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Research Article

Stability Analysis of Anchored Soil Slope Based on Finite Element Limit Equilibrium Method

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Under the condition of the plane strain, finite element limit equilibrium method is used to study some key problems of stability analysis for anchored slope. The definition of safe factor in slices method is generalized into FEM. The "true" stress field in the whole structure can be obtained by elastic-plastic finite element analysis. Then, the optimal search for the most dangerous sliding surface with Hooke-Jeeves optimized searching method is introduced. Three cases of stability analysis of natural slope, anchored slope with seepage, and excavation anchored slope are conducted. The differences in safety factor quantity, shape and location of slip surface, anchoring effect among slices method, finite element strength reduction method (SRM), and finite element limit equilibrium method are comparatively analyzed. The results show that the safety factor given by the FEM is greater and the unfavorable slip surface is deeper than that by the slice method. The finite element limit equilibrium method has high calculation accuracy, and to some extent the slice method underestimates the effect of anchor, and the effect of anchor is overrated in the SRM.

1. Introduction

As an effective reinforced measure to slope, anchor rod has the advantages of simple construction, being fast, having less quantity of project, and so forth. It is widely used in protective engineering of landslides and other geological disasters. Therefore, the improvement of slope stability needs to be evaluated accurately and efficiently during the design of slope anchored.

Slice method [1, 2] has the advantage of clear concepts, definite physical meaning, and rich experience, but limitation of the method is equally clear: due to the presumption that the potential sliding mass is considered as a rigid body, the anchoring effect of slice method is reflected on the structure of the shear resistance to the balance of force and torque, rather than the actual potential sliding soil mass deformation constraint or the inner force redistribution. Therefore, it is less able to reflect the substance of soil-anchoring structure interaction.

The finite element strength reduction method (SRM), which can meet equilibrium and compatibility conditions automatically, has more rigorous theories system than slice method without assuming the shape and position of the sliding surface. It has received widespread attention since being proposed by Zienkiewicz et al. in 1975 [3], whereafter Matsui and San [4] verified the theoretical and numerical rationality of SRM for the finite element slope stability analysis. Zheng and Zhao [5] have done some research on stability of slope under the action of prestressed anchor cable with SRM based on the reduction of soil strength. However, they did not study the anchor strength reduction. On the basis of previous research, Wei et al. [6, 7] proposed an anchor rod strength model which can be applied in SRM and recommended that while soil strength is reduced, the reduction of anchor rod should be considered. Shi et al. [8] basing their theory on the direct reduction of cohesion and friction angle presented a discount method for tangent modulus. The intersection point of two lines, one of which is the deformation energy integral curve in potential slip area and the other is reduction coefficient curve, is chosen by the safety factor of slope stability. Isakov and Moryachkov [9] established the relationship equation between comprehensive safety factor

and strength reduction path and proposed the expression of minimum comprehensive safety factor by the shortest strength reduction path. Bai et al. [10] introduced the classical strength reduction method into the double reduction calculation process and have proved that the safety factor of double reduction method is almost always smaller than that of the classical SRM with theoretical derivation and numerical simulation. Xue et al. [11] based their theory on the assumption of soil strength parameters linear attenuation and introduced the nonproportional relationship between the cohesion reduction coefficient and the friction angle reduction coefficient into the traditional SRM, and the comprehensive safety factor is proposed based on shear strength parameters contributing to the resistant shear force. However, these studies have not mentioned any further research and discussion on some key questions, for example, whether the different soil layers should share the same reduction factor for heterogeneous slopes and whether the reinforcement should reduce structure strength for reinforced slope.

