\omega_i\mu_{ij}^k\right) \\ &= -\sum_{i=1}^{n}\sum_{k=1}^{2}\omega_i\mu_{ij}^k\left(\ln\omega_i + \ln\mu_{ij}^k\right) \\ &= -\sum_{i=1}^{n}\sum_{k=1}^{2}\omega_i\mu_{ij}^k\ln\omega_i - \sum_{i=1}^{n}\sum_{k=1}^{2}\omega_i\mu_{ij}^k\ln\mu_{ij}^k \end{aligned}$$

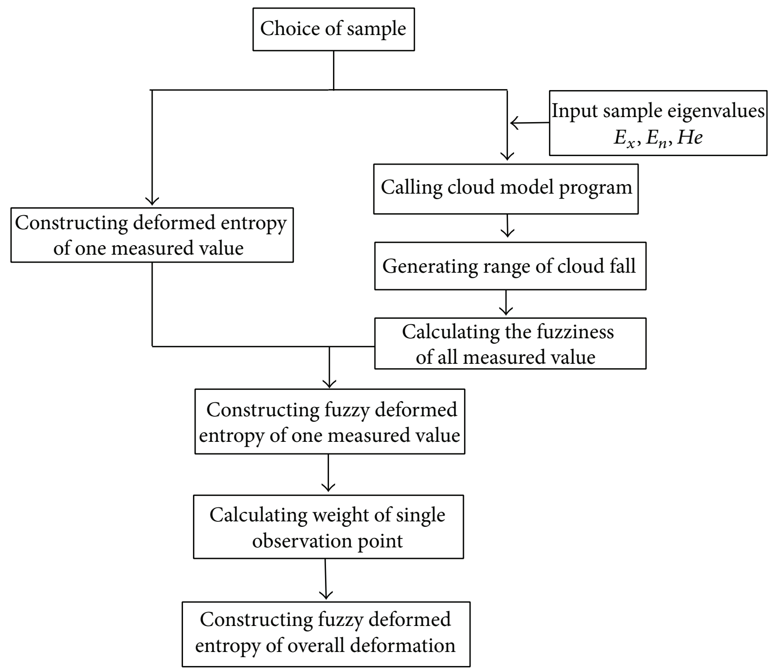

FIGURE 6: Computational process of fuzzy information entropy of overall deformation.

$$
\begin{aligned}
&= -\sum_{i=1}^{n} \omega_i \ln \omega_i \sum_{k=1}^{2} \mu_{ij}^{k} - \sum_{i=1}^{n} \omega_i \sum_{k=1}^{2} \mu_{ij}^{k} \ln \mu_{ij}^{k} \\
&= -\sum_{i=1}^{n} \omega_i \ln \omega_i + \sum_{i=1}^{n} \omega_i S_{ij}.
\end{aligned} \tag{15}
$$

When the dam move deforms upstream, the expression of the information entropy of overall deformation is

$$
S_j = \sum_{i=1}^{m} \sum_{k=1}^{2} \omega_i \mu_{ij}^{k} \ln\left(\omega_i \mu_{ij}^{k}\right) = \sum_{i=1}^{n} \omega_i \ln \omega_i + \sum_{i=1}^{n} \omega_i S_{ij}. \tag{16}
$$

Therefore, the expression of information entropy of overall deformation is defined as follows:

$$
S_j = \begin{cases} -\sum_{i=1}^{n} \omega_i \ln \omega_i + \sum_{i=1}^{n} \omega_i S_{ij}, & S_{ij} \geq 0 \\ \sum_{i=1}^{n} \omega_i \ln \omega_i + \sum_{i=1}^{n} \omega_i S_{ij}, & S_{ij} < 0. \end{cases} \tag{17}
$$

The absolute value of the information entropy of the overall deformation measures the danger level of the deformation; that is, a smaller absolute value means a higher danger level; positive and negative values stand for the direction of the deformation: a positive value means downstream deformation, whereas a negative value means upstream deformation.

Considering the influences of random factors, the fuzzy information entropy of x_{ij} is $[S_{ij}^0, S_{ij}^1]$; the fuzzy information entropy of overall deformation can be illustrated through (17). Computational process of fuzzy information entropy of overall deformation is shown in Figure 6.

5. Proposed Multistage Fuzzy Information Entropy of Overall Deformation Warning Indicators

Horizontal displacement of dam crest changes in an annual cycle: "upstream and downstream switch." Therefore, this displacement should be in a certain scale and be controlled under some monitoring indicators for the safe dam operation.

In the case of downstream displacement, the primary fuzzy warning indicator δ_1' is defined as $\delta_1' = (\delta_1^0, \delta_1^1)$. δ_1^0 is the lower limit of this indicator, and δ_1^1 is the upper limit; the secondary indicator δ_2' is $\delta_2' = (\delta_2^0, \delta_2^1)$, where δ_2^0 is the lower limit of this indicator and δ_2^1 is the upper limit. When $\delta_1^1 > \delta_2^0$, a cross phenomenon appears in the primary indicator and secondary indicator, when both of them should be categorized according to the membership of displacement measured. δ^* was introduced because the membership of displacement at this point is the same. Figure 7 shows diagram of multistage fuzzy information entropy warning indicators.

The primary fuzzy warning indicator is $\delta_1' = (\delta_1^0, \delta^*)$, and the secondary is $\delta_2' = (\delta^*, \delta_2^1)$.

If the deformation value is in (δ_1^0, δ_1^1) or (δ_1^0, δ^*), the dam is in the state of primary warning; if the value is in (δ_2^0, δ_2^1) or (δ_2^0, δ^*), the dam is in the state of secondary warning.

The time sequence of deformation at each observation point was analyzed by using the above theoretical method. The lower and upper limits of the fuzzy information entropy of overall deformation affected by Δ_{ij} will be $\{S_j^0\}$ and $\{S_j^1\}$. Considering the dam's long-term service, when the dam moves downstream, the lower limit $\{S_{mj}^0\}$ and upper limit

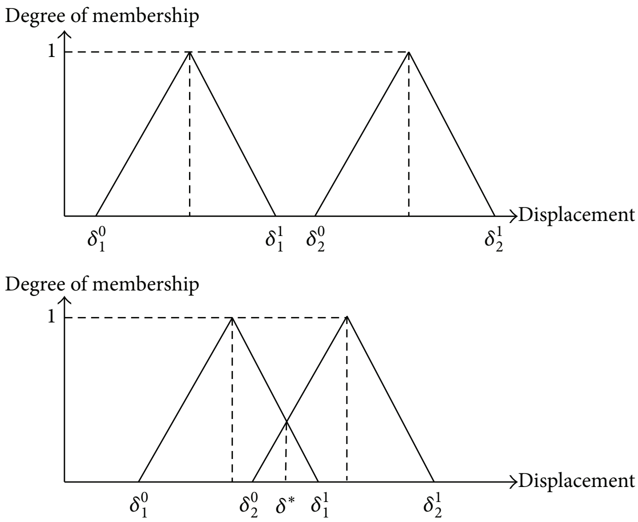

FIGURE 7: Diagram of multistage fuzzy information entropy warning indicators.

$\{S_{mj}^1\}$ were selected and when the dam moves upstream, the lower limit $\{R_{mj}^0\}$ and upper limit $\{R_{mj}^1\}$ were selected. $\{S_{mj}^0\}$, $\{S_{mj}^1\}$, $\{R_{mj}^0\}$, and $\{R_{mj}^1\}$ are random variables, and four subsample spaces with the sample size of N can be obtained by the following:

$$\begin{aligned} S^0 &= \left\{S_{m1}^0, S_{m2}^0, \ldots, S_{mm}^0\right\} \\ S^1 &= \left\{S_{m1}^1, S_{m2}^1, \ldots, S_{mm}^1\right\} \\ R^0 &= \left\{R_{m1}^0, R_{m2}^0, \ldots, R_{mm}^0\right\} \\ R^1 &= \left\{R_{m1}^1, R_{m2}^1, \ldots, R_{mm}^1\right\}. \end{aligned} \tag{18}$$