For the first time, Brown and King [12] introduced finite element stress field and homologous sliding surfaces determination method to analyze slope stability. Since then, many domestic and foreign scholars have been making thorough research and developing it. Naylor [13] defined the safety factor on circular slip surface as the ratio of the sum of antislide force to the sum of sliding force for the whole sliding surface. The stress of calculated points is provided by the finite element stress field. Shao and Li [14] who have proposed a proved sufficient and necessary condition to define the safety factor on any sliding surface is using the ratio of shearing resistance integral to shearing stress integral. This method is based on the theoretic foundation for finite element limit equilibrium method. Zhao [15] used an interface element to simulate the interaction between soil nails and surrounding soil and then analyzed the stability of foundation pit soil-nailing supporting engineering with finite element limit equilibrium method. Based on the limit equilibrium principle, Zhu et al. [16] took the anchor load as the analytical elastic stress distribution in an infinite wedge approximating the slope with the anchor load acting on the apex. And then the normal stress on the slip surface for the anchor-reinforced slope is assumed to be the linear combination of two normal stresses, where one exists before the application of anchor and another is induced by the anchor load. Zhuang et al. [17] compared and analyzed the differences between slice method and finite element limit equilibrium method on the shape and position of slice surface, value of safety factor, and anchoring effect, which are based on a detailed study for anchored slope finite element model.

Finite element limit equilibrium method combines the advantages of limit equilibrium method and finite element method organically, avoiding the controversy caused by using SRM. Therefore, it is widely accepted and applied in stability analysis of natural slopes, embankment and excavation slope, tailings dam, reinforced slope, and research on ultimate bearing capacity of soil structure [18–21] in recent years. Many satisfactory engineering results have been obtained by this method. In this paper, the author uses finite element limit equilibrium method to evaluate the stability of anchored slope directly and to explore some of the key issues. The law obtained in this study can provide reference and experience for correlational studies.

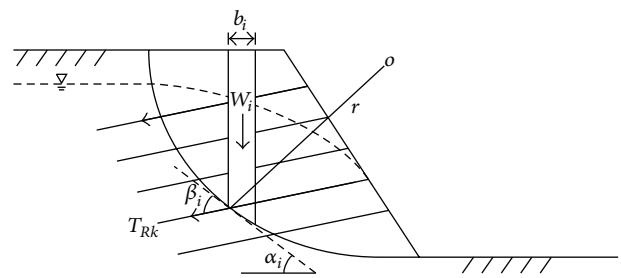


Figure 1: Stability analysis for anchored soil slope using slice method.

2. Anchoring Slope Stability Analysis Approach

2.1. The Limit Equilibrium Method. According to national standards "Construction Side Slope Engineering Technology Standard," Sweden arc method is suitable for stability analysis of soil slopes, as shown in Figure 1. Contributions made by anchor structures to antislide force (torque) can be expressed as a single variable discrete function:

$$F_R(\beta_i) = \frac{T_k}{S_{hk}}(\sin\beta_i \tan\varphi_i + \cos\beta_i), \tag{1}$$

where T_k is the maximum resistance of the first k rows of anchor rod section, S_{hk} is the horizontal spacing of the first k rows of anchor rod, β_i is the included angle between the first k rows of anchor rod and tangent of arch, and φ_i is the friction angle of slice i. Considering the effect of anchoring structure, we give out the expression of the safety factor calculation formula of anchoring slope:

$$F_s = \frac{\sum(W_i \cos\alpha_i \tan\varphi_i + c_i l_i) + \sum F_R(\beta_i)}{\sum(W_i \sin\alpha_i)}, \tag{2}$$

where W_i is soil weight and surface loads of slice i, l_i is the length of slip surface, α_i is the intersection angle between tangent of the first i slice arch failure surface and horizontal plane, and c_i is the cohesion force of slice i.

2.2. Finite Element Strength Reduction Method. In anchored slope stability analysis using SRM, shear strength index, cohesion c, and friction angle φ can be reduced by stability coefficient F_s through (3). Then we use ideal elastic-plastic stress-strain model and the Mohr-Coulomb yield criterion with iterative calculation based on the nonassociated flow rules and take the nonconvergence of finite element calculation as instability criterion:

$$\begin{aligned} c' &= \frac{c}{F_s}, \\ \varphi' &= \arctan\left(\frac{\tan\varphi}{F_s}\right). \end{aligned} \tag{3}$$

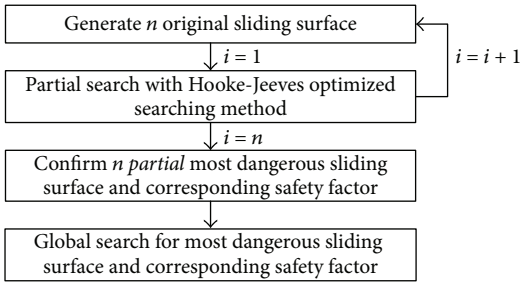


Figure 2: Searching flow chart of the most dangerous sliding surface.