Shapiro-Wilk test and Kolmogorov-Smirnov test can both test whether the samples obey normal distribution or not. But the Kolmogorov-Smirnov test is applicable to fewer samples. It can not only test if the samples are subject to normal distribution, but also test if samples are subject to other distributions. The basic idea of the K-S test is to compare the cumulative frequency of the observed value ($F_n(x)$) with the assumed theoretical probability distribution ($F_x(x)$) to construct statistics.

According to the method of empirical distribution function, segmented cumulative frequency is obtained by using the following formula:

$$F_n(x) = \begin{cases} 0, & x < x_i \\ \dfrac{i}{n}, & x_i \le x < x_{i+1} \\ 1, & x \ge x. \end{cases} \tag{19}$$

In the formula, $x_1, x_2, \ldots, x_n$ is sample data after arrangement. The sample size is n.

In the full range of random variable X, the maximum difference between $F_n(x)$ and $F_x(x)$ is

$$D_n = \max\left|F_x(x) - F_n(x)\right| < D_n^\beta. \tag{20}$$

In the formula, D_n is a random variable whose distribution depends on n. D_n^β is critical value for a significant level β. It is considered that the distribution to be used at a significant level β cannot be resisted; otherwise, it should be rejected.

The distribution form was tested through the K-S method to determine the probability density function. Fuzzy warning indicator was then determined with different significant level. In dam safety evaluation, significant level α is the probability of the dam failure. Supposing S_m is the extreme of the information entropy of the upstream overall deformation, if $S > S_m$, the probability of the dam failure is $P(S > S_m) = \alpha = \int_{S_m}^{\infty} f(x)dx$ and the reliability index of dam failure is $1 - \alpha$. According to the dam importance, different failure probability is set and the multistage warning indicators were identified.

6. Example Analysis

6.1. Project Profile. One flat-slab deck dam built with reinforced concrete is an important part of one river basin cascade exploitation. The elevation of this dam crest is 137.70 m, and the height of biggest part is about 43 m; the crest runs 225.0 m in length and is made of 27 flat-slab buttresses with a span of 7.5 m. The space between the left side of 2# buttress and the right side of 9# buttress is the joint part; the overflow buttress is located from the 9# buttress to the 20# buttress; the rest is the water-retaining buttress. The workshop buttress is located from the 5# buttress to the 8# buttress. In this dam, the level of dead water is 122.0 m, the normal high water level is 131.0 m (in practice, it is 129.0 m), the design flood level is 136.7 m, and the check flood level is 137.5 m. To monitor the displacement of this dam, a direct plumb line and an inverted plumb line were arranged in four buttresses: 4#, 9#, 21#, and 24#. There is a crushed zone under the dam foundation where occurrence is N20°~25°W, SW∠70°~80°, the maximum width is about 3 m, and the narrowest place is about 1 m. There is an elevation clip joint mud at level 91 m.

The deformation field characterized by the observation point at 21# should be typical and is a key point because it is in the riverbed. Thus, the observation point G21 at the height of 134 m along the direct plumb line at 21# was analyzed, as well as point 27 at the height of 118 m along this line and point 28 at the height of 107 m. All these values measured were transformed into absolute displacement. The arrangement of each point is shown in Figure 8. The daily monitoring data series is from January 1, 2003, to December 31, 2013.

In this paper, two-stage warning indicators were set according to the practical running of this project and danger: $\alpha = 5\%$ is the primary warning that is mainly used to discriminate and handle early dangerous case and the reliability index of dam failure is 95%, whereas $\alpha = 1\%$ is the secondary warning that is mainly used to determine grave danger and prevent urgent danger and the reliability index of dam failure is 99%.