2.3. Finite Element Limit Equilibrium Method

(1) Safety Factor Definition. Consulting the approach of anchoring slope stability analysis with slice method and combining it with the finite element stress analysis, we define the anchored slope safety factor F_s as

$$F_s = \frac{\int_0^l \tau_f \, dl + \sum_1^m F_R(\beta_i)}{\int_0^l \tau \, dl}, \tag{4}$$

where τ_f is the allowable shear strength on slip surface, the Mohr-Coulomb yield criterion generally is usually used, and τ is the actual shearing stress on slip surface.

(2) The Search for Most Dangerous Slip Surface. Finite element slope stability analysis can be seen as generalized mathematical programming problems with constraints condition. It is stated that, in the region with known stress field and with a set of specific nodes of which x coordinates have been known, the corresponding y coordinate is to be solved to make sure that the curve which is determined by those node coordinates is corresponding to F_s which is smallest and calculated by (4). The calculation of mathematical programming problem is the search process of most dangerous sliding surface. In order to solve the above problem, many intelligent algorithms are introduced into solving process. Manouchehrian et al. [22] proposed an evolutionary algorithm based on genetic algorithm which was used to develop a regression model for prediction of factor of safety for circular mode failure under the finite element stress field. The root mean squared error is used as the fitness function and searches among a large number of possible regression models to choose the best for estimation of safety factor. Malkawi et al. [23] got the critical slip surface with Monte Carlo method and the control of line element angle which is on slip surface. Based on the finite element limit equilibrium method, the most dangerous slip surface can be determined by particle swam optimization algorithm by Liu et al. [24] and Li et al. [25].

In this paper, its process is shown in Figure 2.

(3) Realization Steps of Finite Element Limit Equilibrium Method. Firstly, the stress of Gauss Points in the soil elements can be determined based on elastic-plastic finite element analysis, and they can be used to extrapolate node stress with the superconvergence stress slicing covering technology. Secondly, many initial feasible sliding surfaces are given in a certain search range, and the soil elements which are intersected by sliding surface can be determined. Then, the stress of control nodes on the sliding surfaces can be calculated with weighted average method. According to (4), the safety factor of sliding surfaces can be gained. Finally, search optimization is conducted with Hooke-Jeeves method until the most dangerous slip surface and the corresponding minimum safety factor are found.

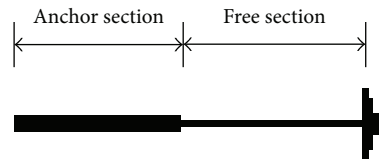


Figure 3: Structure diagram of anchorage.

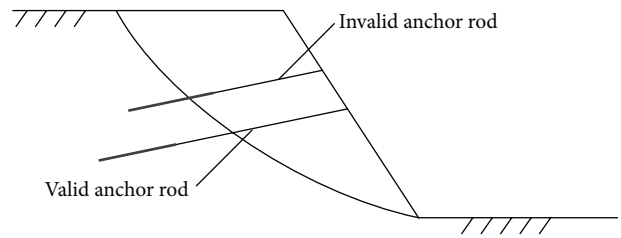


Figure 4: Schematic diagram of anchorage failure criterion.

3. Finite Element Model of Anchor and Failure Criterion

As shown in Figure 3, the anchor rod usually consists of free section and anchor section. Element: point-anchor pile element and fully grouted bolt beam element can be simulated separately. For free section without grouting, the interaction between free section and surrounding soil can be ignored. Therefore, only the contribution of nodes at both ends of the pile element is considered in the finite element calculation. Meanwhile, since there is no apparent sliding between anchor section and the surrounding soil, continuum model with common-node and different material property is used to simulate the interaction.

The pull-out resistance of anchor section F_R is accumulated through the grout along the anchoring length to achieve the designed value, which is $F_R = q_s l_a \pi d$, where q_s is the shear resistance of grout, d is the diameter of anchor section, and l_a is the length of anchor section. As shown in Figure 4, when sliding surfaces and anchor section intersect, that is, l_a is less than the design length, F_R will be less than the design value or lose the pull-out resistance. Thus, in the search process of most dangerous slip surface, if such above situation in which $F_R = 0$ is satisfied appears, the anchor structure will fail.