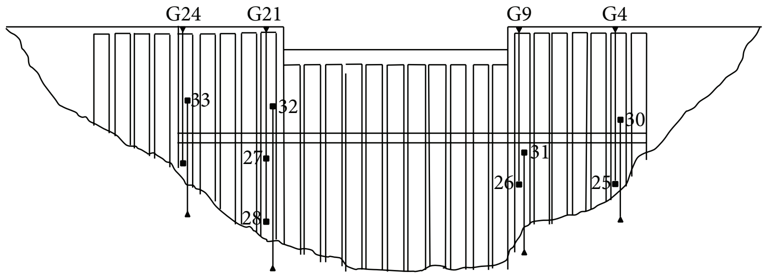

Figure 8: The arrangement of observation point.

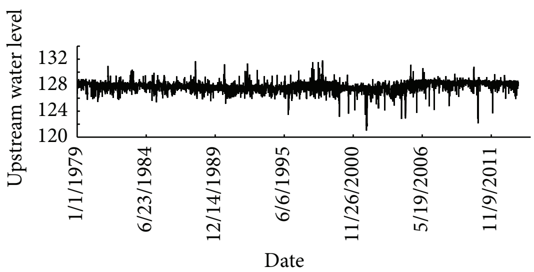

Figure 9: The process line of upstream water level.

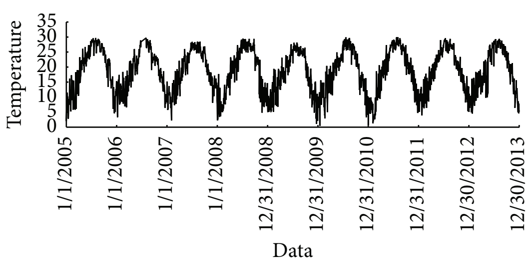

Figure 10: The process line of temperature.

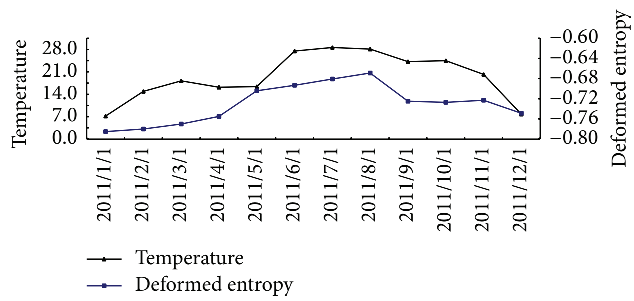

Figure 11: The relationship between information entropy of overall deformation and temperature in 2011.

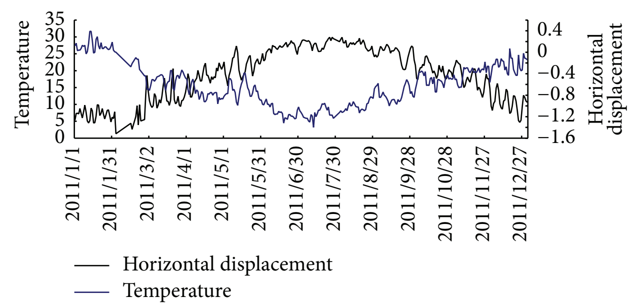

Figure 12: The relationship between horizontal displacement and temperature of observation point G21 in 2011.

6.2. Calculating the Contribution of Deformation at the Observation Point to the Overall Deformation. Figures 9-10 show the upstream stage hydrograph and temperature stage hydrograph, respectively. The water stage remained unchanged, whereas the temperature changed in the annual cycle. Figure 11 shows the relationship between information entropy of overall deformation and temperature in 2007. A negative correlation exists between the overall deformation of 21# buttress and temperature; that is, an increase in temperature corresponds to the decrease in upstream or downstream displacement and a decrease in temperature means an increase in the upstream or downstream displacement. Figure 12 shows the correlation between the displacement at observation point G21 and the temperature. The overall deformation law is roughly identical with that at observation point G21.

Figure 12 reveals that the temperature obviously influenced overall deformation; that is, the temperature can change the contribution of single observation point to the overall deformation. Temperature change was divided into the stage of temperature rise and stage of temperature drop. The weight of deformation at observation point in two stages was calculated. The results are shown in the Table 1.

6.3. Results of Information Entropy of Overall Deformation. Range of cloud fall of the displacement at G21, 27, and 28 is shown in Figure 13. The boundary values of most dangerous information entropy from 2003 to 2013 are shown in Tables 2–5. The absolute value means the degree of danger; downstream is set as negative value. In the significance level $\alpha = 0.05$, 7 kinds of common distribution (lognormal distribution, normal distribution, uniform distribution, triangular distribution, exponential distribution, γ distribution, and β distribution) were used to carry on hypothesis test for $\{S_{mj}^{0}\}$, $\{S_{mj}^{1}\}$, $\{R_{mj}^{0}\}$, and $\{R_{mj}^{1}\}$ with K-S method. The maximum D_n of each sequence was obtained and compared with the critical value $D_n^{0.05}$ of the significant level 0.05 to determine the type

Table 1: The weight table of observation point.

Observation point	Altitude	The period of temperature rise	The period of temperature drop
G21	134 m	0.357	0.392
27	118 m	0.331	0.316
28	107 m	0.312	0.292

Table 2: The lower limit of the most dangerous information entropy of downstream overall deformation.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Information entropy	0.4784	0.4837	0.4767	0.4784	0.4969	0.4887	0.4969	0.5102	0.5082	0.4799	0.4739

Table 3: The upper limit of the most dangerous information entropy of downstream overall deformation.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Information entropy	0.4791	0.4845	0.4770	0.4786	0.4971	0.4899	0.4974	0.5186	0.5161	0.4817	0.4769

Table 4: The lower limit of the most dangerous information entropy of upstream overall deformation.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Information entropy	−0.5433	−0.5491	−0.4773	−0.5083	−0.5067	−0.4784	−0.5080	−0.4839	−0.4836	−0.4825	−0.5020

Table 5: The lower limit of the most dangerous information entropy of upstream overall deformation.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Information entropy	−0.5339	−0.5440	−0.4770	−0.5079	−0.5055	−0.4770	−0.5040	−0.4822	−0.4806	−0.4754	−0.4766

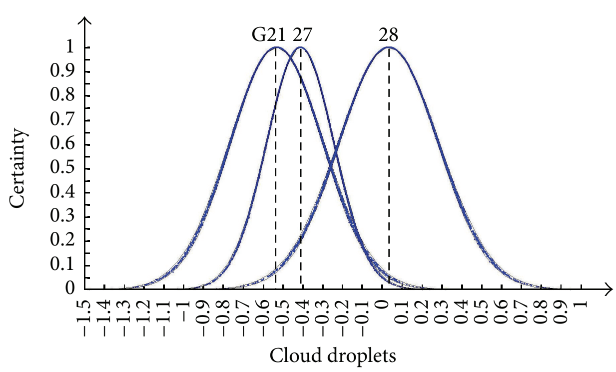

Figure 13: Range of cloud fall of the displacement at G21, 27, and 28.

of the best distribution. K-S test results are shown in Table 6. Multistage fuzzy warning values are presented in Table 7.

K-S test shows that $\{S^0_{mj}\}$, $\{S^1_{mj}\}$, $\{R^0_{mj}\}$, and $\{R^1_{mj}\}$ satisfy normal distribution.

The probability density function of the sequence is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{\sigma}\right). \quad (21)$$

Parameter values of (21) are presented in Table 7.