4. Finite Element Stability Analysis on Anchored Slope

4.1. General Slope Stability Results Comparative Analysis. We suppose a simple soil slope without pore water, of which

Table 1: Material parameters of soil.

Elastic modulus/MPa	Unit weight/kN·m^{-3}	Poisson ratio	Cohesion/kPa	Friction angle/°
10	20.2	0.3	3	19.6

Table 2: Safety factors without anchoring.

	Solution method	Safety factor
Slice method	Bishop	0.990
FEM	Finite element limit equilibrium method	1.006
	SRM	1.006

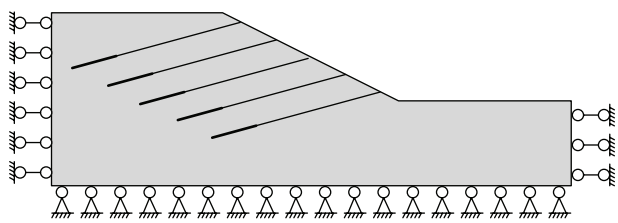


Figure 5: Schematic diagram of anchorage and constraint condition.

height is 10 m and slope ratio is 1 : 2. In order to avoid influence of boundary, the foot and top of slope are extended by 20 m. The slope is fixed at the bottom. Both side boundaries are constrained in the horizontal direction and free in other directions. Soil mechanical parameters are shown in Table 1.

According to the relevant requirements of the technical code for building slope engineering [26], the strengthening scheme is shown in Figure 5. From 1 m under the slope crest, the 5 rows of anchor rods are laid, whose horizontal angle is 15°, vertical spacing is 2 m, and horizontal spacing is 1 m. Each anchor rod, with 5 m anchor section and 0.1 m diameter, is 20 m long. The anchor section slip casting uses M30 cement mortar. The shear strength of the grouting is about 47 kPa.

(1) Natural Slope Stability Analysis. Firstly, three methods including slice method, finite element limit equilibrium method, and SRM are used in stability analysis of slope without anchor. From Table 2 which shows the result of stability analysis, we can find that the safety factors obtained from three methods are consistent, with the maximum difference about 1.6% only.

The sliding surfaces from slice method and finite element limit equilibrium method and expressed by shear strain increment from SRM are shown in Figure 6. The positions of sliding in and slipping out from three methods are almost consistent, and the shapes of sliding surfaces are the same. The sliding surfaces gained from two FEM are slightly deeper than those which are gained from slice method.

(2) Stability Analysis of Anchored Slope. The anchorage treatment is conducted in the case of above natural slope. Safety factors obtained from slice method, finite element limit equilibrium method, and SRM are shown in Table 3. As

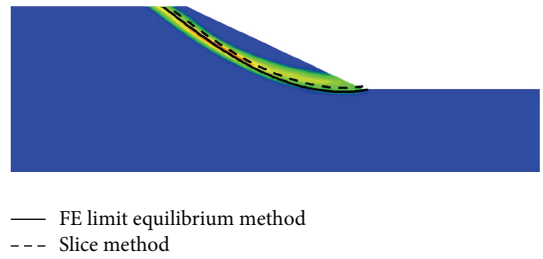


Figure 6: Sliding surface of natural slope.

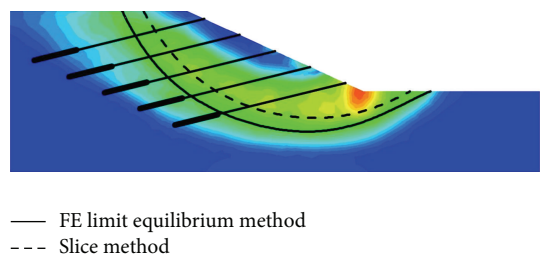


Figure 7: Sliding surface of anchored soil slope.

Table 3: Safety factors with anchoring.