In downstream deformation, if $\alpha = 5\%$, the primary warning indicator is $(0.4763, 0.4775)$; if $\alpha = 1\%$, the secondary warning indicator is $(0.4757, 0.4763)$. δ^* is used as the boundary value when two indicators overlap. In the case of upstream deformation, if $\alpha = 5\%$, the primary warning indicator is $(-0.4779, -0.4768)$; if $\alpha = 1\%$, the secondary warning indicator is $(-0.4768, -0.4763)$ (Table 8).

6.4. The Structure Calculation of Monitoring Index of Dam Horizontal Displacement. According to the actual situation, three-dimensional finite element model of the dam is established. According to the structure and basic geological conditions of 21# dam section, the scope of finite element calculation model can be got as follows: taking 1.5 times as high dam in the upstream direction, taking 1.5 times as high dam in the upstream direction, and taking 1 time as high dam below the dam foundation. The model is constituted of 11445 nodes and 8571 units. The unit type is 6 sides 8 nodes isoparametric element. The deformation observation data analysis shows the dam under the condition of low temperature and high water level: there is larger displacement in the downstream when in high temperature, and in low water level, there is larger displacement in the upstream (Table 9, Figure 14). In view of the actual situation of the dam observation and data analysis results, the load condition selection is as follows (Table 10, Figures 15 and 16):

Working condition 1: normal water level 129.0 m and maximum temperature drop.

Working condition 2: dead water level 122.0 m and maximum temperature rise.

Table 6: K-S test results.

Probability distribution	S_{mj}^0	S_{mj}^1	R_{mj}^0	R_{mj}^1
Lognormal distribution	0.26	0.11	0.23	0.28
Normal distribution	0.11	0.08	0.26	0.22
Uniform distribution	0.74	1.55	0.88	0.68
Triangular distribution	0.53	0.54	0.59	0.61
Exponential distribution	0.41	0.42	0.55	0.35
γ distribution	0.37	0.36	0.31	0.33
β distribution	0.68	0.73	0.86	0.63
$D_n^{0.05}$	0.34	0.34	0.29	0.29
The most reasonable probability distribution	Normal distribution	Normal distribution	Normal distribution	Normal distribution

Table 7: Parameter values of the probability density function.

Data series	Parameter values	
	μ	σ^2
$\{S_{mj}^0\}$	0.488355	0.0129
$\{S_{mj}^1\}$	0.490627	0.0151
$\{R_{mj}^0\}$	−0.50210	0.0249
$\{R_{mj}^1\}$	−0.49674	0.0245

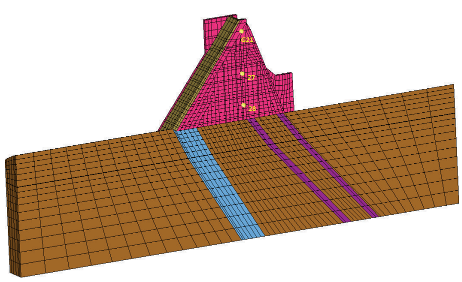

Figure 14: Finite element model of the dam.

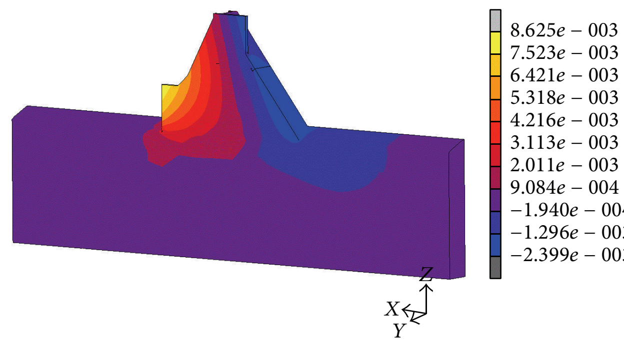

Figure 15: The results of finite element calculation of working condition 1.

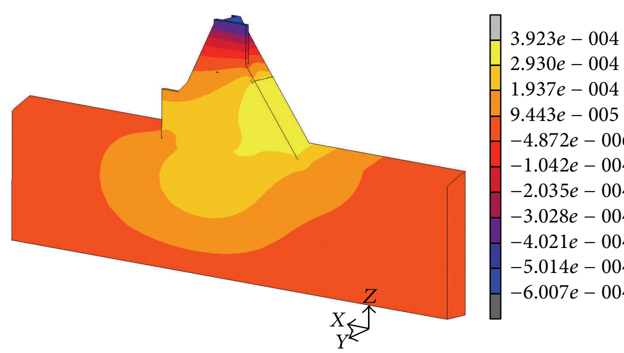

Figure 16: The results of finite element calculation of working condition 2.

The primary warning indicators of concrete dam were obtained by calculation methods for structures. If the information entropy of the overall deformation reached 0.4770, the dam moved downstream and in the state of primary warning. If the information entropy of the overall deformation reached −0.4773, the dam moved upstream and in the state of primary warning (Table 11). They all fell into intervals calculated by fuzzy methodology. The analysis shows that the method brought up in this paper is reasonable and scientific. Also, the analysis shows the physical meaning of the fuzzy warning index. Under the action of the unfavorable load combination and the influence of the complex random factors, the maximum entropy and minimum information entropy of the overall deformation lie in this interval. Considering the influences of random factors, multistage fuzzy warning values were receptive and safe.

In this paper, the overall deformation of 21# buttress was analyzed through the theoretical method proposed. Influences of random factors on the warning value were considered and multistage fuzzy warning values were determined. If the information entropy of overall deformation is in (0.4763, 0.4775), the dam moved downstream and in the state of primary warning. If the information entropy of overall deformation is in (0.4757, 0.4763), the dam moved downstream and in the state of secondary warning. If the information entropy of overall deformation is in (−0.4779, −0.4768), the dam moved upstream and in the state of primary warning. If the information entropy of the overall deformation is in (−0.4768, −0.4763), the dam moved upstream and in the state of secondary warning.

7. Conclusion

This paper presented multistage warning indicators of concrete dam space and considered the influences of complex

Table 8: Multistage fuzzy warning values of concrete dam.

The direction of deformation	Confidence level	
	The primary warning indicator	The secondary warning indicator
Downstream	(0.4763, 0.4775)	(0.4757, 0.4763)
Upstream	(−0.4779, −0.4768)	(−0.4768, −0.4763)

Table 9: Material parameter of the dam.

Structure	Density (kg/m)	Poisson's ratio	Elastic modulus (GPa)
Concrete face slab	2400	0.167	24
Buttress	2400	0.167	24
Partition wall	2400	0.167	24
Reinforced concrete block	2400	0.160	22
Foundation rock mass	2700	0.175	12
Diorite-dyke	2000	0.3	1.15
Crushed zone	2000	0.3	0.29
Horizontal joints	2000	0.3	0.7

Table 10: The results of finite element calculation of displacement of observation point (mm).

Observation point	Working condition 1	Working condition 2
G21	0.785	−0.599
27	0.654	−0.386
28	0.356	0.114

Table 11: The primary warning indicators of concrete dam.

The direction of deformation	The primary warning indicator
Downstream	0.4770
Upstream	−0.4773

random factors. The results of the specific studies are as follows:

(1) Influences of fuzziness and randomness of random factors on the long-term service of dam were discussed; a fuzziness indicator that measures the influence of random factors on monitoring value was constructed through cloud model.

(2) Equivalent model of overall deformation was proposed. In the model, the overall deformation of dam was regarded as a deformation system where each observation point had different contributions (weights) and affected one another. Based on entropy theory, a space information entropy that can measure the overall deformation condition was established.

(3) Multistage warning indicators against overall deformation of concrete dam under the influences of fuzziness and randomness were determined and nondeterministic optimal control of the indicator was achieved to improve the competence of warning against the deformation of concrete dam.

Competing Interests

No conflict of interests exits in the submission of this paper.