	Solution method	Safety factor
Slice method	Bishop	1.645
FEM	Finite element limit equilibrium method	1.715
	SRM	1.750

shown in this table, compared with the safety factors from slice method, the factor from SRM increases approximately by 6.38%; safety factors from finite element limit equilibrium methods increase approximately by 4.25%.

As Figure 7 shows, compared with Bishop, the sliding surfaces from two types of FEM apparently slide downwards into the depths of soil under the effect of anchor rod. The end position of sliding surface from SRM has a slight movement to the toe of slope.

4.2. Anchored Slope Stability Results Comparative Analysis under Seepage Effect. One backfill soil slope with two-stage filling: From bottom to top, the slope ratio is 1 : 0.8 and 1 : 0.75, respectively. Because of the reduction of stability which is caused by seepage effect, the reinforcement needs to be conducted. Soil mechanical parameters are shown in Table 4.

From 6 m under the slope, with horizontal angle of 30°, vertical spacing of 6 m and horizontal spacing of 1 m lay the anchor rods in 2 rows. Each anchor rod size is 0.3 m diameter on anchor section and 0.00109 m^2 section area on free section. Anchor rod on the first row has 6 m in anchor

Table 4: Material parameters of soil.

Elastic modulus/MPa	Unit weight/kN·m^{-3}	Poisson ratio	Cohesion/kPa	Friction angle/°	Permeability coefficient/m·sec^{-1}
10	18	0.3	18.5	25	$1e-7$

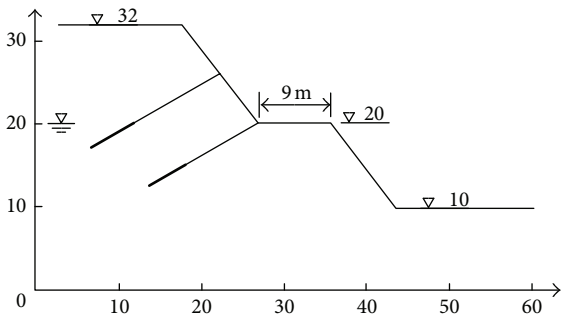


Figure 8: Computational profile of slope.

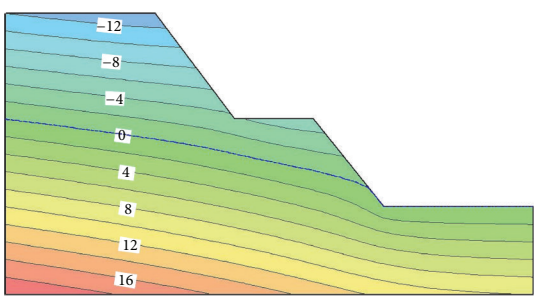


Figure 9: The distribution diagram of pore-water pressure (unit: m).

Table 5: Factors without anchoring under seepage.

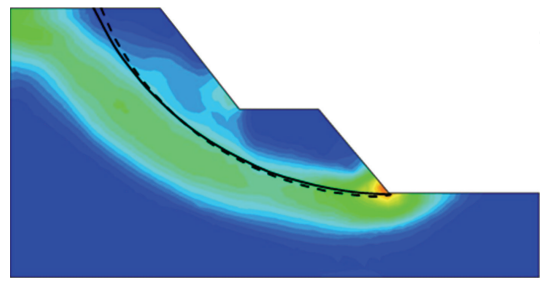
	Solution method	Safety factor
Slice method	Bishop	1.256
FEM	Finite element limit equilibrium method	1.306
	SRM	1.310

section length and 18 m in total length, while on the second it has 5 m in anchor section length and 15 m in total length. The structure dimension and anchor situation are shown in Figure 8.

Under the steady flow assumption, the distribution of pore-water pressure is shown in Figure 9.

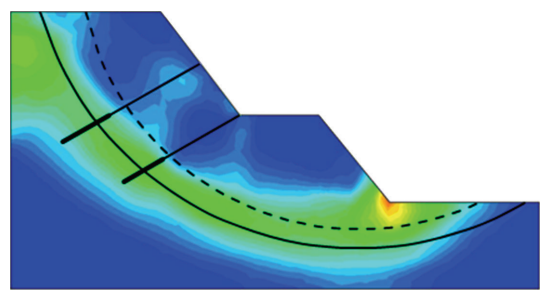
(1) The Stability Analysis for Slope without Anchor under Seepage Effect. The safety factors of slope without anchor under seepage effect are shown in Table 5. The safety factors from FEM are slightly larger than the Bishop, but the maximal difference is only 4.3%. Their sliding surfaces are basically identical to the Bishop (as Figure 10 showed).

(2) The Stability Analysis for Anchored Slope under Seepage Effect. As shown in Table 6 which shows the safety factor of anchored slope, the safety factors from two kinds of FEM are



—— FE limit equilibrium method
- - - Slice method

Figure 10: Sliding surface without anchoring under seepage.



—— FE limit equilibrium method
- - - Slice method

Figure 11: Sliding surface with anchoring under seepage.

Table 6: Safety factors with anchoring under seepage.

	Solution method	Safety factor
Slice method	Bishop	1.351
FEM	Finite element limit equilibrium method	1.361
	SRM	1.410

little larger than slice method that is identical to unanchored situation with the maximal difference of 4.36% only.

But from Figure 11 the most dangerous sliding surface given by slice method is quite different from FEM. The locations of sliding surfaces from two types of FEM apparently are lower than from Bishop. The end position of sliding surface from SRM is much closer to the toe of slope relative to example of general slope.

4.3. Excavation Slope Stability Results Comparative Analysis. Take a slope with 4 m excavation as an example for further study on verification of the suggested method. The slope ratio of excavation face is 1 : 0.3. The process of excavation can

Table 7: Material parameters of soil.

Material number	Elastic modulus/MPa	Unit weight/kN·m^{-3}	Poisson ratio	Cohesion/kPa	Friction angle/°
①	10	18	0.3	8	15
②	30	18	0.3	10	15

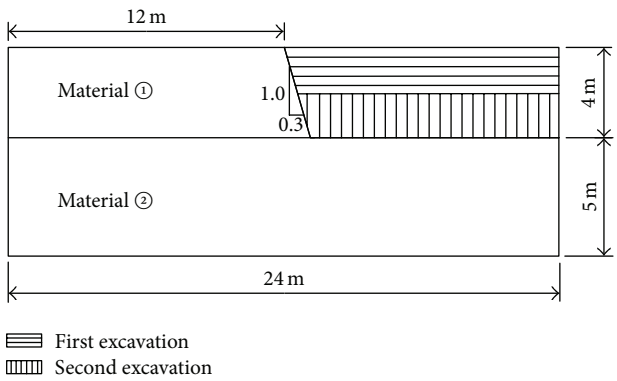


Figure 12: Geometric model of slope and excavation process.

Table 8: Excavation slope of safety factors without anchoring.

	Solution method	Safety factor
Slice method	Bishop	0.923
FEM	Finite element limit equilibrium method	0.976
	SRM	1.005

divide into two steps and excavation depth of each step is 2 m; specific excavation process is shown in Figure 12. Slop soil consists of two layers. Soil mechanical parameters are shown in Table 7.

(1) The Excavation Stability Analysis for Slope without Anchor. The safety factors of foundation pit are summarized in Table 8. The excavation effect can be considered in the finite element analysis but in the slice method. Because the calculation of safety factor with FEM depends on the stress field, with the increases of gradient, the stress-concentration phenomenon will appear, which has an impact on the accuracy of result to a certain degree. Meanwhile, it can be seen from the stability analysis results that the safety factors obtained from three methods are different. Compared with the Bishop, the safety factor from finite element limit equilibrium method increases approximately by 5.7%, and SRM's result increases approximately by 8.9%.

As can be seen from Figure 13, results of two FEM are almost coincident, while the start position of sliding surface from slice method is much closer to the excavation face. The results from three methods are accordant in the end position and shape of sliding surface.

(2) The Excavation Stability Analysis for Slope with Anchor. During the excavation process, a row of anchors is used to support the slope with horizontal angle of 15°, which are placed 1 m under the ground with horizontal spacing of 1 m.

Table 9: Excavation slope of safety factors with anchoring.