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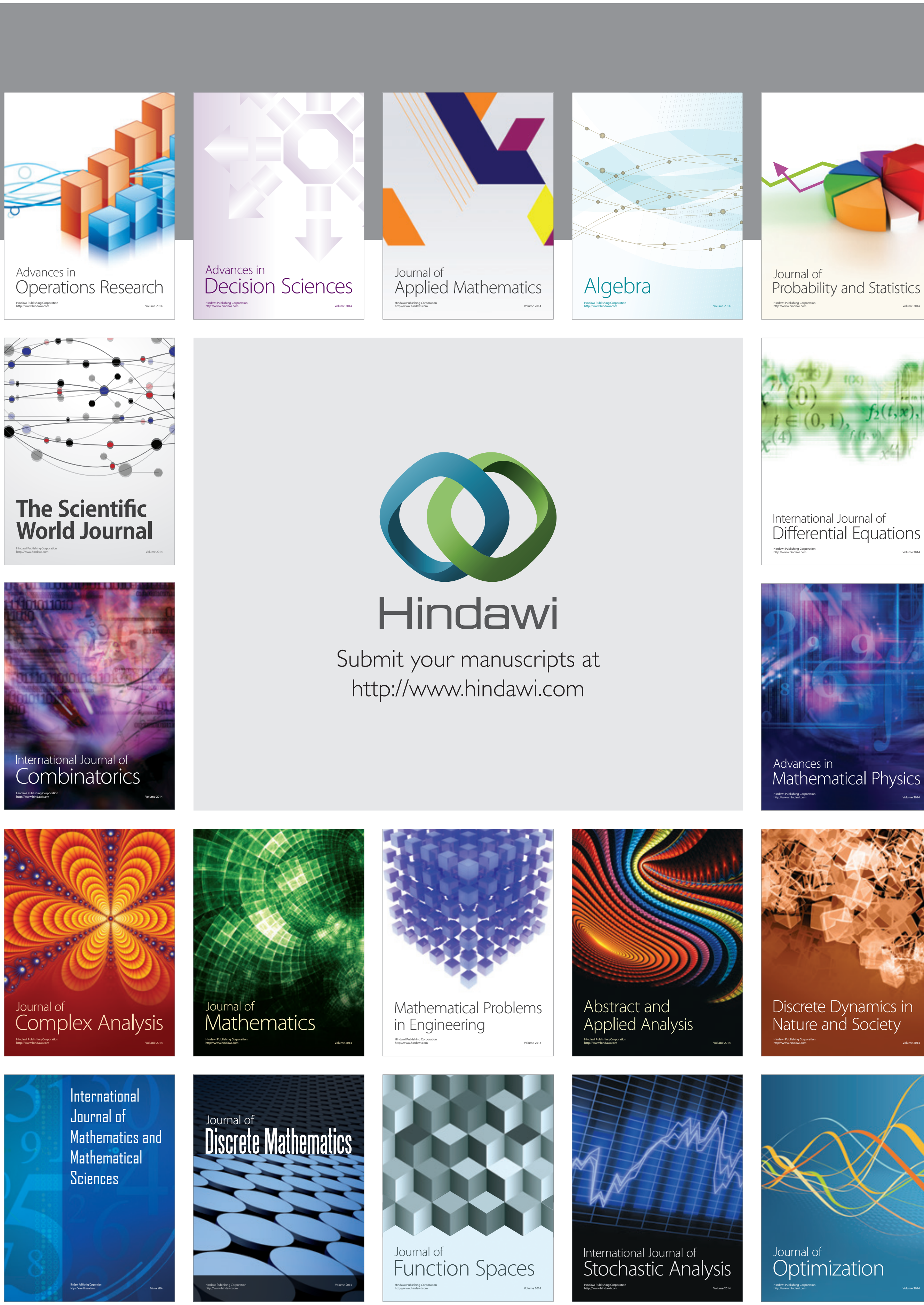
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