	Solution method	Safety factor
Slice method	Bishop	2.161
FEM	Finite element limit equilibrium method	2.422
	SRM	2.510

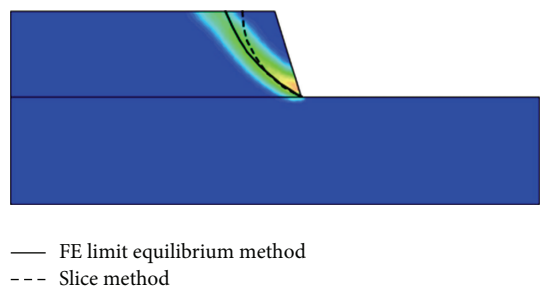


Figure 13: Sliding surface of excavation slope without anchoring.

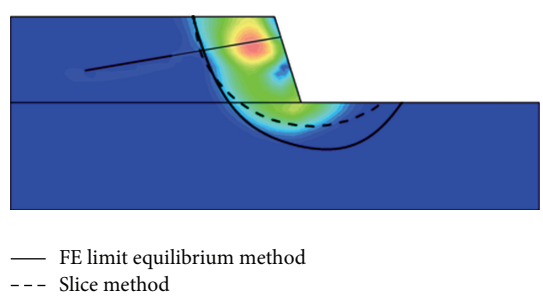


Figure 14: Sliding surface of excavation slope with anchoring.

Each anchor rod, with 4 m anchor section, 0.4 m diameter, and M30 cement mortar casting, is 9 m long. The hole will be drilled after the first excavation and installation of anchors finish at the same time. The assumption that the stress of anchor followed the finish of slip casting will not affect the calculation result under construction interval short enough.

From Table 9 which lists the safety factors of anchored slope from three methods, the results of FEM are greater than Bishop. As same as the natural slope, there is further increase in the gap between safety factors of excavation slope from different methods. Compared to Bishop, the factors of finite element limit equilibrium method increase approximately by 12.07%; the factors of the SRM increase approximately by 14.41%.

As shown in Figure 14, the parts of sliding surface from three methods which are behind excavation face are almost coincided, when the parts located at toe are different because of reasons such as stress release and bottom heave. The sliding

surfaces from two types of FEM are deeper than that from slice method. Compared with example of general slope and anchored slope under seepage effect, with the increases of gradient, the end position of sliding surface from SRM is closer to the toe more pronouncedly.

4.4. Difference Analysis between Slice Method and FEM in Anchor Effect. In the stability analysis for anchored slope, the effect of anchor structure is considered in slice method, finite element limit equilibrium method, and SRM. However, the approaches of the three methods are essentially different.

Anchoring force is hailed as the concentrated load in stability analysis with slice method and finite element limit equilibrium method. However, in the slice method, the effect of anchoring force is limited in term $\sum F_R(\beta_i)$ of (4), while, in the finite element limit equilibrium method, the anchor structure participates in the calculation of stress field and changes the stress distribution of soil. Its effect is not confined to term $\sum F_R(\beta_i)$. The shearing stress term $\int \tau\, dl$ and shearing resistance $\int \tau_f\, dl$ of (4) can show the combined action of both anchor structure and soil.

From (3), the safety factor reduction is only aimed at shearing resistance c and φ, taking nonconvergence of force and displacement as an instability criterion in the stability analysis with SRM. Because of no reduction for anchor structure at same time, the anchor plays a more important role in stability analysis, which may lead to overestimating on the anchored effect of anchorage structure at a certain extent.

The representation of the above reasons in the stability result is as follows: the safety factors from two types of FEM are larger and the sliding surfaces are deeper.

5. Conclusion

(1) The results show that combined with the definition of safety factor in the slice method of anchored slope, finite element limit equilibrium method can be used to evaluate the stability of anchored slope.

(2) When the slice method is used to analyze anchored slope stability, the safety factor is smaller and the slip surface is higher than FEM due to the inconsiderate inner slice force assumption of reinforcement effect and anchoring structure.

(3) When SRM is applied on anchored slope for stable analysis, compared with the other two methods, overestimation of the anchored effect of anchorage structure at a certain extent will be obtained due to considering only reduction of soil body strength parameter without the reduction of anchorage structure strength. Compared with the results of slice method and finite element limit equilibrium method, SRM has the maximum safety factor and the deepest slip surface.

Competing Interests

The authors declare that they have no competing interests.

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